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FRANCISCO JAVIER CONTRERAS BUSTOS

REGULATING WITH MARKET INFORMATION

Recife
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Thesis presented to the Graduate Program in Economics of the Universidade Federal de Pernambuco as a requirement for obtaining the degree of Doctor in Economics.

Advisor: *Prof. Rafael Coutinho Costa Lima*

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Dedicated first of all to God, my wife Amanda, my daughter Ana Luíza, my parents Daniel and Patricia, and my brothers. Without their support I could not have made it this far.

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Yours, Lord, is the greatness and the power and the glory and the majesty and the splendor, for everything in heaven and earth is yours. Yours, Lord, is the kingdom; you are exalted as head over all. Wealth and honor come from you; you are the ruler of all things. In your hands are strength and power to exalt and give strength to all. Now, our God, we give you thanks, and praise your glorious name.

—1 CHRONICLES 29:11-13 (Bible)

Abstract

Governments often provide goods and services by hiring private firms that are already operating in the market. This is the case of the *Programa Universidade para Todos* (PROUNI), created in 2004 to provide scholarships to low-income students by private institutions of higher education. In exchange, the private institutions participating in the program benefit from tax exemptions. Studies have shown a positive impact of PROUNI on variables such as accessibility, permanence and academic performance. However, studies conducted by the Federal Court of Auditors have demonstrated that the government is paying a higher price than the market median. Traditional theory treats regulated firms as if they exclusively serve the regulator. In reality, firms usually operate in the market. These market operations can provide information for the regulator. This study examines how the regulator can use market information to save information rents and reduce allocative distortions. Assuming that costs are not observed by the government, we follow the theoretical framework implemented by Baron and Myerson, who analyze contracts based on the principal-agent paradigm. The innovation to the regulatory literature is that it uses the firm's own information in the market to reveal its type. Using a model with one monopolistic firm and another with two firms in an oligopolistic market, interesting results are obtained: when the government observes what the firm does in the private market and conditions its behavior on it, optimal regulation requires the quantity of the first-best for the efficient firm and less distortion for the inefficient firm's contract. In addition, the use of this information allows for fiscal savings.

Keywords: Regulation. PROUNI. Higher Education Institutions. Fiscal Renouncement. Public Finance.

Resumo

Os governos geralmente fornecem bens e serviços contratando empresas privadas que já estão operando no mercado. Esse é o caso do Programa Universidade para Todos (PROUNI), criado em 2004 para fornecer bolsas de estudo a estudantes de baixa renda por instituições privadas de ensino superior. Em troca, as instituições privadas que participam do programa se beneficiam de isenções fiscais. Estudos demonstraram um impacto positivo do PROUNI em variáveis como acessibilidade, permanência e desempenho acadêmico. No entanto, estudos realizados pelo Tribunal de Contas da União mostraram que o governo está pagando um preço mais alto do que a mediana do mercado. A teoria tradicional trata as empresas reguladas como se elas atendessem exclusivamente ao órgão regulador. Na realidade, as empresas geralmente operam no mercado. Essas operações de mercado podem fornecer informações para o regulador. Este estudo examina como o órgão regulador pode usar as informações de mercado para economizar rendas informacionais e reduzir as distorções alocativas. Partindo do pressuposto de que os custos não são observados pelo governo, seguimos a estrutura teórica implementada por Baron e Myerson, que analisam contratos com base no paradigma principal-agente. A inovação na literatura regulatória é que o trabalho utiliza as próprias informações da firma no mercado para revelar seu tipo. Usando um modelo com uma firma monopolista e outro com duas firmas em um mercado oligopolista, obtêm-se resultados interessantes: quando o governo observa o que a empresa faz no mercado privado e condiciona seu comportamento a isso, a regulação ótima exige a quantidade do *first-best* para a empresa eficiente e menos distorção para o contrato da empresa ineficiente. Além disso, o uso dessas informações permite economia fiscal.

Palavras-chave: Regulação. PROUNI. Instituições de Ensino Superior. Renúncia Fiscal. Finanças Públicas.

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Summary

CHAPTER 1
Introduction

Regulation theory models frequently view the problem of regulating a firm as one plagued with information asymmetries. The regulator wishes to execute a project, or have goods produced, but does not know the firm's production cost. The solution consists of incentive compatible contracts, that induce the firm to reveal her cost through her choices. The optimal solution is not efficient. Distortions arise when compared to situations without information asymmetries.

This approach implicitly assumes that the regulator does have another channel to learn the firm's cost. That should be the case when the project (or good) is tailor made for the regulator. Or when there's no competing firm on the market. When such firms exist, the regulator benefits from using information from the competing firm's contract on the contract, on what is called yardstick competition (Shleifer, 1985).

For some classes of goods and services, however, the regulated firm already services some consumers on the market. For example, in Brazil, the government contracts private higher education institutions to offer free undergraduate education for poor students. In the program called *Programa Universidade para Todos* (PROUNI), not all students are eligible for the government scholarship, so there is a market demand for these institutions. As result, the same firm serves consumers in the market while contracting a basically identical service with the government. The firm's market operation provides the regulator with additional information about the firm's costs, since it can be easily tracked (by tax records, for example). But should the regulator use this information? If so, how exactly? In this thesis we wish to provide a first take on these questions.

In order to answer these questions, we first present in Chapter 2 the *Programa Universidade para Todos*, the historical context in which it was developed, the conditions that students must meet to obtain this benefit, the evolution of fiscal expenditures resulting from its operation and the literature that justifies its creation, showing the higher cost that the government is paying compared to the market, motivating a theoretical application of our models to the higher education market. Specifically we will analyze the Brazilian government program PROUNI mentioned above.

Models representing different situations will be developed. Chapter 3 of this thesis, presents the first model, and simpler, which assumes a monopolistic firm in the market without being able to discriminate prices. For this purpose, we use the basic model developed by Baron and Myerson (1982), where it is assumed that the regulator is not able to observe the firm's costs, having contracts offering only the amount of grants and the firm's own transfer for this service. By occupying the firm's own information in the market, in order to limit the informational rent, it is necessary to dislocate the scholarship quantity. Also, it is possible to save tax expenses resulting from this new information.

The next step is to incorporate the existence of competition in the market. In Chapter 4, the firm will no longer be the only firm that offers the good or service, existing other firm that provide the same service. In this sense, following the concept of oligopolistic competition presented by Tirole (1988), and assuming two firms in the market, the question to be answered is how we should use the firm's own information interacting in the market in order to regulate the firm that the government is contracting.

CHAPTER 2

Programa Universidade para Todos - PROUNI

The *Programa Universidade para Todos*, in its abbreviation PROUNI, through a system of partial and full scholarships, has made it possible that students from public schools or with limited resources to fulfill their dream of pursuing higher education in various academic areas. Thanks to the integration of socioeconomic criteria and performance on the National High School Exam (*Exame Nacional do Ensino Médio*, or ENEM), the program has provided fair and equitable opportunities for access to education, allowing emerging talents to find their way to higher education.

However, like any large-scale public policy, the PROUNI has also faced challenges and debates surrounding its regulation. Regulation of such a significant program becomes essential to ensure the efficiency, transparency and sustainability of its objectives. Proper regulation must ensure that the resources allocated to the program are used effectively, that the beneficiaries are appropriate, and that continuous evaluation and improvement of the program's social and educational impact is maintained.

In this research, we will explore Brazil's *Programa Universidade para Todos* in depth, analyzing its objectives, targeting mechanisms, scope and achievements to date. We will also examine the importance of careful regulation for PROUNI, considering aspects such as equity in the distribution of scholarships, academic quality of the private institutions involved, and accountability in the use of public resources.

The regulation of the *Programa Universidade para Todos* is fundamental to safeguard its continuity and success in the mission of democratizing higher education in Brazil. By analyzing its results and challenges, we will be able to establish recommendations for efficient regulation to further strengthen this transformative program and maintain its positive impact on the nation's social and economic development.

2.1 The PROUNI

The *Programa Universidade para Todos* was created in 2004, with the publication of Provisional Measure 213, later converted into Law 11,096 on January 13, 2005 (BRASIL, 2005a). It was developed with the aim of granting scholarships in face-to-face and distance learning undergraduate courses at private institutions to low-income students who do not have a higher education degree, associated with the adoption of affirmative policies and the improvement of the qualification of public school teachers. On the other hand, private higher education institutions participating in the program benefited from the new tax waiver, although the law promotes differentiated rules according to the institutional model (C. H. A. d. Carvalho, 2011), as explained below.

The program is aimed at the most vulnerable portions of the population, by offering full scholarships (100%) for beneficiaries who prove to have a gross per capita family income of up to one and a half minimum wages, and partial scholarships of 50% for those with a gross per capita family income of up to three minimum wages[1]. The scholarship is destined for[2]:

- Students who have completed their secondary education in public schools or in private institutions as full scholarship students;
- Students with disabilities[3], according to the law;
- Teacher of the public school system, exclusively for degree and pedagogy courses, aimed at training the teaching staff of basic education. In this case, the income limit required for other candidates is not applied..
- In 2022, Law 14.350/2022 (BRASIL, 2022) extends access to students from private schools, even without a scholarship.

In addition, the candidate must meet the following requirements: i) obtain at least 450 points in the average of the five areas of knowledge in the *Exame Nacional do Ensino Médio* and ii) not obtain a zero score in the essay. Lastly, the final selection stage is carried out by the higher education institution according to its own criteria, which will also be responsible for checking the information provided by the candidate[4].

Institutions that join the program are exempt from the following taxes[5]: Corporate Income Tax (*Imposto de Renda Pessoa Jurídica*, or IRPJ), Social Contribution on Net Income (*Contribuição Social sobre Lucro Líquido*, or CSLL), Contribution to Social Security Financing (*Contribuição para o Financiamento da Seguridade Social*, or COFINS) and Contribution to the Social Integration Program (*Contribuição para o Programa de Integração Social*, or PIS). Under the previous benefit, institutions participating in the program are

[1] Art. 1, Law No. 11.096.

[2] Art. 2, Law No. 11.096.

[3] Represent 1% of the 2,859,370 scholarship holders until 2020. Source: Open Data, Ministry of Education. See `https://dadosabertos.mec.gov.br/prouni`.)

[4] Article 3, Law No. 11.096

[5] Art. 8, Law No. 11.096

required to offer a percentage of scholarships in the number of paying and regularly enrolled students, in all their courses and shifts. This percentage may vary, under the terms of the law[6], depending on the institution's option to join and its legal nature.

According to Art. 5 of the law that creates PROUNI, the private for-profit or nonprofit non-benefit higher education institution that joins the program must offer at least 1 full scholarship for the equivalent of 10.7 students regularly paying and duly enrolled at the end of the corresponding previous academic period, excluding the number corresponding to the full scholarships granted by PROUNI or by the institution itself, in courses effectively installed in it. Alternatively, in lieu of the previous requirement, the for-profit or nonprofit non-benefit higher education institution may offer 1 full scholarship for every 22 students regularly paying and duly enrolled in courses effectively installed in it, provided that it additionally offers partial scholarships of 50% in the proportion necessary for the sum of the benefits granted to reach the equivalent of 8.5% of the annual revenue of the school periods that already had PROUNI scholarship holders.

Finally, in the case of the social assistance charitable entity, for the purpose of granting or renewing the certification, the higher education institution must grant 1 full scholarship for every 9 paying students.

In exchange for these scholarships, the institutions participating in the program are exempt from certain taxes according to their institutional status.

Table 2.1 Aliquots and base of calculation of federal taxes by institutional status

Tributes	For-profit		Non-profit			
			Confessional / Community		Philanthropic	
	wo PROUNI	w PROUNI	wo PROUNI	w PROUNI	wo PROUNI	w PROUNI
IRPJ	25% x profit	-	-	-	-	-
CSLL	9% x profit	-	-	-	-	-
COFINS	7.6% x revenue	-	3% x revenue	-	-	-
PIS	1.65% x revenue	-	1% x leaf	-	1% x leaf	-
INSS	20% x leaf	20% x profit	20% x leaf	20% x leaf	-	-

Source: (C. H. A. Carvalho & Lopreato, 2005). "w" for with and "wo" for without.

For-profit institutions are the most benefited by being exempt from practically all taxes, in addition to representing 62% of the number of program's scholarships until 2020 according to data from PROUNI's own site[7] (34% belongs to non-profit institutions).

[6]Art. 7, Law N° 11.096.

[7]See https://dadosabertos.mec.gov.br/prouni.

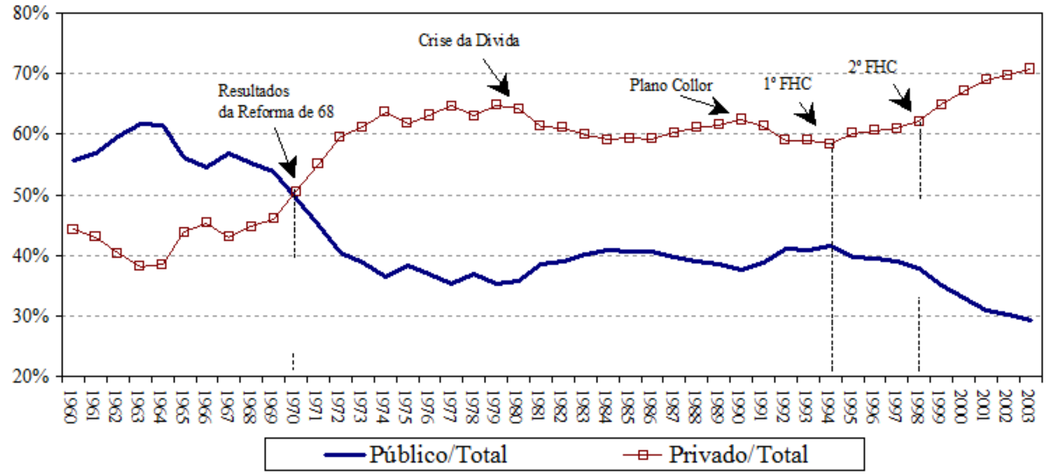

Figure 2.1 Participation in enrolments between public and private institutions (1960-2003). Source: Carvalho (2005)

2.2 Historical context of its establishment

To understand its creation, it is important to understand the historical context in which PROUNI was designed. C. H. A. Carvalho and Lopreato (2005) and C. H. A. d. Carvalho (2005) give a brief history of tax waivers. They report that financing mechanisms through tax exemptions is not something new, tax exemptions had already been used in the late 1960s, where the military government used these tax incentives to support private action in the national development project, benefiting a wide range of sectors, with higher education being the central factor in financing the private segment. Law No. 5.172, of October 25, 1966, of the National Tax Code (BRASIL, 1966), in harmony with the Federal Constitution of 1967 (?, ?), determined the exemption of taxes on income, assets and services of educational establishments of any nature, thus showing that since its creation private higher education institutions have benefited from tax immunity. This mechanism was fundamental for the intensive growth over the last three decades of these institutions, as well as in the expansion of private enrollment as shown in Figure 2.1, helping the continuity of the educational company's activity in the period of crisis by reducing the impact on costs and expenses related to its service provision activity.

In the second half of the 1990s, there was a significant legislative change, inserted by Art. 20 of the Law of Guidelines and Bases of National Education (BRASIL, 1996), where the institutional differentiation within the private segment was defined, with higher education institutions being classified as private for-profit and non-profit (confessional, community and philanthropic), thus implying that the former ceased to benefit directly from public resources and indirectly from tax waivers, making it possible to add the collection of the Union and municipalities. Another relevant fact in the second half of the 1990s is the change in fiscal policy based on the *public deficit* as an indicator of the fiscal situation, moving to a fiscal result based on the *debt sustainability*, where the government's interest is that the market believes that, in the expected scenario for a certain time in the future, there will be no risk of debt default. This fiscal health index became definitively implemented in Brazil after the agreement with the International Monetary Fund (IMF) in 1998, requiring in the second government of Fernando Henrique Cardoso (FHC) a primary surplus that would ensure debt sustainability whatever the exchange rate and interest rate on the stock of public debt. This situation of maintaining a high primary surplus, which is fixed in advance and the amount of spending is adjusted to the behavior of budget revenue, forced the government to cut spending, mainly in the Capital and Costing Budget, explaining the wage squeeze, the cut in funding for health, education and others, as well as the reduction in investment spending (C. H. A. Carvalho & Lopreato, 2005). This adjustment took place through increases in the tax burden and significant cuts in public spending. This same debt sustainability indicator was the context that characterized the fiscal rule that Lula's government had to comply with when the law that instructed PROUNI was created and published, a government where public investments were the lowest in recent brazilian history (C. H. A. Carvalho & Lopreato, 2005).

At the time of the creation of PROUNI, the demand for higher education was not being met due to the fact that public higher education institutions did not have the physical capacity to meet this demand, and private higher educaction institutions did not allow students from low-income families to enter because they did not have the economic conditions to pay tuition fees, a situation that the FIES was not able to complement. In addition, private higher education institutions had an idle capacity of places, which had grown faster than demand in the period 1995-2004[8] (IPEA, 2015).

Given the above, C. H. A. d. Carvalho (2005) points out that PROUNI appears with a discourse of social justice, being its main indicator the low net schooling[9], which according to data from the National Institute of Educational Studies and Research Anísio Teixeira (INEP) and the Ministry of Education (MEC), in 2003 only 9% of the population between 18 and 24 years old attended higher education, a fact that also presents the Union Court of Auditors (*Tribunal de Contas da União*, or TCU)[10] in a report made to PROUNI to measure its impact, being for the year 2007 the percentage of 13%, a very low rate for Brazil compared to the average rate of Latin America and the Caribbean. But in fact,

[8] Social Policies: monitoring and analysis n° 23, 2015 (IPEA).

[9] Defined by IBGE as: Indicator that identifies the percentage of the population in a certain age group enrolled in the level of education appropriate to that age group.

[10] Report, Vote and Judgment 2043 - Plenary; Secretariat of Higher Education, Ministry of Education. TC: 004.379/2009-9.

this discourse masks the pressure of associations representing the interests of the private segment, as narrated by C. H. A. Carvalho and Lopreato (2005), and Catani, Hey, and Gilioli (2006), where they make a study and monitoring of the social program from the first proposal of law presented until the law that sanctioned it. Specifically C. H. A. Carvalho and Lopreato (2005) expose that the private higher education institutions were in a framework of uncertainty by the increase of the idleness of the vacancies and the degree of default/dropout. Thus, the emergence of PROUNI came as an excellent opportunity for institutions threatened by the weight of excessive vacancies. As mentioned above, the law created differentiated rules according to the institutional model, which was reflected by the pressure of the private for-profit sector, and by overly benefiting the profitable institutions in the exemption of taxes.

2.3 Impact of PROUNI

The program has been criticized as a measure of social inclusion and democratization of higher education, due to the fact that the final formulation of the law was mainly according to the preferences of representatives of for-profit institutions, but it is important to note the impact that the program has generated on its target audience: afro-descendant, indigenous and low-income scholarship holders. According to PROUNI statistics[11], there were 2,859,370 scholarship holders until 2020, of which 42.9% declared themselves white, 41.7% brown, 12.7% black, 1.8% yellow, 0.8% did not inform and 0.1% declared themselves indigenous. Of the total number of scholarship recipients, only 1% are elementary school teachers, and only 1% of the total has a disability condition. From the above data it can be inferred that the policy is fulfilling the objective of affirmative action of racial quotas, but it would not yet be inclusive for people with disabilities, as well as the objective of improving the quality of teachers in the public education network. Another aspect of criticism of the program, on the one hand it expands access to higher education to low-income people, but Faceira (2008) reports that it is necessary to understand that democratizing access also means guaranteeing the permanence and quality of the process of professional training of scholarship holders, being necessary to provide sufficient conditions for students to continue their education, and thus reduce the possibilities of evasion. Saraiva and Nunes (2011) emphasize this point in an approach based on individual interviews with scholarship holders in Minas Gerais. As a measure to solve the problem of permanence, the government in June 2005 announced a "package of goodies" for the educational area, as mentioned in Catani, Hey, and Gilioli (2007), which would include the creation, through a Provisional

[11] See `https://dadosabertos.mec.gov.br/prouni`

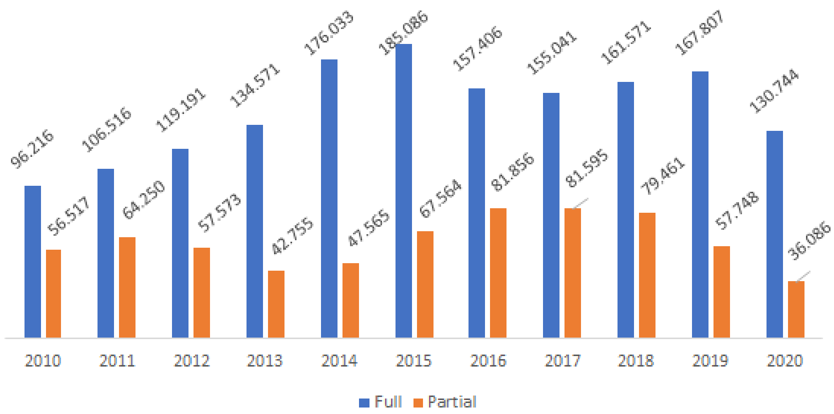

Figure 2.2 Scholarships holders. Source: Open Data, PROUNI. Own elaboration.

Measure, of the scholarship-permanence. Law 11.180, Article No. 11, on September 23, 2005 (BRASIL, 2005b), created the R$ 300,00 maintenance scholarship for full PROUNI scholarship holders enrolled in full-time courses. Currently, the monthly value is R$ 900,00 for indigenous and *quilombolas* and R$ 400,00 for other beneficiaries.

Aiming at the quality of the courses of the higher education institutions that adhere to the program, in a report on the monitoring of PROUNI made by the TCU (2013)[12], shows that in the legislation that regulates the program there was no action to seek the proportionality of the tax exemption also linked to the quality of the courses offered. Likewise, it highlights an important fact of the Law, in paragraph 4, article 7, in which the MEC must remove from the program the course considered insufficient, according to performance criteria of the National System for the Evaluation of Higher Education (*Sistema Nacional de Avaliação da Educação Superior*, or SINAES), for two consecutive evaluations. A SINAES evaluation lasts about three years, so for a course to be excluded from PROUNI the process will take six years for its realization, a period in which several students will graduate from low quality courses.

In terms of accessibility to higher education under PROUNI, the government has increased the number of scholarships holders each year through 2015, and has remained at those levels, as shown in Figure 2.2, which is important in the perspectives of the scholarship holders themselves. Oliveira, Contarine, and Cury (2012) conducted a documentary analysis, semi-structured interviews in focus groups and questionnaires made to scholarship holders of the Coração Eucarístico campus of PUC Minas Gerais. In the testimonies, the scholarship holders show the appreciation of PROUNI as a public policy for entering higher education, but even so they report that many students are still socially and educationally excluded, as they do not achieve good performance in ENEM. They also indicate that they maintain

[12]Monitoring. Judgments 816/2009 and 2043/2010. Both of the Plenary. TC: 000.997/2013-7.

good relationships with classmates and teachers, but in some cases they have felt differentiated from non-scholarship colleagues associated with the most socially and economically favored social classes. From teachers, 87% evaluated the program positively, 7% negatively, and 6% did not know how to evaluate it; of the evaluation for the scholarship holders, the teachers evaluate them positively in their majority (78%), 15% show that the scholarship holders had difficulties and 7% did not know how to evaluate. Saraiva and Nunes (2011) also conclude that, according to the scholars, the program is effective as it meets their immediate expectations of access to higher education, but on the other hand, they show performance problems by failing to achieve, in fact, access to education as provided for in the Federal Constitution. The authors also cite that the proliferation of private higher education institutions is being stimulated, in contradiction with citizen demands for an expansion of the offer of places in public universities.

Finally, in an impact assessment carried out by the TCU's Pitágoras Project in 2009[13], PROUNI has had an impact on access to higher education, permanence and performance of scholarship students. In the first case, regarding the group of students who benefit from the scholarship, the estimate of the average impact of the treatment on those treated (ATT) was 36.1% in relation to the control group, demonstrating a positive and statistically significant effect on the probability of students entering higher education. In the case of dropout/permanence, the estimates indicated that the full scholarship holder has a probability of dropout 8 percentage points lower than a partial scholarship holder. In relation to student performance, using the score in the National Student Performance Exam (*Exame Nacional de Desempenho de Estudantes*, or ENADE) as the variable of interest, among incoming and outgoing students, for incoming students the estimated impact was 1.4%, but for outgoing students the impact was null. The program also reserves scholarships for people who declare themselves indigenous, brown and black, provided they meet the selection criteria. The percentage of scholarships allocated to quota holders depends on the number of black, brown and indigenous citizens in each state according to the latest IBGE census. Regarding the increase in the possibilities of access to higher education, the impact for quota holders was 10 percentage points higher when compared to non-quota holders.

2.4 PROUNI costs

As of 2006, the Federal Revenue Secretariat (*Secretaria da Receita Federal*, or SRF) publishes the estimated tax expenditures of PROUNI in the Demonstrative of Indirect

[13]Report, Vote and Judgment 2043 - Plenary; Secretariat of Higher Education, Ministry of Education. TC: 004.379/2009-9.

Government Expenditures of a Tax Nature. The estimate for a certain period is made at the beginning of each year, and the tax exemption for each tax in which higher education institutions are exempt is detailed. The analysis in the case of this study begins with 2011, the year in which PROUNI's legislation [14] changed the calculation of the exemption, becoming a function of the effective proportion of scholarships due, and not of the total scholarships offered. As shown in Table 2.2, tax expenditures have been increasing over the years, growing by an average of 12% per year. By 2023, tax expenditures reaches 0.03% of GDP from 0.02% in 2011.

The government faces misrepresentation of information from both students and educational institutions. Curiously, in Brazil, the government is paying a fee that is higher than the median fee charged in the private sector. This fact is in accordance with the Bennett hypothesis, a premise that will be explained later, which we will assume to be true, so the models will not test it in the next chapters. This excess cost that the government faces in the goods or services motivates the application of these models to the case of PROUNI.

Table 2.2 Distribution of estimated tax expenditures of PROUNI, by type of tax (valued in R$ million at constant prices based on Sep 2022)

Year	IRPJ	CSLL	COFINS	PIS	Total
2011	43.02%	15.39%	34.14%	7.45%	1,035
2012	37.28%	17.23%	37.39%	8.10%	1,396
2013	42.09%	14.55%	35.64%	7.72%	1,350
2014	34.82%	10.41%	45.01%	9.75%	1,020
2015	40.72%	12.89%	38.12%	8.26%	1,548
2016	46.07%	16.42%	30.82%	6.68%	1,844
2017	46.07%	16.42%	30.82%	6.68%	1,799
2018	46.07%	16.42%	30.82%	6.68%	1,813
2019	54.27%	21.17%	19.98%	4.59%	2,752
2020	49.58%	18.78%	26.01%	5.63%	3,242
2021	48.06%	17.14%	28.61%	6.20%	3,135
2022	45.04%	15.54%	32.41%	7.02%	2,766
2023	49.39%	13.07%	30.85%	6.69%	3,203

Source: Prepared with data from the SFR.

As shown in Table 2.1, the exemption of IRPJ and CSLL corresponds exclusively to for-profit higher education institutions, representing on average 61% of total tax expenditures, while COFINS, which in Carvalho's analysis (2011) was the most representative tax, reduces the tax costs of for-profit and non-profit higher education institutions, reaching 32% of total tax expenditures. Finally, PIS affects the three types of higher education institutions, but it only represent 7%.

In general, to obtain the annual or monthly cost of PROUNI per student, the procedure used by C. H. A. d. Carvalho (2011), or other authors such as Corbucci (2007), is to

[14] Law No. 12.431 of 2011

divide the total annual tax expenditure by the number of scholarships offered in that same year, without distinguishing between partial or full scholarships. But this procedure does not represent the right value of a scholarship for the government, because it accumulates the expenses of the different types of higher education institutions and their scholarship holders, and takes into account the number of scholarships offered, and not those actually occupied, considering that the idle places have been on average 31.5% in 11 years of the program (2005 to 2015)[15], causing the cost per student to decrease, as the number of scholarship holders increases and the same amount of tax waiver remains. In fact, C. H. A. d. Carvalho (2011) mentions that the monthly cost to the government per student is well below the market prices of school fees, as the calculation is made according to this methodology.

The right thing to do would be to calculate the average cost per PROUNI student, separating the analysis by type of participating institution. From the data available at SRF and PROUNI, it is not possible to make such segmentation. In the operational audit carried out by TCU (2009) to PROUNI, the SRF was asked to elaborate more concrete data on the average cost of the program and by type of higher education institution. In the calculation procedure, they created an annual equivalence index of the number of students participating in the program in each type of institution, which considers the fact that there are scholars with different percentage of scholarships and who remain in the program for different periods throughout the year, leaving four types of possible situations: partial scholarships, full scholarships, effective in the first semester and effective in the second semester. The calculation is made for the year 2006, pointing out that the monthly cost of the PROUNI scholarship in for-profit institutions was less than half of the cost of the PROUNI scholarship in non-profit charitable institutions. At 2016 prices, the monthly cost to the government of PROUNI per student in a for-profit institution was R$ 903, while in non-profit the monthly cost per student was R$ 1,905 and R$ 1,673, for the charitable and non-charitable ones respectively. These "monthly fees" can be compared with the median monthly fees for the year 2006, in Figure 2.3. As can be seen, the median tuition fee for all face-to-face undergraduate courses at all higher education institutions in the country was R$ 780. In other words, the government on average has been paying a higher tuition fee for scholars compared to the median market tuition fee.

In terms of demand-side regulation, the government has been able to reduce the highest cost for cases of students with irregularities, i.e., who according to the PROUNI grant criteria should not be receiving a scholarship. For example, TCU monitoring in 2013 excluded 15,581 scholarship holders from 2012, much more than the 1,766 scholarship

[15] Report of Globo of 31/03/0216. Available at http://g1.globo.com/educacao/noticia/2016/03/prouni-tem-315-de-vagas-ociosas-em-11-anos-de-programa.html.

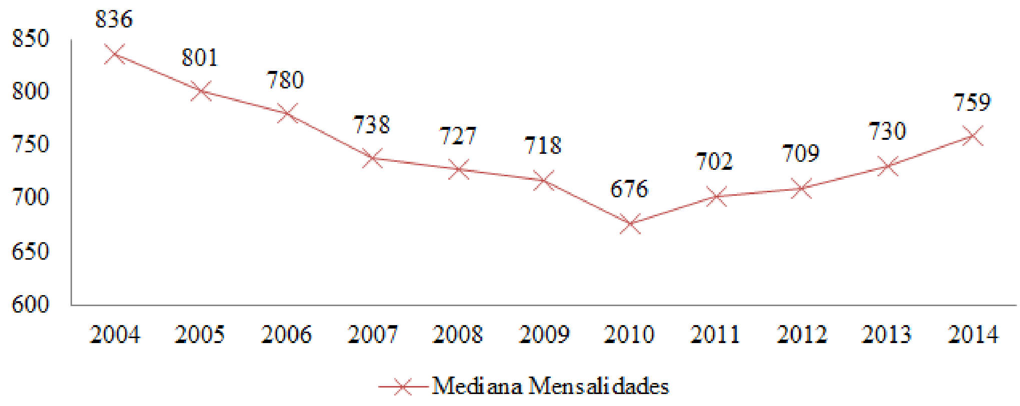

Figure 2.3 Evolution of Tuition Fees in Private Higher Education. Source: "Sectoral Analysis of Private Higher Education - Brazil 2014", Hopper Educação (in R$ 2016).

holders excluded in 2009 and the 4,253 excluded in 2010. This increase was due to the MEC having created a module for SISPROUNI, the Scholarship Holder Supervision module, where data are cross-checked annually with the following databases: Annual Social Information Report (*Relação Anual de Informações Sociais*, or RAIS), National Registry of Motor Vehicles (RENAVAM), Integrated Platform for Management of Federal Higher Education Institutions (PINGIFES) and with 14 databases of large state universities.

Therefore, it is necessary to apply the theory of incentives in PROUNI, justified by the higher cost that is burdening the government by scholarship students in private higher education institutions, compared to students in the same institutions who are not scholarship students, characterized by the monthly fees paid in the market.

2.5 Related literature

There is general agreement that higher education is associated with a number of benefits at both the individual and societal levels, which justify the government's action to provide higher education to low-income students. In individual terms, higher education graduates tend to have more stable jobs, less dependence on public assistance, better health and higher levels of general well-being, as well as better employment opportunities and higher wages (Delaney & Yu, 2013). In addition, there is evidence that those who graduate from higher education have attitudes of greater engagement with politics, environmental issues, and more positive attitudes toward migrants. At the social level, evidence associates higher educational levels with higher overall well-being, lower levels of unemployment and poverty, higher tax revenues, and greater civic participation (Tentsho, McNeil, & Tongkumchum, 2019).

The human capital literature studies the different themes in the higher education process. Part of this literature studies the gains from higher education (Heller (2013); Kane and Rouse (1995); Levy and Murnane (1992); Murnane, Willett, and Levy (1995); Murnane et al. (1995); Zucker (2001)). The incremental premium for obtaining a degree in higher education - the additional gain for a degree compared to completing high school - has been improving in recent decades. This has affected the demand for school graduates as well as adults with low levels of education, corresponding to an increase in enrollment in higher education institutions (Heller, 2013).

But the increase in demand for higher education has been impeded, in the case of the low-income population, by the cost of tuition fees. The literature justifying government financial aid, whether through grants or loans, shows that the credit market presents frictions to investment in education because future income cannot be pledged as collateral. Moreover, low-income students rarely have assets that would allow them to use them as collateral (Becker, 1967). This intermediation imperfection leads to an underinvestment in human capital, which also goes against a normative point of view with the right to education by the entire population, being the duty of the state and the family to provide this right (Article 205 of the Constitution of the Federative Republic of Brazil of 1988 (BRASIL, 1988); Pavan and Grando (2012)), and against the goal of the National Education Plan (PNE - Law No. 11. 172/2001, BRASIL (2001)) of increasing the proportion of young people aged 18 to 24 enrolled in higher education to 30% by 2010 (Catani et al., 2006).

There is a large literature that studies tuition prices over the years, mostly focusing on the impact of tuition increases on college enrollment, student retention or attendance, and graduation. Dynarski (1999) examines whether financial aid has a positive effect on attendance and degree completion. He takes advantage of a change in the law in 1982 that eliminated a social security program (Social Security Student Benefit Program), which benefited 18-22 year olds, children or dependents of beneficiaries who died while still in their childhood, disabled, or retired from Social Security, with monthly payments while they were enrolled full-time in college. Using the death of a parent as a proxy for being an insurance beneficiary, she finds that offering US$ 1,000 in aid increases the probability of attending university by 4%.

Using a discontinuous regression design, Kane (2003) studies the impact of the CalGrant financial aid program in California on college enrollment. To participate in the program students must fit the minimum threshold of the following three characteristics: income, assets, and high school GPA. He finds that being chosen for the program has a positive impact of 3% to 4% on college enrollment, where the largest impact was on choices of private four-year universities.

Few studies have focused on the role of government financial aid programs in increasing tuition prices, with different results. In this area of the literature, this effect is known as the "Bennett Hypothesis", thanks to a 1987 publication in The New York Times by then-U.S. Secretary of Education William Bennett (Bennett, 1987), where his theory was that the availability of federal loans-particularly subsidized loans offered at a below-market interest rate and paying interest as the student is enrolled-allows universities to raise their prices because students can offset the price increase, or at least a portion of that increase, with federal loans (Heller, 2013).

McPherson and Schapiro (1991) tested this theory, using the Department of Education's Integrated Postsecondary Education Data System (IPEDS) database for the years 1978 through 1985, finding that increases in government aid are accompanied by increases in scholarship spending at private universities, contrary to Bennett's predictions. In contrast, Li (1999) finds some evidence for Bennett's Hypothesis when she uses Pell Grant Information System data to track program recipients and their respective college enrollment levels. Long (2004) argues that a possible reason for this contradiction comes from the difficulty in isolating the effect of government aid on tuition from other factors. Long (2004) also finds no evidence of a response in tuition for higher education tax benefits between private and public four-year colleges, and limited evidence on tuition increases between public two-year colleges. Cellini and Goldin (2014) use data from three states - Florida, Michigan, and Wisconsin - to analyze tuition at two types of for-profit institutions, those accredited by government-recognized agencies, allowing them to participate in federal *Title IV* programs, and those that are not, but offer similar educational programs. They find that institutions (especially those offering certificates) that are chosen by *Title IV* charge tuition rates 78% higher than those charged by comparable programs at non-participating institutions, lending some credence to Bennett's hypothesis.

For Brazil, Duarte, de Mello, et al. (2016) test whether the increased availability of loans for students increases tuition costs. In early 2010, there was an operational and normative change in the legislation of the Student Financing Fund (*Fundo de Financiamento Estudantil*, or FIES), where the interest rate of the program was reduced from 6.5% to 3.5% per year. The change created heterogeneity in access to funds, which added to the strong increase in FIES, creates a quasi-experiment. From the authors' analysis, using differences-in-differences, they find that facilitating access to students causes an increase in tuition fees. They also estimated a structural demand model, and show that relaxing the credit constraint reduces the price elasticity of demand. Thus the mechanism behind the increase in tuition fees in Brazil is an increase in tuition insensitivity, at least in part.

As observed in the literature, increases in government aid, either through subsidized loans or through grants themselves, which is the case of the PROUNI, are accompanied by increases in tuition fees. In the case of Brazil, as shown by Duarte et al. (2016), increases in tuition fees are not being explained solely by increases in marginal costs or increases in the returns to education, but rather by the fact that there is greater availability of student credit, so it is necessary to regulate higher education institutions so that they are not overcharging the government.

Finally, the literature related to the theory of incentives and regulation has studied the pharmaceutical industry, nuclear plants, incentive schemes in corporations, among others, as mentioned by Laffont and Tirole in their book Laffont and Tirole (1993), but to my knowledge the theory has not been applied to the education market. In addition, in general the works done study monopolies serving a certain market, such as Gagnepain and Ivaldi (2002) who use the same standard model of Tirole and Laffont for the public transportation system in France, but do not use some market price information, information that will be used in this study.

The main reason for applying the incentive theory arises from the background that the cost that the government is assuming for PROUNI scholarship students is higher than the cost of tuition fees in the market. In 2006 (at constant 2016 prices) the average tuition paid by the government per scholarship student was R$ 1,435, but the median market tuition in the same year was R$ 780, just over half of what the government pays for the program. Thus, the idea proposed in this thesis is to use the information on tuition fees paid in the private market to regulate what the higher education institution can charge the government for PROUNI scholarship students. To this end, the government presents a pricing rule that the higher education institution must comply with, which will consider market information.

Therefore, this study adds to the literature an application of incentive theory in the private higher education market, specifically in PROUNI, as well as includes in the analysis a pricing rule controlling with the information of tuition fees in the market, both cases having not been studied so far.

CHAPTER 3

Regulating a Monopolist with market information

3.1 Introduction

The following chapter aims to take a first look at the regulation of a single firm in the market that provides a service for the government. The regulation theory presents different models of firms with different assumptions regarding the information asymmetries that the regulator presents, giving solutions through incentive-compatible contracts and their respective distortions.

Often, for some products or services for which the government wishes to provide, as in the case of PROUNI, there is already a private firm providing this service in the market. Therefore, the same firm serves both consumers in the market and the government. Given their operation in the market, this allows the regulator additional information about the profits they are making, the price and the quantity they produce. By having this information, we will seek to create an optimal regulation for the regulator by giving an answer to whether or not to use this information, as well as how to use it.

This chapter develops a model where a monopolist already operates in the market and is hired by regulator to provide the same service offered in the market. Assuming two firm efficiency cases to represent the asymmetry of information, a linear cost function in the amount of service offered and private and social demands known by the regulator and the firm, the chapter extends the standard regulatory models by including the monopolist's own profit in the market within the incentive compatibility constraints, having an additional tool to induce the firm to choose the contract designed for her. Comparing our results with those of the standard models, we show that market information has an impact on fiscal savings through the reduction of informational rents by improving the government's bargaining power, achieving an optimal regulatory policy that improves the consumer net surplus. In addition, comparing with the case without regulation we obtain the same results of improvement in allocation efficiency as the models that do not incorporate market information.

The chapter is related to the line of research that studies how to regulate a monopolist under different assumptions of uncertainty. Each regulatory mechanism is basically based on providing incentives, without the regulator incurring many costs by decreasing the monopolist's informational rent, so that the firm expands its output beyond its levels in the absence of regulation. Loeb and Magat (1979) design a mechanism where the monopolist receives a subsidy equal to the increase in consumer surplus by increasing its output and decreasing its price. They note that the subsidy needed to implement their mechanism must be large, in addition to needing to tax consumers, leading to decreasing efficiency. In response to this problem, Loeb and Magat (1979) resolve to auction the right to be a monopolist, and the revenues from the auction should be equal to the profits that the monopolist would obtain.

Baron and Myerson (1982), authors of the model which is our basis, study how to regulate a monopolistic firm when the regulator doesn't know the costs. They use the revelation principle (Dasgupta, Hammond, & Maskin, 1979), limiting themselves to incentive compatible disclosure mechanisms, requiring the monopoly firm to report its unknown cost information and ensuring that it has no incentive to misrepresent the information. Their optimal price regulatory policy is induced to be above marginal cost for all cost realizations other than the lowest. These higher cost realizations help reduce the firm's incentive to exaggerate costs by reducing inefficiently production levels and then decreasing firm's cost advantage. Our results are a bit different we have an additional dislocation for looking at the market concerning the first-best.

Laffont and Tirole (1986) incorporate moral hazard in their model. Contrary to the case of cost unobservability, they find an effort distortion for a given output that is more than offset from a welfare point of view by the lower price distortion, demonstrating the trade-off between inducing revelation and inducing effort (they have a partial sharing of cost). Our approach adds market information in the analysis, having relatively similar results in the optimal quantity of the good or service, following the same rule as under complete information. But to delimit the informational rent of the efficient type, as our model does not observe cost, the higher dislocation of the inefficient firm's output indirectly affects effort.

Lewis and Sappington (1988) analyze the case when the regulator is imperfectly informed about both the firm's cost and demand functions. As the results from the last models, the optimal regulated price will be higher, the higher are the firm's costs and the greater is demand, and this distorts efficient production level to reduce the firm's rents. For the smallest demand and cost realization, the output will be supplied at the efficient level. But the firm can exaggerate costs and understate demand at the same time. In this case, the regulator may find optimal to set a price below realized marginal cost. In our model we will assume that the government perfectly knows the demand for the service, therefore we will not have to worry about cases where prices have to be set below the marginal cost.

Finally, Faure-Grimaud (2002) performs a regulation scheme using the stock price information of the regulated firm itself. Assuming that the stock price signals the marginal cost of the firm and a high monitoring cost for the regulator, this mechanism improves the quality of regulation and welfare. Our model, in contrast, occupies exact information from the firm itself of her operations with the market.

In general, our results will be in accordance with the literature, having the standard trade-off between allocative output and informational rent extraction. But our model in comparison with the standard models of regulation will also have an additional distortion in the inefficient firm.

3.2 The Model

There is a service (project) for which consumers have a positive surplus $S(q)$, which is known by the regulator and the firm, where q is the quantity offered. To keep the problem mathematically tractable, we shall assume that the firm's cost is linear in q of the form:

$$C(q,e) = (\beta - e)q + \psi(e) + \alpha \tag{3.1}$$

where β is a private information cost parameter, e is the effort the firm places on cost reduction and α is the fixed cost. β can be either high or low, $\beta \in \{\underline{\beta}, \overline{\beta}\}$, representing the efficient and inefficient firm, respectively. Effort reduces the firm's utility. The disutility from effort is denoted by $\psi(e)$. This disutility grows with effort $\psi'(e) > 0$ for $e > 0$, at an increasing rate $\psi(e)'' > 0$, and satisfies $\psi(0) = 0$, $\lim_{e \to \beta} \psi(e) = +\infty$. The government can make transfers t to compensate the firm. There's a shadow price of governmental funds, denoted by λ. It means that raising costs is costly for the society.

The chapter will follow Baron and Myerson's model, where the regulator contracts a quantity (q) and gives a transfer (t). The contract does not specify effort. In practice, this means the firm chooses the effort and the regulator does not obverse it. This implies that the firm chooses effort and the quantity she offers in the market. The regulator chooses quantity and transfers. The timing is such that regulator offers before the firm's choice.

3.2.1 Baron and Myerson

Before regulating the firm serving the government and the private market, we will present the basic Baron and Myerson's model just giving the service to the government.

Traditional theory views the total cost as unobservable [1], so the marginal and total costs are self-reported by the firm to the regulator, and the costs are reimbursed by the government. To accept the relationship with the government, the firm must be compensated by a transfer $t \in \{\underline{t}, \bar{t}\}$ that pays the total cost. The firm's preferences are given by

$$U_G(q,t,e) = t - (\beta - e)q - \psi(e) - \alpha \tag{3.2}$$

The government cares about consumer surplus plus the firm's profits. But transferring money from consumers to the firm has an implicit shadow cost (λ). The government's preferences are given by

$$V(q,e,t) = S(q) - (1+\lambda)t + U_G \tag{3.3}$$

We can eliminate transfers, substituting (3.2) in (3.3)

$$V(q,e) = S(q) - (1+\lambda)\left[(\beta - e)q + \psi(e) + \alpha\right] - \lambda U_G \tag{3.4}$$

That is, social welfare is the difference between the consumer surplus attached to the project and the total cost of the project as perceived by the taxpayers, plus the firm's rent above its reservation utility times the shadow cost of public funds. The important feature is that the regulator dislikes leaving a rent to the firm.

3.2.1.1 Complete information

This section gives the first-best of the regulatory policy without the market, that will be one of our benchmarks. In its relationship with the regulator, the firm must obtain at least its reservation utility, defined exogenously by Π^o.

The timing of the relationship between the firm and the government is as follows:

i. Nature determines the type of firm, and the firm learns it.

ii. The government offers the contract, detailing the transfer and the quantity required, $\{t, q_G\}$.

iii. With the contract offered, the firm obtains its reaction curve based on the parameters of the contract.

iv. The government determines the level of production that maximizes social welfare.

[1] When observed, as in the Laffont and Tirole's model, it is developed in Appendix A.

The importance of the timing is that the government acts as a leader in a Stackelberg model. Before signing the contract, it informs what it wants through the contract offered. Then the firm maximizes its profit with this contract. Considering this optimization, the regulator makes a take-it-or-leave-it offer, choosing the desired quantity, q_G, that maximizes social welfare. This timing is the one that will follow all the problems in this thesis.

Therefore, given the contract offered $\{t, q_G\}$, where t is the transfer and q_G the quantity required by the government, the firm chooses effort

$$\max_{e} \mathrm{t} - (\beta - \mathrm{e})\mathrm{q_G} - \psi(\mathrm{e}) - \alpha$$

The first order condition is

$$q_G = \psi'(e) \tag{3.5}$$

Lemma 1. *Optimal decision of a monopolistic firm under complete information that serves the government, is characterized by (3.5).*

i. the marginal disutility of effort, $\psi'(e)$, must be equal to marginal cost savings with the government and the market.

From equation (3.5) we obtain the reaction curve of the firm's variable with respect to the regulator's contract: $e(\beta, q_G)$. Notice that the effort is decreasing with respect to β, that is, higher β leads to lower q_G which induces less effort, and also increasing with respect to q_G. So the regulator's problem is

$$\max_{q_G} \mathrm{S(q_G)} - (1+\lambda)\mathrm{t} + \mathrm{U_G}$$

subject to

$$U_G = t - (\beta - e(\beta, q_G))q_G - \psi(e(\beta, q_G)) - \alpha \geq \Pi^o$$

This program becomes

$$\max_{q_G} \mathrm{S(q_G)} - (1+\lambda)\left[(\beta - \mathrm{e}(\beta, \mathrm{q_G}))\mathrm{q_G} + \psi(\mathrm{e}(\beta, \mathrm{q_G})) + \alpha\right] - \lambda \mathrm{U_G}$$

Notice that the optimal solution involves $U_G = \Pi^o$. So the transfer is adjusted as to make the firm indifferent between accepting or rejecting the contract. With complete information this implies in no distortion. The first order condition is given by [2]

$$\Longrightarrow S'(q_G) = (1+\lambda)(\beta - e(\beta, q_G)) \tag{3.6}$$

[2] All the maximization problems of this chapter are in Appendix B.

Lemma 2. *Optimal regulation of a monopoly under complete information and the structure of the Baron and Myerson's model, is characterized by (3.6).*

i. the marginal social value of the project is equal to the marginal cost for taxpayers

3.2.1.2 Asymmetric information with two-type case

This section assumes that the regulator knows that β can be $\underline{\beta}$ or $\overline{\beta}$, with probability v or $1-v$, respectively. Since there are two types of firms, there will be two participation constraints and two incentive compatibility constraints. Such participation constraints are given by

$$\underline{U} = \underline{t} - (\underline{\beta} - \underline{e})\underline{q}_G - \psi(\underline{e}) - \alpha \geq 0 \tag{3.7}$$

$$\overline{U} = \overline{t} - (\overline{\beta} - \overline{e})\overline{q}_G - \psi(\overline{e}) - \alpha \geq 0 \tag{3.8}$$

The incentive compatibility constraints, which require that the contract designed for each firm be the preferred one for its type, are as follows

$$\underline{t} - (\underline{\beta} - \underline{e})\underline{q}_G - \psi(\underline{e}) \geq \overline{t} - (\underline{\beta} - \underline{e})\overline{q}_G - \psi(\underline{e}) \tag{3.9}$$

$$\overline{t} - (\overline{\beta} - \overline{e})\overline{q}_G - \psi(\overline{e}) \geq \underline{t} - (\overline{\beta} - \overline{e})\underline{q}_G - \psi(\overline{e}) \tag{3.10}$$

From (3.9) and (3.8), we can obtain the participation constraint of the efficient firm, as shown below by having $\overline{\beta} > \underline{\beta}$ and $\psi(e)$ is increasing

$$\begin{aligned} \underline{U} &\geq \overline{t} - (\underline{\beta} - \underline{e})\overline{q}_G - \psi(\underline{e}) \\ &\geq (\overline{\beta} - \overline{e})\overline{q}_G + \psi(\overline{e}) + \alpha - (\underline{\beta} - \underline{e})\overline{q}_G - \psi(\underline{e}) \\ &\geq \left[(\overline{\beta} - \overline{e}) - (\underline{\beta} - \overline{e})\right]\overline{q}_G + \psi(\overline{e}) - \psi(\underline{e}) + \alpha \\ &\geq 0 \end{aligned} \tag{3.11}$$

i.e., since the efficient type can always emulate the inefficient type at a lower cost, we can ignore (3.7). The incentive compatibility constraint of the inefficient firm will be ignored for the moment, and then we will see that the solution of the problem considering (3.8) and (3.9) satisfies (3.10). This is true for all maximization problems in this thesis, that is, the participation constraint of the inefficient firm and the incentive compatibility of the efficient firm will be considered.

The efficient firm may pass as inefficient, that means choose the inefficient firm's contract. That implies that the incentive compatibility constraint is

$$\max_e \underline{t} - (\underline{\beta} - e)\underline{q}_G - \psi(e) = \max_e \bar{t} - (\underline{\beta} - e)\bar{q}_G - \psi(e)$$

Following the same timing as in the previous section, whereby the regulator offers before the firm's choice, and considering the reservation utility for both types, $\underline{\Pi}^o$ and $\overline{\Pi}^o$, we obtain

$$\underline{t} - (\underline{\beta} - e(\underline{\beta}, \underline{q}_G))\underline{q}_G - \psi(e(\underline{\beta}, \underline{q}_G)) = \bar{t} - (\underline{\beta} - e(\underline{\beta}, \bar{q}_G))\bar{q}_G - \psi(e(\underline{\beta}, \bar{q}_G)) \tag{3.12}$$

But we also have that the participation constraint of the inefficient type is binding. So

$$\bar{t} = \overline{\Pi}^o + (\overline{\beta} - e(\overline{\beta}, \bar{q}_G))\bar{q}_G + \psi(e(\overline{\beta}, \bar{q}_G)) + \alpha \tag{3.13}$$

The incentive compatibility constraint of the efficient firm (3.12) and the participation constraint of the inefficient firm (3.13) are binding because given the regulator's preferences, what it wants is to reduce as much as possible their profits. Therefore, it leaves them indifferent between participating or not.

Combining (3.12) with (3.13), this leads to the informational rent of the efficient type

$$\underline{U}_G = \overline{\Pi}^o + \bar{q}_G \left[(\overline{\beta} - e(\overline{\beta}, \bar{q}_G)) - (\underline{\beta} - e(\underline{\beta}, \bar{q}_G)) \right] + \psi(e(\overline{\beta}, \bar{q}_G)) - \psi(e(\underline{\beta}, \bar{q}_G)) + \alpha \tag{3.14}$$

Then, the utility of the regulator becomes

$$\begin{aligned} &v \left\{ S(\underline{q}_G) - (1+\lambda) \left[(\underline{\beta} - e(\underline{\beta}, \underline{q}_G))\underline{q}_G + \psi(e(\underline{\beta}, \underline{q}_G)) + \alpha \right] - \lambda \underline{U}_G \right\} \\ (1-v) &\left\{ S(\bar{q}_G) - (1+\lambda) \left[(\overline{\beta} - e(\overline{\beta}, \bar{q}_G))\bar{q}_G + \psi(e(\overline{\beta}, \bar{q}_G)) + \alpha \right] - \lambda \overline{\Pi}^o \right\} \end{aligned}$$

with $\underline{U}_G$ from (3.14).

From the first order conditions we obtain

$$S'(\underline{q}_G) = (1+\lambda)(\underline{\beta} - e(\underline{\beta}, \underline{q}_G)) \tag{3.15}$$

and

$$\begin{aligned} S'(\bar{q}_G) = {} & (1+\lambda)(\overline{\beta} - e(\overline{\beta}, \bar{q}_G)) \\ & + \frac{\lambda v}{1-v} \underbrace{\left\{ (\overline{\beta} - e(\overline{\beta}, \bar{q}_G)) - (\underline{\beta} - e(\underline{\beta}, \bar{q}_G)) \right\}}_{>0} \end{aligned} \tag{3.16}$$

Lemma 3. *Optimal regulation of a monopoly under asymmetric information and the structure of the Baron and Myerson's model, is characterized by* (3.15) *and* (3.16)*.*

i. an efficient level of q_G, given to equal the marginal social value of the project to the marginal cost for taxpayers, and a positive informational rent for type $\underline{\beta}$.

ii. underproduction and no informational rent for type $\overline{\beta}$.

Note that constraint (3.10) is satisfied by this solution. The incentive compatibility constraint of the inefficient firm can be written

$$\overline{U} \geq \underline{U} - \left[(\overline{\beta} - \overline{e}) - (\underline{\beta} - \underline{e})\right] \underline{q}_G - [\psi(\overline{e}) - \psi(\underline{e})] + \alpha$$

or

$$\overline{U} \geq - \left[(\overline{\beta} - \overline{e}) - (\underline{\beta} - \underline{e})\right] \left[\underline{q}_G - \overline{q}_G\right] + \alpha$$

which is true sinse $\overline{e} < \underline{e}$ and $\overline{q}_G < \underline{q}_G$ from **Lemma 1**, (3.15) and (3.16), then (3.10) is a neglected constraint.

As can be seen from equation (3.15), the optimum for the efficient firm is not distorted. But the contract for the inefficient firm, looking at equation (3.16), there is an additional term that distort the allocation of the quantity that the government will offer to the inefficient firm. Since effort decreases with respect to β, and both effort terms are reacting to the same quantity $\overline{q}_G$, the marginal cost of the inefficient firm increases and the marginal cost of the efficient firm decreases, resulting in this term being positive. Seeing that the cost is unobserved, the regulator will make an adjustment depending on the differential with what the firm reports. This distortion decreases the quantity offered to the inefficient firm, indirectly decreases its effort, and decreases the informational rent of the efficient firm.

3.2.2 Serving the government and the market

However, for some classes of goods and services, the regulated firm already serves consumers in the market. The model of this chapter, and its contribution to the literature, is now presented. It assumes that the firm serves both the government and the private market, where the monopolist's own market information will be used to regulate it.

The private market is composed of those consumers who are not beneficiaries of the government's offer, in the case of PROUNI consists of those students who do not receive scholarships. This implies that the firm also gets a revenue in the private market, denoted by

$$P_m(q_m)q_m$$

where q_m is the number of paying students. The cost structure is assumed to be the same as for the government service. In other words

$$C(q_m, e) = (\beta - e)q_m + \psi(e) + \alpha$$

Then, the preferences of the firm if it offers the service for the regulator and the market, are given by

$$U = t + P_m(q_m)q_m - (\beta - e)(q_G + q_m) - \psi(e) - \alpha \tag{3.17}$$

The government cares about beneficiaries surplus plus the firm's profits. But transferring money from consumers to the firm has an implicit shadow cost (λ). The government's preferences are given by

$$V = S(q_G) - (1+\lambda)t + U \tag{3.18}$$

We can eliminate transfers, substituting (3.17) in (3.18)

$$V = S(q_G) - (1+\lambda)\left[-P(q_m)q_m + (\beta - e)(q_G + q_m) + \psi(e) + \alpha\right] - \lambda U \tag{3.19}$$

3.2.2.1 Complete information

In this subsection, the regulator knows the type of firm. In its relationship with the regulator, the firm must obtain at least as much utility as outside the relationship. When the firm only serves the market, it solves

$$\max_{q_m, e} \mathrm{P(q_m)q_m} - (\beta - \mathrm{e})\mathrm{q_m} - \psi(\mathrm{e}) - \alpha$$

The first order conditions are given by

$$\begin{aligned} P'(q_m^o)q_m^o + P(q_m^o) - (\beta - e^o) &= 0 \qquad (3.20) \\ q_m^o - \psi'(e^o) &= 0 \qquad (3.21) \end{aligned}$$

This gives rise to a profit Π_m^o, representing its reservation utility and profit being outside the relationship with the government.

Given the offer $\{t, q_G\}$ the firm chooses market quantity and effort

$$\max_{q_m, e} \mathrm{t} + \mathrm{P(q_m)q_m} - (\beta - \mathrm{e})(\mathrm{q_m} + \mathrm{q_G}) - \psi(\mathrm{e}) - \alpha$$

The first order conditions are

$$\begin{aligned} P'(q_m)q_m + P(q_m) - (\beta - e) &= 0 &(3.22)\\ q_G + q_m - \psi'(e) &= 0 &(3.23)\end{aligned}$$

Lemma 4. *Optimal decision of a monopolistic firm under complete information that serves the market and the government, is characterized by* (3.22) *and* (3.23)*.*

i. the firm's marginal revenue is equal to the marginal cost of the quantity offered on the market.

ii. the marginal disutility of effort, $\psi'(e)$, must be equal to marginal cost savings with the government and the market.

From equations (3.22) and (3.23) we obtain reaction curves of the firm's variables with respect to the regulator's contract: $q_m(\beta, q_G)$ and $e(\beta, q_G)$. Notice that now the amount the firm offers on the market, q_m, will depend on the amount it serves to the government q_G. Also q_m is decreasing with respect to β, and increasing with respect to q_G, because more q_G induces more effort, which reduces cost, which makes him want to produce more. On the other hand, the effort is decreasing with respect to β, i.e. higher β leads to lower q_m which induces less effort, and also increasing with respect to q_G.

So the regulator's problem is

$$\max_{t,q_G} \mathrm{S(q_G) - (1+\lambda)t + U}$$

subject to

$$U = t + P(q_m(\beta, q_G))q_m(\beta, q_G) - (\beta - e(\beta, q_G))(q_m(\beta, q_G) + q_G) - \psi(e(\beta, q_G)) - \alpha \geq \Pi_m^o$$

Notice that the optimal solution involves $U = \Pi_m^o$. So the transfer is adjusted as to make the firm indifferent between accepting or rejecting the contract. In perfect information this implies in no distortion. The first order condition is given by

$$\Longrightarrow S'(q_G) = (1+\lambda)(\beta - e(\beta, q_G)) \tag{3.24}$$

Proposition 1. *Optimal regulation under complete information of a monopoly that serves the market and the government, is characterized by* (3.24)*.*

i. an efficient level of q_G, given to equal the marginal social value of the project to the marginal cost for taxpayers, i.e., as the same rule given by (3.6)*.*

3.2.2.2 Asymmetric information with two-type case: without looking the market

Now the regulator knows that β can be $\underline{\beta}$ or $\overline{\beta}$, with probability v or $1-v$, respectively. We consider the incentive compatibility constraint of the efficient firm, for the case when the regulator does not consider in its regulatory policy the amount produced in the market, so the monopolist can choose freely q_m if it wants to lie, as can be seen in the right-hand side of the following constraint. The efficient firm may pass as inefficient, which means choosing the inefficient firm's contract. That implies

$$\max_{q_m,e} \underline{\mathrm{t}}+\mathrm{P}(\mathrm{q_m})\mathrm{q_m}-(\underline{\beta}-\mathrm{e})(\mathrm{q_m}+\underline{\mathrm{q}}_{\mathrm{G}})-\psi(\mathrm{e})=\max_{q_m,e} \bar{\mathrm{t}}+\mathrm{P}(\mathrm{q_m})\mathrm{q_m}-(\underline{\beta}-\mathrm{e})(\mathrm{q_m}+\bar{\mathrm{q}}_{\mathrm{G}})-\psi(\mathrm{e})$$

That becomes

$$\begin{aligned}&\underline{t}+P(q_m(\underline{\beta},\underline{q}_G))q_m(\underline{\beta},\underline{q}_G)-(\underline{\beta}-e(\underline{\beta},\underline{q}_G))(q_m(\underline{\beta},\underline{q}_G)+\underline{q}_G)-\psi(e(\underline{\beta},\underline{q}_G))=\\&\bar{t}+P(q_m(\underline{\beta},\bar{q}_G))q_m(\underline{\beta},\bar{q}_G)-(\underline{\beta}-e(\underline{\beta},\bar{q}_G))(q_m(\underline{\beta},\bar{q}_G)+\bar{q}_G)-\psi(e(\underline{\beta},\bar{q}_G))\end{aligned} \tag{3.25}$$

But we also have that the participation constraint of the inefficient type is binding, with a reservation utility $\overline{\Pi}^o_m$ from the outside option. So

$$\bar{t}=\overline{\Pi}^o_m-P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G)+(\overline{\beta}-e(\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G)+\bar{q}_G)+\psi(e(\overline{\beta},\bar{q}_G))+\alpha \tag{3.26}$$

This leads to

$$\begin{aligned}\underline{t}=&-P(q_m(\underline{\beta},\underline{q}_G))q_m(\underline{\beta},\underline{q}_G)+(\underline{\beta}-e(\underline{\beta},\underline{q}_G))(q_m(\underline{\beta},\underline{q}_G)+\underline{q}_G)+\psi(e(\underline{\beta},\underline{q}_G))\\&+\overline{\Pi}^o_m-P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G)+(\overline{\beta}-e(\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G)+\bar{q}_G)+\psi(e(\overline{\beta},\bar{q}_G))\\&+P(q_m(\underline{\beta},\bar{q}_G))q_m(\underline{\beta},\bar{q}_G)-(\underline{\beta}-e(\underline{\beta},\bar{q}_G))(q_m(\underline{\beta},\bar{q}_G)+\bar{q}_G)-\psi(e(\underline{\beta},\bar{q}_G))\end{aligned} \tag{3.27}$$

This incentive compatibility is long, but most of the derivatives of the reaction curves cancel out. The result is dual here. It resembles Baron and Myerson, since the distortion will appear on the quantity. There's an indirect effect over effort and over the market quantity.

Therefore, the regulator's maximization problem is to search for the optimal $\underline{q}_G$ and $\bar{q}_G$ of its expected preferences given by

$$\begin{aligned}&v\Big\{S(\underline{q}_G)-(1+\lambda)\Big[-P(q_m(\underline{\beta},\underline{q}_G))q_m(\underline{\beta},\underline{q}_G)+(\underline{\beta}-e(\underline{\beta},\underline{q}_G))(q_m(\underline{\beta},\underline{q}_G)+\underline{q}_G)+\psi(e(\underline{\beta},\underline{q}_G))+\alpha\Big]-\lambda\underline{U}\Big\}\\&+(1-v)\Big\{S(\bar{q}_G)-(1+\lambda)\Big[-P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G+(\overline{\beta}-e(\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G)+\bar{q}_G)+\psi(e(\overline{\beta},\bar{q}_G))+\alpha\Big]-\lambda\overline{\Pi}^o_m\Big\}\end{aligned}$$

with $\underline{U}$ being

$$\underline{U} = \overline{\Pi}_m^o - P(q_m(\overline{\beta},\overline{q}_G))q_m(\overline{\beta},\overline{q}_G) + (\overline{\beta} - e(\overline{\beta},\overline{q}_G))(q_m(\overline{\beta},\overline{q}_G) + \overline{q}_G) + \psi(e(\overline{\beta},\overline{q}_G))$$
$$+P(q_m(\underline{\beta},\overline{q}_G))q_m(\underline{\beta},\overline{q}_G) - (\underline{\beta} - e(\underline{\beta},\overline{q}_G))(q_m(\underline{\beta},\overline{q}_G) + \overline{q}_G) - \psi(e(\underline{\beta},\overline{q}_G)) + \alpha$$

From the first order conditions we obtain [3]

$$S'(\underline{q}_G) = (1+\lambda)(\underline{\beta} - e(\underline{\beta},\underline{q}_G)) \tag{3.28}$$

and

$$\begin{aligned} S'(\overline{q}_G) &= (1+\lambda)(\overline{\beta} - e(\overline{\beta},\overline{q}_G)) \\ &+ \frac{\lambda v}{1-v} \underbrace{\left\{(\overline{\beta} - e(\overline{\beta},\overline{q}_G)) - (\underline{\beta} - e(\underline{\beta},\overline{q}_G))\right\}}_{>0} \end{aligned} \tag{3.29}$$

Proposition 2. *Optimal regulation under asymmetric information of a monopoly serving the market and the government, the regulator without looking at the market, is characterized by* (3.28) *and* (3.29)*.*

i. an efficient level of q_G, given to equal the marginal social value of the project to the marginal cost for taxpayers, i.e., as the same rule given by (3.6)*, and a positive informational rent for type $\underline{\beta}$.*

ii. underproduction and no informational rent for type $\overline{\beta}$. The distortion is the same as (3.16)*.*

As in the case when the firm only serves the government, the allocation of the efficient firm does not present distortions with respect to the first best, if we compare (3.15) with (3.28). Comparing (3.16) with (3.29), we have the same additional term. Since the firm also serves the market and the regulator is not observing the market, she is free to choose q_m.

3.2.2.3 Asymmetric information with two-type case: looking the market

Now we consider the incentive compatibility constraint when the regulator looks at the market. In this case, the firm needs to move the market quantity to get to lie for the regulator, leading to the production of inefficient firm.

[3] For more details on the maximization problem, see Appendix B.

The efficient firm may pass as inefficient, that means choose the inefficient firm's contract. As the regulator now observes its behavior in the market, this implies that now the efficient firm cannot choose the quantity it wants on the right-hand side of the incentive compatibility constraint, having to produce the same as the inefficient one, leaving only the option to choose its effort. On the left-hand side of the constraint, we see that the firm can choose the quantity and effort it wants when choosing the contract that is offered for the efficient firm. This implies

$$\max_{q_m,e} \underline{\mathrm{t}} + \mathrm{P}(\mathrm{q_m})\mathrm{q_m} - (\underline{\beta} - \mathrm{e})(\mathrm{q_m} + \underline{\mathrm{q}}_{\mathrm{G}}) - \psi(\mathrm{e}) = \max_{e} \bar{\mathrm{t}} + \mathrm{P}(\mathrm{q_m})\mathrm{q_m} - (\underline{\beta} - \mathrm{e})(\mathrm{q_m} + \overline{\mathrm{q}}_{\mathrm{G}}) - \psi(\mathrm{e})$$

with $\overline{q}_m = q_m(\overline{\beta}, \overline{q}_G)$ in the right-hand side. That becomes

$$\begin{aligned}&\underline{t} + P(q_m(\underline{\beta}, \underline{q}_G))q_m(\underline{\beta}, \underline{q}_G) - (\underline{\beta} - e(\underline{\beta}, \underline{q}_G))(q_m(\underline{\beta}, \underline{q}_G) + \underline{q}_G) - \psi(e(\underline{\beta}, \underline{q}_G)) = \\ &\bar{t} + P(q_m(\overline{\beta}, \overline{q}_G))q_m(\overline{\beta}, \overline{q}_G) - (\underline{\beta} - e(\underline{\beta}, \overline{\beta}, \overline{q}_G))(q_m(\overline{\beta}, \overline{q}_G) + \overline{q}_G) - \psi(e(\underline{\beta}, \overline{\beta}, \overline{q}_G))\end{aligned} \tag{3.30}$$

where e on the right-hand side corresponds to the efficient firm when it is passed as inefficient, so it is also dependent on $\underline{\beta}$.

But we also have that the participation constraint of the inefficient type is binding. So

$$\bar{t} = \overline{\Pi}^o_m - P(q_m(\overline{\beta}, \overline{q}_G))q_m(\overline{\beta}, \overline{q}_G) + (\overline{\beta} - e(\overline{\beta}, \overline{q}_G))(q_m(\overline{\beta}, \overline{q}_G) + \overline{q}_G) + \psi(e(\overline{\beta}, \overline{q}_G)) + \alpha \tag{3.31}$$

This leads to

$$\begin{aligned}\underline{t} = &-P(q_m(\underline{\beta}, \underline{q}_G))q_m(\underline{\beta}, \underline{q}_G) + (\underline{\beta} - e(\underline{\beta}, \underline{q}_G))(q_m(\underline{\beta}, \underline{q}_G) + \underline{q}_G) + \psi(e(\underline{\beta}, \underline{q}_G)) \\ &+\overline{\Pi}^o_m - P(q_m(\overline{\beta}, \overline{q}_G))q_m(\overline{\beta}, \overline{q}_G) + (\overline{\beta} - e(\overline{\beta}, \overline{q}_G))(q_m(\overline{\beta}, \overline{q}_G) + \overline{q}_G) + \psi(e(\overline{\beta}, \overline{q}_G)) \\ &+P(q_m(\overline{\beta}, \overline{q}_G))q_m(\overline{\beta}, \overline{q}_G) - (\underline{\beta} - e(\underline{\beta}, \overline{\beta}, \overline{q}_G))(q_m(\overline{\beta}, \overline{q}_G) + \overline{q}_G) - \psi(e(\underline{\beta}, \overline{\beta}, \overline{q}_G))\end{aligned} \tag{3.32}$$

or

$$\begin{aligned}\underline{t} = &- P(q_m(\underline{\beta}, \underline{q}_G))q_m(\underline{\beta}, \underline{q}_G) + (\underline{\beta} - e(\underline{\beta}, \underline{q}_G))(q_m(\underline{\beta}, \underline{q}_G) + \underline{q}_G) + \psi(e(\underline{\beta}, \underline{q}_G)) \\ &+ \overline{\Pi}^o_m + \left[(\overline{\beta} - e(\overline{\beta}, \overline{q}_G)) - (\underline{\beta} - e(\underline{\beta}, \overline{\beta}, \overline{q}_G))\right] \left[q_m(\overline{\beta}, \overline{q}_G) + \overline{q}_G\right] + \psi(e(\overline{\beta}, \overline{q}_G)) - \psi(e(\underline{\beta}, \overline{\beta}, \overline{q}_G))\end{aligned}$$

The result is also dual here. It resembles Baron and Myerson, since the distortion will appear on the quantity. There's an indirect effect over effort and over the market quantity.

Therefore, the regulator's maximization problem is to search for the optimum of $\underline{q}_G$ and $\overline{q}_G$ of his expected preferences given by

$$\begin{aligned}&v\left\{S(\underline{q}_G) - (1+\lambda)\left[-P(q_m(\underline{\beta}, \underline{q}_G))q_m(\underline{\beta}, \underline{q}_G) + (\underline{\beta} - e(\underline{\beta}, \underline{q}_G))(q_m(\underline{\beta}, \underline{q}_G) + \underline{q}_G) + \psi(e(\underline{\beta}, \underline{q}_G))\right] - \lambda \underline{U}\right\} \\ +&(1-v)\left\{S(\overline{q}_G) - (1+\lambda)\left[-P(q_m(\overline{\beta}, \overline{q}_G))q_m(\overline{\beta}, \overline{q}_G) + (\overline{\beta} - e(\overline{\beta}, \overline{q}_G))(q_m(\overline{\beta}, \overline{q}_G) + \overline{q}_G) + \psi(e(\overline{\beta}, \overline{q}_G))\right] - \lambda \overline{\Pi}^o_m\right\}\end{aligned}$$

with $\underline{U}$ being

$$\underline{U} = \overline{\Pi}_m^o + \left[(\overline{\beta} - e(\overline{\beta}, \overline{q}_G)) - (\underline{\beta} - e(\underline{\beta}, \overline{\beta}, \overline{q}_G))\right]\left[q_m(\overline{\beta}, \overline{q}_G) + \overline{q}_G\right] + \psi(e(\overline{\beta}, \overline{q}_G)) - \psi(e(\underline{\beta}, \overline{\beta}, \overline{q}_G)) + \alpha$$

From the first order conditions we obtain [4]

$$S'(\underline{q}_G) = (1+\lambda)(\underline{\beta} - e(\underline{\beta}, \underline{q}_G)) \tag{3.33}$$

and

$$\begin{aligned} S'(\overline{q}_G) &= (1+\lambda)(\overline{\beta} - e(\overline{\beta}, \overline{q}_G)) \\ &+ \frac{\lambda \nu}{1-\nu} \underbrace{\left\{(\overline{\beta} - e(\overline{\beta}, \overline{q}_G)) - (\underline{\beta} - e(\underline{\beta}, \overline{\beta}, \overline{q}_G))\right\}}_{>0} \underbrace{\left(\frac{\partial q_m(\overline{\beta}, \overline{q}_G)}{\partial \overline{q}_G} + 1\right)}_{>0} \end{aligned} \tag{3.34}$$

Proposition 3. *Optimal regulation under asymmetric information of a monopoly serving the market and the government, the regulator looking at the market, is characterized by* (3.33) *and* (3.34)*.*

i. *an efficient level of q_G, given to equal the marginal social value of the project to the marginal cost for taxpayers, i.e., as the same rule given by* (3.6)*, and a positive informational rent for type $\underline{\beta}$.*

ii. *underproduction and no informational rent for type $\overline{\beta}$. The distortion is greater than* (3.16) *and* (3.29)*.*

Since the government now observes what the firm does in the market, it has a new way of limiting informational rent, given the effect the government has through the contract it offers, $\{t, q_G\}$, on the market quantity q_m. The second parenthetical term in the second line of (3.34) summarizes this. This term is positive because by increasing q_G, it induces more effort, reducing costs, and hence wanting to produce more in the market. As mentioned previously, the first parenthetical term of the second line is also positive, because effort decreases with respect to β, and both effort terms are reacting to the same quantity $\overline{q}_G$, then the marginal cost of the inefficient firm increases and the marginal cost of the efficient firm decreases, resulting in this term being positive.

Indirectly the effort will also be affected. Looking at the second point of **Lemma 4**, a decrease in the quantity produced to the government by the inefficient firm implies also a decrease in its effort, a distortion that is along the same lines as the as the Baron and Myrson traditional theory but is even larger, since it now has market information.

[4]For more details on the maximization problem, see Appendix B.

3.3 Discussion

As we can see from Lemma 2 and 3 with Propositions 1 to 3, there are similarities with respect to the basic model of Baron and Myerson and the extension developed in this chapter, where the use of the firm's own information in the market is incorporated in the regulator's analysis.

As discussed in Chapter 1, the government is paying a more expensive price than the average in the private market as a result of the PROUNI scholarships, so it is necessary to create a regulatory mechanism that helps to make tax expenditures more efficient. This is essential to ensure equity in access to higher education, transparency in the allocation of resources, efficiency in the administration of funds and the promotion of academic performance.

Since the model assumes that costs are unobserved, the mechanism that generates the incentives is implemented through the amount of scholarships that the government will offer in the contract for the inefficient firm, and will indirectly affect the firm's effort and the amount offered in the market. Prior the government began to observe the firm's operation in the market, the monopolist had the option to deviate from the contract designed for his type by being able to lie without the government having a mechanism designed to punish the deviation, gaining a higher informational rent for lying. What the government does is transferred to the firm according to the self-reported costs.

With the inclusion of the firm's own information in the market, that is, when the government starts to observe what the firm does in the market, what is achieved is that the deviation is tied up both with the regulator and in the private market. This is reflected in incentive compatibility (3.30), where in order to lie for the government it must be fulfilled that the quantity offered in the market is that of the inefficient type.

This brings a number of benefits in favor of the regulator. First, if we compare Proposition 2 with 3, in the latter case a higher distortion in the quantity of the inefficient firm is necessary, decreasing more $\overline{q}_G$, as well as the negative effect on effort and output in the market will be greater. This also has an effect on the amount of the transfer that the government offers for both types of firms. By also having the market information in the incentive compatibilities, a smaller transfer becomes necessary, decreasing the incentive for the efficient firm to reveal its type. This can be seen by comparing (3.27) with (3.32), while (3.26) with (3.31) are the same. By comparing the transfers for the efficient type, we have a tax cost saving

$$P(q_m(\underline{\beta},\overline{q}_G))q_m(\underline{\beta},\overline{q}_G) - (\underline{\beta} - e(\underline{\beta},\overline{q}_G))q_m(\underline{\beta},\overline{q}_G) - P(q_m(\underline{\beta},\underline{q}_G))q_m(\underline{\beta},\underline{q}_G)$$
$$+(\underline{\beta} - e(\underline{\beta},\overline{\beta},\overline{q}_G))(q_m(\overline{\beta},\overline{q}_G) + \overline{q}_G) + \psi(e(\underline{\beta},\overline{\beta},\overline{q}_G)) - \psi(e(\underline{\beta},\overline{q}_G))$$

That is, by incorporating the firm's own information in the market into the incentive compatibility and participation constraints, this helps to save in the transfer what the firm earns in the market. In the case of the efficient firm, it also corrects for the deviating gain.

3.4 Conclusion

The present chapter looks for many industries where a regulated firm serves the same product or service for the market and for the government. It analyzes the relationship between the market information available to the regulator and how he performs the regulatory policies. In our knowledge, a theoretical model that adds the exact and own firm information in a simultaneous moral hazard and adverse selection model was not yet studied in the literature. As well, a specific application in the private higher education market was not studied either.

Knowing that our model is quite simple, it adds important insights in comparison with the standard regulation models. First, we find an additional distortion in the output of the inefficient firm. When the government wants to mitigate the informational rent of the efficient type, as it is related to $\overline{q}_G$, the government will distort the inefficient output from the first-best allocation to control this rent. Secondly, with the added market information the government does not distort the efficient output of the first-best. Third, with this additional information the regulator will also have a saving on the tax expenses necessary to disclose the type of firm, in comparison to the case when the firm is in the private market but does not observe what it does.

Taking into account the overcharging the government is paying, when the regulator begins to look for the information available in the market to carry out his regulatory policies, specifically the monopolist's profit, what is being generated is a Bayesian update on their beliefs, and in turn a decrease in the information asymmetry that the government faces with respect to the costs of the firm, achieving greater negotiation power when dealing the trade-off between allocative output, indirectly the effort allocation and informational rent with the firm.

As we did many assumptions regarding the model's environment, many questions remain for future research. One of them will see to look situations where the monopolist can discriminate prices, because in reality there is a menu of prices and costs can be inferred through that heterogeneity. Also, when more firms are operating in the market, how to incorporate the information of other companies participating in the market, and not only the regulated firm's own information, a case that will be discussed in the following chapter.

CHAPTER 4

Regulating a Oligopoly with market information

4.1 Introduction

The existence of markets dominated by a small number of firms has become increasingly common in recent times. Their presence has been seen in industries such as automotive, electronics, telecommunications, higher education, among others. Increased international expansion, the quest to benefit from economies of scale and the growth of firms in these industries has resulted in mergers and acquisitions that result in a few firms dominating in such markets. Often, these firms adopt non-cooperative competitive strategies, which can lead to collusive behavior that negatively affects consumers and competition in general.

As can be seen, the factors contributing to the growth of these firms' market power vary. One argument suggests the relaxation of antitrust policies as responsible for this trend (Van Reenen, 2018). Other papers attribute this result to the emergence of superstar firms due to the market power of their network economies (Van Reenen & Patterson, 2017).

In this context, there are opportunities when the regulator wishes to provide a product or service to consumers who are not being able to acquire it, looking within the firms that are already offering the product in the market, but fails to find out the firms' production costs. As in the case of a monopoly, the solution is to offer incentive compatible contracts, inducing firms to disclose their costs through their chosen options. But the solution will also be inefficient, having distortions when comparing the full information case.

This chapter develops a model where a firm that already participates in an oligopolistic market is contracted by the regulator to provide the same service or product offered in the market. The idea is to use the firm's own information in the market so that the regulator can identify the type of the firm, assuming in the model the case of two types.

The study is related to the line of research that studies how to characterize a monopoly and how to regulate it. The work that begins with the characterization of an oligopoly, a model that is studied in all economics courses around the world, is Cournot's model. Augustin Cournot contributed to economic theory from the formulation of the concept of the demand function to the analysis of price determination (Vives, 1989). In his paper, Cournot (1838) was the first to construct a model of a market where a few firms control the demand supplied and the price of the products traded. His model is able to correctly estimate the conditions necessary to find an equilibrium, following the assumptions of competing in an undifferentiated product, the decisions of how much to produce are taken simultaneously, there are two firms competing in the product and there is no cooperation between them, responding rationally to each other and trying to maximize their profits.

Another line of related literature is that which analyzes different markets in a mixed oligopoly, where supplier firms can be private and public. Cremer and Maldonado (2013) study oligopolistic competition in the school education market, where schools can be both private and public, examining how quality, tuition fees and welfare are affected by the presence of public schools and by their relative position in quality range, as well as by the presence of the peer group effect, a measure that reflects the students with the highest ability of a specific group. Without this effect, efficiency is achieved with at least one public school; with the presence of the peer group effect, the effectiveness of public schools as a regulatory tool is reduced. Therefore, this result is important because in many countries with mixed markets, regulatory tools are limited to the introduction of public institutions. Rey Canteli and Estevan (2020), inspired by the Brazilian university system, where education is provided by public and private institutions, explore the effects of subsidiary public policies for the incorporation of students in private universities, with no reaction or variation in public universities. In the case of private university fees, they initially remain constant, then decrease, and finally increase steadily as the subsidy percentage increases. This chapter adds to this literature by considering a slightly different model, where two private firms are already in the oligopolistic market, and the government contracts one of them to provide the product or service.

A third line of research comes from the literature on regulatory mechanisms when the regulator faces information asymmetries. Gradstein (1994) designs a regulatory mechanism in an oligopoly with imperfect cost information and shows that the informational rent that should be offered to oligopolistic firms can be zero balanced, i.e., marginal cost pricing can always be implemented independent of the definition of social welfare. But since such rents cannot be individually split for any of the regulated firms, it is difficult to implement a policy that forces oligopolistic firms to participate. Saglam (2017) proposes a solution to the above participation problem by equating the aggregate fixed cost of the oligopolistic industry to the transfer proposed by Gradstein (1994), ensuring that the regulation mechanism satisfies the industry participation constraint. Evrenk and Zenginobuz (2010) design a regulation mechanism where firms conduct a revenue competition among firms, inducing firms to expand their output above the level they would produce without being regulated. The mechanism is self-financing, because the losing firms pay the winners, without having to charge taxes on consumers, in addition to not having to incur higher costs by saving the shadow price of tax-financed policies. In several circumstances, the mechanism manages to increase social welfare by improving the efficiency of the imperfectly competitive industry.

Finally, there is the literature that justifies the regulation of oligopolies because of their effect on the inequitable distribution of income. Kumar and Stauvermann (2020) study the negative effect of oligopolies on income distribution, as well as the consequences of encouraging more competitive markets. For this, using an overlapping generations growth model, lobbies and political corruptions are the most important reasons for losing competition in markets. Moreover, it leads to a more unequal intragenerational distribution and a redistribution of income from the older to the younger generation, the impact on economic growth is generally ambiguous, and depends specifically on the cost of lobbying.

4.2 The Model

There is a service (project) for which consumers have a positive surplus $S(q)$, where q is the quantity offered. Now, we will assume that there are two firms in the market that can provide the service. They are a oligopolistic market that compete in q.

As in the monopoly problem, to keep the problem mathematically tractable, we shall assume that each firm's cost is linear in q_i of the form:

$$C_i(q_i, e_i) = (\beta_i - e_i)q_i + \psi(e_i) + \alpha \tag{4.1}$$

where β_i is a private information cost parameter of firm i, e_i is the effort the firm i places on cost reduction and α is the fixed cost equal for both. β_i can be either high or low, $\beta \in \{\underline{\beta}, \overline{\beta}\}$, representing the efficient and inefficient firm, respectively. Effort reduces the firm's utility. The disutility from effort is denoted by $\psi(e_i)$. This disutility grows with effort $\psi'(e_i) > 0$ for $e_i > 0$, at an increasing rate $\psi(e_i)'' > 0$, and satisfies $\psi(0) = 0$, $\lim_{e \to \beta} \psi(e_i) = +\infty$. To find the solutions, we will assume the following function

$$\psi(e_i) = \gamma e_i^2 \tag{4.2}$$

with $\gamma > 0$. The government can make transfers t to compensate the firm. There's a shadow price of governmental funds, denoted by λ. It means that raising costs is costly for the society.

The chapter will follow Cournot model, where the regulator contracts a quantity (q_G) and gives a transfer (t). We will now assume that firm 1 serves the government. The timing is such that regulator offers before the firm's choice.

In order to find the optimal quantities for each firm, as well as the quantity that the government will require, we will assume the following inverse demand functions for the scholarship holders and for the private market.

$$P(q_G) = d - fq_G \tag{4.3}$$

$$P(q_1 + q_2) = a - b(q_1 + q_2) \tag{4.4}$$

where a, b, d and f are positive constants. The demand of scholarship students represents a small market, but they still have a willingness to pay for higher education.

4.2.1 Cournot Model

Before regulating one of the firms serving the government and the private market, we will introduce the basic Cournot model with two firms presented by Tirole (1988). Using the general reduced form for the profit function $\Pi_i(q_1, q_2)$ for $i = \{1, 2\}$:

$$\Pi_i(q_1, q_2) = P(q_1 + q_2)q_i - C_i(q_i) \tag{4.5}$$

Each firm maximizes its profit given the quantity chosen by the other firm. Assuming that the profit function is strictly concave in q_i, the first order condition of the maximization problem is

$$\frac{\partial \Pi_i(q_1, q_2)}{\partial q_i} = \frac{\partial P(q_1 + q_2)}{\partial q_i} q_i + P(q_1 + q_2) - \frac{\partial C_i(q_i)}{\partial q_i} = 0 \tag{4.6}$$

The last two terms represent the profit of an extra unit of q_i, while the first term shows the negative effect on the price $P(q_1 + q_2)$ of an extra unit, which affects the q_i units already produced. This implies that the firm considers the adverse effect on price when it changes its own output, rather than the effect on market output.

Then, at the optimum, each firm will equal the marginal benefit of an extra unit of q_i with the negative price effect of selling one more unit.

$$P(q_1^* + q_2^*) - \frac{\partial C_i(q_i^*)}{\partial q_i^*} = -\frac{\partial P(q_1^* + q_2^*)}{\partial q_i^*} q_i^* \tag{4.7}$$

Therefore, to find the optimum, we have the following lemma

Lemma 5. *Nash equilibrium will exist if* $\Pi_1(q_1^*, q_2^*) \geq \Pi_1(q_1, q_2^*)$ *for all* q_1, *and* $\Pi_2(q_1^*, q_2^*) \geq \Pi_2(q_1^*, q_2)$ *for all* q_2.

Given the inverse demand functions (4.3) and (4.4), together with the disutility of effort (4.2), the optimal quantities of each firm are

$$q_1^* = \frac{2\gamma}{(4b\gamma-1)^2-4b^2\gamma^2}[a(2b\gamma-1)+2b\beta_2\gamma-(4b\gamma-1)\beta_1] \tag{4.8}$$

and

$$q_2^* = \frac{2\gamma}{(4b\gamma-1)^2-4b^2\gamma^2}[a(2b\gamma-1)+2b\beta_1\gamma-(4b\gamma-1)\beta_2] \tag{4.9}$$

4.2.1.1 Complete information

It is assumed that firm 1 will provide the service for the government. In its relationship with the regulator, firm 1 must obtain at least its reservation utility, defined exogenously by Π_1^o.

Given the contract defined by $\{t, q_G\}$, where t is the transfer and q_G the quantity required by the government, and if it only serves the government, firm 1 chooses effort

$$\max_{e_1} \mathrm{t} - (\beta_1 - \mathrm{e}_1)\mathrm{q_G} - \psi(\mathrm{e}_1) - \alpha$$

The first order condition is

$$q_G = \psi'(e_1) \tag{4.10}$$

result described by Lemma 1.

From equation (4.10) we obtain the firm 1's reaction curve of the effort with respect to the regulator's contract: $e_1(\beta_1, q_G)$. Remembering that the effort is decreasing with respect to β_1, and also increasing with respect to q_G, then the regulator's problem that he faces is

$$\max_{q_G} \mathrm{S(q_G)} - (1+\lambda)\mathrm{t} + \mathrm{U}_1$$

subject to

$$U_1 = t - (\beta_1 - e(\beta_1, q_G))q_G - \psi(e_1(\beta_1, q_G)) - \alpha \geq \Pi_1^o$$

This program becomes

$$\max_{q_G} \mathrm{S(q_G)} - (1+\lambda)\left[(\beta_1 - \mathrm{e}(\beta_1, \mathrm{q_G}))\mathrm{q_G} + \psi(\mathrm{e}(\beta_1, \mathrm{q_G})) + \alpha\right] - \lambda \mathrm{U}_1$$

Notice that the optimal solution involves $U_1 = \Pi_1^o$. So the transfer is adjusted as to make the firm 1 indifferent between accepting or rejecting the contract. In complete information this implies in no distortion. The first order condition is given by [1]

[1] All the maximization problems of this chapter are in Appendix C.

$$\Longrightarrow S'(q_G) = (1+\lambda)(\beta_1 - e_1(\beta_1, q_G)) \tag{4.11}$$

Lemma 6. *Optimal regulation of an oligopolistic firm under complete information and the structure of the Cournot model, is characterized by (4.11).*

i. the marginal social value of the project is equal to the marginal cost for taxpayers

Since $P(q_G) = S'(q_G)$, in addition to (4.10) and (4.11), the optimal quantity demanded by the government is

$$q_G = \frac{2\gamma}{2d\gamma - (1+\lambda)}[d - (1+\lambda)\beta_1] \tag{4.12}$$

As there is no interaction in the private market, and firm 1 is only serving the government, there is no effect on firm 2 through q_G. But, since $q_1 = 0$, it implies that the firm 2 will have the entire private market. Then, from the reaction curve of firm 2, (C.4), the optimal quantity of firm 2 will be

$$q_2^* = \frac{2\gamma}{4b\gamma - 1}(a - \beta_2) \tag{4.13}$$

4.2.1.2 Asymmetric information with two-type case

This section assumes that the regulator has some prior beliefs about the type of firm 1. It knows that β_1 can be $\underline{\beta}$ or $\overline{\beta}$, with probability v or $1-v$, respectively. Now we consider the incentive compatibility constraint. The efficient firm may pass as inefficient, that means choose the inefficient firm's contract. That means

$$\max_{e_1} \underline{\mathrm{t}} - (\underline{\beta} - \mathrm{e}_1)\underline{\mathrm{q}}_{\mathrm{G}} - \psi(\mathrm{e}_1) = \max_{e_1} \bar{\mathrm{t}} - (\underline{\beta} - \mathrm{e}_1)\bar{\mathrm{q}}_{\mathrm{G}} - \psi(\mathrm{e}_1)$$

Following the same timing where the firm chooses after government offer, considering the firm 1's reservation utility, as well as serving the regulator, we obtain

$$\underline{t} - (\underline{\beta} - e_1(\underline{\beta}, \underline{q}_G))\underline{q}_G - \psi(e_1(\underline{\beta}, \underline{q}_G)) = \bar{t} - (\underline{\beta} - e_1(\underline{\beta}, \bar{q}_G))\bar{q}_G - \psi(e_1(\underline{\beta}, \bar{q}_G)) \tag{4.14}$$

But we also have that the participation constraint of the inefficient type is binding. So

$$\bar{t} = \overline{\Pi}_1^o + (\overline{\beta} - e_1(\overline{\beta}, \bar{q}_G))\bar{q}_G + \psi(e_1(\overline{\beta}, \bar{q}_G)) + \alpha \tag{4.15}$$

As in the previous chapter, these restrictions are active because the regulator's preferences are to reduce firms' utility. So it decreases them to the point of making them indifferent to participate in the contracts.

Combining (4.14) with (4.15), this leads to the informational rent the efficient type

$$\underline{U}_1 = \overline{\Pi}_1^o + \overline{q}_G \left[(\overline{\beta} - e_1(\overline{\beta}, \overline{q}_G)) - (\underline{\beta} - e_1(\underline{\beta}, \overline{q}_G)) \right] + \psi(e_1(\overline{\beta}, \overline{q}_G)) - \psi(e_1(\underline{\beta}, \overline{q}_G)) + \alpha \quad (4.16)$$

The utility of the regulator becomes when the firm is efficient

$$S(\underline{q}_G) - (1+\lambda) \left[(\underline{\beta} - e_1(\underline{\beta}, \underline{q}_G))\underline{q}_G + \psi(e_1(\underline{\beta}, \underline{q}_G)) + \alpha \right] - \lambda \underline{U}_1$$

with $\underline{U}_1$ from (4.16). And when the principal is low type becomes

$$S(\overline{q}_G) - (1+\lambda) \left[(\overline{\beta} - e_1(\overline{\beta}, \overline{q}_G))\overline{q}_G + \psi(e_1(\overline{\beta}, \overline{q}_G)) + \alpha \right] - \lambda \overline{\Pi}_1^o$$

From the first order conditions we obtain

$$S'(\underline{q}_G) = (1+\lambda)(\underline{\beta} - e_1(\underline{\beta}, \underline{q}_G)) \quad (4.17)$$

and

$$\begin{aligned} S'(\overline{q}_G) &= (1+\lambda)(\overline{\beta} - e_1(\overline{\beta}, \overline{q}_G)) \\ &\quad + \frac{\lambda v}{1-v} \underbrace{\left\{ (\overline{\beta} - e_1(\overline{\beta}, \overline{q}_G)) - (\underline{\beta} - e_1(\underline{\beta}, \overline{q}_G)) \right\}}_{>0} \end{aligned} \quad (4.18)$$

Lemma 7. *Optimal regulation of an oligopolistic firm under asymmetric information and the structure of the Cournot model, is characterized by* (4.17) *and* (4.18)*.*

i. an efficient level of q_G, given to equal the marginal social value of the project to the marginal cost for taxpayers, and a positive informational rent for type $\underline{\beta}$.

ii. underproduction and no informational rent for type $\overline{\beta}$.

As in the monopoly case, it can be seen from equation (4.17), the optimum for the efficient firm is not distorted. But the contract for the inefficient firm, looking at equation (4.18), there is the same additional positive terms that distort the allocation of the quantity that the government will offer to the inefficient firm, comparing with (3.16). Since the cost is unobserved, the regulator will make an adjustment depending on the differential with what the firm reports. The distortion decreases the quantity offered to the inefficient firm, indirectly decreases its effort, and decreases the informational rent of the efficient firm. The difference with respect to the previous chapter is given by the reservation utility, where in the monopoly it is higher, requiring a larger transfer compared to the case of an oligopolistic firm.

To find the optimum offered by the government for each type, we follow (4.17) and (4.18). For (4.17), since it has no distortions, it follows (4.12).

$$\underline{q}_G = \frac{2\gamma}{2d\gamma - (1+\lambda)}\left[d - (1+\lambda)\underline{\beta}\right] \tag{4.19}$$

On the other hand, for the inefficient firm, we have

$$\overline{q}_G = \frac{2\gamma}{2d\gamma - (1+\lambda)}\left[d - (1+\lambda)\overline{\beta} - \frac{\lambda v}{1-v}(\overline{\beta} - \underline{\beta})\right] \tag{4.20}$$

having a smaller quantity than the solution of the first-best, and for firm 2 the same solution given by (4.13).

4.2.2 Serving the government and the market

The case of PROUNI is represented by another reality, where the same firm receives both paying and scholarship students, in a market where there are more than one higher education institution in the market. Now our model, it is assumed that firm 1 serves both the government and the private market, and the regulatory mechanism will occupy the firm's own information in the market.

The demand for higher education is composed of those students who are not beneficiaries of the government's offer, in the case of PROUNI consists of those students who do not receive scholarships. This implies that the firm 1 also gets a revenue in the private market, denoted by

$$P(q_1 + q_2)q_1$$

where q_1 is the number of paying students of firm 1, and the price is given by (4.4). Since there is competition between two firms, the price depends on the quantity they both produce, q_1 and q_2. The cost structure is assumed to be the same as for the government service. That is

$$C_1(q_1, e_1) = (\beta_1 - e_1)q_1 + \psi(e_1) + \alpha$$

Then the preferences of the firm 1, if it offers the service for the regulator and the market, are given by

$$U_1 = t + P_1(q_1 + q_2)q_1 - (\beta_1 - e_1)(q_G + q_1) - \psi(e_1) - \alpha \tag{4.21}$$

The government cares about beneficiaries surplus plus the firm's profits. But transferring money from consumers to the firm 1 has an implicit shadow cost (λ). The government's preferences are given by

$$V = S(q_G) - (1+\lambda)t + U_1 \tag{4.22}$$

We can eliminate transfers, substituting (4.21) in (4.22)

$$V = S(q_G) - (1+\lambda)\left[-P(q_1+q_2)q_1 + (\beta_1 - e_1)(q_G+q_1) + \psi(e_1) + \alpha\right] - \lambda U_1 \tag{4.23}$$

4.2.2.1 Complete information

The regulator knows the type of firm. The outside option, i.e., the reservation utility of firm 1 comes from the profit that it earns only by serving the private market. When the firm 1 only serves the market, it solves

$$\max_{q_1,e_1} \mathrm{P(q_1+q_2)q_1 - (\beta_1 - e_1)q_1 - \psi(e_1) - \alpha}$$

The first order conditions are given by

$$\begin{aligned} P'(q_1^o+q_1^o)q_1^o + P(q_1^o+q_1^o) - (\beta_1 - e_1^o) &= 0 \qquad (4.24)\\ q_1^o - \psi'(e_1^o) &= 0 \qquad (4.25)\end{aligned}$$

This gives rise to a profit $\Pi^o_{1,C}$, q_1^o the quantity offered by firm 1 on the market and it respects the **Lemma** 5. Now, firm 1 serves the market and the government. Given the contract defined by $\{t, q_G\}$, firm 1 chooses market quantity and effort

$$\max_{q_1,e_1} \mathrm{t + P(q_1+q_2)q_1 - (\beta_1 - e_1)(q_1+q_G) - \psi(e_1) - \alpha}$$

The first order conditions are

$$\begin{aligned} P'(q_1+q_2)q_1 + P(q_1+q_2) - (\beta_1 - e_1) &= 0 \qquad (4.26)\\ q_G + q_1 - \psi'(e_1) &= 0 \qquad (4.27)\end{aligned}$$

These results are described in a similar way in Lemma 4, but in this case when firm 1 in a duopoly is regulated. From equations (4.26) and (4.27) we obtain firm 1 reaction curves of the firm's variables with respect to the regulator's contract $q_1(\beta_1, q_G, q_2)$ and $e_1(\beta_1, q_G, q_2)$. As can be seen, the reaction curves also depend on the quantity that firm 2 produces in the market, given by q_2. So the regulator's problem that he faces is

$$\max_{t,q_G} \mathrm{S(q_G) - (1+\lambda)t + U_1}$$

subject to

$$U_1 = t + P(q_1(\beta_1,q_G,q_2)+q_2)q_1(\beta_1,q_G,q_2) - (\beta_1 - e_1(\beta_1,q_G,q_2))(q_1(\beta_1,q_G,q_2)+q_G) \\ -\psi(e_1(\beta_1,q_G,q_2)) - \alpha \geq \Pi^o_{1,C}$$

The first order condition with respect to q_G is given by

$$\implies S'(q_G) = (1+\lambda)(\beta_1 - e_1(\beta_1,q_G,q_2)) \tag{4.28}$$

Proposition 4. *Optimal regulation under complete information of an oligopolistic firm (firm 1) that serves the market and the government, is characterized by* (4.28)*.*

i. the marginal social value of the project is equal to the marginal cost for taxpayers.

*ii. having q_G, we can determine the quantity offered in the market by firm 1, $q_1(\beta_1,q_G,q_2)$, fulfilling **Lemma** 5.*

Because firm 1 now interacts with the private market and the government, the quantity that the government asks for in the contract will affect the decisions of firm 2. For this, using (4.26) and (4.27), plus the inverse demands (4.3) and (4.4), it is obtained that the quantity that firm 1 will offer in the market is

$$\hat{q}_1 = \eta q_1^* + \mu \tag{4.29}$$

with q_1^* from the result obtained in (C.3)[2]. If conditions (C.33) and (C.34) are met, $\hat{q}_1$ will be greater than in the case where the firm only caters to the private market, q_1^*. This implies that it would affect firm 2, resulting in a lower quantity offered, q_2^*.

4.2.2.2 Asymmetric information with two-type case: without looking the market

This section assumes that the regulator knows that β_1 can be $\underline{\beta}$ or $\overline{\beta}$, with probability v or $1-v$, respectively. As in the previous chapter, it is shown the case when the regulator does not observe what firm 1 is doing in the market. The incentive compatibility constraint of firm 1 shows this when the firm can choose the quantity to be produced in the market at the time of selecting the contract with the government. The efficient firm may pass as inefficient, that means choose the inefficient firm's contract. That means

$$\max_{q_1,e_1} \underline{\mathrm{t}} + \mathrm{P}(\mathrm{q}_1+\mathrm{q}_2)\mathrm{q}_1 - (\underline{\beta} - \mathrm{e}_1)(\mathrm{q}_1 + \underline{\mathrm{q}}_\mathrm{G}) - \psi(\mathrm{e}_1) = \max_{q_1,e_1} \bar{\mathrm{t}} + \mathrm{P}(\mathrm{q}_1+\mathrm{q}_2)\mathrm{q}_1 - (\underline{\beta} - \mathrm{e}_1)(\mathrm{q}_1 + \bar{\mathrm{q}}_\mathrm{G}) - \psi(\mathrm{e}_1)$$

[2]Appendix C details η and μ in the expressions (C.31) and (C.32).

Following the same timing as in the previous section, considering the decision out of the relationship with the government, as well as serving the regulator, we obtain

$$\begin{aligned}&\underline{t}+P(q_1(\underline{\beta},\underline{q}_G,q_2)+q_2)q_1(\underline{\beta},\underline{q}_G,q_2)-(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2))(q_1(\underline{\beta},\underline{q}_G,q_2)+\underline{q}_G)-\psi(e_1(\underline{\beta},\underline{q}_G,q_2))=\\&\bar{t}+P(q_1(\underline{\beta},\overline{q}_G,q_2)+q_2)q_1(\underline{\beta},\overline{q}_G,q_2)-(\underline{\beta}-e_1(\underline{\beta},\overline{q}_G,q_2))(q_1(\underline{\beta},\overline{q}_G,q_2)+\overline{q}_G)-\psi(e_1(\underline{\beta},\overline{q}_G,q_2))\end{aligned} \tag{4.30}$$

But we also have that the participation constraint of the inefficient type is binding. So

$$\bar{t}=\overline{\Pi}^o_{1,C}-P(q_1(\overline{\beta},\overline{q}_G,q_2)+q_2)q_1(\overline{\beta},\overline{q}_G,q_2)+(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))(q_1(\overline{\beta},\overline{q}_G,q_2)+\overline{q}_G)+\psi(e_1(\overline{\beta},\overline{q}_G,q_2))+\alpha \tag{4.31}$$

Combining (4.30) with (4.31), this leads to

$$\begin{aligned}\underline{U}_1=\overline{\Pi}^o_{1,C}&-P(q_1(\overline{\beta},\overline{q}_G,q_2)+q_2)q_1(\overline{\beta},\overline{q}_G,q_2)+(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))(q_1(\overline{\beta},\overline{q}_G,q_2)+\overline{q}_G)+\psi(e_1(\overline{\beta},\overline{q}_G,q_2))\\&+P(q_1(\underline{\beta},\overline{q}_G,q_2)+q_2)q_1(\underline{\beta},\overline{q}_G,q_2)-(\underline{\beta}-e_1(\underline{\beta},\overline{q}_G,q_2))(q_1(\underline{\beta},\overline{q}_G,q_2)+\overline{q}_G)-\psi(e_1(\underline{\beta},\overline{q}_G,q_2))+\alpha\end{aligned} \tag{4.32}$$

Then, the regulator's maximization problem is to search for the optimal $\underline{q}_G$ and $\overline{q}_G$ of its expected preferences given by

$$\begin{aligned}&v\left\{S(\underline{q}_G)-(1+\lambda)\left[-P(q_1(\underline{\beta},\underline{q}_G,q_2)+q_2)q_1(\underline{\beta},\underline{q}_G,q_2)+(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2))(q_1(\underline{\beta},\underline{q}_G,q_2)+\underline{q}_G)+\psi(e_1(\underline{\beta},\underline{q}_G,q_2))+\alpha\right]-\lambda\underline{U}\right\}\\&+(1-v)\left\{S(\overline{q}_G)-(1+\lambda)\left[-P(q_1(\overline{\beta},\overline{q}_G,q_2)+q_2)q_1(\overline{\beta},\overline{q}_G,q_2)+(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))(q_1(\overline{\beta},\overline{q}_G,q_2)+\overline{q}_G)+\psi(e_1(\overline{\beta},\overline{q}_G,q_2))+\alpha\right]-\lambda\overline{\Pi}^o_{1,C}\right\}\end{aligned}$$

with $\underline{U}_1$ from (4.32)

From the first order conditions we obtain

$$S'(\underline{q}_G)=(1+\lambda)(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2)) \tag{4.33}$$

and

$$\begin{aligned}S'(\overline{q}_G)=&(1+\lambda)(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))\\&+\frac{\lambda v}{1-v}\underbrace{\left\{(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))-(\underline{\beta}-e_1(\underline{\beta},\overline{q}_G,q_2))\right\}}_{>0}\end{aligned} \tag{4.34}$$

Proposition 5. *Optimal regulation under asymmetric information of an oligopolistic firm (firm 1) that serves the market and the government, the regulator without looking at the market, is characterized by* (4.33) *and* (4.34)*.*

i. an efficient level of q_G, given to equal the marginal social value of the project to the marginal cost for taxpayers, and a positive informational rent for type $\underline{\beta}$.

ii. *underproduction and no informational rent for type $\overline{\beta}$. The distortion is the same as (4.18).*

iii. *having q_G, we can determine the quantity offered in the market by firm 1, $q_1(\beta_1, q_G, q_2)$, fulfilling **Lemma** 5.*

The optimum for the efficient firm is not distorted, given by (4.33). But the contract for the inefficient firm, looking at equation (4.34), the same distortions as our monopoly model are present. Additionally, the Nash equilibrium conditions in a 2-firm oligopoly must be respected, following **Lemma** 5.

As the efficient firm has no distortion in its contract, the quantity it will offer in the market respects (C.30), i.e.

$$\hat{\underline{q}}_1 = \eta \underline{q}_1^* + \mu \tag{4.35}$$

But firm 1 when it is inefficient, given the distortion, she will offer the following quantity in the private market[3]

$$\hat{\overline{q}}_1 = \eta \overline{q}_1^* + \mu - \sigma(\overline{\beta} - \underline{\beta}) \tag{4.36}$$

As the quantity that firm 1 offers in the market decreases, firm 2 will increase its quantity in the private market with respect to the case of complete information.

4.2.2.3 Asymmetric information with two-type case: looking the market

Assuming the government has the same beliefs about β, now firm 1 needs to signal correctly in order to lie for the regulator, because it is now auditing what it does in the marketplace. The incentive compatibility constraint of firm 1 shows this when now it cannot choose the quantity to be produced in the market at the time of selecting the contract with the government. The efficient firm may pass as inefficient, that means choose the inefficient firm's contract. Then

$$\max_{q_1, e_1} \underline{\mathrm{t}} + \mathrm{P}(\mathrm{q}_1 + \mathrm{q}_2)\mathrm{q}_1 - (\underline{\beta} - \mathrm{e}_1)(\mathrm{q}_1 + \underline{\mathrm{q}}_\mathrm{G}) - \psi(\mathrm{e}_1) = \max_{e_1} \overline{\mathrm{t}} + \mathrm{P}(\mathrm{q}_1 + \mathrm{q}_2)\mathrm{q}_1 - (\underline{\beta} - \mathrm{e}_1)(\mathrm{q}_1 + \overline{\mathrm{q}}_\mathrm{G}) - \psi(\mathrm{e}_1)$$

with $q_1 = q_1(\overline{\beta}, \overline{q}_G, q_2)$ in the right-hand side. That becomes

$$\begin{aligned}
&\underline{t} + P(q_1(\underline{\beta}, \underline{q}_G, q_2) + q_2)q_1(\underline{\beta}, \underline{q}_G, q_2) - (\underline{\beta} - e_1(\underline{\beta}, \underline{q}_G, q_2))(q_1(\underline{\beta}, \underline{q}_G, q_2) + \underline{q}_G) - \psi(e_1(\underline{\beta}, \underline{q}_G, q_2)) = \\
&\overline{t} + P(q_1(\overline{\beta}, \overline{q}_G, q_2) + q_2)q_1(\overline{\beta}, \overline{q}_G, q_2) - (\underline{\beta} - e_1(\underline{\beta}, \overline{\beta}, \overline{q}_G, q_2))(q_1(\overline{\beta}, \overline{q}_G, q_2) + \overline{q}_G) - \psi(e_1(\underline{\beta}, \overline{\beta}, \overline{q}_G, q_2))
\end{aligned} \tag{4.37}$$

But we also have that the participation constraint of the inefficient type is binding. So

$$\overline{t} = \overline{\Pi}^o_{1,C} - P(q_1(\overline{\beta}, \overline{q}_G, q_2) + q_2)q_1(\overline{\beta}, \overline{q}_G, q_2) + (\overline{\beta} - e_1(\overline{\beta}, \overline{q}_G, q_2))(q_1(\overline{\beta}, \overline{q}_G, q_2) + \overline{q}_G) + \psi(e_1(\overline{\beta}, \overline{q}_G, q_2)) + \alpha \tag{4.38}$$

[3] Appendix C details σ in the expresssion (C.42).

Combining (4.37) with (4.38), this leads to

$$\underline{U}_1 = \overline{\Pi}^o_{1,C} - P(q_1(\overline{\beta},\overline{q}_G,q_2)+q_2)q_1(\overline{\beta},\overline{q}_G,q_2) + (\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))(q_1(\overline{\beta},\overline{q}_G,q_2)+\overline{q}_G) + \psi(e_1(\overline{\beta},\overline{q}_G,q_2))$$
$$+P(q_1(\overline{\beta},\overline{q}_G,q_2)+q_2)q_1(\overline{\beta},\overline{q}_G,q_2) - (\underline{\beta}-e_1(\underline{\beta},\overline{\beta},\overline{q}_G,q_2))(q_1(\overline{\beta},\overline{q}_G,q_2)+\overline{q}_G) - \psi(e_1(\underline{\beta},\overline{\beta},\overline{q}_G,q_2)) + \alpha \tag{4.39}$$

Then, the regulator's maximization problem is to search for the optimal $\underline{q}_G$ and $\overline{q}_G$ of its expected preferences given by

$$v\left\{S(\underline{q}_G) - (1+\lambda)\left[-P(q_1(\underline{\beta},\underline{q}_G,q_2)+q_2)q_1(\underline{\beta},\underline{q}_G,q_2) + (\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2))(q_1(\underline{\beta},\underline{q}_G,q_2)+\underline{q}_G) + \psi(e_1(\underline{\beta},\underline{q}_G,q_2)) + \alpha\right] - \lambda\underline{U}\right\}$$
$$+(1-v)\left\{S(\overline{q}_G) - (1+\lambda)\left[-P(q_1(\overline{\beta},\overline{q}_G,q_2)+q_2)q_1(\overline{\beta},\overline{q}_G,q_2) + (\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))(q_1(\overline{\beta},\overline{q}_G,q_2)+\overline{q}_G) + \psi(e_1(\overline{\beta},\overline{q}_G,q_2)) + \alpha\right] - \lambda\overline{\Pi}^o_{1,C}\right\}$$

being with $\underline{U}_1$ from (4.39), after removing the terms that are cancelled out and ordering

$$\begin{aligned}\underline{U}_1 =& \overline{\Pi}^o_{1,C} + \left[(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2)) - (\underline{\beta}-e_1(\underline{\beta},\overline{\beta},\overline{q}_G,q_2))\right]\left[q_1(\overline{\beta},\overline{q}_G,q_2)+\overline{q}_G\right] \\ &+ \psi(e_1(\overline{\beta},\overline{q}_G,q_2)) - \psi(e_1(\underline{\beta},\overline{\beta},\overline{q}_G,q_2)) + \alpha\end{aligned} \tag{4.40}$$

From the first order conditions we obtain

$$S'(\underline{q}_G) = (1+\lambda)(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2)) \tag{4.41}$$

and

$$\begin{aligned}S'(\overline{q}_G) =& (1+\lambda)(\overline{\beta}-e(\overline{\beta},\overline{q}_G,q_2)) \\ &+ \frac{\lambda v}{1-v}\underbrace{\left\{(\overline{\beta}-e(\overline{\beta},\overline{q}_G,q_2)) - (\underline{\beta}-e(\underline{\beta},\overline{\beta},\overline{q}_G,q_2))\right\}}_{>0}\left(\frac{\partial q_1(\overline{\beta},\overline{q}_G,q_2)}{\partial \overline{q}_G}+1\right)\end{aligned} \tag{4.42}$$

Proposition 6. *Optimal regulation under asymmetric information of an oligopolistic firm (firm 1) that serves the market and the government, the regulator looking at the market, is characterized by* (4.41) *and* (4.42)*.*

i. an efficient level of q_G, given to equal the marginal social value of the project to the marginal cost for taxpayers, and a positive informational rent for type $\underline{\beta}$.

ii. underproduction and no informational rent for type $\overline{\beta}$. The distortion is greater than (4.34)*.*

*iii. having q_G, we can determine the quantity offered in the market by firm 1, $q_1(\beta_1, q_G, q_2)$, fulfilling **Lemma** 5.*

The optimum for the efficient firm is not distorted, given by (4.41). But the contract for the inefficient firm, looking at equation (4.42), the same distortions as our monopoly model are present. Additionally, the Nash equilibrium conditions in a 2-firm oligopoly must be respected, following **Lemma** 5.

Then, when firm 1 is efficient, what it offers in the market is the same as in the previous case.

$$\underline{\hat{q}}_1 = \eta \underline{q}_1^* + \mu \tag{4.43}$$

while when it is inefficient it will offer

$$\hat{\overline{q}}_1 = \eta \overline{q}_1^* + \mu - \sigma \frac{2f\gamma}{1+\lambda}(\overline{\beta} - \underline{\beta}) \tag{4.44}$$

i.e., by having a greater decrease in the quantity offered in the private market, firm 2 will have a greater market share than in the previous case, when the government doesn't look the private market.

4.3 Discussion

The market structure developed in this chapter is more realistic than that of a monopoly. It is more common to find situations where a small number of firms compete with each other rather than a single firm controlling the entire market. With more than one firm, there is some competition among them. This results in having incentives to reduce prices, improve quality or innovate to differentiate their product in order to gain a larger market share. This generates a greater variety of options and benefits for consumers compared to a monopoly, by having a lower price to pay and a greater quantity of goods in the market.

Given the above, the incentive compatibility and participation restrictions already start from a lower requirement for the regulator, since the reserve utility that firms have is lower than in the case of a monopoly, making it less expensive to generate incentives for the regulator. This implies in having Π_m^o of Chapter 2, higher than the profit of an oligopolistic firm given by $\Pi_{1,C}^o$. In addition, $q_1 + q_2$ will be greater than the monopoly quantity, given by q_m, and the price to the consumer in this chapter, $P(q_1 + q_2)$, will be less than the price paid for a monopoly $P(q_m)$.

Looking at Lemma 6 and 7 with Propositions 4 to 6, and comparing them with those of the previous chapter, it can be seen that they are very similar, where the optimal regulation in each situation comes from demanding an efficient level in the number of scholarships from the market for the firm with lower costs, while for the inefficient firm a lower level than the first-best is demanded in its contract. This comes from the fact of limiting the informational rent of the efficient firm, which depends on the inefficient quantity.

The condition of having a higher distortion in the inefficient firm is maintained by looking at the information of the firm in the market with the Baron and Myerson's model. As the regulator gets to look at the firm's quantity, if the firm deviates from the contract it has chosen, the optimal regulation ties the distortion to what the firm declares. When comparing Proposition 5 with Proposition 6, i.e. when the regulator does not look for the market and when it does, the distortion necessary to reveal the firm's type will be higher. As in the case of monopoly, this will imply in having a higher negative spillover effect on the effort and quantity offered in the market.

All this, assuming also that the Nash equilibrium of Lemma 3 is respected, where both firms in the market act simultaneously with respect to each other, having no other choice that maximizes their profit. This is reflected in firm 1, the one with which the regulator obtains the PROUNI scholarships, that all the reaction functions depend on the type of the firm, given by β, by the amount demanded by the government q_G, and by the number of paying students that firm 2 has in the private market, q_2.

Finally, added to the lower amount of the transfer that the government must pay as a result of using the firm's own information in the market, a result that was also noted in the previous chapter, the breadth of the contracts is lower than in the case when the private market is not observed. This is because when looking at the reservation utility when the regulator does not look for the market as compared to when it does, i.e. (4.32) vs (4.40), the reserve utility in the former case is higher in

$$P(q_1(\underline{\beta},\overline{q}_G,q_2)+q_2)q_1(\underline{\beta},\overline{q}_G,q_2)-P(q_1(\overline{\beta},\overline{q}_G,q_2)+q_2)q_1(\overline{\beta},\overline{q}_G,q_2)$$

This implies that the regulator has to offer a higher transfer in the first case in order to respect the incentive compatibility constraint (4.30).

4.4 Conclusion

This chapter extends the model by assuming an oligopolistic market, which better represents the situation of the higher education market in most countries.

Using a model with two firms acting in the market, we find interesting results. First, comparing the results when the firm only serves the government, including the market leads to higher distortions. Since cost is not observed, the distortions come from decreasing the contract quantity for the inefficient firm, thus decreasing the informational rent of the efficient firm. But when comparing the situation with the case when the government does not look at the market, by looking at how the firm acts in the market and forcing it to act according to the contract it chooses, it is implicitly tying it down. This allows for less distortion than when the regulator only considers self-reported information. In addition, it allows for tax savings.

The implementation of regulations on increased transfers paid by a government is based on the need to balance the economic and social impact of such transfers. While these measures can have positive effects by reducing poverty, stimulating consumption and promoting equality, it is also crucial to consider their long-term sustainability. Regulation becomes an essential tool to ensure that these transfers are distributed fairly and efficiently, avoiding the risk of market distortions and fiscal imbalances. Ultimately, regulation seeks to optimize the positive impact of transfers on society and ensure that they contribute sustainably to overall welfare without compromising economic stability and intergenerational equity.

The model is very simple, so there are unresolved questions. First, differentiating by type of consumer, when the regulator pays for full and partial scholarships. Another unresolved question is how to use de information of firm 2. Although indirectly the reaction functions are conditional on q_2, one could see how the analysis changes when observing the behavior of another firm in the market, as does Shleifer (1985).

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APPENDIX A

Monopoly regulation under Laffont and Tirole Model

There is a service (project) for which consumers have a positive surplus $S(q)$, where q is the quantity offered. There is a firm that can provide this service at a cost $C(q) = (\beta - e)q + \alpha$, where β is a private information cost parameter, e is the effort the firm places on cost reduction and α is the fixed cost. β can be either high or low ($\beta \in \{\underline{\beta}, \overline{\beta}\}$). Effort reduces the firm's utility. The disutility from effort is denoted by $\psi(e)$. This disutility function have the same properties as in the Chapter 3 and Chapter 4, i.e., increasing at increasing rates. The government can make transfers t to compensate the firm. There's a shadow price of governmental funds, denoted by λ. It means that raising costs is costly for the society.

Laffont and Tirole (1993) view the total cost as observable, so the government compensates the firm for the production and also gives an additional transfer to compensate for effort. The firm's preferences are given by

$$U_G(q,e,t) = t - (\beta - e)q - \psi(e) - \alpha \tag{A.1}$$

The government cares about consumer surplus plus the firm's profits. But transferring money from consumers to the firm has an implicit shadow cost. The government's preferences are given by

$$V(q,e,t) = S(q) - (1+\lambda)t + U_G \tag{A.2}$$

We can eliminate transfers

$$V(q,e) = S(q) - (1+\lambda)\left[(\beta - e)q + \psi(e) + \alpha\right] - \lambda U_G \tag{A.3}$$

A.0.1 Complete information

This section gives the first-best of the regulatory policy, that will be one of our benchmarks. In its relationship with the regulator the firm must obtain at least as much utility as its reservation utility, given exogenously by Π^o.

The regulator searches for the optimal effort and quantity, this is obtained finding the first order conditions for q and e in A.3. The two conditions are

$$q = \psi'(e) \tag{A.4}$$

and

$$S'(q) = (1+\lambda)(\beta - e) \tag{A.5}$$

Lemma 8. *Optimal regulation of a monopoly under complete information and the structure of the Laffont and Tirole model, is characterized by* A.4 *and* A.5.

i. the benefit of effort equals its marginal disutility.

ii. the marginal social value of the project is equal to the marginal cost for taxpayers.

iii. The regulator leaves no rents, $U_G = 0$.

A.0.2 Asymmetric information with two-type case

The regulator does not know the firm's type, but has its prior beliefs. Let v be the probability of $\beta = \underline{\beta}$. The inefficient firm gets zero rents, because her participation constraint is binding.

$$\overline{U}_G = \bar{t} - \overline{cq} - \psi(\overline{\beta} - \bar{c}) - \alpha = 0 \tag{A.6}$$

For the efficient type the incentive compatibility constraint is binding, so

$$\underline{t} - \underline{cq} - \psi(\underline{\beta} - \underline{c}) = \bar{t} - \overline{cq} - \psi(\underline{\beta} - \bar{c}) \tag{A.7}$$

Substituting A.6 into A.7, we get

$$\underline{U}_G = \overline{U}_G + \psi(\overline{\beta} - \bar{c}) - \psi(\underline{\beta} - \bar{c}) = \psi(\bar{e}) - \psi(\bar{e} - \Delta\beta) = \Phi(\bar{e}) + \alpha \tag{A.8}$$

i.e., the informational rent that the efficient firm obtains for lying and taking the contract from the inefficient firm.

The optimal contract involves

$$\begin{aligned}
S'(\underline{q}) &= (1+\lambda)(\underline{\beta} - \underline{e}) && \text{(A.9)}\\
\psi'(\underline{e}) &= \underline{q} && \text{(A.10)}\\
S'(\bar{q}) &= (1+\lambda)(\overline{\beta} - \bar{e}) && \text{(A.11)}\\
\psi'(\bar{e}) &= \bar{q} - \frac{\lambda}{1+\lambda}\frac{v}{1-v}\Phi'(\bar{e}) && \text{(A.12)}
\end{aligned}$$

Lemma 9. *Optimal regulation of a monopoly under asymmetric information and the structure of the Laffont and Tirole model, is characterized by* A.9 *to* A.12.

i. an efficient level of production and effort, and a positive informational rent for type $\underline{\beta}$.

ii. undereffort and efficient level of production for type $\overline{\beta}$.

iii. The regulator leaves no rents for type $\overline{\beta}$*, i.e.,* $\overline{U}_G = 0$.

As can be seen, in order to decrease the informational rent of the efficient firm, the regulator must offer a contract with a sub-optimal effort for the inefficient type, increasing the cost. So, providing incentives is costly and the full cost reimbursement is not efficient.

A.1 Serving the government and the market

Now the firm serves the market and the government. The value of the service to the government is $S(q)$ as in the Laffont and Tirole model. As this takes interpretation of a revenue, we have $S'(q) = P(q)$. The government still compensates the firm's costs.

The firm has a production cost given by

$$C = (\beta - e)Q - \alpha = (\beta - e)(q_m + q_G) - \alpha \tag{A.13}$$

which is the same for the government and the market and a disutility of effort $\psi(e)$, q_G is the quantity offered to the government and q_m on the market. But the firm algo gets a revenue in the private market, denoted by

$$P_m(q_m)q_m \tag{A.14}$$

So the firm's utility is given by

$$U = t + P_m(q_m)q_m - (\beta - e)(q_m + q_G) - \psi(e) - \alpha = U_G + U_m \tag{A.15}$$

The regulator cares about the firm's utility, but has a marginal cost for funds. So its preferences are given by

$$V = S(q_G) - (1+\lambda)t + U \tag{A.16}$$

Replacing t, we get

$$V = S(q_G) - (1+\lambda)\left[-P_m(q_m)q_m + (\beta - e)(q_m + q_G) + \psi(e) + \alpha\right] - \lambda[U_G + U_m] \tag{A.17}$$

A.1.1 Complete information

At this stage, if the solution were to be similar to Laffont and Tirole model, the results would be similar, in perfect or private information. That is, if the optimal transfer set $U_G = 0$ then the only difference would stem from the fact that effort would be greater since the firm would be serving two markets, as observed from first order conditions

$$S'(q_G) = (1+\lambda)(\beta - e) \tag{A.18}$$
$$\psi'(e) = q_G + q_m \tag{A.19}$$

So the transfer are set to be

$$t = (\beta - e)q_G - \psi(e) - \alpha \tag{A.20}$$

So now we have to work with

$$U_m = P_m(q_m)q_m - (\beta - e)q_m - \alpha \tag{A.21}$$

And the regulator's utility would be given by

$$V = S(q_G) - (1+\lambda)\left[(\beta - e)q_G + \psi(e) + \alpha\right] + P_m(q_m)q_m - (\beta - e)q_m \tag{A.22}$$

Now, the solution to this optimization problem is

$$S'(q_G) = (1+\lambda)(\beta - e) \tag{A.23}$$
$$\psi'(e) = q_G + \frac{q_m}{1+\lambda} \tag{A.24}$$

Proposition 7. *Optimal regulation under complete information of a monopoly that serves the market and the government, is characterized by* A.22 *and* A.24*.*

i. an efficient level of production.

ii. a greater effort than the Laffont and Tirole model A.5*, by serving both the market and the government..*

iii. The regulator leaves no rents, $U_G = 0$.

A.1.2 Asymmetric information

The regulator does not know the firm's type, but has its prior beliefs. Let v be the probability of $\beta = \underline{\beta}$. Now we take the binding incentive compatibility constraint for the efficient firm and the binding participation constraint for the high type. The incentive constraint is given by

$$\underline{t}-\psi(\underline{\beta}-\underline{c})+P_m(\underline{q}_m)\underline{q}_m-\underline{c}(\underline{q}_m+\underline{q}_G)\geq \bar{t}-\psi(\underline{\beta}-\bar{c})+P_m(\bar{q}_m)\bar{q}_m-\bar{c}(\bar{q}_m+\bar{q}_G) \quad \text{(A.25)}$$

with either

$$\overline{U}_G=0 \implies \bar{t}=\overline{cq}_G+\psi(\overline{\beta}-\bar{c})+\alpha \quad \text{(A.26)}$$

or

$$\overline{U}^r=\overline{U}_G+\overline{U}_m=\overline{U}_m \implies \bar{t}=\overline{U}^r-P_m(\bar{q}_m)\bar{q}_m+\bar{c}(\bar{q}_m+\bar{q}_G)+\psi(\overline{\beta}-\bar{c})+\alpha \quad \text{(A.27)}$$

In case (A.26) we get

$$\underline{U}=\overline{cq}_G+\psi(\overline{\beta}-\bar{c})-\overline{cq}_G-\psi(\underline{\beta}-\bar{c})+P_m(\bar{q}_m)\bar{q}_m-\overline{cq}_m-\alpha=\Phi(\bar{e})+\overline{U}_m \quad \text{(A.28)}$$

In case (A.27) we get

$$\underline{U}=\overline{U}^r+\overline{cq}_G+\psi(\overline{\beta}-\bar{c})-\overline{U}_m-\overline{cq}_G-\psi(\underline{\beta}-\bar{c})+\overline{U}_m-\alpha=\Phi(\bar{e})+\overline{U}^r \quad \text{(A.29)}$$

But $\overline{U}_m=\overline{U}^r$ because the reserve profit will be the same as it would be if the market alone. Then both restrictions are the same. Now we replace this transfer in the welfare function.

This second case we are setting the transfer such that $\overline{U}=\overline{U}^r$. So everything is the same as in the complete information case, except that there's now a new information rent term $\Phi(\bar{e})+\overline{U}^r$. So, if the regulator extracts the profits from the market, there is now new effects in the model. The effort choices are similar to the case where there's no market operation.

The utility of the regulator becomes when the firm is efficient

$$v\left\{S(\underline{q}_G)-(1+\lambda)\left[(\underline{\beta}-\underline{e})\underline{q}_G+\psi(\underline{e})-P_m(\underline{q}_m)\underline{q}_m+(\underline{\beta}-\underline{e})\underline{q}_m+\alpha\right]-\lambda\left[\Phi(\bar{e})+\overline{U}_m\right]\right\}$$

and when the principal is low type becomes

$$(1-v)\left\{S(\bar{q}_G)-(1+\lambda)\left[(\overline{\beta}-\bar{e})\bar{q}_G+\psi(\bar{e})+\alpha\right]+\overline{U}_m\right\}$$

The first order conditions are

$$\begin{aligned}
S'(\underline{q}_G) &= (1+\lambda)(\underline{\beta}-\underline{e}) && \text{(A.30)}\\
S'(\overline{q}_G) &= (1+\lambda)(\overline{\beta}-\overline{e}) && \text{(A.31)}\\
\psi'(\underline{e}) &= \underline{q}_G+\underline{q}_m && \text{(A.32)}\\
\psi'(\overline{e}) &= \overline{q}_G+\frac{\overline{q}_m}{1+\lambda}-\frac{\nu}{1-\nu}\frac{\lambda}{1+\lambda}\left(\Phi'(\overline{e})+\overline{q}_m\right) && \text{(A.33)}
\end{aligned}$$

Proposition 8. *Optimal regulation under asymmetric information of a monopoly serving the market and the government, is characterized by* A.30 *to* A.30.

i. an efficient level of production and a greater effort, with a positive informational rent for type $\underline{\beta}$.

ii. a lower undereffort than Laffont and Tirole modelo A.12 *and efficient level of production for type $\overline{\beta}$, by serving both the market and the government.*

iii. The regulator leaves no rents for type $\overline{\beta}$, i.e., $\overline{U}_G = 0$.

In the last condition comes the effect of observing the market. Since the regulator knows that the informational rent of the efficient firm also depends on the profit of the inefficient firm in the market, it conditions the inefficient effort on the quantity supplied that it observes in the market, offering a contract with an even lower effort than that of the Laffont and Tirole model.

APPENDIX B

Maximization problems - Regulating a Monopolist with market information

B.1 Baron and Myerson model

In its relationship with the regulator the firm must obtain at least its reservation utility, defined exogenously by Π^o. Under complete information, the following maximization problem is the one that results in **Lemma 2**.

Firm's decision

Given the offer $\{t, q_G\}$ the firm chooses effort

$$\max_{e} \mathrm{t} - (\beta - \mathrm{e})\mathrm{q_G} - \psi(\mathrm{e}) - \alpha$$

The first order condition is

$$q_G = \psi'(e) \tag{B.1}$$

From this first order conditions we obtain the reaction curve of the firm's variable with respect to the regulator's contract: $e(\beta, q_G)$. So the regulator's problem that he faces is

$$\max_{q_G} \mathrm{S(q_G)} - (1+\lambda)\mathrm{t} + \mathrm{U_G}$$

subject to

$$U_G = t - (\beta - e(\beta, q_G))q_G - \psi(e(\beta, q_G)) - \alpha \geq \Pi^o$$

This program becomes

$$V = S(q_G) - (1+\lambda)\left[U_G + (\beta - e(\beta, q_G))q_G + \psi(e(\beta, q_G)) + \alpha\right] + U_G$$

or

$$\max_{q_G} \mathrm{S(q_G)} - (1+\lambda)\left[(\beta - \mathrm{e}(\beta, \mathrm{q_G}))\mathrm{q_G} + \psi(\mathrm{e}(\beta, \mathrm{q_G})) + \alpha\right] - \lambda \mathrm{U_G}$$

Notice that the optimal solution involves $U_G = \Pi^o$.
The first order condition is given by

$$S'(q_G) - (1+\lambda)\left\{ -\frac{\partial e(\beta, q_G)}{\partial q_G} \underbrace{\left[q_G - \frac{\partial \psi(e(\beta, q_G))}{\partial e(\beta, q_G)} \right]}_{\text{(0 by (B.1))}} + (\beta - e(\beta, q_G)) \right\} = 0$$

$$\Longrightarrow S'(q_G) = (1+\lambda)(\beta - e(\beta, q_G))$$

This is the result of **Lemma 2**.

Asymmetric information with two-type case

The following problem shows the result of **Lemma 3**. Now we consider the incentive compatibility constraint. The efficient firm may pass as inefficient, that means choose the inefficient firm's contract. That means

$$\max_e \underline{\mathrm{t}} - (\underline{\beta} - \mathrm{e})\underline{\mathrm{q}}_{\mathrm{G}} - \psi(\mathrm{e}) = \max_e \bar{\mathrm{t}} - (\underline{\beta} - \mathrm{e})\bar{\mathrm{q}}_{\mathrm{G}} - \psi(\mathrm{e})$$

That becomes

$$\underline{t} - (\underline{\beta} - e(\underline{\beta}, \underline{q}_G))\underline{q}_G - \psi(e(\underline{\beta}, \underline{q}_G)) = \bar{t} - (\underline{\beta} - e(\underline{\beta}, \bar{q}_G))\bar{q}_G - \psi(e(\underline{\beta}, \bar{q}_G))$$

But we also have that the participation constraint of the inefficient type is binding, with a reservation utility $\overline{\Pi}^o$. So

$$\bar{t} = \overline{\Pi}^o + (\overline{\beta} - e(\overline{\beta}, \bar{q}_G))\bar{q}_G + \psi(e(\overline{\beta}, \bar{q}_G)) + \alpha$$

This leads to

$$\underline{U_G} = \overline{\Pi}^o + \overline{q_G}\left[(\overline{\beta} - e(\overline{\beta}, \bar{q}_G)) - (\underline{\beta} - e(\underline{\beta}, \bar{q}_G)) \right] + \psi(e(\overline{\beta}, \bar{q}_G)) - \psi(e(\underline{\beta}, \bar{q}_G)) + \alpha \quad \text{(B.2)}$$

The utility of the regulator becomes when the firm is efficient

$$v\left\{ S(\underline{q}_G) - (1+\lambda)\left[(\underline{\beta} - e(\underline{\beta}, \underline{q}_G))\underline{q}_G + \psi(e(\underline{\beta}, \underline{q}_G)) + \alpha \right] - \lambda \underline{U} \right\}$$

with $\underline{U}$ from (B.2). And when the principal is low type becomes

$$(1-v)\left\{S(\overline{q}_G)-(1+\lambda)\left[(\overline{\beta}-e(\overline{\beta},\overline{q}_G))\overline{q}_G+\psi(e(\overline{\beta},\overline{q}_G))+\alpha\right]-\lambda\overline{\Pi}^o\right\}$$

From the first order conditions with respect to $\underline{q}_G$, we obtain

$$S'(\underline{q}_G)-(1+\lambda)\left\{-\frac{\partial e(\underline{\beta},\underline{q}_G)}{\partial \underline{q}_s}\underbrace{\left[\underline{q}_G-\frac{\partial\psi(e(\underline{\beta},\underline{q}_G))}{\partial e(\underline{\beta},\underline{q}_G)}\right]}_{\text{(0 by (B.1))}}+(\underline{\beta}-e(\underline{\beta},\underline{q}_G))\right\}=0$$

$$\Longrightarrow S'(\underline{q}_G)=(1+\lambda)(\underline{\beta}-e(\underline{\beta},\underline{q}_G)) \tag{B.3}$$

From the first order conditions with respect to $\overline{q}_G$, we obtain

$$\begin{aligned}
&- \quad \lambda v\left\{\left[(\overline{\beta}-e(\overline{\beta},\overline{q}_G))-(\underline{\beta}-e(\underline{\beta},\overline{q}_G))\right]\right\}\\
&- \quad \lambda v\left\{-\frac{\partial e(\overline{\beta},\overline{q}_G)}{\partial\overline{q}_G}\underbrace{\left[\overline{q}_G-\frac{\partial\psi(e(\overline{\beta},\overline{q}_G))}{\partial e(\overline{\beta},\overline{q}_G)}\right]}_{\text{(0 by (B.1))}}+\frac{\partial e(\underline{\beta},\overline{q}_G)}{\partial\overline{q}_G}\underbrace{\left[\overline{q}_G-\frac{\partial\psi(e(\underline{\beta},\overline{q}_G))}{\partial e(\underline{\beta},\overline{q}_G)}\right]}_{\text{(0 by (B.1))}}\right\}\\
&+ \quad (1-v)\left\{S'(\overline{q}_G)-(1+\lambda)\left\{-\frac{\partial e(\overline{\beta},\overline{q}_G)}{\partial\overline{q}_G}\underbrace{\left[\overline{q}_G-\frac{\partial\psi(e(\overline{\beta},\overline{q}_G))}{\partial e(\overline{\beta},\overline{q}_G)}\right]}_{\text{(0 by (B.1))}}+(\overline{\beta}-e(\overline{\beta},\overline{q}_G))\right\}\right\}=0
\end{aligned}$$

Rearranging the terms, we arrive at the equation 3.16, in Chapter 3.

$$\begin{aligned}
\Longrightarrow S'(\overline{q}_G)&=(1+\lambda)(\overline{\beta}-e(\overline{\beta},\overline{q}_G))\\
&\quad+\frac{\lambda v}{1-v}\underbrace{\left\{(\overline{\beta}-e(\overline{\beta},\overline{q}_G))-(\underline{\beta}-e(\underline{\beta},\overline{q}_G))\right\}}_{>0}
\end{aligned} \tag{B.4}$$

These are the results of **Lemma 3**.

B.2 Serving the government and the market

Now our model, where the firm serves both the government and the private market. The following problems will result in our contributions to the literature, obtaining **Proposition 1**, **Proposition 2** y **Proposition 3**.

Outside option

When the firm only serves the market, it solves

$$\max_{q_m,e} \mathrm{P(q_m)q_m - (\beta - e)q_m - \psi(e) - \alpha}$$

The first order conditions are given by

$$\begin{aligned} P'(q_m^o)q_m^o + P(q_m^o) - (\beta - e^o) &= 0 && \text{(B.5)} \\ q_m^o &= \psi'(e^o) && \text{(B.6)} \end{aligned}$$

This gives rise to a profit Π_m^o.

Firm's decision

Given the offer $\{t, q_G\}$ the firm chooses effort and market quantities

$$\max_{q_m,e} \mathrm{t + P(q_m)q_m - (\beta - e)(q_m + q_G) - \psi(e) - \alpha}$$

The first order conditions are

$$\begin{aligned} P'(q_m)q_m + P(q_m) - (\beta - e) &= 0 && \text{(B.7)} \\ q_G + q_m - \psi'(e) &= 0 && \text{(B.8)} \end{aligned}$$

These last two equations result in **Lemma 4**. From these first order conditions we obtain reaction curves of the firm's variables with respect to the regulator's contract $q_m(\beta, q_G)$ and $e(\beta, q_G)$. So the regulator's problem is that he faces is

$$\max_{t,q_G} \mathrm{S(q_G) - (1+\lambda)t + U}$$

subject to

$$U = t + P(q_m(\beta,q_G))q_m(\beta,q_G) - (\beta - e(\beta,q_G))(q_m(\beta,q_G)+q_G) - \psi(e(\beta,q_G)) - \alpha \geq \Pi_m^o$$

This program becomes

$$V = S(q_G) - (1+\lambda)\left[U - P(q_m(\beta,q_G))q_m(\beta,q_G) + (\beta - e(\beta,q_G))(q_m(\beta,q_G)+q_G) + \psi(e(\beta,q_G)) + \alpha\right] + U$$

or

$$\max_{q_G} \mathrm{S}(\mathrm{q_G}) - (1+\lambda)\left[-\mathrm{P}(\mathrm{q_m}(\beta,\mathrm{q_G}))\mathrm{q_m}(\beta,\mathrm{q_G}) + (\beta - \mathrm{e}(\beta,\mathrm{q_G}))(\mathrm{q_m}(\beta,\mathrm{q_G})+\mathrm{q_G}) + \psi(\mathrm{e}(\beta,\mathrm{q_G})) + \alpha\right] - \lambda U$$

Notice that the optimal solution involves $U = \Pi_m^o$. So the transfer is adjusted as to make the firm indifferent between accepting or rejecting the contract. In complete information this implies in no distortion. The first order condition is given by

$$S'(q_G) - (1+\lambda)\left\{ -\frac{\partial q_m}{\partial q_G}\underbrace{\left[\frac{\partial P(q_m)}{\partial q_m}q_m + P(q_m) - (\beta - e)\right]}_{(0 \text{ by } (B.7))} - \frac{\partial e}{\partial q_G}\underbrace{\left[q_m + q_G - \frac{\partial \psi(e)}{\partial e}\right]}_{(0 \text{ by } (B.8))} + (\beta - e) \right\} = 0$$

$$\Longrightarrow S'(q_G) = (1+\lambda)(\beta - e(\beta,q_G)) \tag{B.9}$$

This is the result of **Proposition 1**.

Asymmetric information with two-type case: without looking the market

Now we consider the incentive compatibility constraint. The efficient firm may pass as inefficient, that means choose the inefficient firm's contract. That means

$$\max_{q_m,e} \underline{\mathrm{t}} + \mathrm{P}(\mathrm{q_m})\mathrm{q_m} - (\underline{\beta} - \mathrm{e})(\mathrm{q_m} + \underline{\mathrm{q}}_{\mathrm{G}}) - \psi(\mathrm{e}) = \max_{q_m,e} \bar{\mathrm{t}} + \mathrm{P}(\mathrm{q_m})\mathrm{q_m} - (\underline{\beta} - \mathrm{e})(\mathrm{q_m} + \bar{\mathrm{q}}_{\mathrm{G}}) - \psi(\mathrm{e})$$

That becomes

$$\begin{aligned} \underline{t} &+ P(q_m(\underline{\beta},\underline{q}_G))q_m(\underline{\beta},\underline{q}_G) - (\underline{\beta} - e(\underline{\beta},\underline{q}_G))(q_m(\underline{\beta},\underline{q}_G) + \underline{q}_G) - \psi(e(\underline{\beta},\underline{q}_G)) = \\ \bar{t} &+ P(q_m(\underline{\beta},\bar{q}_G))q_m(\underline{\beta},\bar{q}_G) - (\underline{\beta} - e(\underline{\beta},\bar{q}_G))(q_m(\underline{\beta},\bar{q}_G) + \bar{q}_G) - \psi(e(\underline{\beta},\bar{q}_G)) \end{aligned}$$

But we also have that the participation constraint of the inefficient type is binding. So

$$\bar{t} = \overline{\Pi}^o_m - P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G) + (\overline{\beta} - e(\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G) + \bar{q}_G) + \psi(e(\overline{\beta},\bar{q}_G)) + \alpha$$

This leads to

$$\begin{aligned} \underline{t} = &-P(q_m(\underline{\beta},\underline{q}_G))q_m(\underline{\beta},\underline{q}_G) + (\underline{\beta} - e(\underline{\beta},\underline{q}_G))(q_m(\underline{\beta},\underline{q}_G) + \underline{q}_G) + \psi(e(\underline{\beta},\underline{q}_G)) \\ &+\overline{\Pi}^o_m - P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G) + (\overline{\beta} - e(\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G) + \bar{q}_G) + \psi(e(\overline{\beta},\bar{q}_G)) \\ &+P(q_m(\underline{\beta},\bar{q}_G))q_m(\underline{\beta},\bar{q}_G) - (\underline{\beta} - e(\underline{\beta},\bar{q}_G))(q_m(\underline{\beta},\bar{q}_G) + \bar{q}_G) - \psi(e(\underline{\beta},\bar{q}_G)) \end{aligned}$$

The utility of the regulator becomes when the firm is efficient

$$S(\underline{q}_G) - (1+\lambda)\left[-P(q_m(\underline{\beta},\underline{q}_G))q_m(\underline{\beta},\underline{q}_G) + (\underline{\beta} - e(\underline{\beta},\underline{q}_G))(q_m(\underline{\beta},\underline{q}_G) + \underline{q}_G) + \psi(e(\underline{\beta},\underline{q}_G)) + \alpha\right] - \lambda\underline{U}$$

with $\underline{U}$ being

$$\begin{aligned} \underline{U} = &\overline{\Pi}^o_m - P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G) + (\overline{\beta} - e(\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G) + \bar{q}_G) + \psi(e(\overline{\beta},\bar{q}_G)) \\ &+P(q_m(\underline{\beta},\bar{q}_G))q_m(\underline{\beta},\bar{q}_G) - (\underline{\beta} - e(\underline{\beta},\bar{q}_G))(q_m(\underline{\beta},\bar{q}_G) + \bar{q}_G) - \psi(e(\underline{\beta},\bar{q}_G)) + \alpha \end{aligned}$$

and when the principal is low type becomes

$$S(\bar{q}_G) - (1+\lambda)\left[-P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G) + (\overline{\beta} - e(\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G) + \bar{q}_G) + \psi(e(\overline{\beta},\bar{q}_G)) + \alpha\right] - \lambda\overline{\Pi}^o_m$$

From the first order condition with respect to $\underline{q}_G$, we obtain

$$\begin{aligned} S'(\underline{q}_G) - (1+\lambda)&\left\{-\frac{\partial \underline{q}_m(\underline{\beta},\underline{q}_G)}{\partial \underline{q}_G}\underbrace{\left[\frac{\partial P(\underline{q}_m(\underline{\beta},\underline{q}_G))}{\partial \underline{q}_m(\underline{\beta},\underline{q}_G)}\underline{q}_m(\underline{\beta},\underline{q}_G) + P(\underline{q}_m(\underline{\beta},\underline{q}_G)) - (\underline{\beta} - e(\underline{\beta},\underline{q}_G))\right]}_{\text{(0 by (B.7))}}\right\} \\ -(1+\lambda)&\left\{-\frac{\partial e(\underline{\beta},\underline{q}_G)}{\partial \underline{q}_G}\underbrace{\left[\underline{q}_m(\underline{\beta},\underline{q}_G) + \underline{q}_G - \frac{\partial \psi(e(\underline{\beta},\underline{q}_G))}{\partial e(\underline{\beta},\underline{q}_G)}\right]}_{\text{(0 by (B.8))}} + (\underline{\beta} - e(\underline{\beta},\underline{q}_G))\right\} = 0 \end{aligned}$$

$$\Longrightarrow S'(\underline{q}_G) = (1+\lambda)(\underline{\beta} - e(\underline{\beta},\underline{q}_G)) \qquad \text{(B.10)}$$

From the first order conditions with respect to $\overline{q}_G$, we obtain

$$
\begin{aligned}
&-\lambda v\left\{\left[(\overline{\beta}-e(\overline{\beta},\overline{q}_G))-(\underline{\beta}-e(\underline{\beta},\overline{q}_G))\right]\right\}\\
&-\lambda v\left\{-\frac{\partial q_m(\overline{\beta},\overline{q}_G)}{\partial \overline{q}_G}\underbrace{\left[\frac{\partial P(q_m(\overline{\beta},\overline{q}_G))}{\partial q_m(\overline{\beta},\overline{q}_G)}q_m(\overline{\beta},\overline{q}_G)+P(q_m(\overline{\beta},\overline{q}_G))-(\overline{\beta}-e(\overline{\beta},\overline{q}_G))\right]}_{(0\text{ by (B.7)})}\right\}\\
&-\lambda v\left\{-\frac{\partial e(\overline{\beta},\overline{q}_G)}{\partial \overline{q}_G}\underbrace{\left[q_m(\overline{\beta},\overline{q}_G)+\overline{q}_G-\frac{\partial \psi(e(\overline{\beta},\overline{q}_G))}{\partial e(\overline{\beta},\overline{q}_G)}\right]}_{(0\text{ by (B.8)})}\right\}\\
&-\lambda v\left\{\frac{\partial q_m(\underline{\beta},\overline{q}_G)}{\partial \overline{q}_G}\underbrace{\left[\frac{\partial P(q_m(\underline{\beta},\overline{q}_G))}{\partial q_m(\underline{\beta},\overline{q}_G)}q_m(\underline{\beta},\overline{q}_G)+P(q_m(\underline{\beta},\overline{q}_G))-(\underline{\beta}-e(\underline{\beta},\overline{q}_G))\right]}_{(0\text{ by (B.7)})}\right\}\\
&-\lambda v\left\{\frac{\partial e(\underline{\beta},\overline{q}_G)}{\partial \overline{q}_G}\underbrace{\left[q_m(\underline{\beta},\overline{q}_G)+\overline{q}_G-\frac{\partial \psi(e(\underline{\beta},\overline{q}_G))}{\partial e(\underline{\beta},\overline{q}_G)}\right]}_{(0\text{ by (B.8)})}\right\}\\
&+(1-v)\left\{S'(\overline{q}_G)-(1+\lambda)\left\{-\frac{\partial q_m(\overline{\beta},\overline{q}_G)}{\partial \overline{q}_G}\underbrace{\left[\frac{\partial P(q_m(\overline{\beta},\overline{q}_G))}{\partial q_m(\overline{\beta},\overline{q}_G)}q_m(\overline{\beta},\overline{q}_G)+P(q_m(\overline{\beta},\overline{q}_G))-(\overline{\beta}-e(\overline{\beta},\overline{q}_G))\right]}_{(0\text{ by (B.7)})}\right\}\right\}\\
&-(1-v)(1+\lambda)\left\{-\frac{\partial e(\overline{\beta},\overline{q}_G)}{\partial \overline{q}_G}\underbrace{\left[q_m(\overline{\beta},\overline{q}_G)+\overline{q}_G-\frac{\partial \psi(e(\overline{\beta},\overline{q}_G))}{\partial e(\overline{\beta},\overline{q}_G)}\right]}_{(0\text{ by (B.8)})}+(\overline{\beta}-e(\overline{\beta},\overline{q}_G))\right\}=0
\end{aligned}
$$

Rearranging the terms, we arrive at the equation (3.29), in Chapter 3.

$$
\begin{aligned}
\Longrightarrow S'(\overline{q}_G)&=(1+\lambda)(\overline{\beta}-e(\overline{\beta},\overline{q}_G))\\
&+\frac{\lambda v}{1-v}\underbrace{\left\{(\overline{\beta}-e(\overline{\beta},\overline{q}_G))-(\underline{\beta}-e(\underline{\beta},\overline{q}_G))\right\}}_{>0}
\end{aligned}
\tag{B.11}
$$

These are the results of **Proposition 2**.

Asymmetric information with two-type case: looking the market

Now we consider the incentive compatibility constraint when the regulator looks at the market. In this case, the firm needs to move the market quantity to get to lie for the regulator, leading to the production of inefficient firm.. That means

$$\max_{q_m,e} \underline{t}+P(q_m)q_m-(\underline{\beta}-e)(q_m+\underline{q}_G)-\psi(e)=\max_{e} \bar{t}+P(q_m)q_m-(\underline{\beta}-e)(q_m+\bar{q}_G)-\psi(e)$$

with $q_m=q_m(\overline{\beta},\bar{q}_G)$ in the right-hand side. That becomes

$$\begin{aligned}\underline{t} &+ P(q_m(\underline{\beta},\underline{q}_G))q_m(\underline{\beta},\underline{q}_G)-(\underline{\beta}-e(\underline{\beta},\underline{q}_G))(q_m(\underline{\beta},\underline{q}_G)+\underline{q}_G)-\psi(e(\underline{\beta},\underline{q}_G))=\\ \bar{t} &+ P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G)-(\underline{\beta}-e(\underline{\beta},\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G)+\bar{q}_G)-\psi(e(\underline{\beta},\overline{\beta},\bar{q}_G))\end{aligned}$$

But we also have that the participation constraint of the inefficient type is binding. So

$$\bar{t}=\overline{\Pi}_m^o-P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G)+(\overline{\beta}-e(\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G)+\bar{q}_G)+\psi(e(\overline{\beta},\bar{q}_G))+\alpha$$

This leads to

$$\begin{aligned}\underline{t}=&-P(q_m(\underline{\beta},\underline{q}_G))q_m(\underline{\beta},\underline{q}_G)+(\underline{\beta}-e(\underline{\beta},\underline{q}_G))(q_m(\underline{\beta},\underline{q}_G)+\underline{q}_G)+\psi(e(\underline{\beta},\underline{q}_G))\\ &+\overline{\Pi}_m^o-P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G)+(\overline{\beta}-e(\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G)+\bar{q}_G)+\psi(e(\overline{\beta},\bar{q}_G))\\ &+P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G)-(\underline{\beta}-e(\underline{\beta},\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G)+\bar{q}_G)-\psi(e(\underline{\beta},\overline{\beta},\bar{q}_G))\end{aligned}$$

The utility of the regulator becomes when the firm is efficient

$$S(\underline{q}_G)-(1+\lambda)\left[-P(q_m(\underline{\beta},\underline{q}_G))q_m(\underline{\beta},\underline{q}_G)+(\underline{\beta}-e(\underline{\beta},\underline{q}_G))(q_m(\underline{\beta},\underline{q}_G)+\underline{q}_G)+\psi(e(\underline{\beta},\underline{q}_G))+\alpha\right]-\lambda\underline{U}$$

with $\underline{U}$ being

$$\underline{U}=\overline{\Pi}_m^o+\left[(\overline{\beta}-e(\overline{\beta},\bar{q}_G))-(\underline{\beta}-e(\underline{\beta},\overline{\beta},\bar{q}_G))\right]\left[q_m(\overline{\beta},\bar{q}_G)+\bar{q}_G\right]+\psi(e(\overline{\beta},\bar{q}_G))-\psi(e(\underline{\beta},\overline{\beta},\bar{q}_G))+\alpha$$

and when the principal is low type becomes

$$S(\bar{q}_G)-(1+\lambda)\left[-P(q_m(\overline{\beta},\bar{q}_G))q_m(\overline{\beta},\bar{q}_G)+(\overline{\beta}-e(\overline{\beta},\bar{q}_G))(q_m(\overline{\beta},\bar{q}_G)+\bar{q}_G)+\psi(e(\overline{\beta},\bar{q}_G))+\alpha\right]-\lambda\overline{\Pi}_m^o$$

From the first order condition with respect to $\underline{q}_G$, we obtain

$$S'(\underline{q}_G) - (1+\lambda)\left\{ -\frac{\partial \underline{q}_m(\underline{\beta},\underline{q}_G)}{\partial \underline{q}_G} \underbrace{\left[\frac{\partial P(\underline{q}_m(\underline{\beta},\underline{q}_G))}{\partial \underline{q}_m(\underline{\beta},\underline{q}_G)} \underline{q}_m(\underline{\beta},\underline{q}_G) + P(\underline{q}_m(\underline{\beta},\underline{q}_G)) - (\underline{\beta} - e(\underline{\beta},\underline{q}_G)) \right]}_{(0 \text{ by (B.7)})} \right\}$$

$$-(1+\lambda)\left\{ -\frac{\partial e(\underline{\beta},\underline{q}_G)}{\partial \underline{q}_G} \underbrace{\left[\underline{q}_m(\underline{\beta},\underline{q}_G) + \underline{q}_G - \frac{\partial \psi(e(\underline{\beta},\underline{q}_G))}{\partial e(\underline{\beta},\underline{q}_G)} \right]}_{(0 \text{ by (B.8)})} + (\underline{\beta} - e(\underline{\beta},\underline{q}_G)) \right\} = 0$$

$$\implies S'(\underline{q}_G) = (1+\lambda)(\underline{\beta} - e(\underline{\beta},\underline{q}_G)) \tag{B.12}$$

From the first order conditions with respect to $\overline{q}_G$, we obtain

$$-\lambda \nu \left\{ \left[(\overline{\beta} - e(\overline{\beta},\overline{q}_G)) - (\underline{\beta} - e(\underline{\beta},\overline{q}_G)) \right] \right\} \left(\frac{\partial q_m(\overline{\beta},\overline{q}_G)}{\partial \overline{q}_G} + 1 \right)$$

$$-\lambda \nu \left\{ -\frac{\partial e(\overline{\beta},\overline{q}_G)}{\partial \overline{q}_G} \underbrace{\left[q_m(\overline{\beta},\overline{q}_G) + \overline{q}_G - \frac{\partial \psi(e(\overline{\beta},\overline{q}_G))}{\partial e(\overline{\beta},\overline{\beta},\overline{q}_G)} \right]}_{(0 \text{ by (B.8)})} \right\}$$

$$-\lambda \nu \left\{ \frac{\partial e(\underline{\beta},\overline{\beta},\overline{q}_G)}{\partial \overline{q}_G} \underbrace{\left[q_m(\overline{\beta},\overline{q}_G) + \overline{q}_G - \frac{\partial \psi(e(\underline{\beta},\overline{\beta},\overline{q}_G))}{\partial e(\underline{\beta},\overline{q}_G)} \right]}_{(0 \text{ by (B.8)})} \right\}$$

$$+(1-\nu)\left\{ S'(\overline{q}_G) - (1+\lambda)\left\{ -\frac{\partial q_m(\overline{\beta},\overline{q}_G)}{\partial \overline{q}_G} \underbrace{\left[\frac{\partial P(q_m(\overline{\beta},\overline{q}_G))}{\partial q_m(\overline{\beta},\overline{q}_G)} q_m(\overline{\beta},\overline{q}_G) + P(q_m(\overline{\beta},\overline{q}_G)) - (\overline{\beta} - e(\overline{\beta},\overline{q}_G)) \right]}_{(0 \text{ by (B.7)})} \right\} \right\}$$

$$-(1-\nu)(1+\lambda)\left\{ -\frac{\partial e(\overline{\beta},\overline{q}_G)}{\partial \overline{q}_G} \underbrace{\left[q_m(\overline{\beta},\overline{q}_G) + \overline{q}_G - \frac{\partial \psi(e(\overline{\beta},\overline{q}_G))}{\partial e(\overline{\beta},\overline{q}_G)} \right]}_{(0 \text{ by (B.8)})} + (\overline{\beta} - e(\overline{\beta},\overline{q}_G)) \right\} = 0$$

Rearranging the terms, we arrive at the equation (3.34), in Chapter 3.

$$\Longrightarrow S'(\overline{q}_G) = (1+\lambda)(\overline{\beta} - e(\overline{\beta}, \overline{q}_G)) + \frac{\lambda v}{1-v} \underbrace{\left\{(\overline{\beta} - e(\overline{\beta}, \overline{q}_G)) - (\underline{\beta} - e(\underline{\beta}, \overline{q}_G))\right\}}_{>0} \left(\frac{\partial q_m(\overline{\beta}, \overline{q}_G)}{\partial \overline{q}_G} + 1 \right) \tag{B.13}$$

These are the results of **Proposition 3**.

Appendix C

Maximization problems - Regulating an Oligopolistic firm with market information

C.1 Cournot model

First, we will solve the Cournot problem, following **Lemma 5**, the inverse demand function (4.4) and the disutility of effort function (4.2).

Firm 1 solves the following problem

$$\max_{q_1,e_1} [\mathrm{a}-\mathrm{b}(\mathrm{q}_1+\mathrm{q}_2)]\mathrm{q}_1-(\beta_1-\mathrm{e}_1)\mathrm{q}_1-\gamma \mathrm{e}_1^2-\alpha$$

The first order conditions are given by

$$\begin{aligned} -bq_1+[a-b(q_1+q_2)]-(\beta_1-e_1^o) &= 0 && \text{(C.1)} \\ q_1-2\gamma e_1 &= \psi'(e_1^o) && \text{(C.2)} \end{aligned}$$

Replacing (C.2) in (C.1) we obtain

$$q_1=\frac{2\gamma}{4b\gamma-1}(a-bq_2-\beta_1) \tag{C.3}$$

and by symmetry

$$q_2=\frac{2\gamma}{4b\gamma-1}(a-bq_1-\beta_2) \tag{C.4}$$

Then, substituting (C.4) in (C.3), we obtain the optimal quantity of firm 1

$$q_1^*=\frac{2\gamma}{(4b\gamma-1)^2-4b^2\gamma^2}\left[a(2b\gamma-1)+2b\beta_2\gamma-(4\gamma-1)-\beta_1\right] \tag{C.5}$$

and by symmetry

$$q_2^*=\frac{2\gamma}{(4b\gamma-1)^2-4b^2\gamma^2}\left[a(2b\gamma-1)+2b\beta_1\gamma-(4\gamma-1)-\beta_2\right] \tag{C.6}$$

As can be seen, the quantity increases if the cost parameter, β, of the other firm increases and vice versa if the firm's own parameter increases. These results summarize **Lemma 5**.

In its relationship with the regulator the firm 1 must obtain at least its reservation utility, defined exogenously by Π_1^o. Under complete information, the following maximization problem is the one that results in **Lemma 6**.

Firm's decision

Given the contract defined by $\{t, q_G\}$, firm 1 chooses effort

$$\max_{e_1} \mathrm{t} - (\beta_1 - \mathrm{e}_1)\mathrm{q}_\mathrm{G} - \psi(\mathrm{e}_1) - \alpha$$

The first order condition is

$$q_G = \psi'(e_1) \tag{C.7}$$

From this first order conditions we obtain the reaction curve of the firm's variable with respect to the regulator's contract: $e(\beta, q_G)$. So the regulator's problem that he faces is

$$\max_{q_G} \mathrm{S}(\mathrm{q}_\mathrm{G}) - (1+\lambda)\mathrm{t} + \mathrm{U}_\mathrm{G}$$

subject to

$$U_1 = t - (\beta_1 - e_1(\beta_1, q_G))q_G - \psi(e_1(\beta_1, q_G)) - \alpha \geq \Pi_1^o$$

This program becomes

$$V = S(q_G) - (1+\lambda)\left[U_G + (\beta_1 - e_1(\beta_1, q_G))q_G + \psi(e_1(\beta_1, q_G)) + \alpha\right] + U_G$$

or

$$\max_{q_G} \mathrm{S}(\mathrm{q}_\mathrm{G}) - (1+\lambda)\left[(\beta_1 - \mathrm{e}_1(\beta_1, \mathrm{q}_\mathrm{G}))\mathrm{q}_\mathrm{G} + \psi(\mathrm{e}_1(\beta_1, \mathrm{q}_\mathrm{G})) + \alpha\right] - \lambda \mathrm{U}_\mathrm{G}$$

Notice that the optimal solution involves $U_G = \Pi_1^o$. So the transfer is adjusted as to make the firm 1 indifferent between accepting or rejecting the contract. In complete information this implies in no distortion. The first order condition is given by

$$S'(q_G) - (1+\lambda)\left\{ -\frac{\partial e_1(\beta_1, q_G)}{\partial q_G} \underbrace{\left[q_G - \frac{\partial \psi(e_1(\beta_1, q_G))}{\partial e_1(\beta_1, q_G)}\right]}_{\text{(0 by (C.7))}} + (\beta_1 - e_1(\beta_1, q_G)) \right\} = 0$$

$$\implies S'(q_G) = (1+\lambda)(\beta_1 - e_1(\beta_1, q_G)) \tag{C.8}$$

This is the result of **Lemma 6**.

Since the firm is only serving the government, the inverse demand function (4.3), together with (C.7) and (C.8), is considered. From (4.3) and (C.8)

$$d - fq_G = (1+\lambda)(\beta_1 - e_1) \tag{C.9}$$

From (4.2) and (C.7)

$$q_G = 2\gamma e_1 \tag{C.10}$$

Substituting (C.10) into (C.9), we obtain the optimal quantity that the government will require

$$q_G^* = \frac{2\gamma}{2f\gamma - (1+\lambda)}[d - (1+\lambda)\beta_1] \tag{C.11}$$

From (C.10)

$$e_1^* = \frac{1}{2f\gamma - (1+\lambda)}[d - (1+\lambda)\beta_1] \tag{C.12}$$

Asymmetric information with two-type case

The following problem shows the result of **Lemma 7**. Now we consider the incentive compatibility constraint. The efficient firm may pass as inefficient, that means choose the inefficient firm's contract. That means

$$\max_{e_1} \mathrm{\underline{t}} - (\underline{\beta}_1 - \mathrm{e}_1)\mathrm{\underline{q}_G} - \psi(\mathrm{e}_1) = \max_{e_1} \bar{\mathrm{t}} - (\underline{\beta}_1 - \mathrm{e}_1)\bar{\mathrm{q}}_\mathrm{G} - \psi(\mathrm{e}_1)$$

That becomes

$$\underline{t} - (\underline{\beta}_1 - e_1(\underline{\beta}_1, \underline{q}_G))\underline{q}_G - \psi(e_1(\underline{\beta}_1, \underline{q}_G)) = \bar{t} - (\underline{\beta}_1 - e_1(\underline{\beta}_1, \bar{q}_G))\bar{q}_G - \psi(e_1(\underline{\beta}_1, \bar{q}_G))$$

But we also have that the participation constraint of the inefficient type is binding. So

$$\bar{t} = \overline{\Pi}_1^o + (\overline{\beta}_1 - e_1(\overline{\beta}_1, \bar{q}_G))\bar{q}_G + \psi(e_1(\overline{\beta}_1, \bar{q}_G)) + \alpha$$

This leads to

$$\underline{U}_G = \overline{\Pi}_1^o + \overline{q_G}\left[(\overline{\beta}_1 - e_1(\overline{\beta}_1, \bar{q}_G)) - (\underline{\beta}_1 - e_1(\underline{\beta}_1, \bar{q}_G))\right] + \psi(e_1(\overline{\beta}_1, \bar{q}_G)) - \psi(e_1(\underline{\beta}_1, \bar{q}_G)) + \alpha \tag{C.13}$$

The utility of the regulator becomes when the firm is efficient

$$v\left\{S(\underline{q}_G)-(1+\lambda)\left[(\underline{\beta}_1-e_1(\underline{\beta}_1,\underline{q}_G))\underline{q}_G+\psi(e_1(\underline{\beta}_1,\underline{q}_G))+\alpha\right]-\lambda\underline{U}_1\right\}$$

with $\underline{U}_1$ from (C.13). And when the principal is low type becomes

$$(1-v)\left\{S(\overline{q}_G)-(1+\lambda)\left[(\overline{\beta}_1-e_1(\overline{\beta}_1,\overline{q}_G))\overline{q}_G+\psi(e_1(\overline{\beta}_1,\overline{q}_G))+\alpha\right]-\lambda\overline{\Pi}_1^o\right\}$$

From the first order conditions with respect to $\underline{q}_G$, we obtain

$$S'(\underline{q}_G)-(1+\lambda)\left\{-\frac{\partial e_1(\underline{\beta}_1,\underline{q}_G)}{\partial \underline{q}_G}\underbrace{\left[\underline{q}_G-\frac{\partial\psi(e_1(\underline{\beta}_1,\underline{q}_G))}{\partial e_1(\underline{\beta}_1,\underline{q}_G)}\right]}_{\text{(0 by (C.7))}}+(\underline{\beta}_1-e_(\underline{\beta}_1,\underline{q}_G))\right\}=0$$

$$\Longrightarrow S'(\underline{q}_G)=(1+\lambda)(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G)) \tag{C.14}$$

From the first order conditions with respect to $\overline{q}_G$, we obtain

$$\begin{aligned}
&-\quad\lambda v\left\{\left[(\overline{\beta}-e(\overline{\beta},\overline{q}_G))-(\underline{\beta}-e(\underline{\beta},\overline{q}_G))\right]\right\}\\
&-\quad\lambda v\left\{-\frac{\partial e(\overline{\beta},\overline{q}_G)}{\partial\overline{q}_G}\underbrace{\left[\overline{q}_G-\frac{\partial\psi(e(\overline{\beta},\overline{q}_G))}{\partial e(\overline{\beta},\overline{q}_G)}\right]}_{\text{(0 by (C.7))}}+\frac{\partial e(\underline{\beta},\overline{q}_G)}{\partial\overline{q}_G}\underbrace{\left[\overline{q}_G-\frac{\partial\psi(e(\underline{\beta},\overline{q}_G))}{\partial e(\underline{\beta},\overline{q}_G)}\right]}_{\text{(0 by (C.7))}}\right\}\\
&+\quad(1-v)\left\{S'(\overline{q}_G)-(1+\lambda)\left\{-\frac{\partial e(\overline{\beta},\overline{q}_G)}{\partial\overline{q}_G}\underbrace{\left[\overline{q}_G-\frac{\partial\psi(e(\overline{\beta},\overline{q}_G))}{\partial e(\overline{\beta},\overline{q}_G)}\right]}_{\text{(0 by (C.7))}}+(\overline{\beta}-e(\overline{\beta},\overline{q}_G))\right\}\right\}=0
\end{aligned}$$

Rearranging the terms, we arrive at the equation (4.18), in Chapter 4.

$$\begin{aligned}
\Longrightarrow S'(\overline{q}_G)=&(1+\lambda)(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G))\\
&+\frac{\lambda v}{1-v}\underbrace{\left\{(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G))-(\underline{\beta}-e_1(\underline{\beta},\overline{q}_G))\right\}}_{>0}
\end{aligned} \tag{C.15}$$

These are the results of **Lemma 7**. Since the efficient firm has no distortions, the quantity offered for it will continue (C.11) and the effort (C.10)

$$\underline{q}_G^* = \frac{2\gamma}{2f\gamma - (1+\lambda)}[d - (1+\lambda)\underline{\beta}] \tag{C.16}$$

$$\underline{e}_1^* = \frac{1}{2f\gamma - (1+\lambda)}[d - (1+\lambda)\underline{\beta}] \tag{C.17}$$

and for the inefficient firm, in compliance with (C.15)

$$d - f\overline{q}_G = (1+\lambda)\left(\overline{\beta} - \frac{\overline{q}_G}{2\gamma} + \frac{\lambda v}{1-v}\left[(\overline{\beta} - \frac{\overline{q}_G}{2\gamma}) - (\underline{\beta} - \frac{\overline{q}_G}{2\gamma})\right]\right)$$

i.e.

$$\overline{q}_G^* = \frac{2\gamma}{2f\gamma - (1+\lambda)}\left[d - (1+\lambda)\overline{\beta} - \frac{\lambda v}{1-v}(\overline{\beta} - \underline{\beta})\right] \tag{C.18}$$

and the effort (C.10)

$$\overline{e}_1^* = \frac{1}{2f\gamma - (1+\lambda)}\left[d - (1+\lambda)\overline{\beta} - \frac{\lambda v}{1-v}(\overline{\beta} - \underline{\beta})\right] \tag{C.19}$$

C.2 Serving the government and the market

Now our model, where the firm 1 serves both the government and the private market. The following problems will result in our contributions to the literature, obtaining **Proposition 4**, **Proposition 5** y **Proposition 6**.

Outside option

When the firm 1 only serves the market, it solves

$$\max_{q_1, e_1} \mathrm{P}(\mathrm{q}_1 + \mathrm{q}_2)\mathrm{q}_1 - (\beta_1 - \mathrm{e}_1)\mathrm{q}_1 - \psi(\mathrm{e}_1) - \alpha$$

The first order conditions are given by

$$\begin{aligned} P'(q_1^o + q_2^o)q_1^o + P(q_1^o + q_2^o) - (\beta_1 - e_1^o) &= 0 && \text{(C.20)} \\ q_1^o &= \psi'(e_1^o) && \text{(C.21)} \end{aligned}$$

This gives rise to a profit $\Pi_{1,C}^o$ and q_1^o.

Firm's decision

Given the offer $\{t, q_G\}$, firm 1 chooses market quantity and effort

$$\max_{q_1,e_1} \mathrm{t} + \mathrm{P}(\mathrm{q}_1 + \mathrm{q}_2)\mathrm{q}_1 - (\beta_1 - \mathrm{e}_1)(\mathrm{q}_1 + \mathrm{q}_\mathrm{G}) - \psi(\mathrm{e}_1) - \alpha$$

The first order conditions are

$$P'(q_1+q_2)q_1 + P(q_1+q_2) - (\beta_1 - e_1) = 0 \quad \text{(C.22)}$$

$$q_G + q_1 = \psi'(e_1) \quad \text{(C.23)}$$

From these first order conditions we obtain reaction curves of the firm's variables with respect to the regulator's contract and the firm 2: $q_1(\beta, q_G, q_2)$ and $e_1(\beta, q_G, q_2)$. So the regulator's problem is that he faces is

$$\max_{t,q_G} \mathrm{S}(\mathrm{q}_\mathrm{G}) - (1+\lambda)\mathrm{t} + \mathrm{U}_1$$

subject to

$$U_1 = t + P(q_1(\beta_1, q_G, q_2) + q_2)q_1(\beta_1, q_G, q_2) - (\beta_1 - e_1(\beta_1, q_G, q_2))(q_1(\beta_1, q_G, q_2) + q_G)$$
$$- \psi(e_1(\beta_1, q_G, q_2)) - \alpha \geq \Pi^o_{1,C}$$

This program becomes

$$V = S(q_G) + U_1$$
$$-(1+\lambda)\left[U_1 - P(q_1(\beta, q_G, q_2))q_1(\beta, q_G, q_2) + (\beta - e_1(\beta, q_G, q_2))(q_1(\beta, q_G, q_2) + q_G) + \psi(e_1(\beta, q_G, q_2)) + \alpha\right]$$

or

$$\max_{q_G} S(q_G) - \lambda U_1$$
$$-(1+\lambda)\left[-P(q_1(\beta, q_G, q_2))q_1(\beta, q_G, q_2) + (\beta - e_1(\beta, q_G, q_2))(q_1(\beta, q_G, q_2) + q_G) + \psi(e_1(\beta, q_G, q_2)) + \alpha\right]$$

Notice that the optimal solution involves $U = \Pi^o_{1,C}$. So the transfer is adjusted as to make the firm 1 indifferent between accepting or rejecting the contract. In complete information this implies in no distortion. The first order condition is given by

$$S'(q_G) - (1+\lambda)\left\{ -\frac{\partial q_1}{\partial q_G}\underbrace{\left[\frac{\partial P(q_1+q_2)}{\partial q_1}q_1 + P(q_1+q_2) - (\beta_1 - e_1)\right]}_{\text{(0 by (C.22))}} - \frac{\partial e_1}{\partial q_G}\underbrace{\left[q_1 + q_G - \frac{\partial \psi(e_1)}{\partial e_1}\right]}_{\text{(0 by (C.23))}} + (\beta_1 - e_1) \right\} = 0$$

$$\implies S'(q_G) = (1+\lambda)(\beta_1 - e_1(\beta, q_G, q_2)) \tag{C.24}$$

This is the result of **Proposition 4**. Therefore, from (C.22), (C.23) and (C.24) we have

$$-bq_1 + [a + b(q_1 + q_2)] - (\beta_1 - e_1) = 0 \tag{C.25}$$

$$q_G + q_1 = 2\gamma e_1 \tag{C.26}$$

$$d - fq_G = (1+\lambda)(\beta_1 - e_1) \tag{C.27}$$

Replacing (C.27) in (C.26)

$$e_1 = \frac{d - (1+\lambda)\beta_1 + q_1 f}{2 - (1+\lambda)} \tag{C.28}$$

Replacing (C.4) and (C.28) in (C.25)

$$\hat{q}_1 = \frac{1}{\frac{6b^2\gamma - 2b}{4b\gamma - 1} - \frac{f}{2f\gamma - (1+\lambda)}} \left\{ \frac{1}{4b\gamma - 1} [a(2b\gamma - 1) + 2b\beta_2 - (4b\gamma - 1)\beta_1] + \frac{d - (1+\lambda)\beta_1}{2f\gamma - (1+\lambda)} \right\} \tag{C.29}$$

or

$$\hat{q}_1 = \eta q_1^* + \mu \tag{C.30}$$

with

$$\eta = \frac{1}{\frac{6b^2\gamma - 2b}{4b\gamma - 1} - \frac{f}{2f\gamma - (1+\lambda)}} \frac{1}{4b\gamma - 1} \frac{(4b\gamma - 1)^2 - 4b^2\gamma^2}{2\gamma} \tag{C.31}$$

and

$$\mu = \frac{1}{\frac{6b^2\gamma - 2b}{4b\gamma - 1} - \frac{f}{2f\gamma - (1+\lambda)}} \frac{d - (1+\lambda)\beta_1}{2f\gamma - (1+\lambda)} \tag{C.32}$$

In (C.30), q_1^* corresponds to that of (C.5). The conditions for $\hat{q}$ to be greater than q_1^* are $\eta > 1$ and $\mu > 0$.

$$\eta > 1 \implies (4b\gamma - 1)[4f\gamma - (1+\lambda)] > 0 \tag{C.33}$$

and

$$\mu > 0 \implies d - (1+\lambda)\beta_1 > 0 \tag{C.34}$$

Asymmetric information with two-type case: without looking the market

Now we consider the incentive compatibility constraint in this case. The efficient firm may pass as inefficient, that means choose the inefficient firm's contract. That means

$$\max_{q_1, e_1} \underline{\mathrm{t}} + \mathrm{P}(\mathrm{q}_1 + \mathrm{q}_2)\mathrm{q}_1 - (\underline{\beta} - \mathrm{e}_1)(\mathrm{q}_1 + \underline{\mathrm{q}}_{\mathrm{G}}) - \psi(\mathrm{e}_1) = \max_{q_1, e_1} \bar{\mathrm{t}} + \mathrm{P}(\mathrm{q}_1 + \mathrm{q}_2)\mathrm{q}_1 - (\underline{\beta} - \mathrm{e}_1)(\mathrm{q}_1 + \bar{\mathrm{q}}_{\mathrm{G}}) - \psi(\mathrm{e}_1)$$

That becomes

$$\underline{t}+P(q_1(\underline{\beta},\underline{q}_G,q_2)+q_2)q_1(\underline{\beta},\underline{q}_G,q_2)-(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2))(q_1(\underline{\beta},\underline{q}_G,q_2)+\underline{q}_G)-\psi(e_1(\underline{\beta},\underline{q}_G,q_2))=$$
$$\bar{t}+P(q_1(\underline{\beta},\bar{q}_G,q_2)+q_2)q_1(\underline{\beta},\bar{q}_G,q_2)-(\underline{\beta}-e_1(\underline{\beta},\bar{q}_G,q_2))(q_1(\underline{\beta},\bar{q}_G,q_2)+\bar{q}_G)-\psi(e_1(\underline{\beta},\bar{q}_G,q_2))$$

But we also have that the participation constraint of the inefficient type is binding. So

$$\bar{t}=\overline{\Pi}^o_{1,C}-P(q_1(\underline{\beta},\underline{q}_G,q_2)+q_2)q_1(\underline{\beta},\underline{q}_G,q_2)+(\overline{\beta}-e_1(\overline{\beta},\bar{q}_G,q_2))(q_1(\underline{\beta},\underline{q}_G,q_2)+\bar{q}_G)$$
$$+\psi(e_1(\overline{\beta},\bar{q}_G,q_2))+\alpha$$

This leads to

$$\underline{t}=-P(q_1(\underline{\beta},\underline{q}_G,q_2)+q_2)q_1(\underline{\beta},\underline{q}_G,q_2)+(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2))(q_1(\underline{\beta},\underline{q}_G,q_2)+\underline{q}_G)+\psi(e_1(\underline{\beta},\underline{q}_G,q_2))$$
$$+\overline{\Pi}^o_{1,C}-P(q_1(\overline{\beta},\bar{q}_G,q_2)+q_2)q_1(\overline{\beta},\bar{q}_G,q_2)+(\overline{\beta}-e_1(\overline{\beta},\bar{q}_G,q_2))(q_1(\overline{\beta},\bar{q}_G,q_2)+\bar{q}_G)+\psi(e_1(\overline{\beta},\bar{q}_G,q_2))$$
$$+P(q_1(\underline{\beta},\bar{q}_G,q_2)+q_2)q_1(\underline{\beta},\bar{q}_G,q_2)-(\underline{\beta}-e_1(\underline{\beta},\bar{q}_G,q_2))(q_1(\underline{\beta},\bar{q}_G,q_2)+\bar{q}_G)-\psi(e_1(\underline{\beta},\bar{q}_G,q_2))$$

The utility of the regulator becomes when the firm is efficient

$$S(\underline{q}_G)-\lambda\underline{U}_1$$
$$-(1+\lambda)\left[-P(q_1(\underline{\beta},\underline{q}_G,q_2)+q_2)q_1(\underline{\beta},\underline{q}_G,q_2)+(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2))(q_1(\underline{\beta},\underline{q}_G,q_2)+\underline{q}_G)+\psi(e_1(\underline{\beta},\underline{q}_G,q_2))+\alpha\right]$$

with $\underline{U}_1$ being

$$\underline{U}_1=\overline{\Pi}^o_{1,C}-P(q_1(\overline{\beta},\bar{q}_G,q_2)+q_2)q_1(\overline{\beta},\bar{q}_G,q_2)+(\overline{\beta}-e_1(\overline{\beta},\bar{q}_G,q_2))(q_1(\overline{\beta},\bar{q}_G,q_2)+\bar{q}_G)+\psi(e_1(\overline{\beta},\bar{q}_G,q_2))$$
$$+P(q_1(\underline{\beta},\bar{q}_G,q_2)+q_2)q_1(\underline{\beta},\bar{q}_G,q_2)-(\underline{\beta}-e_1(\underline{\beta},\bar{q}_G,q_2))(q_1(\underline{\beta},\bar{q}_G,q_2)+\bar{q}_G)-\psi(e_1(\underline{\beta},\bar{q}_G,q_2))+\alpha$$

and when the principal is low type becomes

$$S(\bar{q}_G)-\lambda\overline{\Pi}^o_{1,C}$$
$$-(1+\lambda)\left[-P(q_1(\overline{\beta},\bar{q}_G,q_2)+q_2)q_1(\overline{\beta},\bar{q}_G,q_2)+(\overline{\beta}-e_1(\overline{\beta},\bar{q}_G,q_2))(q_1(\overline{\beta},\bar{q}_G,q_2)+\bar{q}_G)+\psi(e_1(\overline{\beta},\bar{q}_G,q_2))+\alpha\right]$$

From the first order condition with respect to $\underline{q}_G$, we obtain

$$S'(\underline{q}_G) - (1+\lambda)\left\{ -\frac{\partial \underline{q}_1}{\partial \underline{q}_G} \underbrace{\left[\frac{\partial P(\underline{q}_1 + q_2)}{\partial \underline{q}_1} \underline{q}_1 + P(\underline{q}_1 + q_2) - (\underline{\beta} - e_1(\underline{\beta}, \underline{q}_G, q_2)) \right]}_{\text{(0 by (C.22))}} \right\}$$

$$-(1+\lambda)\left\{ -\frac{\partial e_1(\underline{\beta}, \underline{q}_G, q_2)}{\partial \underline{q}_G} \underbrace{\left[\underline{q}_1 + \underline{q}_G - \frac{\partial \psi(e_1(\underline{\beta}, \underline{q}_G, q_2))}{\partial e_1(\underline{\beta}, \underline{q}_G, q_2)} \right]}_{\text{(0 by (C.23))}} + (\underline{\beta} - e_1(\underline{\beta}, \underline{q}_G, q_2)) \right\} = 0$$

where $\underline{q}_1 = q_1(\underline{\beta}, \underline{q}_G, q_2)$. Then

$$\Longrightarrow S'(\underline{q}_G) = (1+\lambda)(\underline{\beta} - e_1(\underline{\beta}, \underline{q}_G, q_2)) \tag{C.36}$$

From the first order conditions with respect to $\overline{q}_G$, we obtain

$$
\begin{aligned}
&-\lambda v\left\{\left[(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))-(\underline{\beta}-e_1(\underline{\beta},\overline{q}_G,q_2))\right]\right\}\\
&-\lambda v\left\{-\frac{\partial\overline{q}_1}{\partial\overline{q}_G}\underbrace{\left[\frac{\partial P(\overline{q}_1+q_2)}{\partial\overline{q}_1)}\overline{q}_1+P(\overline{q}_1+q_2)-(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))\right]}_{\text{(0 by (C.22))}}\right\}\\
&-\lambda v\left\{-\frac{\partial e_1(\overline{\beta},\overline{q}_G,q_2)}{\partial\overline{q}_G}\underbrace{\left[\overline{q}_1+\overline{q}_G-\frac{\partial\psi(e_1(\overline{\beta},\overline{q}_G,q_2))}{\partial e_1(\overline{\beta},\overline{q}_G,q_2)}\right]}_{\text{(0 by (C.23))}}\right\}\\
&-\lambda v\left\{\frac{\partial\hat{q}_1}{\partial\overline{q}_G}\underbrace{\left[\frac{\partial P(\hat{q}_1+q_2)}{\partial\hat{q}_1}\hat{q}_1+P(\hat{q}_1+q_2)-(\underline{\beta}-e_1(\underline{\beta},\overline{q}_G,q_2))\right]}_{\text{(0 by (C.22))}}\right\}\\
&-\lambda v\left\{\frac{\partial e_1(\underline{\beta},\overline{q}_G,q_2)}{\partial\overline{q}_G}\underbrace{\left[\hat{q}_1+\overline{q}_G-\frac{\partial\psi(e_1(\underline{\beta},\overline{q}_G,q_2))}{\partial e_1(\underline{\beta},\overline{q}_G,q_2)}\right]}_{\text{(0 by (C.23))}}\right\}\\
&+(1-v)\left\{S'(\overline{q}_G)-(1+\lambda)\left\{-\frac{\partial\overline{q}_1}{\partial\overline{q}_G}\underbrace{\left[\frac{\partial P(\overline{q}_1+q_2)}{\partial\overline{q}_1}\overline{q}_1+P(\overline{q}_1+q_2)-(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))\right]}_{\text{(0 by (C.22))}}\right\}\right\}\\
&-(1-v)(1+\lambda)\left\{-\frac{\partial e_1(\overline{\beta},\overline{q}_G,q_2)}{\partial\overline{q}_G}\underbrace{\left[\overline{q}_1+\overline{q}_G-\frac{\partial\psi(e_1(\overline{\beta},\overline{q}_G,q_2))}{\partial e_1(\overline{\beta},\overline{q}_G,q_2)}\right]}_{\text{(0 by (C.23))}}+(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))\right\}=0
\end{aligned}
$$

where $\overline{q}_1 = q_1(\overline{\beta},\overline{q}_G,q_2)$ and $\hat{q}_1 = q_1(\underline{\beta},\overline{q}_G,q_2)$. Rearranging the terms, we arrive at the equation (4.34), in Chapter 4

$$
\begin{aligned}
S'(\overline{q}_G) &= (1+\lambda)(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))\\
&\quad+\frac{\lambda v}{1-v}\underbrace{\left\{(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))-(\underline{\beta}-e_1(\underline{\beta},\overline{q}_G,q_2))\right\}}_{>0}
\end{aligned}
\tag{C.37}
$$

These are the results of **Proposition 5**. Thus, for the efficient firm follows (C.30)

$$\hat{\underline{q}}_1 = \eta \underline{q}_1^* + \mu \tag{C.38}$$

But from (C.37), now we have

$$d - f\overline{q}_G = (1+\lambda)(\overline{\beta} - e_1) + \frac{\lambda v}{1-v}(\overline{\beta} - \underline{\beta}) \tag{C.39}$$

Replacing (C.39) in (C.26)

$$\overline{e}_1 = \frac{d - (1+\lambda)\overline{\beta} + \overline{q}_1}{2f\gamma - (1+\lambda)} - \frac{\lambda v}{1-v}\frac{1}{2f\gamma - (1+\lambda)}(\overline{\beta} - \underline{\beta}) \tag{C.40}$$

Replacing (C.40) in (C.25)

$$\hat{\overline{q}}_1 = \eta \overline{q}_1^* + \mu - \sigma(\overline{\beta} - \underline{\beta}) \tag{C.41}$$

with

$$\sigma = \frac{1}{\frac{6b^2\gamma - 2b}{4b\gamma - 1} - \frac{f}{2f\gamma - (1+\lambda)}} \frac{\lambda v}{1-v} \frac{1}{2f\gamma - (1+\lambda)} \tag{C.42}$$

Asymmetric information with two-type case: looking the market

Now we consider the incentive compatibility constraint when the regulator looks what the firm 1 is doing in the market. The efficient firm may pass as inefficient, that means choose the inefficient firm's contract. That means

$$\max_{q_1, e_1} \underline{\mathrm{t}} + \mathrm{P}(\mathrm{q}_1 + \mathrm{q}_2)\mathrm{q}_1 - (\underline{\beta} - \mathrm{e}_1)(\mathrm{q}_1 + \underline{\mathrm{q}}_{\mathrm{G}}) - \psi(\mathrm{e}_1) = \max_{e_1} \overline{\mathrm{t}} + \mathrm{P}(\mathrm{q}_1 + \mathrm{q}_2)\mathrm{q}_1 - (\underline{\beta} - \mathrm{e}_1)(\mathrm{q}_1 + \overline{\mathrm{q}}_{\mathrm{G}}) - \psi(\mathrm{e}_1)$$

with $q_1 = q_1(\overline{\beta}, \overline{q}_G, q_2)$ in the right-hand side.

That becomes

$$\begin{aligned} &\underline{t} + P(q_1(\underline{\beta}, \underline{q}_G, q_2) + q_2)q_1(\underline{\beta}, \underline{q}_G, q_2) - (\underline{\beta} - e_1(\underline{\beta}, \underline{q}_G, q_2))(q_1(\underline{\beta}, \underline{q}_G, q_2) + \underline{q}_G) - \psi(e_1(\underline{\beta}, \underline{q}_G, q_2)) = \\ &\overline{t} + P(q_1(\overline{\beta}, \overline{q}_G, q_2) + q_2)q_1(\overline{\beta}, \overline{q}_G, q_2) - (\underline{\beta} - e_1(\underline{\beta}, \overline{\beta}, \overline{q}_G, q_2))(q_1(\overline{\beta}, \overline{q}_G, q_2) + \overline{q}_G) - \psi(e_1(\underline{\beta}, \overline{\beta}, \overline{q}_G, q_2)) \end{aligned} \tag{C.43}$$

But we also have that the participation constraint of the inefficient type is binding. So

$$\begin{aligned} \overline{t} = \overline{\Pi}^o_{1,C} - P(q_1(\underline{\beta}, \underline{q}_G, q_2) + q_2)q_1(\underline{\beta}, \underline{q}_G, q_2) + (\overline{\beta} - e_1(\overline{\beta}, \overline{q}_G, q_2))(q_1(\underline{\beta}, \underline{q}_G, q_2) + \overline{q}_G) \\ + \psi(e_1(\overline{\beta}, \overline{q}_G, q_2)) + \alpha \end{aligned}$$

This leads to

$$\begin{aligned}
\underline{t} = & -P(q_1(\underline{\beta},\underline{q}_G,q_2)+q_2)q_1(\underline{\beta},\underline{q}_G,q_2)+(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2))(q_1(\underline{\beta},\underline{q}_G,q_2)+\underline{q}_G)+\psi(e_1(\underline{\beta},\underline{q}_G,q_2)) \\
& +\overline{\Pi}^o_{1,C}-P(q_1(\overline{\beta},\overline{q}_G,q_2)+q_2)q_1(\overline{\beta},\overline{q}_G,q_2)+(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))(q_1(\overline{\beta},\overline{q}_G,q_2)+\overline{q}_G)+\psi(e_1(\overline{\beta},\overline{q}_G,q_2)) \\
& +P(q_1(\overline{\beta},\overline{q}_G,q_2)+q_2)q_1(\overline{\beta},\overline{q}_G,q_2)-(\underline{\beta}-e_1(\underline{\beta},\overline{q}_G,q_2))(q_1(\underline{\beta},\overline{q}_G,q_2)+\overline{q}_G)-\psi(e_1(\underline{\beta},\overline{q}_G,q_2))
\end{aligned}$$

The utility of the regulator becomes when the firm is efficient

$$\begin{aligned}
& S(\underline{q}_G)-\lambda\underline{U}_1 \\
& -(1+\lambda)\left[-P(q_1(\underline{\beta},\underline{q}_G,q_2)+q_2)q_1(\underline{\beta},\underline{q}_G,q_2)+(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2))(q_1(\underline{\beta},\underline{q}_G,q_2)+\underline{q}_G)+\psi(e_1(\underline{\beta},\underline{q}_G,q_2))+\alpha\right]
\end{aligned}$$

with $\underline{U}_1$ being

$$\begin{aligned}
\underline{U}_1 = & \overline{\Pi}^o_{1,C}+\left[(\overline{\beta}-e(\overline{\beta},\overline{q}_G,q_2))-(\underline{\beta}-e(\underline{\beta},\overline{\beta},\overline{q}_G,q_2))\right]\left[q_m(\overline{\beta},\overline{q}_G,q_2)+\overline{q}_G\right] \\
& +\psi(e(\overline{\beta},\overline{q}_G,q_2))-\psi(e(\underline{\beta},\overline{\beta},\overline{q}_G,q_2))+\alpha
\end{aligned}$$

and when the principal is low type becomes

$$\begin{aligned}
& S(\overline{q}_G)-\lambda\overline{\Pi}^o_{1,C} \\
& -(1+\lambda)\left[-P(q_1(\overline{\beta},\overline{q}_G,q_2)+q_2)q_1(\overline{\beta},\overline{q}_G,q_2)+(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))(q_1(\overline{\beta},\overline{q}_G,q_2)+\overline{q}_G)+\psi(e_1(\overline{\beta},\overline{q}_G,q_2))+\alpha\right]
\end{aligned}$$

From the first order condition with respect to $\underline{q}_G$, we obtain

$$\begin{aligned}
& S'(\underline{q}_G)-(1+\lambda)\left\{-\frac{\partial\underline{q}_1}{\partial\underline{q}_G}\underbrace{\left[\frac{\partial P(\underline{q}_1+q_2)}{\partial\underline{q}_1}\underline{q}_1+P(\underline{q}_1+q_2)-(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2))\right]}_{\text{(0 by (C.22))}}\right\} \\
& -(1+\lambda)\left\{-\frac{\partial e_1(\underline{\beta},\underline{q}_G,q_2)}{\partial\underline{q}_G}\underbrace{\left[\underline{q}_1+\underline{q}_G-\frac{\partial\psi(e_1(\underline{\beta},\underline{q}_G,q_2))}{\partial e_1(\underline{\beta},\underline{q}_G,q_2)}\right]}_{\text{(0 by (C.23))}}+(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2))\right\}=0
\end{aligned}$$

where $\underline{q}_1 = q_1(\underline{\beta},\underline{q}_G,q_2)$. Then

$$\Longrightarrow S'(\underline{q}_G)=(1+\lambda)(\underline{\beta}-e_1(\underline{\beta},\underline{q}_G,q_2)) \tag{C.44}$$

From the first order conditions with respect to $\overline{q}_G$, we obtain

$$
\begin{aligned}
&-\lambda v\left\{\left[(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))-(\underline{\beta}-e_1(\underline{\beta},\overline{\beta},\overline{q}_G,q_2))\right]\right\}\left(\frac{\partial\overline{q}_1}{\partial\overline{q}_G}+1\right)\\
&-\lambda v\left\{-\frac{\partial e_1(\overline{\beta},\overline{q}_G,q_2)}{\partial\overline{q}_G}\underbrace{\left[\overline{q}_1+\overline{q}_G-\frac{\partial\psi(e_1(\overline{\beta},\overline{q}_G,q_2))}{\partial e_1(\overline{\beta},\overline{q}_G,q_2)}\right]}_{\text{(0 by (C.23))}}\right\}\\
&-\lambda v\left\{\frac{\partial e_1(\underline{\beta},\overline{\beta},\overline{q}_G,q_2)}{\partial\overline{q}_G}\underbrace{\left[\overline{q}_1+\overline{q}_G-\frac{\partial\psi(e_1(\underline{\beta},\overline{\beta},\overline{q}_G,q_2))}{\partial e_1(\underline{\beta},\overline{\beta},\overline{q}_G,q_2)}\right]}_{\text{(0 by (C.23))}}\right\}\\
&+(1-v)\left\{S'(\overline{q}_G)-(1+\lambda)\left\{-\frac{\partial\overline{q}_1}{\partial\overline{q}_G}\underbrace{\left[\frac{\partial P(\overline{q}_1+q_2))}{\partial\overline{q}_1}\overline{q}_1+P(\overline{q}_1+q_2)-(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))\right]}_{\text{(0 by (C.22))}}\right\}\right\}\\
&-(1-v)(1+\lambda)\left\{-\frac{\partial e_1(\overline{\beta},\overline{q}_G,q_2)}{\partial\overline{q}_G}\underbrace{\left[\overline{q}_1+\overline{q}_G-\frac{\partial\psi(e_1(\overline{\beta},\overline{q}_G,q_2))}{\partial e_1(\overline{\beta},\overline{q}_G,q_2)}\right]}_{\text{(0 by (C.23))}}+(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))\right\}=0
\end{aligned}
$$

where $\overline{q}_1 = q_1(\overline{\beta},\overline{q}_G,q_2)$. Rearranging the terms, we arrive at the equation (4.42), in Chapter 4.

$$
\begin{aligned}
\Longrightarrow S'(\overline{q}_G)&=(1+\lambda)(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))\\
&+\frac{\lambda v}{1-v}\underbrace{\left\{(\overline{\beta}-e_1(\overline{\beta},\overline{q}_G,q_2))-(\underline{\beta}-e_1(\underline{\beta},\overline{\beta},\overline{q}_G,q_2))\right\}}_{>0}\left(\frac{\partial q_1(\overline{\beta},\overline{q}_G,q_2)}{\partial\overline{q}_G}+1\right)
\end{aligned}
\tag{C.45}
$$

These are the results of **Proposition 6**. Since the efficient firm has no distortions, its optimal remains the same as in the previous sections, given by

$$\hat{\underline{q}}_1=\eta\underline{q}_1^*+\mu \tag{C.46}$$

Finally, the expression (C.45) is not the same as (C.37). From (C.26) and (C.27) we have

$$d - f\overline{q}_G = (1+\overline{\beta}) - (1+\lambda)\frac{\overline{q}_G + \overline{q}_1}{2\gamma}$$

or

$$\overline{q}_1 = \frac{2f\gamma - (1+\lambda)}{1+\lambda}\overline{q}_G - 2\gamma[d - (1+\lambda)\overline{\beta})] \tag{C.47}$$

Therefore, the last expression of (C.45) is

$$\left(\frac{\partial q_1(\overline{\beta}, \overline{q}_G, q_2)}{\partial \overline{q}_G} + 1\right) = \frac{2f\gamma - (1+\lambda)}{1+\lambda} + 1 = \frac{2f\gamma}{1+\lambda} \tag{C.48}$$

Hence, the quantity of the inefficient firm is

$$\hat{\overline{q}}_1 = \eta \overline{q}_1^* + \mu - \sigma\frac{2f\gamma}{1+\lambda}(\overline{\beta} - \underline{\beta}) \tag{C.49}$$