FISCAL AND MONETARY POLICY INTERACTIONS: A GAME THEORETICAL APPROACH

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Resumo

Este trabalho estuda as interações entre as políticas monetária e fiscal numa abordagem de teoria dos jogos. A coordenação entre essas duas políticas é essencial, uma vez que certas decisões tomadas por uma instituição podem ter efeitos desastrosos sobre a outra instituição, resultando em perda de bem estar social. Nesse sentido, foram derivadas as políticas monetária e fiscal ótimas em três contextos de coordenação ou formas de interação: quando as duas instituições minimizam sua perda apenas levando em conta seus objetivos, essa solução é conhecida como solução de Nash; quando uma instituição escolhe primeiro como proceder e a outra segue, num mecanismo conhecido como solução de Stackelberg; quando as instituições se comportam de forma cooperativa, buscando objetivos em comum. No caso brasileiro, as simulações dos modelos derivados nesses regimes de coordenação apontam uma perda mínima quando há uma solução de Stackelberg, mais especificamente quando a política monetária é a líder. Esse resultado é corroborado por outros tralhados.

Palavras-chave: Política Fiscal e Monetária, Teoria dos Jogos, Equilíbrio de Nash, Equilíbrio de Stackelberg, Cooperação, Brasil.

Abstract

This work attempts to study the interaction between fiscal and monetary policy in a game theoretic approach. The coordination between these two policies is essential, since certain decisions taken by one institution may have disastrous effects on the other one, resulting in loss of social welfare. In this sense, we derived optimal monetary and fiscal policies in three contexts of coordination or forms of interaction: when the two institutions minimize their losses only taking into account their objectives, a solution known as the Nash solution; when an institution moves first and the other follows, in a mechanism known as the Stackelberg solution; when institutions behave cooperatively, seeking common goals. In the Brazilian case, the simulations demonstrated that a minimal loss is found when there is a Stackelberg solution, particularly when monetary policy is the leader. This result is corroborated by other works.

Keywords: Fiscal and Monetary Policy, Game Theory, Nash Equilibrium, Stackelberg Equilibrium, Cooperation, Brazil.

Résumé

Ce travail envisage les interactions entre les politiques monétaire et fiscale, axées sur la Théorie des Jeux. La coordination entre ces deux politiques est essentielle, une fois que certaines décisions prises par une instituition peuvent produire des effets désastreux sur une autre institution et cela peut eventiellement résulter en perte de bien-être sociale. En ce sens, notre recherche va dans le sens des dérivées des politiques monétaire e fiscale optimales dans trois contextes de coordination ou de formes d'intéraction. Soit: quand les deux institutions minimisent leurs pertes, prennant en compte, tout simplement, leurs objectifs – cette solution est connue par Solution de Nash; soit quand une institution choisit d'abord sa procédure et l'autre la suit, dans un mécanisme connu par Solution de Stackelberg; soit lorsque les institutions agissent de manière cooperative, tout en cherchant ensembles un but commun. Dans le cas brésilien, les simulations des modèles derivés de ces régimes de coordination indiquent une perte minimale lorsqu'on employe la Solution de Stackelberg, plus spécifiquement si la politique monétaire la conduit. Ce résultat est confirmé par d'autres travaux de même genre.

Mots-cléf: Politique Fiscale et Monétaire, Théorie des Jeux, Équilibre de Nash, Équilibre de Stackelberg, Coopération, Brésil.

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CHAPTER 1

Introduction

1.1 Resumo

Nesta dissertação estudamos as interações entre as políticas monetária e fiscal numa abordagem de teoria dos jogos. Muitos trabalhos têm sido desenvolvidos nesse sentido, tanto do ponto de vista acadêmico quanto institucional [Dixit & Lambertini (2000), Dixit (2001), van Aarle et al. (2002), Kohler (2002), Muscatelli et al. (2004), Kirsanova et al. (2005), Fatum (2006), entre outros]. Essa introdução faz um breve apanhado sobre as áreas e ferramentas envolvidas no processo de interação, i.e. políticas fiscal e monetária e teoria dos jogos, respectivamente, e sobre os principais objetivos desse estudo.

A política monetária representa a dimensão da política econômica voltada principalmente para a controle do nível de preços e aumento do nível de emprego da economia, atuando basicamente com três instrumentos, a saber, recolhimento compulsório, redesconto de liquidez e operações de mercado aberto.

Recolhimento compulsório são depósitos, em termos percentuais dos depósitos à vista, que os bancos comercias[1] são obrigados a manter junto ao banco central. Note que quando

[1] As contas em que os bancos comercias mantêm os depósitos compulsórios são denominadas “reservas bancárias”.

os agentes fazem depósitos em um banco comercial, esse banco usa parte desse depósito para emprestar a outro agente, assim esse mecanismo permite à autoridade monetária, banco central, reduzir (aumentar) a capacidade de empréstimos e de criação de moeda, ou seja, interferir no chamado multiplicador monetário. O compulsório, assim, permite à autoridade monetária controlar a quantidade de moeda na economia e consequentemente o nível de preços.

O redesconto permite à autoridade monetária conceder empréstimos, na forma de crédito em reservas bancárias para cobertura de deficiências momentâneas de caixa. Desse modo, se o banco central cobra uma taxa pequena e fornece um prazo longo aos bancos, os mesmos podem se expor a riscos maiores, e aumentar a quantidade de empréstimos, aumentando o montande de moeda em circulação, e possivelmente o nível de preços.

As operações de mercado aberto permitem à autoridade monetária administrar a taxa de juros e controlar a oferta monetária. Essas operações são realizadas através da compra e venda de títulos públicos por parte do banco central. O efeito da taxa de juros sobre o nível de preços pode ser estabelecido pela demanda agregada. Um aumento (diminuição) na taxa de juros reduz (aumenta) a demanda agregada, isso por sua vez diminiu a demanda por moeda por parte dos agentes, diminuindo o nível de preços.

A política fiscal, por outro lado, é a administração dos gastos e receitas do governo de maneira a influenciar a economia. Esses gastos e receitas (impostos, emissão de dívida e outras fontes) são os principais instrumentos da autoridade fiscal. O principal objetivo da política fiscal é manter o crescimento econômico com estabilidade no nível de preços. A política fiscal pode assumir três posturas, a saber, neutra, expansionista e contracionista. Uma política é dita neutra quando o orçamento do governo está equilibrado, ou seja, receitas se igualam às despesas. Por outro lado, é dita expansionista, se o governo gasta mais do que arrecada, recorrendo à venda de títulos públicos[2], senhoriagem[3], e outras fontes, para financiar suas despesas. Finalmente, é dita contracionista se os gastos são

[2]Por exemplo, títulos da dívida pública.
[3]Receitas pela impressão de papel moeda.

menores que as despesas. Assim, uma política expansionista é associada a um orçamento deficitário e um política contracionista a um orçamento superavitário.

Teoria dos jogos é uma ferramenta matemática criada para melhorar o entendimento ou interpretação da maneira a qual interações estratégias entre jogadores racionais produzem consequências com respeito às preferênicas dos jogadores.

A história da teoria dos jogos está relacionada a uma carta escrita por James Waldegrave para Nicolas Bernoulli. Waldegrave analisa o jogo de cartas *le Her* e fornece uma solução que é um equilíbrio em estratégia mista. Posteriormente, com as contribuições de von Neumann & Morgenstern (1944), Nash Jr. (1950b), Nash Jr. (1951) e Nash Jr. (1950a), a teoria dos jogos ganhou grande destaque em várias áreas, principalmente na economia.

Com os conceitos de políticas fiscal e monetária e teoria dos jogos em mente, os principais objetivos são: estudar os efeitos, em termos de otimalidade, de três regimes de coordenação ou interação entre as políticas monetária e fiscal, as quais são a solução não cooperativa (solução de Nash), solução com cooperação parcial (liderança de Stackelberg) e solução cooperativa; fornecer o melhor esquema de coordenação dos citados acima. Observa-se que o modelo é calibrado para a economia brasileira, portanto mostraremos resultados ótimos para o Brasil.

1.2 Introduction

Fiscal and monetary policy interactions have been the focus of intense attention both academically and institutionally [Dixit & Lambertini (2000), Dixit (2001), van Aarle et al. (2002), Kohler (2002), Muscatelli et al. (2004), Kirsanova et al. (2005), Fatum (2006), among others]. The interactions and their consequences have been crucial to comprehend what kind of policies the monetary and fiscal authorities should take in order to improve social welfare. The purpose of this work is to shed some light on this area, analyzing mainly the interactions through a game theoretical perspective. It is evident that before introducing the analysis, a brief overview of the tools which will be used throughout this work is made necessary. Hence, the next topics introduce the reader to monetary policy, fiscal policy, and game theory. Posteriorly, specific objectives are presented.

1.2.1 Monetary policy

Monetary policy represents the dimension of economic policy focused mainly on the control of the price level and oriented towards the growth and stability of the economy. The institution which is incumbent to execute these objectives is the central bank, and his basic instruments are:

- compulsory deposit requirements;
- discount window;
- open market operations.

Compulsory deposit requirements are deposits (in percentage terms) which commercial banks must hold in reserve with the central bank. This measure aims to reduce the risk of banks overextending themselves and suffering from bank runs. In addition, compulsory deposits regulate the money multiplier, which measures the capacity of monetary expansion. The idea is that a trite deposit may have leverage effects, since the bank that received the deposits may lend part of this money to other clients. Hence, the control of compulsory deposit requirements will ultimately control the money supply in the economy. This money supply can be contractionary or expansionary, a low money multiplier rate could expand the money supply possibly increasing the inflation rate, and a high money multiplier could contract the money supply possibly reducing the inflation rate.

The banks may eventually borrow money in order to meet temporary shortages of liquidity caused by internal or external disruptions, and the central bank, in this scenario, becomes the lender of last resort. That assistance is called discount window and may help the central bank to reduce or expand the money supply. The interest rate charged on such loans by the central bank is denominated the discount rate. The central bank may, for example, lend (long-term) money charging high discount rates, discouraging banks to borrow money which further may reduce the money supply and consequently the inflation rate. Alternatively, the central bank may lend (short-term) money charging low interest rate, resulting in a reduction in the inflation rate.

The open market operations are an agile instrument which allow for adjustment of the daily money supply and interest rate. These operations consist of sells and/or buys of public bonds by the central bank. When the central bank buys public bonds, exchanging money for the public bonds, it raises the money supply and consequently the inflation rate. Conversely, selling of public bonds lowers the money supply and consequently the inflation rate. The operations also determines the short-term interest rate. An increase (decrease) in the interest rate decreases (increases) the aggregate demand, which in turn pushes down (up) the inflation rate.

The central banks have many available regimes to pursue price stability with economic growth. An important monetary regime, and that has gained quite visibility, is the inflation targeting (IT) regime. Many countries all over the world have adopted this regime, among them is Brazil. IT is a regime in which the main goal is to keep inflation within a predefined band, following a reaction function where the interest rate responds to changes in both inflation and output, namely, involves stabilizing inflation around an inflation target and stabilizing the real economy, represented by the output gap. Minella et al. (2002), Minella et al. (2004) and Bogdanski et al. (2000) analyze the implementation of IT regime in Brazil and the challenges faced by Brazil in order to overcome confidence crisis. They constructed a framework to estimate a reaction function for the Central Bank, concluding a forward-looking stance in the period analyzed, and a confidence in responses for variation in inflation and output, corroborating the importance of the estimations. Although there has been highlighted that the central banks are not good predictors of the output gap in real time.

1.2.2 Fiscal policy

The fiscal policy is the use of government spending and revenue collection to influence the economy. The policy involves the definition and application of the tax burden upon economic agents, as well as the definition of government spending, which has as base collected taxes and issued bonds. The fiscal policy has received considerable attention [Lopreato (2006), Toye (2006), Tanzi & Schuknecht (2003), Perotti (1999), Kneller et al. (1999)], mainly due to his relevance and impact on other areas, e.g. monetary policy. Thus both fiscal and monetary policy try to keep the economy in a straight line. In contrast with monetary instruments, i.e. control of the interest rates and the supply of money, the three main instruments of the fiscal policy are government spending, taxation, and debt issuance. A fiscal deficit is often funded by issuing treasury bills, which are bonds issued by a government.

A fiscal policy can basically assume three stances, videlicet neutral, expansionary and contractionary. A neutral stance of fiscal policy implies a balanced budged where $G = T$ (Government spending = Tax revenue). An expansionary stance of fiscal policy involves an unbalanced budget where net government spending is higher than taxation ($G > T$). This situation could be due to rises in government spending, a fall in taxation revenue (e.g. an income tax cut), or both. A contractionary fiscal policy ($G < T$) occurs when net government spending is reduced through either higher taxation revenue or reduced government spending, or both. Consequently, an expansionary policy is related to a budget deficit and a contractionary policy is associated with a budget surplus.

The government can inject (spend) money into the economy through several channels. For instance, through public services like police, education, healthcare, transfer payments (e.g. the Brazilian Social Program, "Bolsa Família", see Hall (2006) for more conceptual details and debate), and so on. These expenditures can come from taxation, seigniorage (printing money), consumption of fiscal reserves, sale of assets, government bonds (treasury bills or consols), etc.

Fiscal policy is used by governments to influence the level of aggregate demand in the economy, in an effort to achieve economic objectives of price stability, full employment and economic growth. Monetary and fiscal objectives converge but frequently there are incongruences, for example, the implementation of fiscal stimulus can have inflationary effects driven by increased demand. Thus, a coordination between these two polices is essential to keep price stability and economic growth.

1.2.3 Game theory

The game theory is a mathematical tool created to improve the understanding or to interpret the manner which strategic interactions among rational players produce outcomes with respect to the preferences (or utilities) of those players, none of which might have been intended by any of them.

The history of game theory is related with the first known discussion which occurred in a letter wrote by James Waldegrave to Nicolas Bernoulli. Waldegrave analyzes the card game 'le Her' and provides a minimax mixed strategy solution. In 1838 Augustin Cournot's work (see Cournot (1938)) considered a duopoly and presents a solution that is a restricted version of the Nash equilibrium. The benchmark came though with the mathematician John von Neumann whom published a series of papers in 1928. However, only in 1944 with "The Theory of Games and Economic Behavior" book, by von Neumann and Oskar Morgenstern (see von Neumann & Morgenstern (1944)), game theory invaded the economics and the applied mathematics, since others experts also decided to contribute for the theory development, which had as first objective to determine mathematical bases for a economic theory. This profound work contains the method for finding mutually consistent solutions for two-person zero-sum games. In 1950, the mathematician John Forbes Nash Junior published four important articles in the game theory field. In "*Equilibrium Points in n-Person Games*" (Nash Jr. (1950b)) and "Non-cooperative Games" (Nash Jr. (1951)), Nash proposed an equilibrium in mixed strategies for non-cooperative games and suggested an approach for cooperative games based on non-cooperative ones. In the "The

Bargaining Problem" (Nash Jr. (1950a)) and "Two Person Cooperative Games" (Nash Jr. (1953)) articles, Nash created a bargain theory and proved that the bargain problem had a solution.

Many other concepts appeared and the game theory experienced a flurry of activity in the 1950s, during which time the concepts of the core, the extensive form game, fictitious play, repeated games, and the Shapley value were developed. In addition, the first applications of game theory to philosophy and political science occurred during this time.

The main contribution of these authors was to display the importance to use mathematical models to formulate hypothesis in distinct economic areas.

Game theory can be applied in several areas of knowledge, among them we highlight:

- Biology: to explain the evolution (and stability) of the approximate 1 : 1 sex ratios; emergence of animal communication (Harper & Maynard Smith (2003)); to explain many seemingly incongruous phenomena in nature, such as biological altruism (Maynard Smith & Price (1973) and Maynard Smith (1976));

- Political science: focused on the overlapping areas of fair division, political economy, public choice, positive political theory, and social choice theory. In each of these areas, researchers have developed game theoretic models in which the players are often voters, states, special interest groups, and politicians (Ordeshook (1986) and Aliprantis & Chakrabarti (2000));

- Sociology: in order to identify situations of conflict between an individual and the collective (Souza (2003));

- Law: in order to interpret and apply some laws (Chung & Fortnow (2007));

- Computer Science and Logic: to model interactive computations and to provide a theoretical basis to the field of multi-agent systems (Messie & Oh (2004), Parsons & Wooldridge (1999), and Yao (1977));

among others.

1.3 Objectives

The debate between fiscal and monetary policies has tended to dissociate the possible influences that fiscal policy could exert on price, for instance, the fiscal policy impact on the monetary policy capacity to control inflation rate [Sargent & Wallace (1981) and Drazen & Helpman (1990)], mainly through the expectation of future seignorage that a deficitary fiscal policy may generate, is not a well-accepted theme among macroeconomists. More recently, other mechanisms, which provoke the same phenomena (fiscal dominance), have been studied as well, for instance, the fiscal policy effect on the probability of default and consequently upon the exchange rate, impacting directly the inflation and the inflationary expectations, however no consensus has been reached as well. Thereby, an explicit mechanism which considers the effects of both monetary and fiscal policy interactions, is beyond question quite noteworthy.

Following this line of objective, the present work seeks:

- to study the effects, in terms of optimality, of three policy regimes which commonly arise between the fiscal and monetary authorities, respectively represented by the treasury and the central bank, that is, the three alternative policy regimes in a stylized dynamic model: (i) non-cooperative monetary and fiscal policies (Nash solution), (ii) partial cooperation (Stackelberg leaderhip), and (iii) cooperative solution;

- to identify from welfare perspective, the best coordination scheme between fiscal and monetary policies, among those introduced at the anterior item.

- to check whether Brazil[4] was under fiscal or monetary dominance in the period after the implementation of the Real Plan;

[4]We use the model calibrated for the Brazilian economy.

1.4 Organization

This work unfolds in the following way: there are four chapters, taking into account this introduction. Chapter 2, entitled “Fiscal and Monetary Policy Interactions”, as the name suggests, studies the interactions of fiscal and monetary policy, focusing on three distinct game scenarios: (i) non-cooperative monetary and fiscal policies (Nash solution); (ii) partial cooperation (Stackelberg leadership); (iii) full cooperation (cooperative solution). Chapter 3, entitled “Numerical Approach”, applies the framework derived in Chapter 2 to the Brazilian economy. Finally, Chapter 5 shows the conclusions achieved and points out future researches.

CHAPTER 2

Fiscal and Monetary Policy Interactions

2.1 Resumo

Esse capítulo tem por objvetivo derivar regras ótimas de política monetária e fiscal a partir de três regimes de coordenação ou interação, a saber, (1) quando as instituições estabelecem seus instrumentos simultaneamente e sem cooperação numa solução conhecida como Nash, (2) quando a autoridade monetátia (fiscal) move primeiro, como um líder, antecipando a resposta da autoridade fiscal (monetária) numa solução conhecida como Stackelberg, (3) quando as autoridades fiscal e monetária estabelecem seus instrumentos simultaneamente e cooperativamente na busca de um objetivo comum.

A política macroeconômica tem experimentado grandes mudanças ao longo dos últimos cinquenta anos. O advento de regras, a despeito de uma política estritamente discricionária, ganhou grande importância principamente na política monetária. Nesse aspecto, pode-se destacar a relevância que o regime de metas de inflação tem desempenhado em vários países, dentre eles o Brasil. Resultados obtidos por Barro & Gordon (1983a) e Barro & Gordon (1983b) monstram que comprometimento a aderir certas políticas fazem a economia responder otimamente a choques que estão por vir. O comprometimento leva a uma inflação ainda menor.

O modelo para descrever a economia consiste de uma curva IS intertemporal de demanda aggregada, o qual vem da relação intertemporal entre investimento e poupança, uma equação de oferta aggregada, também conhecida como curva de Phillips novo keynesiana, e uma restrição orçamentária intertemporal a qual o governo deve obdecer. O modelo é fechado com uma regra para a política monetária (regra para a taxa de juros) e uma regra para a política fiscal (regra para os gastos do governo).

As interações entre as políticas monetária e fiscal acontecem num cenário de teoria dos jogos. Dentre as várias formas de representar um jogo, podemos destacar duas, a saber, forma normal e forma extensiva. A primeira forma será utilizada para modelar a solução de Nash, ao passo que a extensiva será usada para modelar a solução de Stackelberg. No tocante ao caso cooperativo, há também várias formas de interpretação, destacamos duas, o problema de barganha e o problema de bem estar social. A rigor, o segundo problema é de relativamente fácil implementação, portanto, para fins de aplicação será utilizado nesse trabalho.

Uma vez apresentado o instrumental teórico, segue-se na obtenção das políticas monetária e fiscal ótimas, as quais são obtidas pelas técnicas de Lagrange discutidas em Woodford (2003). Ambas autoridades (monetária e fiscal) minimizam uma função perda sujeita às restrições da economia (equações descritas anteriormente). Destaca-se que essas técnicas de Lagrange são temporalmente consistentes e atemporais. A primeira característica é devido ao comprometimento, ao passo que a segunda baseia-se no fato que não é necessário minimizar as perdas esperadas a partir de $t = t_0$ para frente. As regras ótimas resultantes são apresentadas na seção 2.6.

2.2 Introduction

The macroeconomic policy has experiencing huge changes in the past fifty years. The rule-based-policymaking approach has taken the scene mainly in the monetary policy all over the world. Woodford (2003), for instance, calls our attention to changes emphasizing the adoption of straightforward rules for the control of inflation which was carried out by sound basis for real economic performance. The actual relevance of commitment with explicit rules can be strengthened by the adoption of inflation targeting regimes as a policy rule by many central banks, e.g. the Central Bank of Brazil, the Bank of England, the Reserve Bank of New Zealand, the Swedish Riksbank, etc.

A long time ago, relatively speaking, Friedman (1968) stressed the relevance of rules and the consequences stemmed from inappropriate policy-rules, which could be source of economic instabilities. He highlighted the relevance of lags (the difference between actions and effects) as being that source.

An illustration of applied rules in order to pursue given objectives can be pointed out by another rule, the Quantity Theory of Money (QTM)[1][Friedman (1956)]. Following this rule, an economy with stable transactions' velocity of money[2] and regular and foreseeable growth rate of output will generate a regular relation between quantity of money and level of prices, hence promoting price stability and sustainable growth of per capita output. Therefore, the control of some monetary aggregate was important to control the level of prices. However, the 1970s experience of several countries has shown that the transactions' velocity of money is not stable.

Following the stream, we can point out the article "Rules Rather than Discretion: The Inconsistency of Optimal Plans" wrote by Kydland & Prescott (1977), which studies the sequential choice of policies, such as tax rates or monetary policy instruments. An interesting result is that governments unable to make binding commitments regarding future policies will face a credibility problem. Despite the announced government policy, the public will realize that the future government policy will not necessarily match the

[1]The quantity theory of money builds upon the following definitional relationship.

$$M \cdot V_T = \sum_i (p_i \cdot q_i) = \mathbf{p^T} \cdot \mathbf{q}$$

where: M is the total amount of money in circulation on average in an economy during a certain period; V_T is the transactions' velocity of money; p_i and q_i are the price and quantity of the i-th transaction.

[2]The transactions' velocity of money measures the frequency on average which the money changes hands.

announced one, unless the plan already encompass incentives for future policy change. Thus, they demonstrated that the outcome in a rational-expectations equilibrium with discretionary policy results in lower welfare than in an equilibrium where the government can commit, that is, the results corroborated the necessity of employing more rules on economic policies.

Barro & Gordon (1983a) and Barro & Gordon (1983b) implemented supply shocks and stabilization policy into the positive theory[3] of monetary policy. They showed that a prior commitment to adhere to a certain policy makes policy respond optimally to upcoming macroeconomics shocks. This commitment leads to better macroeconomic outcomes than discretionary policy, i.e. it provides lower inflation and the same average unemployment rate, as well as the same extent of macroeconomic stabilization.

Subsequent researches by Backus & Driffill (1985) and Tabellini (1985) used the theory of repeated games to demonstrate that, under certain conditions, equilibria with low inflation could also appear under discretionary (absence of commitment) policymaking. The argument is that when monetary policymakers perform a combat against inflation by good reputation, they consequently influence private-sector expectations concerning future inflation rates. This result indicates that even under discretion the government needs to demonstrate certain quantity of commitment (good reputation).

As the ruled-type policy rapidly gained wide acceptance, the development of an alternative and operational macroeconomic framework was called for. From this new perspective appeared the Taylor rule which is a monetary-policy rule that stipulates how much the central bank would or should change the nominal interest rate in response to divergences of actual inflation rates from target inflation rates and of actual Gross Domestic Product (GDP) from potential GDP[4]. Taylor (1993) identified the bases for a contemporaneous analysis of the monetary policy. In sequence, several works have tried to rationalize this

[3]Inflation here is higher when equilibrium unemployment is higher relative to the unemployment target that policymakers try but are not able to attain.

[4]Potential output measures the capacity production (maximum sustainable level of output) of the economy and output gap is just the percentage difference between actual output and potential output.

rule.

This chapter aims to derive optimal monetary and fiscal policy rules in three different regimes, which results from three different forms of interactions between the policy authorities:

(i) The first mechanism is that which define the equilibrium when monetary and fiscal policy-makers set their instruments simultaneously and without cooperation in a Nash game. The two institutions minimize their loss according to economic constraints and only taking into account their own decisions;

(ii) In the second, the equilibrium arises when the fiscal/monetary authority moves first, as a Stackelberg leader, anticipating the response from the fiscal/monetary authority;

(iii) In the third, the monetary and fiscal policy-makers set their instruments simultaneously but they cooperate with each other in pursuit of a common objective.

The next section sheds some light on the recent literature of monetary and fiscal interaction. Later on the model will be derived and implemented.

2.3 Previous research

Many works have been done on monetary and fiscal policy interactions, mainly in the past 10 years. This section describes some of these works.

A coordination between two different government branches, i.e. fiscal and monetary policy, arises conflicting questions since every policymaker is primarily interested in solving the policy issues of his own branch. The induced economic spillovers and externalities - in this scenario - become very important. Engwerda (1998a), Engwerda et al. (1999) and Engwerda et al. (2002) model dynamic games among monetary and fiscal policymakers.

Dixit (2001) builds various models of the EMU (European Economic and Monetary Union) and ECB (European Central Bank) in order to analyze different countries' points of view concerning monetary and fiscal policies. He finds that the voting mechanism[5] achieves moderate and stable inflation. From a Walsh's contract[6] metaphor and common interest standpoint, the author obtains a monetary policy rule that implements the unweighted average of the countries' most-preferred policy rules. In case of a repeated game the ECM should foresee eventual member drawback and overcome this perturbation. The author emphasizes the dangerous role of unconstrained national fiscal policies which can undermine the ECB's monetary commitment.

van Aarle et al. (2002) implement a framework involving monetary and fiscal policies in the EMU according to Engwerda et al. (2002) EMU's model. The idea is to study the various interactions, spillovers and externalities involving macroeconomic policies considering three policy regimes. Numerical examples are illustrated for various coalitions and it is interesting to highlight that in various simulations full cooperation does not induce a Pareto improvement for the ECB.

Kirsanova et al. (2005) extend the three-equation Taylor-rule macroeconomics, which has been used to model the stabilization of an economy by monetary policy alone. The idea is to extend the set-up to a five-equation system in order to describe what they have called a good fiscal policy. In this simple set-up, the fiscal policy rule, for instance, feeds back on the level of debt and helps the monetary authority to stabilize output, providing more space to the monetary authority to stabilize inflation. The authors, then, investigate the outcomes of optimal monetary and fiscal policies to study the common effort to stabilize the economy against shocks in three scenarios: (i) non-cooperative policy, (ii) partial cooperative policy, and (iii) benevolent policy. The results demonstrate that: (i) if fiscal and monetary authorities are benevolent and cooperate, the monetary

[5]This is a mechanism of decision-making in the European Central Bank (ECB). Basically, the ECB has two decision making bodies, the Governing Council and the Executive Board. Th first one consists of the Governors of the national central banks of the countries that are members and the second one consists of the President and Vice-President. The Governing Council is in charge of monetary policies such that each body makes its decisions by simple majority voting.

[6]Walsh (1995) and Person & Tabellini (1993) propose a central banker contract whereby the inflationary bias and the trade-off between inflation reduction and stabilization disappear.

authority will bear all of the burden of stabilizing the economy; (ii) Nash equilibrium will result in large welfare losses when monetary authorities are benevolent, if fiscal authority discounts too much the future or aims for an excessive level of output; (iii) if we have the (ii) circumstances, and there is a regime of fiscal leadership, then the result will be very similar to (i). The last part is dedicated to relate the papers to the existing literature.

The Blanchard and Fisher's quote

> *"in an economy in which fluctuations are partly due to the combination of aggregate demand effects and nominal rigidities, fiscal policy also* [additionally to monetary policy] *has the potential to reduce fluctuations in aggregate demand"* (Blanchard & Fisher (1989), page 583),

inspired Lambertini & Rovelli (2003) to study the monetary and fiscal policy coordination in a game theoretic approach. Particularly, they argue that both monetary and fiscal authorities have a decisive impact on aggregate demand, even if their loss functions or structural equations were not considering that aggregate demand. The authors suggest that each policy maker prefers to be the follower in a Stackelberg situation. Moreover, when compared to the Nash solution, both Stackelberg solutions, i.e. fiscal leadership and monetary leadership, are preferable. The authors also argue that fiscal authorities would naturally behave as leaders in a strategic game with monetary authorities, since the policy process underlying monetary decisions can be implemented in a relative short time; on the other hand, the policy process underlying fiscal decisions cannot be easily modified once decisions get the stage of implementation.

Favero (2004) shows that the strategic complementarity or substitutability of fiscal and monetary policy depends on the type of shocks hitting the economy, and that countercyclical fiscal policy can be welfare-reducing if fiscal and monetary policy rules are inertial and not coordinated.

The next section details[7] the aggregate demand block represented by an intertemporal

[7]This work will restrict attention to the final equations, namely the derivation from microeconomic principles will not be described.

IS curve, the aggregate supply block represented by a new Keynesian Phillips curve, and a government budget constraint which should be complied by the government. These equations will serve as the basic constraints of the economy. The IS and Phillips curve equations are derived from optimization problems[8], that is, the representative agent's problem and the pricing decisions of individual firms. Thus, the coefficients in these equations are functions of the underlying structural parameters of the consumer's utility function, the production function and the price-setting process.

2.4 The baseline model

The new keynesian[9] model has been the basic model used to analyze optimal monetary and fiscal policy rules. The main argument to justify the specification of this model refers to the fact that it is a linear approximation[10], in logarithmic form, of the dynamic stochastic general equilibrium (abbreviated DSGE or sometimes SDGE) model with sticky prices. The DSGE methodology attempts to explain aggregate economic phenomena, such as economic growth, business cycles, and the effects of monetary and fiscal policy, on the basis of macroeconomic models derived from microeconomic principles, consequently immune to Lucas critique[11]. The model consists of an aggregate supply equation, also known as new keynesian Phillips curve, an aggregate demand intertemporal forward-looking IS curve, which comes from an intertemporal relationship between investment and savings. Additionally, there are an intertemporal budget constraint which the government should comply and the optimal monetary and fiscal rules. The two rules will be derived later on. The aggregate demand function of the economy represented by an intertemporal IS curve

[8] We point out that these equations will not be derived in this work.

[9] New Keynesian economics is a school of macroeconomics which incorporates microeconomic tools and has two basic assumptions: households and firms have rational expectations; prices and wages are sticky, i.e. they do not adjust instantaneously to changes in economic conditions.

[10] The DSGE models presented here have a reduced form and a log-linearized structure around the steady state values, since the intertemporal maximization of firms and families yields nonlinear equilibrium relationships.

[11] The outstanding economist Robert Lucas [Lucas (1976)] demonstrated that economic policy entirely on the basis of relationships observed in historical data can be misleading, since the parameters of those models are not structural (policy-invariant).

is a result of the equilibrium relationships of the first-order conditions of families. Basically, we have a model which consists of the intertemporal maximization of the sum of the present value expected from the utility function of a representative family. The IS curve can be modeled taking into account the primary deficit, as in Nordhaus (1994), the public debt as in Kirsanova et al. (2005) and Bénassy (2007), or even the level of government expenditures as in Muscatelli et al. (2004). This work amends the IS curve proposed by Woodford (2003) in order to capture the effects of the public debt on aggregate demand. Thus, the set-up here considers the following IS curve[12] in log-linearized form[13]

$$\hat{x}_t = E_t\hat{x}_{t+1} - \sigma(\hat{i}_t - E_t\pi_{t+1}) + \alpha\hat{b}_t + \hat{r}_t^n, \tag{2.1}$$

where $\hat{x}_t = (\widehat{Y}_t - \widehat{Y}_t^n)$ is the output gap (difference between actual and potential output), $\hat{i}_t$ is the nominal interest rate, $\hat{r}_t^n$ is a demand shock, E_t represents the expected value of both inflation rate π_{t+1} and output gap $\hat{x}_{t+1}$, b_t is the real stock of debt, the parameter $\sigma > 0$ represents the intertemporal elasticity of substitution in private spending, and finally the parameter α measures the sensibility of output gap to the debt. Note that the aggregate demand relationship depends mainly on the values expected for short-term variations and not just on present values. Thereby, changes in current variables are less important than changes in expectational variables. On the aggregate supply side (Phillips curve), firms face a decision to choose a price that solves the profit maximization problem. The assumption of price rigidity [Calvo (1983)], in which a fraction $0 < \vartheta < 1$ of goods prices remains fixed each period, is done, then the aggregate inflation rate and aggregate output in any period t must satisfy an aggregate-supply (log-linearized) relation of the form

$$\pi_t = \kappa\hat{x}_t + \beta E_t\pi_{t+1} + \nu_t, \tag{2.2}$$

where the inflation rate at period t (π_t) depends on the expected E_t inflation rate at $t+1$, and the current output gap $\hat{x}$. We allow a supply shock[14] denoted by ν_t. The parameter

[12]Note that this equation is conducted in the context of a closed economy.
[13]The hat notation is used to denote deviations from the steady state in logarithm form.
[14]As in Woodford (2003), ν_t generates the trade-off between inflation and output.

$\kappa > 0$ measures the sensitivity of inflation rate to the output gap, and is determined by the frequency of price adjustment and the marginal cost elasticity in relation to the real level of economic activity. The discount factor of the private sector and policymakers is represented by β, where $0 < \beta < 1$. This parameter measures the sensibility of agents to the inflation rate.

The debt in Equation (2.1) also needs to be modeled. This work models the real stock of debt $\hat{b}_t$ similarly to Kirsanova et al. (2005). Thus, the real stock of debt at the beginning of period t, $\hat{b}_t$, depends on the stock of debt at the last period, $\hat{b}_{t-1}$, added to the flows of interest payments, government spending, and revenues, such that[15]:

$$\hat{b}_t = (1 + i^*)\hat{b}_{t-1} + \bar{b}\hat{i}_t + \hat{g}_t - \varpi\hat{x}_t + \eta_t, \tag{2.3}$$

where i^* is the equilibrium interest rate, $\bar{b}$ accounts for the steady state value of debt, $\hat{i}_t$ is the interest rate anteriorly defined, $\hat{g}_t$ is the government spending, ϖ represents the tax rate, $\hat{x}_t$ is the output gap, and finally η_t is the debt shock. Note that the tax revenues vary with output through the term $\varpi\hat{x}_t$.

The model discussed in this section models the channels of transmission of both monetary policy and fiscal policy via interest rate and government spending, respectively. Through Equation (2.1), one can see that the monetary transmission takes place when an increase (decrease) in the interest rate[16] decreases (increases) the level of activity (aggregate demand) of the economy, which in turn decreases (increases) the inflation rate via Equation (2.2). On the other hand, Equation (2.3) establishes that an increase (decrease) in the government spending increases (descreases) the level of debt, which in turn increases (decreases) the level of activity of the economy via Equation (2.1). The ultimate result is an increase (decrease) in the inflation rate via Equation (2.2). Note also that an increase in the interest rate tends to increase the payments to the private sector, thus boosting the aggregate demand. Additionally, a high inflation rate has corrosive effects

[15]This version is log-linearized around the steady state values.

[16]Notice that the increase (decrease) in the interest rate must be higher (lower) than the expected inflation at period $t + 1$.

on the income yielded from the public bonds.

Equations (2.1), (2.3), and (2.2) constitute the basic equilibrium constraints to implement the optimization problems of section (2.6). The resulting optimization procedures close the model with the behavior of monetary policy (interest rate rule) and fiscal policy (government spending rule). The next section will introduce the reader to game theoretic tools, which also will help to comprehend the regimes of policy coordination.

2.5 Game theoretic approach

As emphasized earlier, this work deals with policy games, i.e. monetary and fiscal authorities interacting with each other in order to minimize their loss functions. Basically, in all game theoretic models the main entity is the player, and he may be faced as an individual, a group of individuals, or a government, making decisions. If players in a game act independently of each other, then we have a non-cooperative model while if they correlate their actions, then we have a cooperative solution.

Despite the types of models, a reliable representation of a game is made necessary. From the non-cooperative point of view, we can highlight two forms, i.e. normal form[17] and extensive form. The first one deals with models in which each player chooses his plan of action once and for all and simultaneously (the players do not know the actions chosen by other players while taking their actions). The second one analyzes cases where decisions are made sequentially, i.e. each player may assess his plan of action whenever he has to take a decision. We notice that the normal form will be used to represent the Nash solution, whereas the extensive form will be utilized to represent the Stackelberg solution.

The cooperative case, among other representations, may be seen as a bargain problem or as a social welfare maximization problem. The present work makes a description synthesizing both bargain problem and social welfare problem. A special attention is

[17] Also known as strategic form.

given to the description of the former, though the cooperative procedure actually applied was the criteria of social welfare. The reason lies basically on the simplicity demonstrated by this criterion. Future works encompassing a bargain problem in such a situation may be very profitable.

All the games which will be described here have two individual players, namely monetary authority (central bank) and fiscal authority (treasury). Each player has his own instrument: the monetary authority controls the interest rate (i); the fiscal authority control the government expenditures (g).

The interaction between these authorities, as emphasized earlier, takes place in three manners:

(1) when monetary and fiscal policymakers set their instruments simultaneously and non-cooperatively in a Nash game; this regime suits the case of a game in normal form;

(2) when the fiscal (monetary) authority moves first, as a Stackelberg leader, anticipating the response from the fiscal (monetary) authority; a game in extensive form is the appropriate manner to represent that coordination;

(3) when monetary and fiscal policymakers set their instruments simultaneously but they cooperate with each other in pursuit of a common objective, one can interpret this as a situation where players coordinate their actions to maximize social welfare.

In the following subsections, we will describe the three types of models (normal, extensive, and bargain problem) according to Osborne & Rubinstein (1994). Additionally, we will incorporate the policy instruments to define the equilibria under each regime of coordination (Nash, Stakelberg leadership, cooperation).

2.5.1 Normal form

The normal form is the simplest manner to represent a game. In order to represent a game in normal form, we only need to know the set of players, the set of strategies of each player, and player's preferences over the set of consequences (or outcomes) that are induced by the strategy profiles of the game.

Formally, we define a game in normal form to be a triple $\Psi = (N, (C_i)_{i \in N}, (u_i)_{i \in N})$, where

- N is the set of players;
- C_i is the set of strategies available to player i;
- u_i is a function which assigns to each strategy profile $c \in \times_{i \in N} C_i$ a real number, namely to each strategy profile $c \in \times_{i \in N} C_i$ the real number $u_i(c)$ represents the expected payoff of player i, if c was the combination of strategies implemented by the players.

A game in normal form Ψ is said to be finite if the set of players N and the sets of strategies C_i are finite.

It is important to point out that differently from extensive games, where the time factor is present, in the analysis of games in normal form time is not considered, since the players choose their strategies simultaneously and once and for all, that is, each player chooses a strategy without knowing the strategies chosen by other players.

Strategies

Definition 2.5.1. *A pure strategy of player i in a normal game $\Psi = (N, (C_i)_{i \in N}, (u_i)_{i \in N})$ is any strategy $c_i \in C_i$.*

A strategy profile of the game is an element of $\times_{i \in N} C_i$.

Nash equilibrium

A strategy profile is a Nash equilibrium if even knowing other players' strategies, a player does not have incentive to change his strategy because it is the best choice to other players' strategies.

Definition 2.5.2. *A profile of pure strategies c^* is a Nash equilibrium of* $\Psi = (N, (C_i)_{i\in N}, (u_i)_{i\in N})$ *iff*[18] $u_i(c^*_{-i}, c^*_i) \geq u_i(c^*_{-i}, c_i)$, $\forall i \in N$.

Application: A Nash game between fiscal and monetary policy-makers

Consider the following equalities:

$$u_M(\vec{i}, \vec{g}) = -L^M,$$

$$u_F(\vec{i}, \vec{g}) = -L^F,$$

where u_j is the utility associated to the monetary M authority or fiscal F authority, $\vec{i}$ and $\vec{g}$ are vectors of interest rate and government expenditure, respectively. L^j represents the loss function related to each authority[19].

Consider still C_j as being the set of strategies available to authority j, such that $C_M = \{(i_0, i_1, i_2, \cdots), i_k \in I\!R^+\}$ and $C_F = \{(g_0, g_1, g_2, \cdots), g_k \in I\!R^+\}$. Then the profile of pure strategies $(\vec{i}^*, \vec{g}^*)$ is a Nash equilibrium iff

$$u_M(\vec{i}^*, \vec{g}^*) \geq u_M(\vec{i}, \vec{g}^*),$$

and

$$u_F(\vec{i}^*, \vec{g}^*) \geq u_F(\vec{i}^*, \vec{g}),$$

$\forall \vec{i} \in C_M$ and $\forall \vec{g} \in C_F$.

2.5.2 Extensive form

An extensive form game is appropriate when we want to take into account the order a game runs. Hence, an extensive games are represented by tree diagrams. The structure of the game suits well to certain situations where decision are taken sequentially, e.g. we

[18] Iff is an abbreviation for *if and only if.*

[19] The fight against macroeconomic fluctuations is translated by specifying a loss function. That function usually penalizes variances in both inflation and output, giving weighs on them. The loss function is a result of microeconomic principles, see Woodford (2003) for details. Section (2.6) will specify the loss of each authority.

can imagine two players: player 1 moves first and chooses an action A, then player 2 sees player 1's move and then chooses an action B.

In this section we provide a formal description of an extensive game with perfect information[20] (hereafter extensive game). Initially, we focus on games in which all important moves are made by the players, i.e. randomness does not intervene . An extensive game is a vector $\Gamma = (N, H, P, \{u_i : i \in N\})$, where

- N is a set of players;
- H is a set made by sequences (finite or infinite) of actions. The set H is closed with respect to prefixes[21], namely $h \in H$ and h' is a prefix of h, then $h' \in H$. Each member of H is a history and each node is characterized by a particular history (sequence of necessary actions to achieve the node). A history is called terminal if it is not a strict prefix to any other history at H. The set of terminal histories is represented by Z.
- $P : (H - Z) \to N$ is a function which assigns to each member of $H - Z$ (set of nonterminal histories) a member of N. $P(h)$ represents the player who moves after history h, thus, if $P(h) = i$ the player i moves after h.
- $u_i : Z \to I\!R$ is a function which assigns to each terminal history a real number which in turn represents a preference relation of player i on any possible game result.

A game in extensive form is said to be finite if the sets N and H are finite.

Throughout this section we stipulate that after any nonterminal history h player $P(h)$ chooses an action from the set $A(h) = \{a : (h, a) \in H\}$.

[20] A perfect information mimics a situation in which each player, when making any decision, has perfect information on all events that previously occurred.

[21] If $h =< a_1, a_2, a_3, \ldots, a_n >$ is a history, then $<>$ and $< a_1, a_2, \ldots, a_k >$, where $1 \leq k \leq n$, are the prefixes of h. If $k \neq n$, then we have a strict prefix.

Strategies

A strategy of a player i can be viewed as a plan that describes all the actions chosen by player i for every history that possibly may happen and i is called upon to player after that history.

Definition 2.5.3. *A pure strategy of player $i \in N$ in an extensive game $\Gamma = (N, H, P, \{u_i : i \in N\})$ is a function that assigns an action in $A(h)$ to each nonterminal history $h \in (H - Z)$ for which $P(h) = i$.*

A strategy profile is a collection of strategies, one for each player of the game. Note that a strategy profile determines a unique path along the game tree and, consequently, a unique terminal history of the game. Let z^s be the terminal history associated with the strategy profile s. We write $u_i(s)$ to denote $u_i(z^s)$.

Nash equilibrium

We say that a strategy profile is a Nash equilibrium iff no player has incentive to change his strategy, in doing so the player will not increase his payoff. Formally, it is possible to define the Nash equilibrium under strategies treated as choices that are made once and for all before play begins.

Definition 2.5.4. *A strategy profile s^* of an extensive game $\Gamma = (N, H, P, \{u_i : i \in N\})$ is a Nash equilibrium if for every player i we have*[22]

$$u(s^*_{-i}, s^*_i) \geq u(s^*_{-i}, s_i)$$

for every pure strategy s_i.

Application: Stackelberg leadership: The case of fiscal leadership

Suppose that the fiscal authority moves first [Figure 2.1], as a Stackelberg leader, anticipating the response from the monetary authority. If the fiscal authority takes into

[22]We use s_{-i} to denote a strategy profile without player's i strategy.

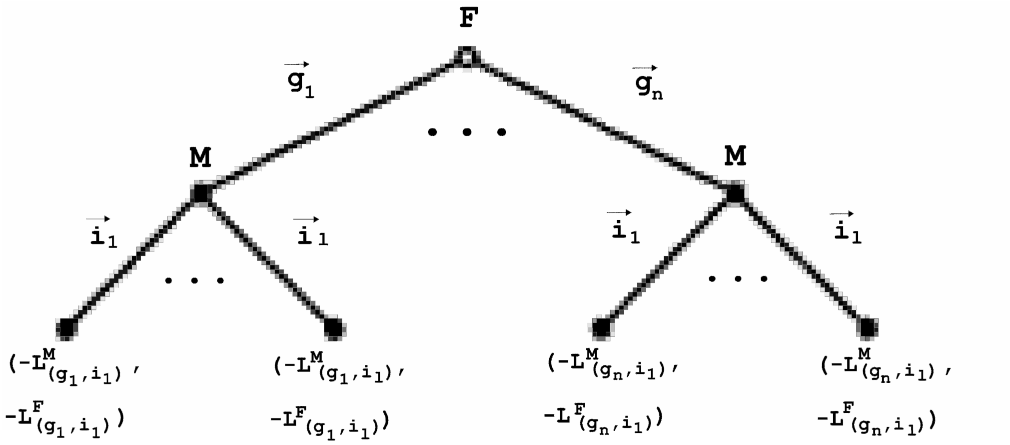

Figure 2.1: An illustrative extensive game that models the fiscal leadership

account the follower's optimal policy, the best fiscal response depends on monetary best response, that is, $\vec{g}^* = \vec{g}^*(\vec{i}^*)$, whereas the follower's optimal policy is the Nash solution.

The utilities of each authority once more can be interpreted as

$$u_M(\vec{i}, \vec{g}) = -L^M,$$

$$u_F(\vec{i}, \vec{g}) = -L^F.$$

However, note that for each $k \in \{1, 2, \ldots, n\}$, $\vec{i}^*_k$ is such that

$$u^M(\vec{i}^*_k, \vec{g}_k) \geq u^M(\vec{i}_j, \vec{g}_k)$$

$\forall j \in \{1, 2, \ldots, l\}$. On the other hand, $\vec{g}^*$ is equal to $\vec{g}_k$ such that

$$u^F(\vec{i}^*_k, \vec{g}_k) \geq u^F(\vec{i}^*_j, \vec{g}_j)$$

$\forall j \in \{1, 2, \ldots, n\}$.

This application follows a similar routine when we have a monetary leadership, being only necessary to invert the variables.

2.5.3 Cooperative games

The models studied in sections 2.5.2 and 2.5.1 (labeled non-cooperative games) assume that the players do not cooperate with each other. In this section we analyze two-person bargaining problems from the perspective of normal games.

Bargain problem

Nash Jr. (1950a) proposed a bargaining problem which divides the gains from agreement according to the relative strengths of the two' parties bargaining position. The inefficient outcome resulted when negotiations break down determines the amount of power a side possesses.

We define a bargaining problem between two players to be a pair (F, v), where F is a set of possible utilities that the two players can jointly achieve, and v is the vector of utilities when the players disagree. It is assumed that F is a compact and convex subset of $\mathbb{R}^2$, $v = (v_1, v_2) \in \mathbb{R}^2$, and the set $F \cap \{(x_1, x_2) : x_1 \geq v_1, x_2 \geq v_2\}$ is non-empty. The convexity is justified if the players agree to randomize jointly the strategies, such that if the vectors x and y are possible and $0 \leq \theta \leq 1$, then the expected utility $\theta x + (1 - \theta)y$ can be fulfilled. We say that a bargaining problem is essential if there exists at least one vector $y \in F$ such that $y_1 > v_1$ and $y_2 > v_2$.

Note that in order to interpret the bargaining problem we need to mimic how the structure of the game would be derived in a two-player normal game context of $\Gamma = (\{1, 2\}, C_1, C_2, u_2, u_2)$. A possible manner to define F is:

$$F = \{u_1(\mu), u_2(\mu)\},$$

where $C = C_1 \times C_2$ and $u_i(\mu) = \sum_{c \in C} \mu(c) u_i(c)$. We also can define F as the set of utility vectors of all the Nash equilibria of game Γ.

The vector v of disagreement outcomes may be defined for player i as the following minimax game:

$$v_1 = \min_{\sigma_2 \in \Delta(C_2)} \max_{\sigma_1 \in \Delta(C_1)} u_1(\sigma_1, \sigma_2),$$

$$v_2 = \min_{\sigma_1 \in \Delta(C_1)} \max_{\sigma_2 \in \Delta(C_2)} u_1(\sigma_1, \sigma_2).$$

The two-players bargaining problem (F, v) aims to find out a vector $\varphi(F, v) \in I\!R^2$, which is result of a cooperative process. The solution of this process hinges on the fulfillment of the following Nash axioms[23]:

PAR. Pareto efficiency (strong). $\varphi(F, v)$ is a vector in F, and for all $x \in F$, if $x \geq \varphi(F, v)$, then $x = \varphi(F, v)$;

IRA. Individual rationality. $\varphi(F, v) \geq v$;

IPAT. Invariance to positive affine transformations: For any numbers λ_1, λ_2, γ_1, and γ_2, such that $\lambda_1 > 0$ and $\lambda_2 > 0$ if

$$G = \{(\lambda_1 x_1 + \gamma_1, \lambda_2 x_2 + \gamma_2) : (x_1, x_2) \in F\},$$

and

$$w = (\lambda_1 v_1 + \gamma_1, \lambda_2 v_2 + \gamma_2),$$

then $\varphi(G, w) = (\lambda_1 \varphi_1(F, v) + \gamma_1, \lambda_2 \varphi_2(F, v) + \gamma_2)$;

IIA. Independence of irrelevant alternatives. For any closed and convex set G, if $G \subseteq F$ and $\varphi(F, v) \in G$, then $\varphi(G, v) = \varphi(F, v)$;

SYM. Symmetry. If $v_1 = v_2$ and $\{(x_2, x_1) : (x_1, x_2) \in F\} = F$, then $\varphi_1(G, v) = \varphi_2(F, v)$.

The PAR axiom imposes the impracticability of inefficient outcomes since it opens space for new agreements that make both players better off. Roughly speaking, given a set F a point $x \in F$ satisfies the strong efficiency (of Pareto) if there does not exist another point $y \in F$ such that $y \geq x$ and $y_i > x_i$ for some player i. On the other hand, the weak efficiency (of Pareto) happens when there does not exist $y \in F$ such that $y > x$.

The IRA axiom states that no player should be in a worse situation in the solution than at the disagreement point. We say that a vector v is individually rational in the

[23] For any two vectors x and y in $I\!R^2$, we write $x \geq y$ iff $x_1 \geq y_1$ and $x_2 \geq y_2$; and $x > y$ iff $x_1 > y_1$ and $x_2 > y_2$.

problem (F, v) if $x \geq v$. IPAT axiom states that if the problem (G, w) may be derived from problem (F, v), through a positive affine transformation, then the solution (G, w) may be obtained making the same transformation in the solution of (F, v).

The IIA axiom asserts that the elimination of viable alternatives which are not solutions do not alter the result. The symmetry (SYM) says that if two players are indistinguishable, then the solution should treat them equally.

The main result obtained by Nash is that there exists only one solution that satisfies PAR, IRA, IPAT, IIA, and SYM, which we call Nash solution of the bargaining problem.

Theorem 2.5.1. *There exists only one solution*[24] $\varphi(\cdot,\cdot)$ *which satisfies the axioms PAR, IRA, IPAT, IIA, and SYM. That solution satisfies, for any two-person bargaining problem* (F, v)*,*

$$\varphi(F, v) \in \text{argmax}_{x \in F, x \geq v}(x_1 - v_1)(x_2 - v_2).$$

Social welfare criterion

The social welfare criterion defines a social welfare function which depends upon both monetary loss and fiscal loss. As noted earlier, this work makes use of that criterion to analyze the cooperative solution. The cooperation occurs indirectly when both authorities associate a positive weight on their instrument variables. The mechanism permits a direct adjustment to ongoing actions taken by the other authority. Basically, the problem is to maximize a social utility (welfare) or, on the other hand, to minimize the social loss function L^S, which is defined by $L^S = L^M + L^F$, that is, the sum of the authorities' losses[25].

Application: Cooperation between policy-makers

Consider now the following definitions:

$$u_M = -L^M,$$

[24] The proof can be found in Osborne & Rubinstein (1994).

[25] That sum does not have microeconomic base. However, it is of great value in a game theoretic approach.

$$u_F = -L^F,$$

where u_j is the utility of authority j. L^j represents the loss function related of authority j.

We say that a cooperative solution[26] takes place between the fiscal and monetary authorities if there exists only one solution $\varphi(\cdot,\cdot)$ which satisfies the axioms PAR, SYM, and IIA, that is, that solutions satisfies the two-authorities bargaining problem (F, v),

$$\varphi(F, v) \in \text{argmax}_{x \in F, x \geq v}(x_1 - v_1)(x_2 - v_2),$$

where $x_1 = u_M$, $x_2 = u_F$, and v_1 and v_2 are the monetary utility and fiscal utility, respectively, under the Nash equilibrium.

Despite the bargain solution, we can interpret the cooperation in terms of social welfare. Thus, the objective is to minimize the social loss function, i.e. $L^S = L^M + L^F$.

2.6 Optimizing fiscal and monetary policy games

Taking the underlying model described in Section (2.4) and following the Lagrangian techniques discussed in Woodford (2003), we derive the optimal reaction functions for different regimes of coordination. Both monetary and fiscal authorities minimize the loss function subject to the equilibrium conditions. The authorities solve the optimization problems once and for all and commit themselves to the optimal policy rules, excluding any incentive to deviate from them.

The Lagrangian techniques [Woodford (2003)] cited above provide relevant properties of time consistency and timelessness to the policy rules. The former characteristic is due to commitment, and the latter hinges on the fact that it is not necessary to minimize expected losses from time $t = t_0$ onwards, depending on the new state of the economy.

[26] As explained earlier, this application is merely illustrative, since the actually implemented procedure is the social welfare criteria.

2.6.1 A Nash game between fiscal and monetary policymakers

The monetary authority is represented by the central bank which tries to minimize a loss function, with positive weights γ_π, γ_x, and γ_i, on the squared deviations of inflation from the inflation target (zero), squared output gap and squared interest rate deviations from the equilibrium (i^*), respectively, such that

$$L_t^M = \gamma_\pi \pi_t^2 + \gamma_x \hat{x}_t^2 + \gamma_i (\hat{i}_t - i^*)^2,$$

subject to the constraints in the economy.

The monetary authority problem is to solve

$$\min E_0 \left\{ \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left(\gamma_\pi \pi_t^2 + \gamma_x \hat{x}_t^2 + \gamma_i (\hat{i}_t - i^*)^2 \right) \right\},$$

subject to

$$\text{(2.1) and (2.2).} \tag{2.4}$$

Constructing the Lagrangian for this problem, we have[27]:

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{l} \frac{1}{2}\gamma_\pi \pi_t^2 + \frac{1}{2}\gamma_x \hat{x}_t^2 + \frac{1}{2}\gamma_i (\hat{i}_t - i^*)^2 \\ +\Lambda_{1,t} \left(\hat{x}_t - \hat{x}_{t+1} + \sigma(\hat{i}_t - \pi_{t+1}) - \alpha \hat{b}_t - \hat{r}_t^n \right) \\ +\Lambda_{2,t} \left(\pi_t - \kappa \hat{x}_t - \beta \pi_{t+1} - \nu_t \right) \end{array} \right] \right\}, \tag{2.5}$$

where $\Lambda_{1,t}$ and $\Lambda_{2,t}$ are the Lagrange multipliers associated with the constraints in period t. The first order conditions yield the following expressions:

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = \gamma_\pi \pi_t - \beta^{-1} \sigma \Lambda_{1,t-1} + \Lambda_{2,t} - \Lambda_{2,t-1} = 0,$$
$$\frac{\partial \mathcal{L}}{\partial \hat{x}_t} = \gamma_x \hat{x}_t + \Lambda_{1,t} - \beta^{-1} \Lambda_{1,t-1} - \kappa \Lambda_{2,t} = 0,$$
$$\frac{\partial \mathcal{L}}{\partial (\hat{i}_t - i^*)} = \gamma_i (\hat{i}_t - i^*) + \sigma \Lambda_{1,t} = 0. \tag{2.6}$$

[27]Note that the dating of the expectations operator captures the idea of the policy maker choosing a rule ex ante which they will follow in the future. As we have a solution under commitment, the Lagrangian is solved for expectations at time zero, which characterizes the time when the rule was defined, thereafter followed without deviations. Thus, we removed the expectations operator on both inflation and output gap at $t + 1$.

Isolating and substituting the Lagrange multipliers we obtain the following optimal nominal interest rate rule[28]:

$$\hat{i}_t = -\Gamma_0 i^* + \Gamma_{i,1}\hat{i}_{t-1} - \Gamma_{i,2}\hat{i}_{t-2} + \Gamma_{\pi,0}\pi_t + \Gamma_{x,0}\hat{x}_t - \Gamma_{x,1}\hat{x}_{t-1}, \tag{2.7}$$

where the coefficients are $\Gamma_0 = \frac{\sigma\kappa}{\beta}$, $\Gamma_{i,1} = \left(\frac{\sigma\kappa}{\beta} + \frac{1}{\beta} + 1\right)$, $\Gamma_{i,2} = \frac{1}{\beta}$, $\Gamma_{\pi,0} = \frac{\gamma_\pi \sigma\kappa}{\gamma_i}$, $\Gamma_{x,0} = \frac{\gamma_x \sigma}{\gamma_i}$, and $\Gamma_{x,1} = \frac{\gamma_x \sigma}{\gamma_i}$.

The rule (2.7), which the central bank commits to follow, has contemporaneous and lagged responses to output gap. Additionally, it encompasses a history dependence since the interest rate responds to past interest rates. The interest rate response to past interest rates depends inversely on the size of β. Thereby, the more importance consumers attach to the future level of the variables, the stronger the monetary policy leverage is. Note that the value of κ affects directly the response of the interest rate, that is, the steeper the slope of the Phillips curve, the stronger the interest rate response to inflation deviations from target. Note also that increases in the weight placed on interest rate γ_i diminishes the interest rate reaction to inflation (from target) and output gap deviations. Moreover, the parameter σ dictates the proportion to which the interest rate responds to the target variables. Thus, when the intertemporal elasticity of substitution in private spending σ is steeper, the interest rate response to deviations of inflation from target, as well as to output gap changes, are stronger.

The fiscal side resembles the monetary one, but here the fiscal authority (treasury) takes into account government spendings so that the loss function takes the form[29]:

$$L_t^F = \rho_\pi \pi_t^2 + \rho_x \hat{x}_t^2 + \rho_g \hat{g}_t^2,$$

where ρ_π, ρ_x, and ρ_g are positive weights on the squared deviations of inflation from the inflation target (zero), squared output gap and squared government expenditures deviations from the equilibrium (zero), respectively. Note that the debt does not enter

[28] This solution coincides with that proposed by Woodford (2003).

[29] Kirsanova et al. (2005) and Dixit & Lambertini (2000) use a similar loss function.

the loss function. The reason relies on the fact that if the fiscal policy feeds back on debt with a large coefficient, then it tends to be welfare-reducing, since the economy will exhibit cycles, and so increasing the volatility of both inflation and output [Kirsanova et al. (2005)].

The fiscal authority problem is to solve

$$\min E_0 \left\{ \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left(\rho_\pi \pi_t^2 + \rho_x \hat{x}_t^2 + \rho_g \hat{g}_t^2 \right) \right\},$$

subject to

$$(2.1), (2.3) \text{ and } (2.2). \tag{2.8}$$

The fiscal authority's Lagrangian is

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{l} \frac{1}{2}\rho_\pi \pi_t^2 + \frac{1}{2}\rho_x \hat{x}_t^2 + \frac{1}{2}\rho_g \hat{g}_t^2 \\ +\Lambda_{1,t} \left(\hat{x}_t - \hat{x}_{t+1} + \sigma(\hat{i}_t - \pi_{t+1}) - \alpha \hat{b}_t - \hat{r}_t^n \right) \\ +\Lambda_{2,t} \left(\pi_t - \kappa \hat{x}_t - \beta \pi_{t+1} - \nu_t \right) \\ +\Lambda_{3,t} \left(\hat{b}_t - (1+i^*)\hat{b}_{t-1} - \overline{b}\hat{i}_t - \hat{g}_t + \varpi \hat{x}_t - \eta_t \right) \end{array} \right] \right\}. \tag{2.9}$$

The associated first order conditions are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \pi_t} &= \rho_\pi \pi_t - \beta^{-1}\sigma \Lambda_{1,t-1} + \Lambda_{2,t} - \Lambda_{2,t-1} = 0, \\ \frac{\partial \mathcal{L}}{\partial \hat{x}_t} &= \rho_x \hat{x}_t + \Lambda_{1,t} - \beta^{-1}\Lambda_{1,t-1} - \kappa \Lambda_{2,t} + \varpi \Lambda_{3,t} = 0, \\ \frac{\partial \mathcal{L}}{\partial \hat{g}_t} &= \rho_g \hat{g}_t - \Lambda_{3,t} = 0, \\ \frac{\partial \mathcal{L}}{\partial \hat{b}_t} &= -\alpha \Lambda_{1,t} + \Lambda_{3,t} - (1+i^*)\beta E_t(\Lambda_{3,t+1}) = 0. \end{aligned} \tag{2.10}$$

Eliminating the Lagrangian multipliers we have the optimal nominal government spending rule:

$$\hat{g}_t = -\Theta_{\pi,0}\pi_t + \Theta_{g,1}\hat{g}_{t-1} - \Theta_{g,2}\hat{g}_{t-2} + \Theta_{g,+1}E_t\hat{g}_{t+1} - \Theta_{x,0}\hat{x}_t + \Theta_{x,1}\hat{x}_{t-1}, \tag{2.11}$$

where the coefficients are: $\Theta_{\pi,0} = \frac{\rho_\pi \alpha \kappa}{\rho_g B}$, $\Theta_{g,1} = \frac{A}{B}$, $\Theta_{g,2} = \frac{1}{\beta B}$, $\Theta_{g,+1} = (1+i^*)\frac{\beta}{B}$, $\Theta_{x,0} = \frac{\rho_x \alpha}{\rho_g B}$, $\Theta_{x,1} = \frac{\rho_x \alpha}{\rho_g B}$. Additionally, $A = \left(\beta^{-1}\sigma\kappa + \frac{1}{\beta} + 1 + (1+i^*)\right)$,
$B = ((1+i^*)(\sigma\kappa\alpha + 1 + \beta) + \varpi\alpha + 1)$.

In this set-up, Equation (2.11), which the treasury commits to follow, fiscal policy feeds back on the level of inflation, output gap, and government spending. The rule encompasses a forward and backward history dependence since the government spending responds to past and future government expenditures. We note that increases in the weight placed on government spending ρ_g diminishes the government spending reaction to inflation (from target) and output gap deviations.

2.6.2 Stackelberg leadership

We now address the equilibrium which emerges when fiscal (monetary) authority moves first, as a Stackelberg leader, anticipating the response from the monetary (fiscal) authority. The leader takes into account the follower's optimal policy, whereas the follower's optimal policy remains as the Nash solution.

Considering primarily the loss function for the fiscal leader, we have the problem:

$$\min E_0 \left\{ \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left(\rho_\pi \pi_t^2 + \rho_x \hat{x}_t^2 + \rho_g \hat{g}_t^2 \right) \right\},$$

subject to

$$(2.1), (2.3), (2.2) \text{ and } (2.7), \tag{2.12}$$

and the corresponding Lagrangian is

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{l} \frac{1}{2}\rho_\pi \pi_t^2 + \frac{1}{2}\rho_x \hat{x}_t^2 + \frac{1}{2}\rho_g \hat{g}_t^2 \\ +\Lambda_{1,t} \left(\hat{x}_t - \hat{x}_{t+1} + \sigma(\hat{i}_t - \pi_{t+1}) - \alpha \hat{b}_t - \hat{r}_t^n \right) \\ +\Lambda_{2,t} \left(\pi_t - \kappa \hat{x}_t - \beta \pi_{t+1} - \nu_t \right) \\ +\Lambda_{3,t} \left(\hat{b}_t - (1+i^*)\hat{b}_{t-1} - \overline{b}\hat{i}_t - \hat{g}_t + \varpi \hat{x}_t - \eta_t \right) \\ +\Lambda_{4,t} \left(\hat{i}_t + \Gamma_0 i^* - \Gamma_{i,1}\hat{i}_{t-1} + \Gamma_{i,2}\hat{i}_{t-2} - \Gamma_{\pi,0}\pi_t - \Gamma_{x,0}\hat{x}_t + \Gamma_{x,1}\hat{x}_{t-1} \right). \end{array} \right] \right\} \tag{2.13}$$

The first order conditions associated are

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \pi_t} &= \rho_\pi \pi_t - \beta^{-1}\sigma\Lambda_{1,t-1} + \Lambda_{2,t} - \Lambda_{2,t-1} - \Gamma_{\pi,0}\Lambda_{4,t} = 0, \\
\frac{\partial \mathcal{L}}{\partial \hat{x}_t} &= \rho_x \hat{x}_t + \Lambda_{1,t} - \beta^{-1}\Lambda_{1,t-1} - \kappa\Lambda_{2,t} + \varpi\Lambda_{3,t} - \Gamma_{\pi,0}\Lambda_{4,t} + \beta\Gamma_{x,1}E_t(\Lambda_{4,t+1}) = 0, \\
\frac{\partial \mathcal{L}}{\partial \hat{g}_t} &= \rho_g \hat{g}_t - \Lambda_{3,t} = 0, \\
\frac{\partial \mathcal{L}}{\partial \hat{b}_t} &= -\alpha\Lambda_{1,t} + \Lambda_{3,t} - (1+i^*)\beta E_t(\Lambda_{3,t+1}) = 0, \\
\frac{\partial \mathcal{L}}{\partial \hat{i}_t} &= \sigma\Lambda_{1,t} - \bar{b}\Lambda_{3,t} + \Lambda_{4,t} - \Gamma_{i,t}\beta E_t(\Lambda_{4,t+1}) = 0.
\end{aligned} \tag{2.14}
$$

The above optimization problem cannot be solved analytically, thereby we implement a numerical solution according to Juillard & Pelgrin (2007). The authors implement a timeless-perspective solution proposed by Woodford (2003). The next chapter will deal with such a problem.

On its turn, the monetary leader aims to minimize:

$$
\min E_0 \left\{ \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left(\gamma_\pi \pi_t^2 + \gamma_x \hat{x}_t^2 + \gamma_i (\hat{i}_t - i^*)^2 \right) \right\},
$$

subject to

$$
(2.1), (2.2) \text{ and } (2.11). \tag{2.15}
$$

The Lagrangian for this problem, is given by

$$
\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{l} \frac{1}{2}\gamma_\pi \pi_t^2 + \frac{1}{2}\gamma_x \hat{x}_t^2 + \frac{1}{2}\gamma_i (\hat{i}_t - i^*)^2 \\ +\Lambda_{1,t} \left(\hat{x}_t - \hat{x}_{t+1} + \sigma(\hat{i}_t - \pi_{t+1}) - \alpha \hat{b}_t - \hat{r}_t^n \right) \\ +\Lambda_{2,t} \left(\pi_t - \kappa \hat{x}_t - \beta \pi_{t+1} - \nu_t \right) \\ +\Lambda_{3,t} \left(\hat{g}_t + \Theta_{\pi,0}\pi_t - \Theta_{g,1}\hat{g}_{t-1} + \Theta_{g,2}\hat{g}_{t-2} - \Theta_{g,+1}E_t\hat{g}_{t+1} + \Theta_{x,0}\hat{x}_t - \Theta_{x,1}\hat{x}_{t-1} \right) \end{array} \right] \right\}. \tag{2.16}
$$

The implied first order conditions are

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = \gamma_\pi \pi_t - \beta^{-1}\sigma \Lambda_{1,t-1} + \Lambda_{2,t} - \Lambda_{2,t-1} + \Theta_{\pi,0}\Lambda_{3,t} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial \hat{x}_t} = \gamma_x \hat{x}_t + \Lambda_{1,t} - \beta^{-1}\Lambda_{1,t-1} - \kappa \Lambda_{2,t} + \Theta_{x,0}\Lambda_{3,t} - \beta \Theta_{x,1} E_t(\Lambda_{3,t+1}) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial (\hat{i}_t - i^*)} = \gamma_i(\hat{i}_t - i^*) + \sigma \Lambda_{1,t} = 0, \tag{2.17}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{g}_t} = \Lambda_{3,t} - \beta \Theta_{g,1} E_t(\Lambda_{3,t+1}) - \beta^{-1}\Theta_{g,1}\Lambda_{3,t-1} = 0. \tag{2.18}$$

Likewise, this problem can not be solved analytically. The numerical solution is discussed in the next chapter.

2.6.3 Cooperation between policymakers

This section analyzes the outcome which occurs if the fiscal and monetary policymakers cooperate with each other in pursuing a common objective. This means that the fiscal (monetary) authority takes into account the monetary (fiscal) reaction function. Under cooperation both fiscal and monetary authorities face a common minimization problem:

$$\min E_0 \left\{ \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left(\xi_\pi \pi_t^2 + \xi_x \hat{x}_t^2 + \xi_i (\hat{i}_t - i^*)^2 + \xi_g \hat{g}_t^2 \right) \right\},$$

subject to

$$(2.1), (2.3) \text{ and } (2.2), \tag{2.19}$$

where $\xi_\pi = \gamma_\pi + \rho_\pi$, $\xi_x = \gamma_x + \rho_x$, $\xi_i = \gamma_i$ and $\xi_g = \rho_g$, that is, the positive weights on the squared deviations of inflation from the inflation target (zero) and squared output gap are the sum of the weights placed by each authority, while the weights on the squared interest rate deviations from the equilibrium (i^*) and squared government spending, remain unchanged.

Constructing the Lagrangian for this problem, we have:

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{l} \frac{1}{2}\xi_\pi \pi_t^2 + \frac{1}{2}\xi_x \hat{x}_t^2 + \frac{1}{2}\xi_i(\hat{i}_t - i^*)^2 + \frac{1}{2}\xi_g \hat{g}_t^2 \\ +\Lambda_{1,t}\left(\hat{x}_t - \hat{x}_{t+1} + \sigma(\hat{i}_t - \pi_{t+1}) - \alpha \hat{b}_t - \hat{r}_t^n\right) \\ +\Lambda_{2,t}\left(\pi_t - \kappa \hat{x}_t - \beta \pi_{t+1} - \nu_t\right) \\ +\Lambda_{3,t}\left(\hat{b}_t - (1+i^*)\hat{b}_{t-1} - \overline{b}\hat{i}_t - \hat{g}_t + \varpi \hat{x}_t - \eta_t\right) \end{array} \right] \right\}, \tag{2.20}$$

with the following first order conditions

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = \xi_\pi \pi_t - \beta^{-1}\sigma\Lambda_{1,t-1} + \Lambda_{2,t} - \Lambda_{2,t-1} = 0,$$
$$\frac{\partial \mathcal{L}}{\partial \hat{x}_t} = \xi_x \hat{x}_t + \Lambda_{1,t} - \beta^{-1}\Lambda_{1,t-1} - \kappa\Lambda_{2,t} + \varpi\Lambda_{3,t} = 0,$$
$$\frac{\partial \mathcal{L}}{\partial (\hat{i}_t - i^*)} = \xi_i(\hat{i}_t - i^*) + \sigma\Lambda_{1,t} - \bar{b}\Lambda_{3,t} = 0,$$
$$\frac{\partial \mathcal{L}}{\partial \hat{g}_t} = \xi_g \hat{g}_t - \Lambda_{3,t} = 0. \quad (2.21)$$

The resulting optimal nominal interest rate rule is given by

$$\hat{i}_t = -\Gamma_0 i^* + \Gamma_{i,1}\hat{i}_{t-1} - \Gamma_{i,2}\hat{i}_{t-2} + \Gamma_{\pi,0}\pi_t + \Gamma_{x,0}\hat{x}_t - \Gamma_{x,1}\hat{x}_{t-1} + \Gamma_{g,0}\hat{g}_t - \Gamma_{g,1}\hat{g}_{t-1} + \Gamma_{g,2}\hat{g}_{t-2}, \quad (2.22)$$

where the coefficients are $\Gamma_0 = \frac{\sigma\kappa}{\beta}$, $\Gamma_{i,1} = \left(\frac{\sigma\kappa}{\beta} + \frac{1}{\beta} + 1\right)$, $\Gamma_{i,2} = \frac{1}{\beta}$, $\Gamma_{\pi,0} = \frac{\xi_\pi\sigma\kappa}{\xi_i}$, $\Gamma_{x,0} = \frac{\xi_x\sigma}{\xi_i}$, and $\Gamma_{x,1} = \frac{\xi_\pi\sigma}{\xi_i}$, $\Gamma_{g,0}\left(\frac{\bar{b}\xi_g}{\xi_i} + \frac{\sigma\varpi\xi_g}{\xi_i}\right)$, $\Gamma_{g,1} = \left(\frac{\sigma\kappa\bar{b}\xi_g}{\beta\xi_i} + \frac{\bar{b}\xi_g}{\beta\xi_i} + \frac{\bar{b}\xi_g}{\xi_i} + \frac{\sigma\varpi\xi_g}{\xi_i}\right)$, $\Gamma_{g,2} = \left(\frac{\bar{b}\xi_g}{\beta\xi_i}\right)$.

The resulting optimal government spending rule is given by

$$\hat{g}_t = -\Theta_{\pi,0}\pi_t + \Theta_{i,0}(\hat{i}_t - i^*) - \Theta_{i,1}(\hat{i}_{t-1} - i^*) + \Theta_{i,2}(\hat{i}_{t-2} - i^*) - \Theta_{x,0}\hat{x}_t \quad (2.23)$$
$$+\Theta_{x,1}\hat{x}_{t-1} + \Theta_{g,1}\hat{g}_{t-1} - \Theta_{g,2}\hat{g}_{t-2},$$

where the coefficients are $\Theta_{\pi,0} = \frac{\kappa\xi_\pi}{C}$, $\Theta_{i,0} = \frac{\xi_i}{\sigma C}$, $\Theta_{i,1} = \left(\frac{\kappa\xi_i}{\beta C} + \frac{\xi_i}{\beta\sigma C} + \frac{\xi_i}{\sigma C}\right)$, $\Theta_{i,2} = \frac{\xi_i}{\beta\sigma C}$, $\Theta_{x,0} = \Theta_{x,1} = \frac{\xi_x}{C}$, $\Theta_{g,1} = \left(\frac{\kappa\bar{b}\xi_g}{\beta C} + \frac{\bar{b}\xi_g}{\beta\sigma C} + \frac{\bar{b}\xi_g}{\sigma C} + \frac{\varpi\xi_g}{C}\right)$, $\Theta_{g,2} = \frac{\bar{b}\xi_g}{\beta\sigma C}$. Finally, $C = \left(\frac{\bar{b}\xi_g}{\sigma} + \varpi\xi_g\right)$.

The above rules resemble Nash ones. However, note that in both the monetary rule and the fiscal rule the instrument variable of the other authority enters the rule, that is, the optimal nominal interest rule responds to current and lagged government spendings, on the other hand, the optimal government spending rule responds to current and lagged interest rates. Thus, the cooperation occurs via those responses. One can also note that the fiscal rule does not respond to future government spending, possibly because of the cooperation.

CHAPTER 3

Numerical Approach

3.1 Resumo

A fim de avaliar os desempenhos das políticas ótimas derivadas para cada regime de coordenação no capítulo anterior, precisamos simular o modelo[1]. Desse modo, para realizar o exercício numérico é necessário que calibremos o modelo[2]. Temos, então, de estipular valores apenas para os parâmetros estruturais [Tabela (3.1)], os quais bastam para que os coeficientes da IS intertemporal, da curva de Phillips, da dívida em relação ao PIB, e das regras fiscal e monetária ótimas sejam obtidos.

A etapa que segue é a simulação de cada regime de coordenação, etapa essa descrita nas seção 3.3. Como precisamos obter soluções para um modelo de expectativas racionais, utilizamos o pacote Dynare para o software MATLAB. Ver Laffargue (1990), Boucekkine (1995), Juillard (1996), Collard & Juillard (2001*a*) e Collard & Juillard (2001*b*) para detalhes. Destaca-se que não há uma solução analítica para obtenção das políticas monetária e fiscal ótimas para a solução de Stackelberg, portanto uma aproximação numérica é utilizada.

[1] Isto se faz necessário porque basicamente precisamos obter as variâncias das trajectórias ótimas das variáveis em cada regime de coordenação, a fim de computar a perda associada.

[2] O modelo é calibrado para o Brasil.

Como destacado anteriormente, precisamos avaliar os desempenhos das políticas ótimas em cada regime de coordenação. A análise é focada principalmente no valor da perda associada a cada regime. Os resultados sugerem que na solução de Stackelberg, quando a autoridade monetária é a líder, temos a menor perda.

Outras duas análises são feitas, a saber, a avaliação gráfica da sensibilidade aos pesos na função perda (tanto no lado monetário quanto fiscal) e a avalição das funções de resposta a impulso. Um resultado interessante na primeira análise, é que as soluções de Nash e cooperativa respondem mais uniformemente as variações nos pesos das variáveis objetivo da função perda. No que concerne à segunda análise, pode-se destacar a magnitude relativamente menor dos impactos dos shocks e também uma velocidade relativamente maior com que as variáveis retornam ao equilíbrio, quando estamos nas soluções de Stackelberg.

Como enfatizado anteriormente, quando a autoridade monetária é a líder, temos a menor perda associada, sugerindo assim que esse regime tem o melhor desempenho. Esse resultado é corroborado pela literatura para trabalhos aplicados ao Brasil, ver Tanner & Ramos (2002), Fialho & Portugal (2005) e Gadelha & Divino (2008). Esses trabalhos mostram que, para o período pós-Real, a economia brasileira estava sob um regime de dominância monetária, a qual pode ser interpretada, em termos de jogos, como um regime em que a autoridade monetária é a líder, e a fiscal a seguidora.

3.2 Introduction

We now turn to the numerical approach. We simulate the model presented and derived in the previous chapter for each regime of coordination. The model is calibrated for the Brazilian economy in the period after the implementation of the Real Plan. Afterwards, we compute the loss associated to each regime (Nash, Stackelberg and cooperation) in order to verify which regime brings the lowest loss. Impulse response functions and sensitivity analysis on the loss are also performed.

3.3 Numerical exercises

In order to evaluate the performance of the optimal monetary and fiscal policies derived for each regime of coordination in the previous chapter, we need to simulate[3] the model, which encompasses the Phillips curve, the IS curve, the government budget constraint, and the optimal monetary and fiscal rules. This section provides simulation details. Additionally, we provide an overview over the losses generated from different monetary and fiscal policy decisions, and also analyze impulse response functions. However, in order to do so it is necessary to calibrate the model. As it is common in the literature, we assume that each period corresponds to a one quarter of a year. The calibrated values are reported in Table 3.1.

Table 3.1: Calibration of parameters

Parameter	Represent	Calibration	Reference
σ	Intertemporal elasticity of substitution in private spending	5.00	Nunes & Portugal (2009).
α	Sensitivity of output gap to the debt	0.20	Pires (2008).
κ	Sensitivity of inflation rate to the output gap	0.50	Gouvea (2007); Walsh (2003).
β	Sensitivity of agents to the inflation rate	0.99	Cavallari (2003); Pires (2008).
i^*	Natural rate of interest	0.07	Barcelos Neto & Portugal (2009).
$\bar{b}$	Steady state debt value	0.20	Kirsanova et al. (2005); Nunes & Portugal (2009).
ϖ	Tax rate	0.26	Kirsanova et al. (2005); Nunes & Portugal (2009).

[3]This is necessary basically because we need to obtain the variances of the optimal trajectories of the variables in each regime of coordination, in order to compute the associated loss.

3.3.1 Simulation of the Nash solution

The model's equilibrium is described by ten equations, five of them are endogenous equations and the remaining ones are exogenous processes. There are ten variables, five endogenous $(\hat{x}_t, \pi_t, \hat{b}_t, \hat{i}_t, \hat{g}_t)$ and five exogenous $(\hat{r}_t^n, \nu_t, \eta_t, \Xi_t, O_t)$.

IS curve: $\hat{x}_t = E_t\hat{x}_{t+1} - \sigma(\hat{i}_t - E_t\pi_{t+1}) + \alpha\hat{b}_t + \hat{r}_t^n$

Phillips curve: $\pi_t = \kappa\hat{x}_t + \beta E_t\pi_{t+1} + \nu_t$

Public debt: $\hat{b}_t = (1 + i^*)\hat{b}_{t-1} + \overline{b}\hat{i}_t + \hat{g}_t - \varpi\hat{x}_t + \eta_t$

Monetary rule: $\hat{i}_t = -\Gamma_0 i^* + \Gamma_{i,1}\hat{i}_{t-1} - \Gamma_{i,2}\hat{i}_{t-2} + \Gamma_{\pi,0}\pi_t + \Gamma_{x,0}\hat{x}_t - \Gamma_{x,1}\hat{x}_{t-1} + \Xi_t$

Fiscal rule: $\hat{g}_t = -\Theta_{\pi,0}\pi_t + \Theta_{g,1}\hat{g}_{t-1} - \Theta_{g,2}\hat{g}_{t-2} + \Theta_{g,+1}E_t\hat{g}_{t+1} - \Theta_{x,0}\hat{x}_t + \Theta_{x,1}\hat{x}_{t-1} + O_t$

Demand shock: $\hat{r}_t^n = \chi_r\hat{r}_{t-1}^n + \varepsilon_r$

Supply shock: $\nu_t = \chi_\nu\nu_{t-1} + \varepsilon_\nu$

Debt shock: $\eta_t = \chi_\eta\eta_{t-1} + \varepsilon_\eta$

Monetary policy shock: $\Xi_t = \chi_\Xi\Xi_{t-1} + \varepsilon_\Xi$

Fiscal policy shock: $O_t = \chi_O O_{t-1} + \varepsilon_O$

The exogenous processes, described by the above shocks, follow $AR(1)$ processes. The reason relies on the fact that an $AR(1)$ process reflects relatively well the persistence that exists in many economic time series. It is also possible to set another process (e.g. $MA(1)$, $AR(2)$, and $ARMA(1)$) but it does not generate significant changes in the dynamics of the economic time series. Moreover, each ε_j is independent and identically distributed ($i.i.d.$) with mean zero and variance σ_j^2. We set $\chi_j = 0.9$ in order to emphasize the high persistence that these shocks usually show, and $\sigma_j^2 = 0.04$, $\forall j$.[4] Note that the effects of a specific shock on the model tend to be contaminated if there exist nonzero correlations. The simulation requires a method which gives solutions for the rational expectations model. To that end a the collection of MATLAB routines (Dynare) was used. Dynare can solve, simulate, and estimate linear and nonlinear models with forward looking variables, thus satisfying our requirement. See Laffargue (1990), Boucekkine (1995), Juillard (1996),

[4]These calibrated values are based on the estimated values using Brazilian data.

Collard & Juillard (2001*a*), and Collard & Juillard (2001*b*) for more details.

3.3.2 Simulation of the Stackelberg solution

The Stackelberg solution is solved numerically under commitment following Juillard & Pelgrin (2007) and Woodford (2003). The structural equations, which represent the constraints on possible equilibrium outcomes under Stackelberg leadership, are assumed to be a system of the form[5]

$$\begin{bmatrix} Z_{t+1} \\ E_t z_{t+1} \end{bmatrix} = A \begin{bmatrix} Z_t \\ z_t \end{bmatrix} + Bu_t + \begin{bmatrix} \varepsilon_{t+1} \\ \mathbf{0}_{n_z \times 1} \end{bmatrix}, \tag{3.1}$$

where z_t is an $n_z \times 1$ vector of nonpredetermined (forward looking) variables, Z_t is an $n_Z \times 1$ vector of predetermined (backward looking) variables, u_t is an $k \times 1$ vector of policy instruments, and ε_{t+1} an $n_Z \times 1$ vector of zero-mean shocks uncorrelated with past values. The number of rows of each matrix then is $n = n_z + n_Z$, and the matrices A and B are functions of some structural parameters.

The representation above helps us deal with all the structural equations in matrix form facilitating the calculus. The intertemporal loss function in matricial form can be written as

$$\frac{1}{2} E_1 \sum_{t=1}^{\infty} \beta^{t-1} y_t^{'} W y_t \tag{3.2}$$

where for sake of simplicity we can define the vector $x_t = (Z_t, z_t)$. Also, $y_t = \left(x_t^{'}, u_t^{'}\right)^{'}$. The matrix W is

$$\begin{pmatrix} W_{xx} W_{xu} \\ W_{xu}^{'} W_{uu} \end{pmatrix}, \tag{3.3}$$

where the matrices W_{xx} and W_{uu} are, without loss of generality, assumed to be symmetric.

The Lagrangian that follows from (3.1) and (3.2) is

$$\mathcal{L} = E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[x_t^{'} W_{xx} x_t + 2x_t^{'} W_{xu} u_t + u_t^{'} W_{uu} u_t + 2\rho_{t+1}^{'} (Ax_t + Bu_t + \xi_{t+1} - x_{t+1}) \right], \tag{3.4}$$

[5]Both fiscal leadership and monetary leadership are written in state space form. See Appendix *A* for details.

where $\xi_{t+1} = (\varepsilon_{t+1}, z_{t+1} - Ez_{t+1})$ and ρ_{t+1} is a vector of multipliers. Subsequently, we should take the first order conditions with respect to ρ_{t+1}, x_t, and u_t, such that

$$\begin{bmatrix} I_n & \mathbf{0}_{nxk} & \mathbf{0}_{nxn} \\ \mathbf{0}_{nxn} & \mathbf{0}_{nxk} & \beta A^{'} \\ \mathbf{0}_{kxn} & \mathbf{0}_{nxk} & -B^{'} \end{bmatrix} \begin{bmatrix} x_{t+1} \\ u_{t+1} \\ E_t\rho_{t+1} \end{bmatrix} = \begin{bmatrix} A & B & \mathbf{0}_{nxn} \\ -\beta W_{xx} & -\beta W_{xu} & I_n \\ W_{xu}^{'} & W_{uu} & \mathbf{0}_{kxn} \end{bmatrix} \begin{bmatrix} x_t \\ u_t \\ \rho_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ \mathbf{0}_{xx1} \\ \mathbf{0}_{kx1} \end{bmatrix}, \tag{3.5}$$

where $\rho_0 = 0$ and x_0 given.

Klein (2000) shows how to solve it via generalized Schur decomposition. However, the optimal Ramsey policy, which was described above, is time inconsistent and Woodford (2003) proposes a new method to surpass this problem. Juillard & Pelgrin (2007) implement empirically that timeless perspective solution through the computation of initial values of the Lagrange multipliers, thus generalizing the timeless perspective solution introduced by Woodford (2003). The time inconsistent drawback comes from the fact that, in the contemporaneous period, the policymaker sets his optimal policy after the private agents' expectations, such that he may have an incentive to re-optimize in the future.

Turning now to a timeless perspective, the Lagrangian acquires a slight modification such that

$$\mathcal{L} = E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[x_t^{'} W x_t + 2\rho_{t+1}^{'}(Ax_t + Bu_t + \xi_{t+1} - x_{t+1}) \right] + \beta^{-1}\rho_0^{'}(y_0 - \overline{y}_0), \tag{3.6}$$

where $y_0 = (x_0^{'}, 0)$. The first order conditions remain as before:

$$\begin{bmatrix} I_n & \mathbf{0}_{nxk} & \mathbf{0}_{nxn} \\ \mathbf{0}_{nxn} & \mathbf{0}_{nxk} & \beta A^{'} \\ \mathbf{0}_{kxn} & \mathbf{0}_{nxk} & -B^{'} \end{bmatrix} \begin{bmatrix} x_{t+1} \\ u_{t+1} \\ E_t\rho_{t+1} \end{bmatrix} = \begin{bmatrix} A & B & \mathbf{0}_{nxn} \\ -\beta W_{xx} & -\beta W_{xu} & I_n \\ W_{xu}^{'} & W_{uu} & \mathbf{0}_{kxn} \end{bmatrix} \begin{bmatrix} x_t \\ u_t \\ \rho_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ \mathbf{0}_{xx1} \\ \mathbf{0}_{kx1} \end{bmatrix}, \tag{3.7}$$

where $\rho_0 \neq 0$ and x_0 given.

Note that the resolution is akin to the anterior Ramsey policy problem. Juillard & Pelgrin (2007) emphasize that the initial values of the Lagrange multipliers of the non-predetermined variables are a complex function of both the past dynamics of the anterior system (Ramsey policy without timeless perspective) and the initial values of the non-predetermined variables. It should be highlighted that the non-predetermined variables of y_0 are selected such that (a) the function of the predetermined variables exists at the initial period and (b) the solution of the optimization problem exists with the following constraint $y_0 = \overline{y}_0$ for $t > 0$. The solution of the Stackelberg problem is then solved with the help of MATLAB.

3.3.3 Simulation of the cooperative solution

The cooperative case is similar to the Nash one. The main difference relies on the optimal solutions (fiscal and monetary). Additional variables entry into these equations modifying thus the optimal responses to other variables.

The equilibrium of the model is described by ten equations, where five of them are endogenous equations and the remaining are exogenous processes. There are ten variables, five endogenous $(\hat{x}_t, \pi_t, \hat{b}_t, \hat{i}_t, \hat{g}_t)$ and five exogenous $(\hat{r}_t^n, \nu_t, \eta_t, \Xi_t, O_t)$.

IS curve: $\hat{x}_t = E_t\hat{x}_{t+1} - \sigma(\hat{i}_t - E_t\pi_{t+1}) + \alpha\hat{b}_t + \hat{r}_t^n$

Phillips curve: $\pi_t = \kappa\hat{x}_t + \beta E_t\pi_{t+1} + \nu_t$

Public debt: $\hat{b}_t = (1 + i^*)\hat{b}_{t-1} + \overline{b}\hat{i}_t + \hat{g}_t - \varpi\hat{x}_t + \eta_t$

Monetary rule: $\hat{i}_t = -\Gamma_0 i^* + \Gamma_{i,1}\hat{i}_{t-1} - \Gamma_{i,2}\hat{i}_{t-2} + \Gamma_{\pi,0}\pi_t + \Gamma_{x,0}\hat{x}_t - \Gamma_{x,1}\hat{x}_{t-1} + \Gamma_{g,0}\hat{g}_t - \Gamma_{g,1}\hat{g}_{t-1} + \Gamma_{g,2}\hat{g}_{t-2} + \Xi_t$

Fiscal rule: $\hat{g}_t = -\Theta_{\pi,0}\pi_t + \Theta_{i,0}(\hat{i}_t - i^*) - \Theta_{i,1}(\hat{i}_{t-1} - i^*) + \Theta_{i,2}(\hat{i}_{t-2} - i^*) - \Theta_{x,0}\hat{x}_t + \Theta_{x,1}\hat{x}_{t-1} + \Theta_{g,1}\hat{g}_{t-1} - \Theta_{g,2}\hat{g}_{t-2} + O_t$

Demand shock: $\hat{r}_t^n = \chi_r\hat{r}_{t-1}^n + \varepsilon_r$

Supply shock: $\nu_t = \chi_\nu\nu_{t-1} + \varepsilon_\nu$

Debt shock: $\eta_t = \chi_\eta\eta_{t-1} + \varepsilon_\eta$

Monetary policy shock: $\Xi_t = \chi_\Xi\Xi_{t-1} + \varepsilon_\Xi$

Fiscal policy shock: $O_t = \chi_O O_{t-1} + \varepsilon_O$

The cooperative solution likewise Nash one has exogenous processes following an $AR(1)$ dynamic, where each ε_j is independent and identically distributed $(i.i.d.)$ with mean zero and variance σ_j^2. The same calibration applies here. The simulation was carried out with the help of Dynare for MATLAB.

3.4 Numerical results

As described earlier, we simulate[6] the monetary and fiscal policy rules jointly with the structural equations. The model consists of an aggregate demand block represented by a dynamic IS curve, an aggregate supply block with a new Keynesian Phillips curve, a government budget constraint, and a block with the derived reaction functions of fiscal and monetary authorities. The model is calibrated for the Brazilian economy in the period after the implementation of the Real Plan.

The main goal of the simulations is to obtain the variances of the variables in their optimal trajectories, in order to compute the expected value of the loss function associated to each regime of coordination. This loss is computed varying two structural parameters.

As a robustness check, we calculate and plot the loss generated from different monetary and fiscal policy decisions, i.e. alternative policies reflected in the weight placed on the target variables. We also analyze impulse response functions, in order to study how the dynamics of the model behaves in the presence of unit shocks of demand, supply, debt, monetary, and fiscal.

Thereby, the analysis will focus on efficient aspects for macroeconomic stabilization. The next subsections will give more details on the points highlighted above.

3.4.1 Loss analysis

In order to assess the relative performance of the optimal policies derived in Section 2.6 and presented in Sections 3.3.1 − 3.3.3, we compare the value of the loss produced under each model.

The loss can be easily calculated by using unconditional variance[7]. Taking, for instance, the monetary loss $L^M = \gamma_\pi \pi_t^2 + \gamma_x \hat{x}_t^2 + \gamma_i (\hat{i}_t - i^*)^2$, it is straightforward to get the expected loss[8], given by:

[6]The main objective of this chapter is to assess the performance of each regime of coordination, namely to provide the variance of target variables in their optimal trajectories to compute the loss in each regime.

[7]See Woodford (2003) for details.

[8]In order to facilitate the notation, this work will not distinguish the loss and the expected loss.

$$L^M = \gamma_\pi^2 \text{var}(\pi_t) + \gamma_x^2 \text{var}(\hat{x}_t) + \gamma_i^2 \text{var}(\hat{i}_t - i^*)$$

The results in Tables 3.2-3.5 show the variance and the losses of each authority for different values of σ and κ, where the former parameter represents the intertemporal elasticity of substitution in private spending and the latter one measures the sensitivity of inflation rate to the output gap and depends on the frequency of price adjustment and marginal cost elasticity in relation to the real level of economic activity. These parameters come from Equations (2.1) and (2.2), namely [$\hat{x}_t = E_t\hat{x}_{t+1} - \sigma(\hat{i}_t - E_t\pi_{t+1}) + \alpha\hat{b}_t + \hat{r}_t^n$] and [$\pi_t = \kappa\hat{x}_t + \beta E_t\pi_{t+1} + \nu_t$]. The reasons for choosing these parameters rely on the effects provoked by them in the economy. Below, we single out some of them:

- if the intertemporal elasticity of substitution in private spending is high ($\sigma > 1$), the agents will postpone consumption, thus decreasing the actual level of economic activity, since the aggregate demand depends on consumption. On the other hand, a low value ($\sigma < 1$) tends to consolidate the actual consumption;
- both higher frequency of price adjustment (the longer the average time interval between price changes) and lower marginal cost elasticity in relation to the real level of economic activity lead to a lower κ (a flatter slope), meaning that large deviations of output from the trend will only result in low levels of inflation in the short-run. The opposite implication also applies.

According to Table 3.2, increases in σ tend to reduce the loss in both monetary and fiscal authorities, except when $\kappa = 0.1$ (upon fiscal authority). As explained earlier, a high intertemporal elasticity of substitution in private spending means a preference for future consumption, namely the agents avoid present consumption and tend to postpone it. The immediate consequence is a reduction in the output, videlicet the aggregate demand experiences a shrinkage and consequently the inflation diminishes too, which in turn pushes interest rate downwards. The overall result should be a decrease in the loss of both monetary and fiscal authorities. However, that phenomenon is not observed for the fiscal case ($\kappa = 0.1$). For example, when $\sigma = 0.50$, the monetary and fiscal losses are $L^M = 12.9905$ and $L^F = 4.2008$, respectively, then for $\sigma = 5.00$, we have $L^M = 8.8767$ and $L^F = 6.6826$. Turning now to the parameter κ, we note that when κ increases both fiscal and monetary losses decrease. For example, when $\sigma = 5.00$ and $\kappa = 0.10$, the losses are $L^M = 8.8767$ and $L^F = 6.6826$, then consecutive increases in κ lead to successive decreases in the loss, namely for $\kappa = 0.50$ and $\kappa = 0.90$, the losses are $L^M = 1.6492$ and $L^M = 1.1882$, and $L^M = 0.6559$ and $L^M = 0.4654$, respectively. The idea behind a raise in parameter κ is that it may reflect some modification in the fraction of goods prices or in the elasticity of real marginal supply cost. Upon another look, an increase in the value of κ tends to increase the sensitivity of inflation to the output gap, having, therefore, a negative effect on the loss. It is interesting to note that when $\sigma = 5.00$ and $\kappa = 0.90$, we obtain the lowest loss ($L^S = 1.1213$), that is, with that combination we maximize the social welfare.

The results reported in Table 3.3 resembles the Nash one when we compare the effects (increases) of the parameter κ. On the other hand, increases in σ do not seem to have clear effects (there are decreases and increases depending on the value of κ for the monetary loss). Notice that the combination $\sigma = 0.50$ and $\kappa = 0.90$ provides the lowest loss ($L^S = 0.0255$). Note also that the losses of the fiscal leadership are lower than the losses of the Nash solution, suggesting that the former is more efficient.

Table 3.4 has a behavior similar to that showed in the fiscal leadership solution. However, increases in σ have lower impacts on the fiscal loss. Once again the pair of parameters $\sigma = 0.50$ and $\kappa = 0.90$ provides the lowest loss ($L^S = 0.0238$). Note that this loss is the global minimum and that a monetary leadership provides globally the lowest losses.

The coordination scheme presented in Table 3.5[9] has characteristics similar to the

[9]Note that the loss function of each institution takes into account only compatible variables with existing goals. The welfare loss reflects just the deviations of the variables from their equilibrium values, thus the established definition of the goals are not considered, if so the loss would be always higher than that one associated to other regimes.

Nash solution, thereby the same analysis can be employed here. The parameters values $\sigma = 2.50$ and $\kappa = 0.50$ lead to the minimum values for the loss ($L^S = 0.3173$).

Ordering the regimes which provide globally the lowest losses, we obtain the following rank: (1) Monetary leadership; (2) Fiscal leadership; (3) Cooperative solution; (4) Nash solution.

Thus, the monetary leadership solution, i.e. when the monetary authority moves first as a Stackelberg leader, is the best scheme of coordination between the authorities. Note that both Stackelberg solutions are superior to the remaining ones, and that the cooperation helps to maximize social welfare despite a non-cooperative solution (Nash solution).

Table 3.2: Loss values for different coefficients under the Nash solution

$L^M = \pi_t^2 + 0.5\hat{x}_t^2 + 0.05(\hat{i}_t - i^*)^2$

$L^F = 0.5\pi_t^2 + \hat{x}_t^2 + 0.3\hat{g}_t^2$

		Variance of							
σ	κ	π_t	$\hat{x}_t$	$\hat{b}_t$	$\hat{i}_t$	$\hat{g}_t$	L^M	L^F	L^S
0.50	0.10	12.7452	0.7057	19.1851	27.5664	3.4317	12.9905	4.2008	17.1913
	0.50	5.2119	0.2723	7.9440	11.3445	1.4225	5.3084	1.7033	7.0117
	0.90	2.7197	0.1369	4.1761	5.9437	0.7486	2.7688	0.8842	3.6500
2.50	0.10	9.6978	2.1892	211.2660	16.3928	11.3685	10.2861	5.6368	15.9229
	0.50	2.3218	0.4530	52.7042	4.0432	2.6607	2.4452	1.2729	3.7181
	0.90	0.9891	0.1819	22.8264	1.7428	1.1210	1.0389	0.5301	1.5690
5.00	0.10	8.1027	2.9793	414.0482	11.6572	18.6392	8.8767	6.6826	15.5593
	0.50	1.5190	0.4980	81.3970	2.2583	3.4490	1.6492	1.1882	2.8374
	0.90	0.6059	0.1908	33.0275	0.9111	1.3681	0.6559	0.4654	1.1213

3.4.2 Weight analysis

As a robustness check, we evaluated the loss[10] generated under the three mechanisms of coordination assuming that the weights in the loss function vary from 0.10 to 1.50, for output gap , inflation, and government spending, and from 0.05 to 1.00, for interest rate. The resulting loss associated to each regime is shown in Figures 3.1-3.4.

[10] It is considered that the economy is hit by a supply shock. The results can change according to the kind of shock applied.

Table 3.3: Loss values for different coefficients under the Stackelberg solution: Fiscal leadership

$$L^M = \pi_t^2 + 0.5\hat{x}_t^2 + 0.05(\hat{i}_t - i^*)^2$$
$$L^F = 0.5\pi_t^2 + \hat{x}_t^2 + 0.3\hat{g}_t^2$$

		Variance of							
σ	κ	π_t	$\hat{x}_t$	$\hat{b}_t$	$\hat{i}_t$	$\hat{g}_t$	L^M	L^F	L^S
0.50	0.10	0.0355	0.0077	0.0318	0.0612	0.0034	0.0376	0.0169	0.0545
	0.50	0.0219	0.0066	0.0183	0.0365	0.0036	0.0236	0.0124	0.0360
	0.90	0.0149	0.0052	0.0120	0.0241	0.0035	0.0163	0.0092	0.0255
2.50	0.10	0.0311	0.0677	0.1868	0.0213	0.0087	0.0481	0.0763	0.1244
	0.50	0.0142	0.0311	0.0238	0.0086	0.0034	0.0220	0.0350	0.0570
	0.90	0.0080	0.0173	0.0071	0.0047	0.0017	0.0123	0.0195	0.0318
5.00	0.10	0.0296	0.1147	0.1973	0.0095	0.0072	0.0583	0.1227	0.1810
	0.50	0.0121	0.0427	0.0148	0.0038	0.0020	0.0228	0.0459	0.0687
	0.90	0.0064	0.0217	0.0057	0.0021	0.0010	0.0118	0.0234	0.0352

Table 3.4: Loss values for different coefficients under the Stackelberg solution: Monetary leadership

$$L^M = \pi_t^2 + 0.5\hat{x}_t^2 + 0.05(\hat{i}_t - i^*)^2$$
$$L^F = 0.5\pi_t^2 + \hat{x}_t^2 + 0.3\hat{g}_t^2$$

		Variance of							
σ	κ	π_t	$\hat{x}_t$	$\hat{b}_t$	$\hat{i}_t$	$\hat{g}_t$	L^M	L^F	L^S
0.50	0.10	0.0325	0.0051	1.1690	0.0001	0.0486	0.0338	0.0176	0.0514
	0.50	0.0143	0.0119	0.3080	0.0018	0.0337	0.0173	0.0185	0.0358
	0.90	0.0073	0.0103	0.1555	0.0050	0.0196	0.0099	0.0139	0.0238
2.50	0.10	0.0325	0.0050	1.1446	0.0013	0.0491	0.0338	0.0175	0.0513
	0.50	0.0132	0.0138	0.2759	0.0009	0.0353	0.0167	0.0203	0.0370
	0.90	0.0055	0.0132	0.1352	0.0001	0.0212	0.0088	0.0165	0.0253
5.00	0.10	0.0326	0.0050	1.1389	0.0015	0.0490	0.0339	0.0176	0.0515
	0.50	0.0133	0.0137	0.2690	0.0018	0.0353	0.0167	0.0202	0.0369
	0.90	0.0055	0.0133	0.1305	0.0006	0.0213	0.0088	0.0166	0.0254

Under Nash solution [Figure 3.1], the monetary loss increases in proportion to the relative importance the central bank attaches to the weights of output gap and interest rate. This can be justified by the fact that increasing the weights on the output gap and the interest rate stabilization reduces the inflation control and so increases the loss. Note that the impact of interest rate on the loss is much more strong. Similarly, the fiscal loss

Table 3.5: Loss values for different coefficients under the cooperative solution

$$L^M = \pi_t^2 + 0.5\hat{x}_t^2 + 0.05(\hat{i}_t - i^*)^2$$
$$L^F = 0.5\pi_t^2 + \hat{x}_t^2 + 0.3\hat{g}_t^2$$

		Variance of							
σ	κ	π_t	$\hat{x}_t$	$\hat{b}_t$	$\hat{i}_t$	$\hat{g}_t$	L^M	L^F	L^S
0.50	0.10	12.1107	0.7355	29.9229	30.4118	4.0704	12.3706	4.1296	16.5002
	0.50	4.4734	0.2630	11.2059	11.3251	1.5229	4.5675	1.5184	6.0859
	0.90	2.2562	0.1300	5.7000	5.7413	0.7742	2.3030	0.7637	3.0667
2.50	0.10	7.8141	1.6191	364.3679	15.3638	11.8087	8.2573	4.6354	12.8927
	0.50	0.1146	0.1094	0.5639	0.1531	0.4110	0.1423	0.1750	0.3173
	0.90	1.6526	0.3492	72.2684	3.1152	2.4879	1.7477	0.9862	2.7339
5.00	0.10	5.6878	1.9734	769.0163	9.0333	17.8931	6.2037	5.0057	11.2094
	0.50	1.0476	0.3632	112.2076	1.4691	3.0751	1.1421	0.9019	2.0440
	0.90	0.4451	0.1524	44.5084	0.6046	1.2813	0.4847	0.3790	0.8637

increases in proportion to the relative importance the treasury attaches to the weights of spending and inflation. These increasing weights weaken output gap stabilization and then increase the loss. Note that the impact of spending is stronger.

Furthermore, the Figure 3.2 shows that the monetary loss (under fiscal leadership) is very sensitive to the relative importance the central bank attaches to the weight of output gap. The fiscal case indicates a relevant sensibility of the loss to the weight of inflation and a small sensibility concerning the weight of spending.

Subsequently, Figure 3.3 demonstrates that, under monetary leadership, the relative importance the central bank attaches to the weights of output gap and interest rate is similar to that showed under fiscal leadership, but in a lower degree. On the other hand, the fiscal loss is more sensitive to the relative weight placed on the spending.

Finally, the losses shown in Figure 3.4 under cooperative solution are quite similar to the Nash ones, but in a lower degree. In global terms, the Nash and cooperative solutions respond more equally to the weights placed on their target variables.

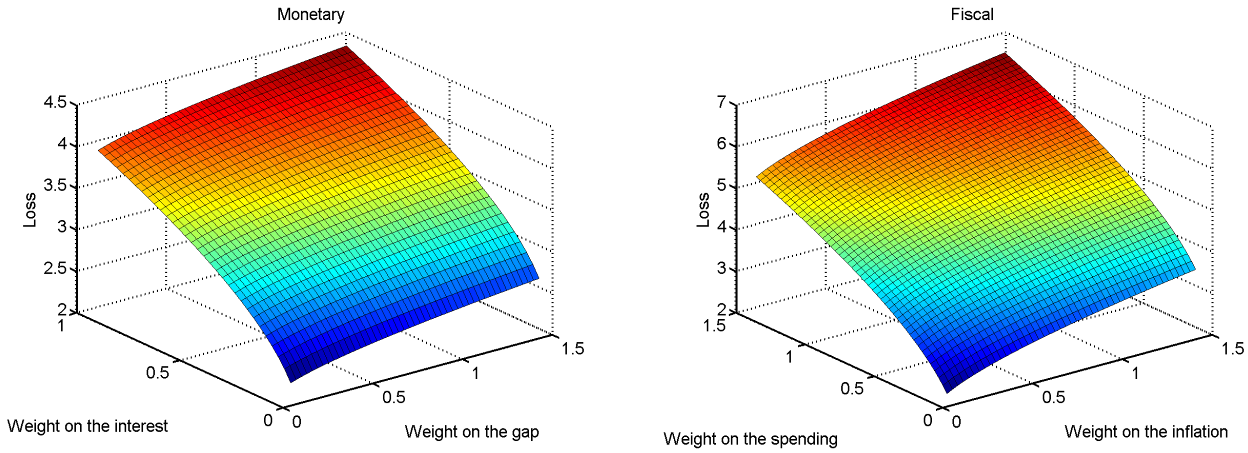

Figure 3.1: Losses for different weights under the Nash solution

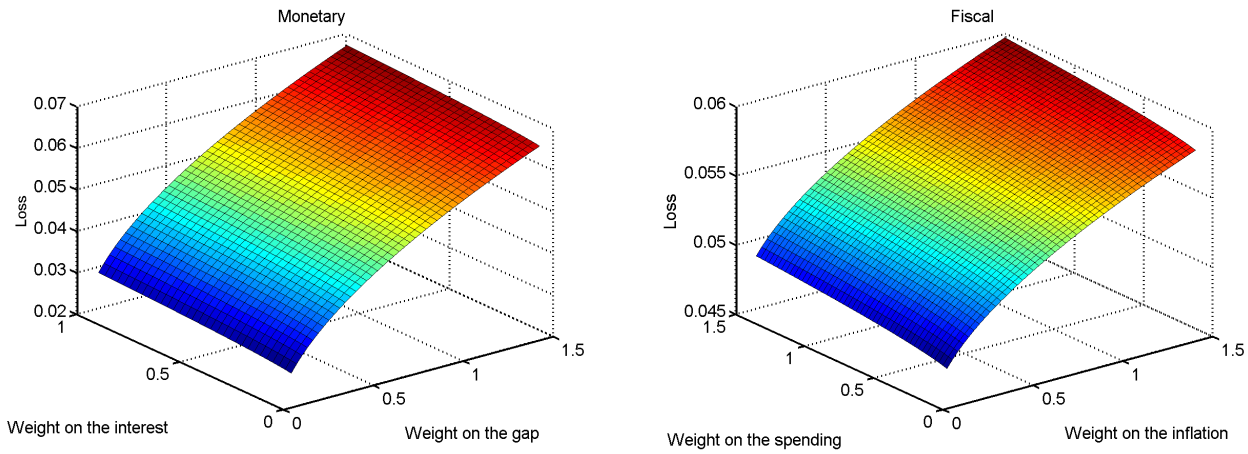

Figure 3.2: Losses for different weights under the fiscal leadership solution

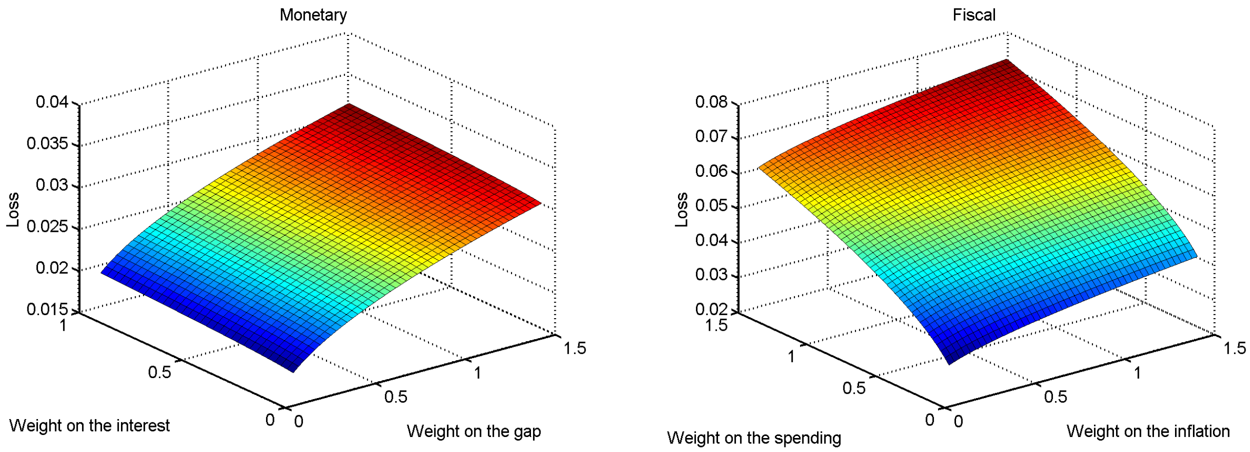

Figure 3.3: Losses for different weights under the monetary leadership solution

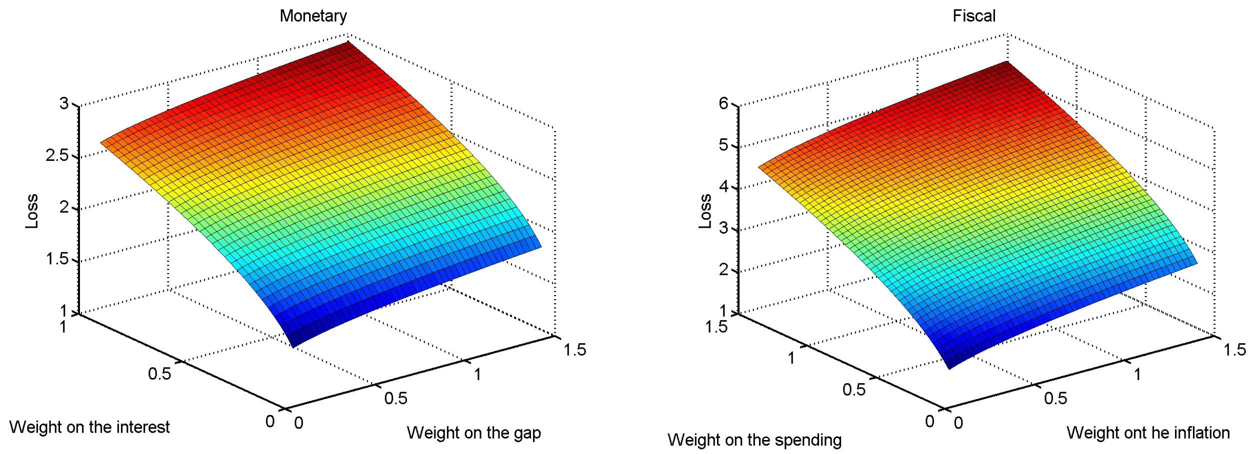

Figure 3.4: Losses for different weights under the cooperative solution

3.4.3 Impulse response analysis

The impulse response function[11] (IRF) describes how the economy reacts over time to shocks (exogenous impulses). The Figures[12] 3.5-3.16 report the impulse response functions of the economy in response to a one unit supply, demand, debt, monetary, or fiscal shock for each regime of coordination.

Figure 3.5 shows the effects of a positive supply shock under Nash solution. Initially, the supply shock pushes inflation upwards, then the monetary authority responds to higher inflation by increasing the interest rate by 60 basis points before slowly coming back to equilibrium. The output gap initially drops down in response to monetary tightening before increasing in response to interest rate loosening. Note that there is a trade-off between inflation and output when the economy is hit by a supply shock. The monetary tightening also leads to an increase of the debt which takes some time to come back to equilibrium. The government expenditure initially increases and immediately decreases in response to a fall in output gap.

A demand shock in Figure 3.6 pushes output gap upwards impacting directly the inflation (demand-pull inflation), then forces the monetary authority to increase the interest rate in order to cool the economy. Note that, as expected, a higher output gap leads to a reduction in the debt which persist for some time before coming back to equilibrium. This result follows directly from the fact that a raise in output gap provokes an increase in the amount of tax revenues diminishing so the amount of debt.

Figure 3.7 shows that following a positive fiscal shock, government spending is higher on impact and then stays below zero until to reach equilibrium. It then generates an increase in output gap and inflation, forcing the monetary authority to increase the interest rate. This increase leads to a raise in the amount of debt, which in turn reduces the capacity of the government to increase future spendings.

[11] See Appendix *B* for details.

[12] The variables are presented in an abbreviated form: pi, inflation; x, output gap; b, debt; i, interest rate; g, government spending; nu, supply shock; rn, demand shock, om, fiscal shock; and xi, monetary shock.

The effects of a supply shock under fiscal leadership [Figure 3.8] have a more discrete impact when compared to the Nash solution. Primarily, the monetary authority responds to the higher inflation by increasing the interest rate by 6 basis points, returning some time later to equilibrium. That response presents a relatively low degree and it may follow from the fact that the fiscal authority takes into account the optimal monetary policy, thus the central bank needs not to act so aggressively since the burden is shared between the authorities. The output gap, after a fall, increases abruptly returning rapidly to equilibrium. The trade-off between inflation and output is also verified. The monetary tightening is not sufficient to maintain the increase in debt, which oscillates until reaching the equilibrium some time later. The government spending follows the oscillations in output gap.

The demand shock in Figure 3.9 pushes output gap upwards but the equilibrium comes back after four quarters. Posteriorly, the monetary authority raises interest rate in response to higher inflation, which in turn increases the debt. The government expenditure initially reduces, later on, it increases (accompanying the output gap) until some time later.

Under fiscal leadership a positive monetary shock[13] [Figure 3.10] increases interest rate and consequently decreases inflation (due to a fall in output gap). As it is well-known, one indicator of inflationary pressures is the output gap, i.e. positive gaps move inflation upwards due to demand pressures and negative gaps sign an underutilization of production capability. Another consequence of a higher interest rate is an increase in debt. On the other hand, government expenditure suffers a fall mainly due to lower output gap.

Figure 3.11 shows a monetary tightening under monetary leadership. The monetary authority responds to higher inflation by increasing the interest rate, reaching a peak of 4.3 basis points in the ninth quarter before slowly coming back to equilibrium. Notice that the interest rate response is lagged. The output gap falls and pushes the government

[13]In the Stackelberg case, impacts of follower's shocks are analyzed on the leader in order to verify their influences.

expenditure to a lower level. The debt response is hump-shaped because it accompanies movements in the interest rate. The peak debt response occurs after six quarters. Globally, the responses here are lower than all the other regimes of coordination.

Under monetary leadership a demand shock in Figure 3.12 has apparently the same effects showed when we have fiscal leadership, but with more persistence in the interest rate. The demand shock pushes output gap upwards, which in turn increases inflation, then the monetary authority raises the interest rate by 0.05 basis points, reaching a peak of 0.08 basis points in the fourth quarter. The output gap stays static over equilibrium from the sixth quarter on. Note that in both fiscal and monetary leaderships no extrinsic persistence is viewed for the demand shock upon both output gap and inflation, namely the shock has relatively a short-lived effect on both. The government expenditures has less volatility when compared to fiscal leadership.

A positive fiscal shock [Figure 3.13] increases government spending and consequently the output gap. The result, as reported anteriorly, is a raise in inflation and an interest rate response which will curb economic heating. The debt accompanies the government spending and reaches equilibrium after some time.

Figure 3.14 shows that following a supply shock under cooperation, inflation is higher on impact and comes back to the equilibrium level after some time. The monetary authority responds to higher inflation by increasing the interest rate by 65 basis points. This response does not have a hump-shape form, indicating that the central bank does not smooth both the level and change of interest rate. The increase in debt stems from the monetary tightening and takes some time to come back to equilibrium. The government spending undergoes a fall mainly due to lower output gap.

The demand shock [Figure 3.15] again pushes the output gap upwards and generates a inflationary process, culminating in an interest rate response by the monetary authority. A higher output gap causes a reduction in the debt and the negative impact on government spending may be seen as a response to a lower amount of debt. The demand shock under cooperation resembles the Nash one, but with more (slightly) volatility, which seems counter-intuitive since the authorities are cooperating.

Finally, Figure 3.16 reports practically the same results presented by the Nash solution when the economy is hit by a fiscal shock. However, inflation, output gap, and interest rate do not show accentuated hump-shaped movements.

The impulse response analysis demonstrated that (1) it is important to identify the kind of shock that is hitting the economy (different shocks have different effects), (2) the dynamic responses of the variables are different, (3) the volatility and convergence velocity (to reach equilibrium) depend on the regime of coordination, in particular, both Stackelberg solutions present less volatility and higher convergence velocity. Moreover, stabilization processes frequently take place under interest rate adjustments (increases).

3.4.4 Preliminary conclusions

The above analysis emphasized that the monetary leadership regime produced (globally speaking) the smallest losses. Thereby, one can associate this superiority of the monetary leadership as the best model to describe the Brazilian economy during the period analyzed, that is, the Brazil was under a monetary leadership regime. That result is supported by other empirical findings. Before mentioning some works it is interesting to explain a regime (considering the modern literature) introduced by Sargent & Wallace (1981), which has been called monetary dominance. Under this regime, the fiscal authority sets primary surpluses[14] aiming to stabilize the relation debt/GDP.[15] It implies that the active monetary authority does not need to monetize the relation debt/GDP, maintaining the control over the price level through the interest rate. On the other hand, in the regime of fiscal dominance the primary surpluses are set independently of the relation debt/GDP, and the passive monetary authority loses the control over price level and consequently is forced to raise an amount of revenue through bond sales and seignorage

[14]Tax revenues minus non-interest expenditures.

[15]It is just the debt to Gross Domestic Product ratio.

to fulfill the solvency of the government. The inflation created here is still motivated by fiscal disequilibria, but it is seen as a monetary phenomenon.

A common and plausible assumption or association that may be postulated is that the regime of monetary leadership resembles monetary dominance, since the relation debt/GDP is faced passively by the fiscal authority.

Some works have been done to test whether or not the Brazilian economy is or is not under monetary dominance. Tanner & Ramos (2002) analyzed Brazilian fiscal data ranging from 1991 to 2000 and found evidence favoring a monetary dominance regime for 1995-97, though not for the 1990s as a whole.

Fialho & Portugal (2005) tested the predominance of fiscal or monetary dominance in Brazil in the post-Real period (1995-2005). The authors followed the fiscal theory of the price level (FTPL)[16] to model a vector autoregression (VAR) framework verifying the relation between the series of debt/GDP and primary surplus/GDP and analyzing impulse response functions. The results indicate a regime of monetary dominance which came from the public debt response to innovations in the primary surplus.

Blanchard (2004) introduced a structural model which models the interaction between the interest rate, the exchange rate, and the probability of default in the Brazilian economy (1995-2004). In this model the monetary policy leads to explosive effects on the public debt under inflating targeting regime. He argues that increases in the real interest rate as a response to inflation that exceeds the target tend to increase the amount of debt to an unsustainable level by impacting positively the debt, the probability of default, and the risk premium. The ultimate effect of an increase in interest rate is an escape of capital and consequently a devaluation of the Real. Since a considerable portion of the debt is indexed by the dollar, a exchange rate depreciation increases the amount of debt and affects the inflation expectations, triggering an inflationary process, characterizing thus a regime of fiscal dominance. Blanchard points out that the economic instability provoked

[16]This theory postulates the role of the fiscal policy on determination of price level. It assumes that the government intertemporal solvency is a necessary condition to obtain price stability.

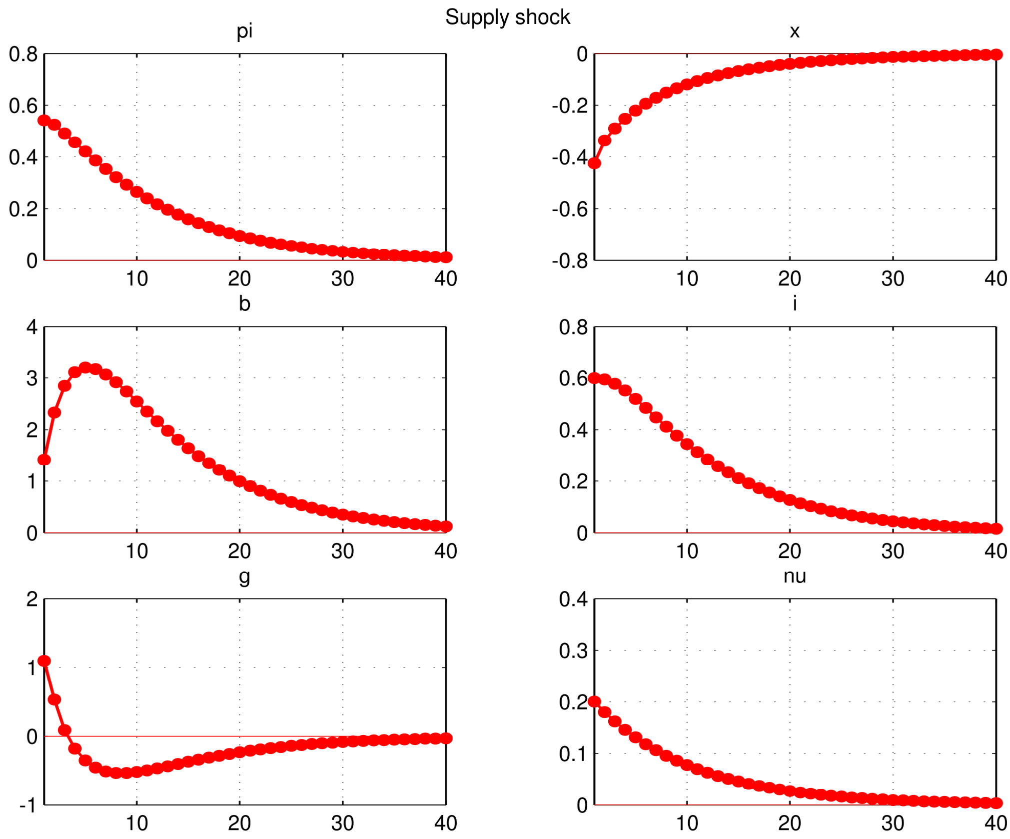

Figure 3.5: Impulse response to a supply shock under the Nash solution

by the 2002 elections is considered as a case of fiscal dominance.

A monetary dominance is also found by Gadelha & Divino (2008). They investigate the long run equilibrium relationship and bivariate and multivariate Granger causality among five key variables, i.e. interest rate, debt to GDP ratio, primary surplus to GDP ratio, real exchange rate and risk premium. Their results show a unidirectional causality from primary surplus to GDP ratio to debt to GDP ratio. It ratifies a regime of monetary dominance.

Thus, the majority of the empirical literature suggests that the Brazilian economy has been under monetary dominance after the Real plan (post 1995 period). The findings here presented are in line with that evidence, corroborating the monetary dominance in the Brazilian economy during the recent period.

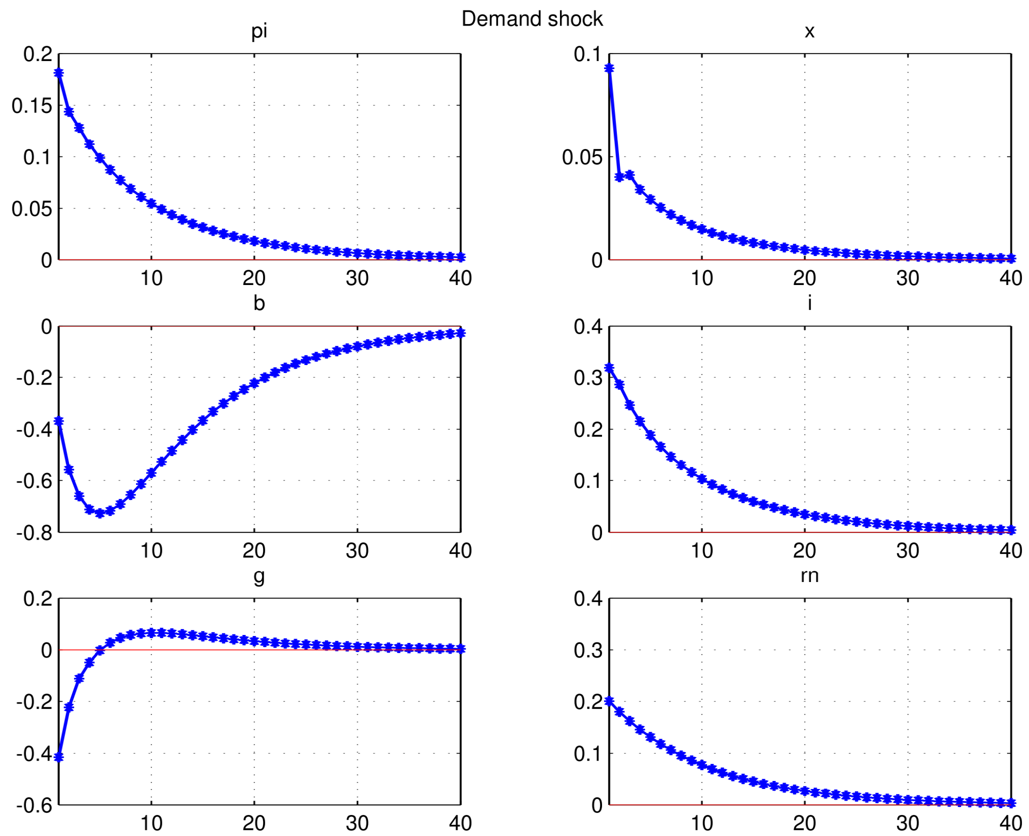

Figure 3.6: Impulse response to a demand shock under the Nash solution

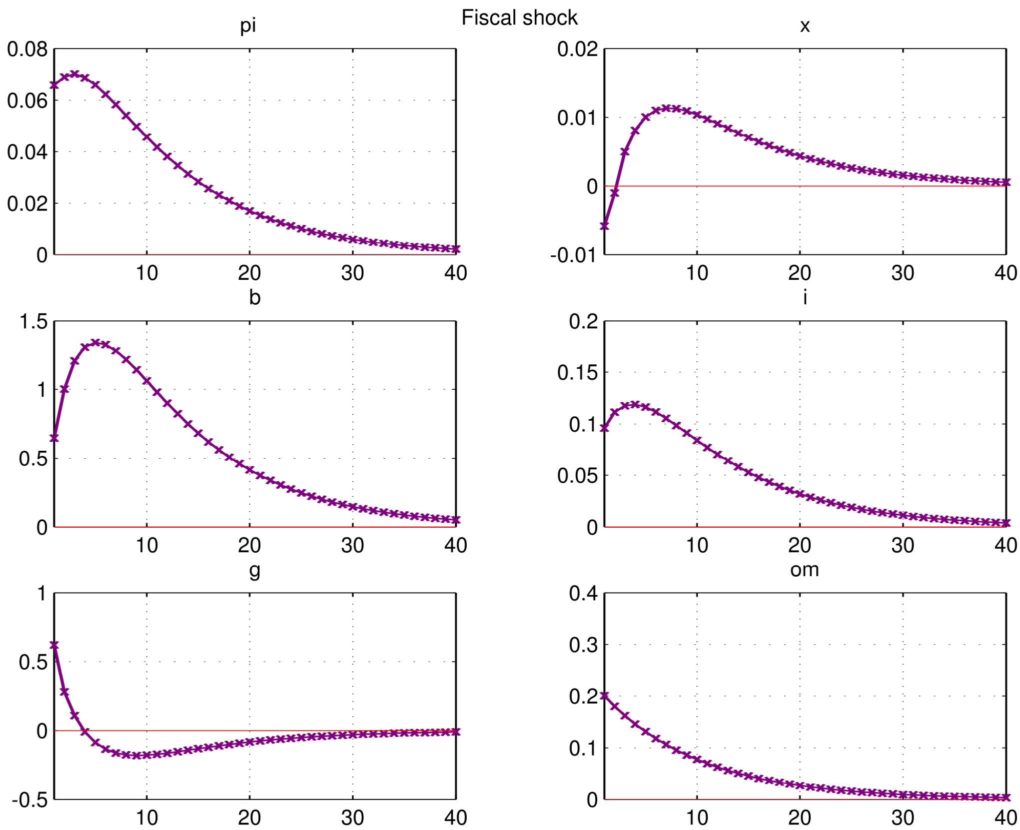

Figure 3.7: Impulse response to a fiscal shock under the Nash solution

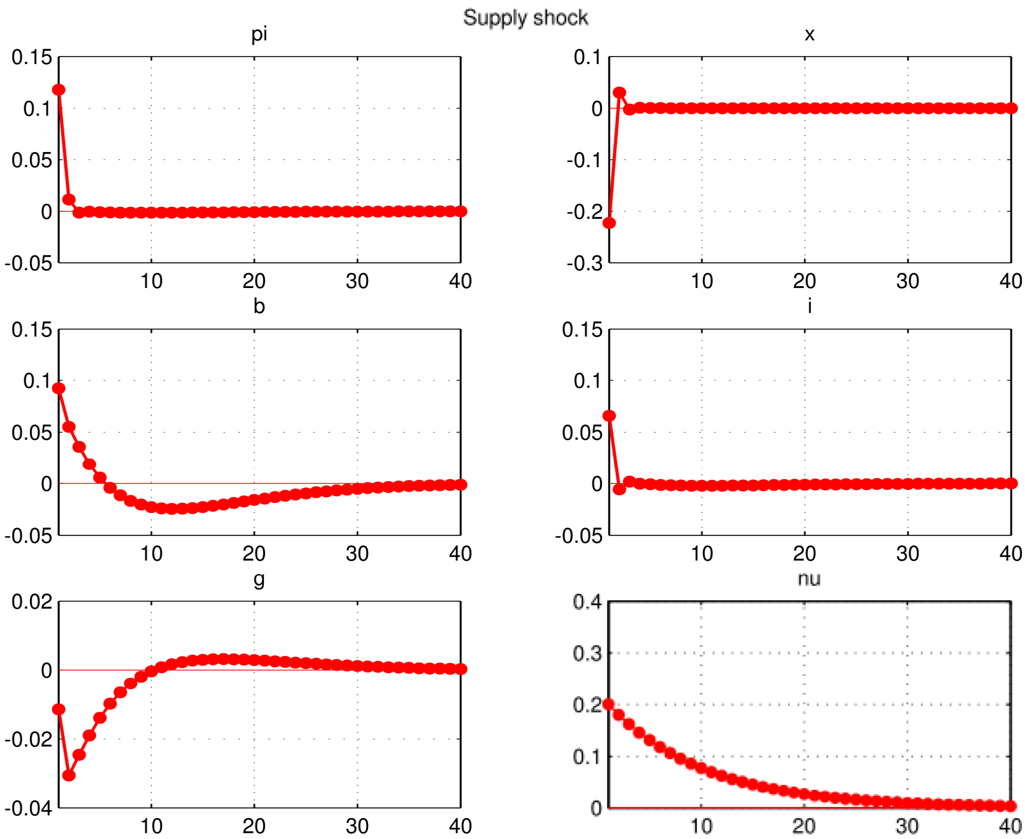

Figure 3.8: Impulse response to a supply shock under the fiscal leadership solution

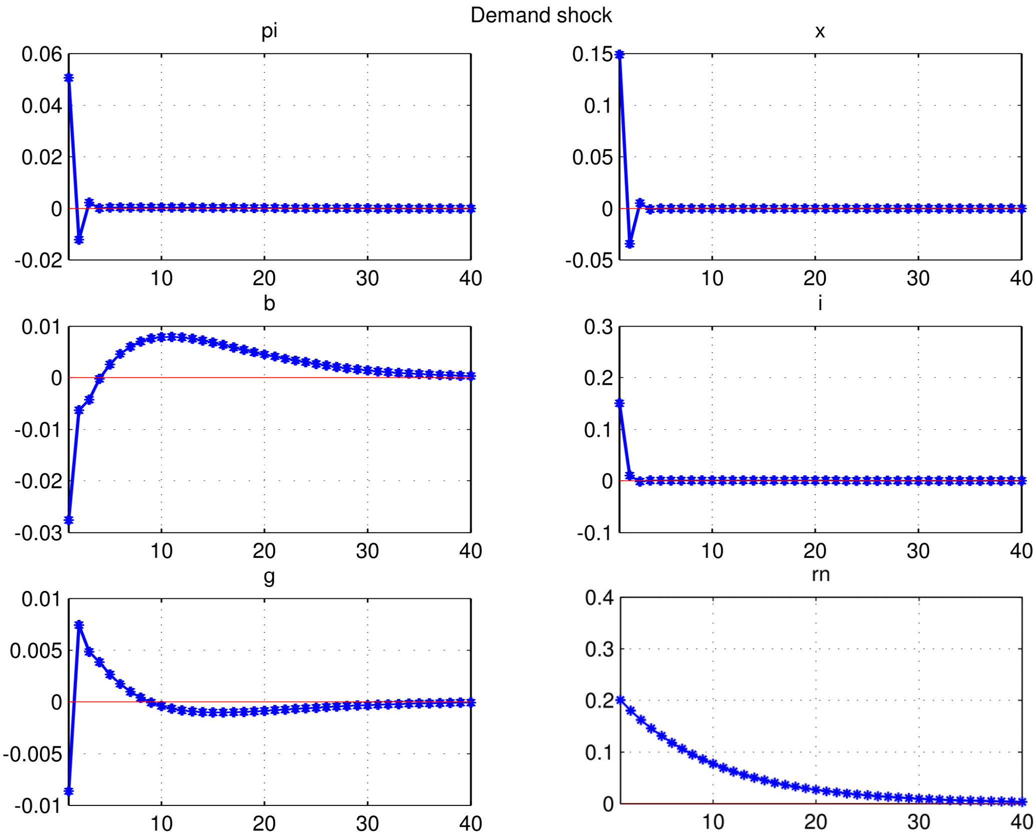

Figure 3.9: Impulse response to a demand shock under the fiscal leadership solution

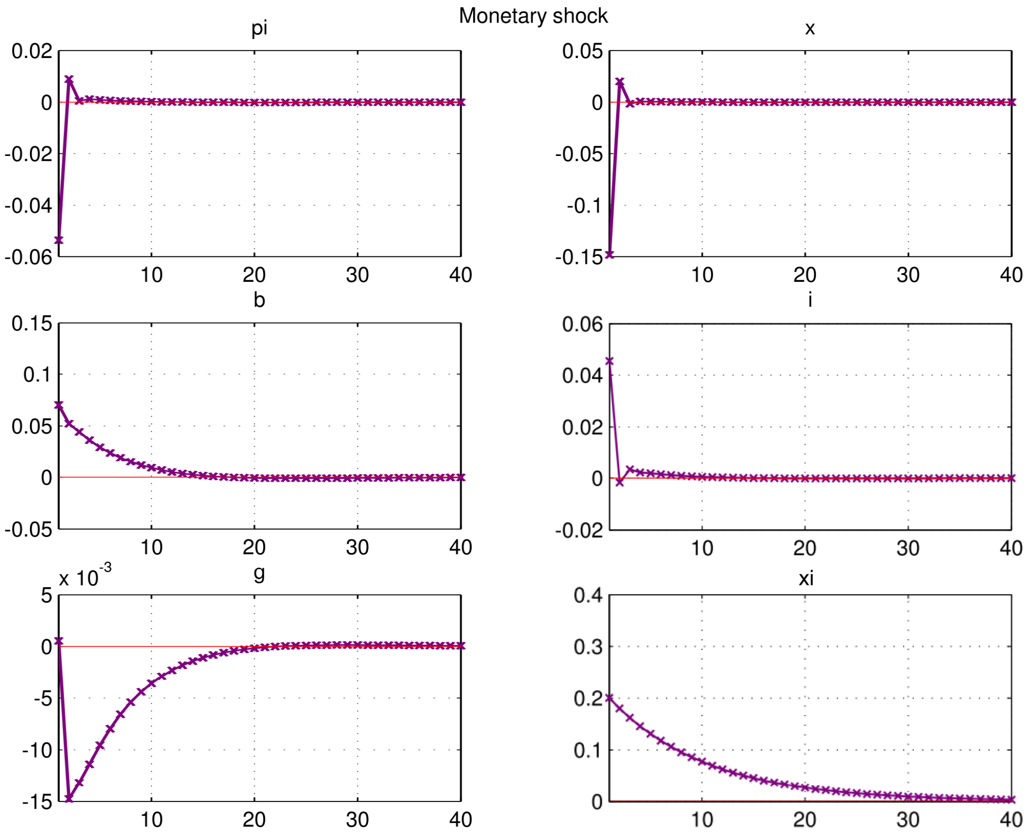

Figure 3.10: Impulse response to a monetary shock under the fiscal leadership solution

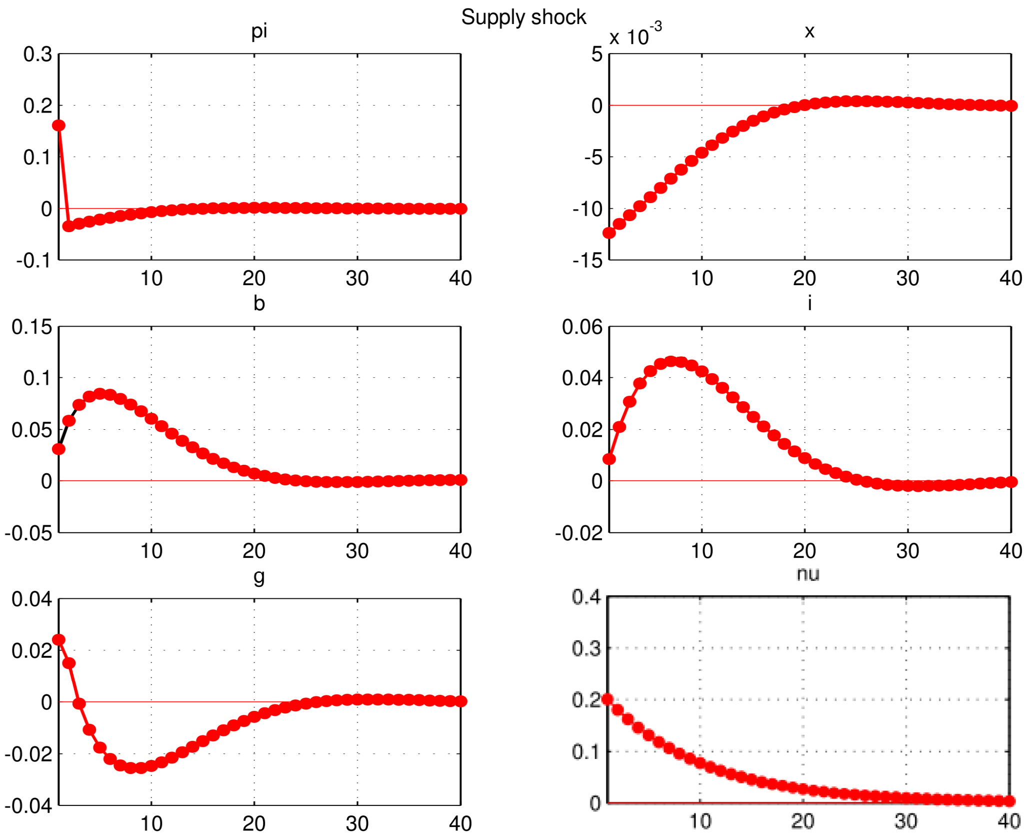

Figure 3.11: Impulse response to a supply shock under the monetary leadership solution

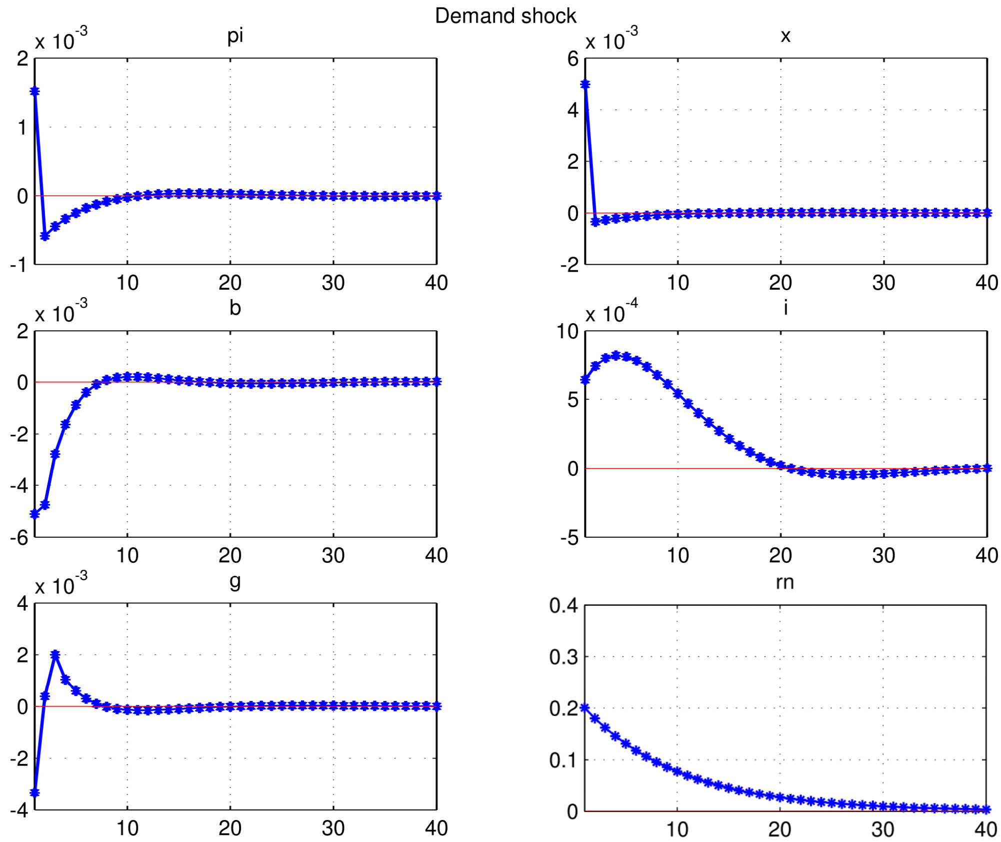

Figure 3.12: Impulse response to a demand shock under the monetary leadership solution

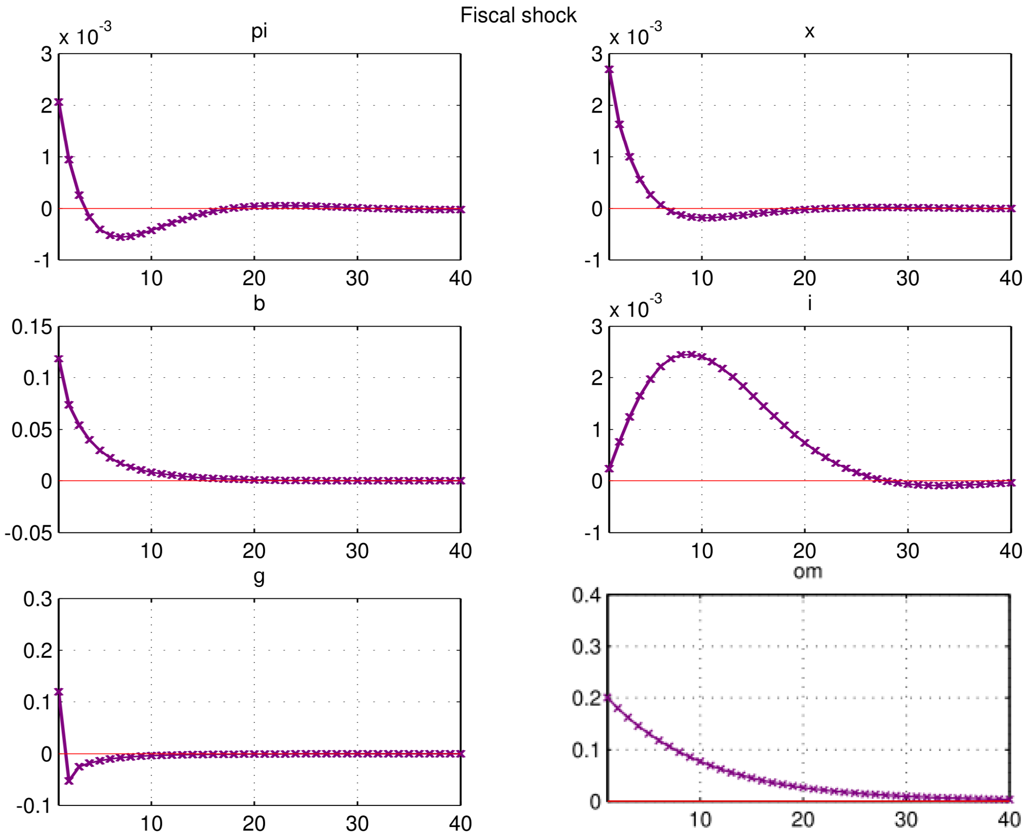

Figure 3.13: Impulse response to a fiscal shock under the monetary leadership solution

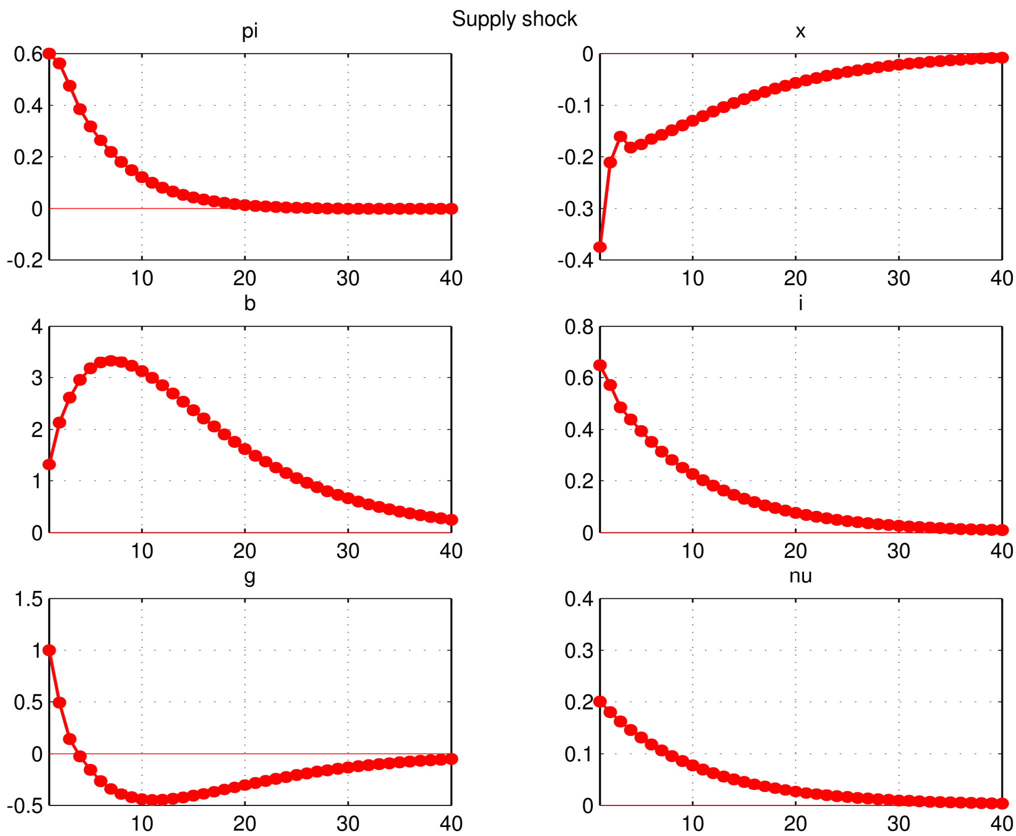

Figure 3.14: Impulse response to a supply shock under the cooperative solution

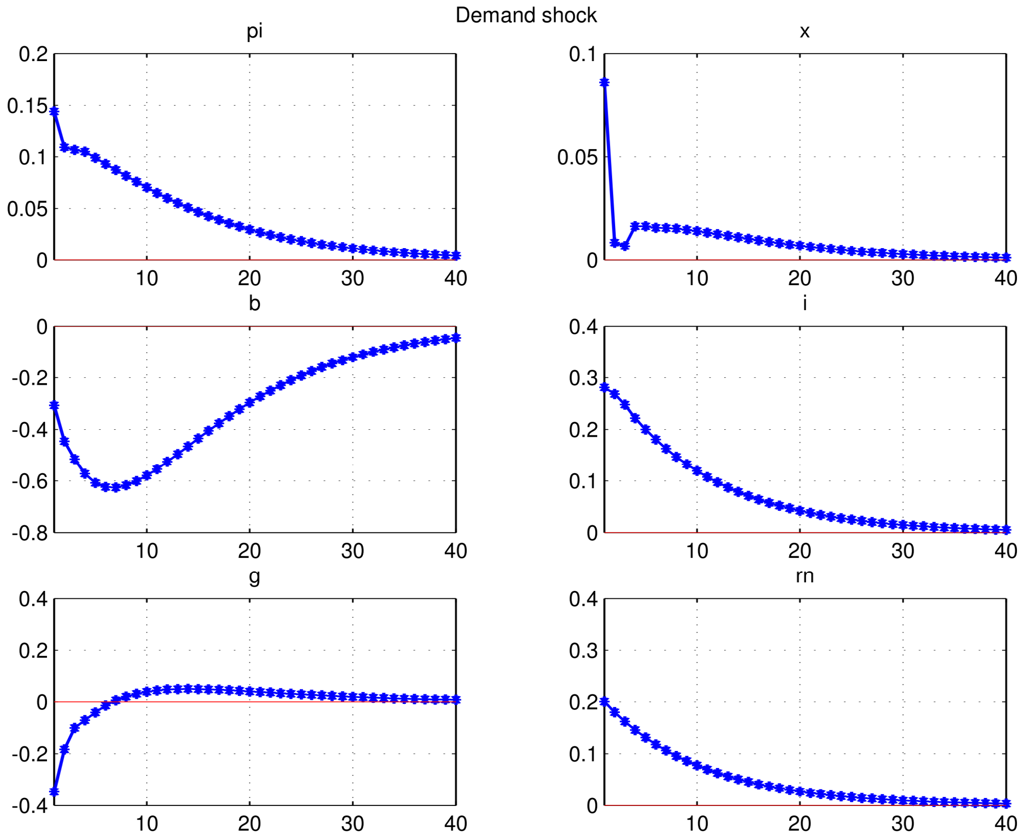

Figure 3.15: Impulse response to a demand shock under the cooperative solution

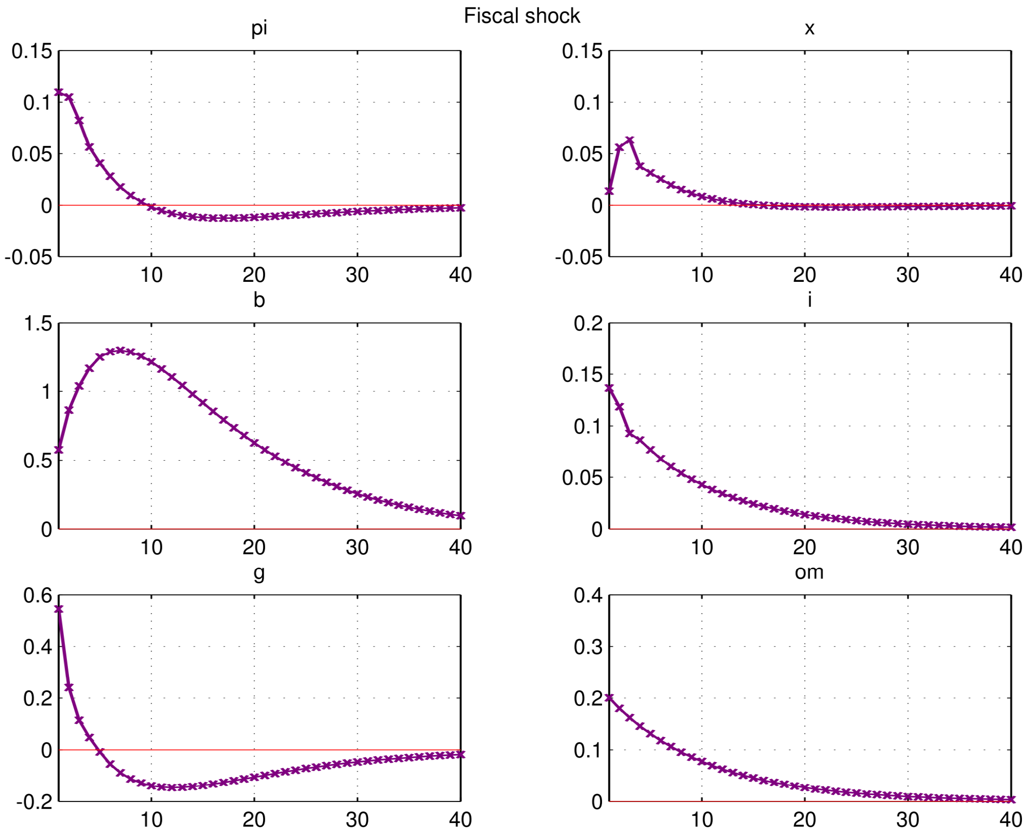

Figure 3.16: Impulse response to a fiscal shock under the cooperative solution

CHAPTER 4

Conclusions and Future Research

4.1 Resumo

As principais contribuições desse trabalho foram (a) fornecer uma interpretação de teoria dos jogos às interações macroeconômicas entre as políticas monetária e fiscal, e (b) fornecer um caminho para formuladores de política, no caso brasileiro, melhorar a análise e implementação das políticas de maneira a aumentar o bem estar social.

Nesse trabalho apresentamos um modelo a qual é composto por uma curva IS, uma curva de Phillips, uma restrição orçamentária, e pelas regras monetária e fiscal. As regras ótimas foram derivadas para cada regime de coordenação, i.e. soluções de Nash, Stackelberg e cooperativa. Os resultados da simulação, considerando parâmetros calibrados para o Brasil, sugerem que na solução de Stackelberg, quando a autoridade monetária é a líder, temos a menor perda, sugerindo também que esse seja o melhor regime/modelo para descrever a economia no período analisado. O regime de liderança monetária (dominância monetária) é corroborado por outros trabalhos aplicados ao Brasil.

No tocante a trabalhos futuros, pode-se destacar a possibilidade de extensão do modelo abordado aqui para um bloco de países, mais especificamente a América do Sul. Outros destaques podem ser dados à implementação do problema de barganha na solução cooperativa e à comparação dos modelos com e sem comprometimento nas regras monetária e fiscal ótimas.

4.2 Conclusions

This work has embedded game theoretical approach coordination schemes (Nash solution, Stackelberg leadership, cooperative solution) into a conventional optimization problem to deliver some quantitative assessment of the losses of each regime of coordination.

Optimal coordination regimes for the Brazilian monetary and fiscal policy were obtained by deriving analytically (except the Stackelberg case, which was obtained numerically) optimal interest rate and public expenditure rules examining the dependence of its coefficients on the parameters.

We applied a numerical approach and found evidence of relative superiority when the regime of coordination is monetary leadership, that is, when the monetary authority moves first, as a Stackelberg leader, taking into account the optimal fiscal policy obtained under the Nash solution. The monetary leadership (monetary dominance) is supported by other empirical findings.

The analysis focused on the loss associated to each regime. In particular, according to the results, the monetary leadership led to the lowest global losses. The weight analysis showed that the Nash and cooperative solutions respond more uniformly to the weights placed on their target variables. The impulse response functions indicated that stabilization processes frequently take place under interest rate adjustments (increases), and that both Stackelberg solutions presented less volatility and higher convergence velocity.

The results presented are of paramount importance, considering the relative difficulty to apply existing models of game theory in macroeconomic models. The model developed in this work will help to improve the analysis done by experts in both macroeconomics and game theory to do optimal choices concerning mainly social welfare, that is, in global terms, the results will help the Brazilian society to increase its welfare.

4.3 Future Research

Fortunately, scientific works do not walk aimlessly. We list below some issues that can be addressed in the future with likely profitable results.

- To apply other structural models to the coordination regimes, and to verify the difference between commitment and discretion (absence of commitment);

- To theoretically analyze other game tools in order to increase the complexity of the approach. For example, the cooperative solution could be implemented through a bargain problem;

- To extend the model to a block of countries, particularly in the South America[1]. A possible study could encompass an optimal arrangement for the area, involving a monetary integration with common fiscal targets.

[1]South America has relatively few works on this theme despite the pile of works that could be found in the European Union case.

APPENDIX A

State Space Representations

A.1 State Space Representation of the Fiscal Leadership

The constraints or structural equations [(2.2), (2.1), (2.3), (2.7), respectively] of the economy can be written as:

$$\pi_{t+1} = \frac{1}{\beta}\pi_t - \frac{\kappa}{\beta}\hat{x}_t - \frac{1}{\beta}\mu_t,$$

$$\hat{x}_{t+1} = \hat{x}_t + \sigma\hat{i}_t - \frac{\sigma}{\beta}\pi_t + \frac{\sigma\kappa}{\beta}\hat{x}_t + \frac{\sigma}{\beta}\mu_t - \alpha\hat{b}_t - \hat{r}_t^n,$$

$$\hat{b}_t = (1 + i^\times)\hat{b}_{t-1} + \overline{b}\hat{i}_t + \hat{g}_t - \varpi\hat{x}_t + \eta_t,$$

$$\hat{i}_t = -\Gamma_0 i^\times + \Gamma_{i,1}\hat{i}_{t-1} - \Gamma_{i,2}\hat{i}_{t-2} + \Gamma_{\pi,0}\pi_t + \Gamma_{x,0}\hat{x}_t - \Gamma_{x,1}\hat{x}_{t-1} + \Xi_t,$$

and their state-space representation is:

$$\mathbf{x_{t+1}} = \mathbf{A}\mathbf{x_t} + \mathbf{B}\mathbf{u_t} + \mathbf{v_{t+1}},$$

that is,

$$\begin{bmatrix} \mu_{t+1} \\ \hat{r}^n_{t+1} \\ \eta_{t+1} \\ \Xi_{t+1} \\ \hat{x}_t \\ \hat{i}_t \\ \hat{i}_{t-1} \\ \hat{b}_t \\ E_t\pi_{t+1} \\ E_t\hat{x}_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -\Gamma_{x,1} & \Gamma_{i,1} & -\Gamma_{i,2} & 0 & \Gamma_{\pi,0} & \Gamma_{x,0} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1+i^*) & 0 & -\varpi \\ -\frac{1}{\beta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ \frac{\sigma}{\beta} & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sigma}{\beta} & (1+\frac{\sigma\kappa}{\beta}) \end{bmatrix} \times$$

$$\begin{bmatrix} \mu_t \\ \hat{r}^n_t \\ \eta_t \\ \Xi_t \\ \hat{x}_{t-1} \\ \hat{i}_{t-1} \\ \hat{i}_{t-2} \\ \hat{b}_{t-1} \\ \pi_t \\ \hat{x}_t \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & \overline{b} & 0 \\ 0 & 0 & 0 \\ 0 & \sigma & -\alpha \end{bmatrix} \times \begin{bmatrix} \hat{g}_t \\ \hat{i}_t \\ \hat{b}_t \end{bmatrix} + \begin{bmatrix} \mu_{t+1} \\ \hat{r}^n_{t+1} \\ \eta_{t+1} \\ \Xi_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Additionally, the target variables are given by:

$$\mathbf{y_t} = \mathbf{C_x x_t} + \mathbf{C_u u_t},$$

which can be written as

$$\begin{bmatrix} \pi_t \\ \hat{x}_t \\ \hat{g}_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \mu_t \\ \hat{r}^n_t \\ \eta_t \\ \Xi_t \\ \hat{x}_{t-1} \\ \hat{i}_{t-1} \\ \hat{i}_{t-2} \\ \hat{b}_{t-1} \\ \pi_t \\ \hat{x}_t \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \hat{g}_t \\ \hat{i}_t \\ \hat{b}_t \end{bmatrix}$$

Following the state-space representation, the loss function becomes

$$\mathbf{L_t} = \mathbf{y_t' K y_t}$$

$$= \begin{bmatrix} \pi_t \\ \hat{x}_t \\ \hat{g}_t \end{bmatrix}' \times \begin{bmatrix} \rho_\pi & 0 & 0 \\ 0 & \rho_x & 0 \\ 0 & 0 & \rho_g \end{bmatrix} \times \begin{bmatrix} \pi_t \\ \hat{x}_t \\ \hat{g}_t \end{bmatrix}$$

A.2 State Space Representation of the Monetary Leadership

The constraints or structural equations [(2.2), (2.1), (2.3), (2.11), respectively] of the economy can be written as:

$$\pi_{t+1} = \frac{1}{\beta}\pi_t - \frac{\kappa}{\beta}\hat{x}_t - \frac{1}{\beta}\mu_t,$$

$$\hat{x}_{t+1} = \hat{x}_t + \sigma\hat{i}_t - \frac{\sigma}{\beta}\pi_t + \frac{\sigma\kappa}{\beta}\hat{x}_t + \frac{\sigma}{\beta}\mu_t - \alpha\hat{b}_t - \hat{r}^n_t,$$

$$\hat{b}_t = (1+i^*)\hat{b}_{t-1} + \overline{b}\hat{i}_t + \hat{g}_t - \varpi\hat{x}_t + \eta_t,$$

$$\hat{g}_{t+1} = \frac{\Theta_{\pi,0}}{\Theta_{g,+1}}\pi_t + \frac{1}{\Theta_{g,+1}}\hat{g}_t - \frac{\Theta_{g,1}}{\Theta_{g,+1}}\hat{g}_{t-1} + \frac{\Theta_{g,2}}{\Theta_{g,+1}}\hat{g}_{t-2} + \frac{\Theta_{x,0}}{\Theta_{g,+1}}\hat{x}_t - \frac{\Theta_{x,1}}{\Theta_{g,+1}}\hat{x}_{t-1} - \frac{1}{\Theta_{g,+1}}O_t$$

and their state-space representation is:

$$\mathbf{x_{t+1}} = \mathbf{Ax_t} + \mathbf{Bu_t} + \mathbf{v_{t+1}},$$

that is,

$$
\begin{bmatrix} \mu_{t+1} \\ \hat{r}^n_{t+1} \\ \eta_{t+1} \\ O_{t+1} \\ \hat{x}_t \\ \hat{g}_t \\ \hat{g}_{t-1} \\ \hat{b}_t \\ E_t\pi_{t+1} \\ E_t\hat{x}_{t+1} \\ E_t\hat{g}_{t+1} \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & (1+i^*) & 0 & -\varpi & 0 \\
-\frac{1}{\beta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 \\
\frac{\sigma}{\beta} & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sigma}{\beta} & (1+\frac{\sigma\kappa}{\beta}) & 0 \\
0 & 0 & 0 & -\frac{1}{\Theta_{g,+1}} & -\frac{\Theta_{x,1}}{\Theta_{g,+1}} & -\frac{\Theta_{g,1}}{\Theta_{g,+1}} & \frac{\Theta_{g,2}}{\Theta_{g,+1}} & 0 & \frac{\Theta_{\pi,0}}{\Theta_{g,+1}} & \frac{\Theta_{x,0}}{\Theta_{g,+1}} & \frac{1}{\Theta_{g,+1}}
\end{bmatrix}
$$

$$
\times \begin{bmatrix} \mu_t \\ \hat{r}^n_t \\ \eta_t \\ \Xi_t \\ \hat{x}_{t-1} \\ \hat{g}_{t-1} \\ \hat{g}_{t-2} \\ \hat{b}_{t-1} \\ \pi_t \\ \hat{x}_t \\ \hat{g}_t \end{bmatrix}
+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & \bar{b} & 0 \\ 0 & 0 & 0 \\ 0 & \sigma & -\alpha \\ 0 & 0 & 0 \end{bmatrix}
\times \begin{bmatrix} \hat{g}_t \\ \hat{i}_t \\ \hat{b}_t \end{bmatrix}
+ \begin{bmatrix} \mu_{t+1} \\ \hat{r}^n_{t+1} \\ \eta_{t+1} \\ O_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.
$$

Additionally, the target variables are given by:

$$\mathbf{y_t} = \mathbf{C_x x_t} + \mathbf{C_u u_t},$$

which can be written as[1]

$$
\begin{bmatrix} \pi_t \\ \hat{x}_t \\ (\hat{i}_t - i^*) \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\times \begin{bmatrix} \mu_t \\ \hat{r}^n_t \\ \eta_t \\ \Xi_t \\ \hat{x}_{t-1} \\ \hat{g}_{t-1} \\ \hat{g}_{t-2} \\ \hat{b}_{t-1} \\ \pi_t \\ \hat{x}_t \\ \hat{g}_t \end{bmatrix}
+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
\times \begin{bmatrix} \hat{g}_t \\ \hat{i}_t \\ \hat{b}_t \end{bmatrix}.
$$

[1] Note that i^* was set equals to zero.

Following the state-space representation, the loss function becomes

$$\mathbf{L_t} = \mathbf{y_t'Ky_t}$$

$$= \begin{bmatrix} \pi_t \\ \hat{x}_t \\ (\hat{i}_t - i^*) \end{bmatrix}' \times \begin{bmatrix} \gamma_\pi & 0 & 0 \\ 0 & \gamma_x & 0 \\ 0 & 0 & \gamma_i \end{bmatrix} \times \begin{bmatrix} \pi_t \\ \hat{x}_t \\ (\hat{i}_t - i^*) \end{bmatrix}.$$

APPENDIX B

The Impulse Response Function

Consider[1] the vector $MA(\infty)$

$$\boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{\epsilon}_t + \boldsymbol{\Psi}_1\boldsymbol{\epsilon}_{t-1} + \boldsymbol{\Psi}_2\boldsymbol{\epsilon}_{t-2} + \cdots , \tag{B.1}$$

where the matrix $\boldsymbol{\Psi}_s$ has the interpretation

$$\frac{\partial \boldsymbol{y}_{t+s}}{\partial \boldsymbol{\epsilon}_t'} = \boldsymbol{\Psi}_s \tag{B.2}$$

namely the row i, column j element of $\boldsymbol{\Psi}_s$ describes the consequences of a one unit increase in the jth variable's innovation at date t (ϵ_{jt}) for the value of the ith variable at time $t+s$ ($y_{i,t+s}$), maintaining all remaining innovations, at all dates, constant.

One can find these dynamic multipliers $\boldsymbol{\Psi}_s$ numerically by simulation. For example, to implement the simulation, set $\boldsymbol{y}_{t-1} = \boldsymbol{y}_{t-2} = \cdots = \boldsymbol{y}_{t-\rho} = \mathbf{0}$. Now, set $\epsilon_{jt} = 1$ and all remaining elements of $\boldsymbol{\epsilon}_t$ to zero, then simulate the following Gaussian vector autoregression,

$$\boldsymbol{y}_t = \boldsymbol{c} + \boldsymbol{\Phi}_1\boldsymbol{y}_{t-1} + \boldsymbol{\Phi}_2\boldsymbol{y}_{t-2} + \cdots + \boldsymbol{y}_\rho\boldsymbol{\epsilon}_{t-\rho} + \boldsymbol{\epsilon}_t, \tag{B.3}$$

where $\boldsymbol{\epsilon}_t \sim$ i.i.d $N(\mathbf{0}, \boldsymbol{\Omega})$, for dates $t, t+1, t+2, \ldots$, with $\boldsymbol{c}$ and $\boldsymbol{\epsilon}_{t+1}, \boldsymbol{\epsilon}_{t+2}, \ldots$ all zero. The value of the vector $\boldsymbol{y}_{t+s}$ at date $t+s$ of this simulation corresponds to the jth column of

[1] This description follows closely Hamilton (1994).

the matrix $\mathbf{\Psi}_s$. Then, all the columns of $\mathbf{\Psi}_s$ can be calculated by simulating separately for impulses to each of the innovations ($j = 1, 2, \ldots, n$). In short, the impulse-response function is plot of the row i, column j element of $\mathbf{\Psi}_s$,

$$\frac{\partial y_{j,t+s}}{\partial \epsilon_{jt}}. \tag{B.4}$$

This function describes the response of $y_{j,t+s}$ to a one time impulse in y_{jt} with all remaining variables dated t or earlier held constant.

APPENDIX C

Computational Platform

The typography system used in the typing of this project was the LaTeX. The system LaTeX, developed by Leslie Lamport in 1985, consists of a series of macros or routines of the system TeX, created by Donald Knuth at Stanford University in 1983, to facilitate the development of text edition . The LaTeX has very comfortable and stylish commands to create tables, indexes, bibliography, references, etc., allowing the user to focus on document content rather than purely technical details. For details on the typography system LaTeX see Grätzer (2007) or the site `http://www.tex.ac.uk/CTAN/latex`.

The simulations and graphs were developed with the help of MATLAB (MATrix LABoratory), which is a high-level language and interactive environment that enables you to perform computationally intensive tasks. Applications include: comprehensive matrix and arrays manipulation in numerical analysis, optimization, symbolic calculus, signal processing, two- and three-dimensional plotting and graphics for different coordinate systems, etc. See Lyshevski (2003) or the site `http://www.mathworks.com` for details. To compute the simulations the number of periods was set equal to 10,000.

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