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MATHEUS VALENÇA CORREIA

**Magnetic skyrmions in synthetic antiferromagnets:
stability, dynamics, and self-propulsion**

Recife

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Thesis presented to the Doctoral Study Program of the Physics Department of the Universidade Federal de Pernambuco (UFPE), as partial requirement to award the joint degree of Doctor in Physics.

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*Eu e o nada, nada não
O vasto, vasto vãõ
Do espaço até o spin.*

(GIL, 1997).

ABSTRACT

Magnetic skyrmions are magnetic textures resembling swirls of magnetization stabilized in ferromagnetic (FM) materials and with growing interest for spintronics applications given their inherent topological stability, nanoscale size, and current-induced motion using very low threshold currents. This thesis aims to investigate the stability, current-induced dynamics, and emergent self-propulsion of these topological textures in synthetic antiferromagnets (SAFs) — a multilayer system based on antiferromagnetically coupled FM layers. For that, we adopt a unified methodology combining micromagnetic simulations of the semiclassical stochastic Landau-Lifshitz-Gilbert (LLG) equation with analytical models based on the quasi-particle description provided by the Thiele equation. Initially, we describe the magnetic phase diagram found for homochiral and heterochiral SAFs — respectively composed by FM layers with parallel and antiparallel DMI vectors — where we focus in the coaxial and non-coaxial skyrmion-skyrmion bound pairs obtained during our study. Particularly, we verify that the increase of the interlayer Ruderman-Kittel-Kasuya-Yosida (RKKY) coupling enables the non-coaxial pair as the preferred bound state of the heterochiral SAF. Then we proceed to investigate the pair dynamics in unbalanced SAFs, featuring asymmetries in the Dzyaloshinskii-Moriya interaction (DMI) and in the magnetic saturation of the FM layers. SAFs are known to provide a vanishing net topological charge and, consequently, the cancellation of the Skyrmion Hall effect (SkHE) — the transverse deflection of skyrmions in motion in a FM layer. The elimination of SkHE is crucial for spintronics applications envisioning the skyrmion as the information carrier, such as the case of racetrack memories. By investigating the persistence of SkHE when skyrmions are set into motion by using the Zhang Li model for the spin-transfer torque (STT) effect, we demonstrate that the cancellation of the SkHE is robust against imbalances in several material parameters such as the relative magnitudes of DMI and anisotropy, but sensitive to asymmetries in the injection of spin-polarized currents into the FM layers and in the magnetic saturations. Furthermore, we reveal that heterochiral SAFs not only stabilize bound pairs in a non-coaxial configuration but also facilitate their experimental detection via stray fields. In a second phase of our study, we theoretically demonstrate that these non-coaxial pairs feature a remarkable behavior, functioning as nanoscale self-propelled, autonomous “swimmers”. By exciting symmetric (SBM) or

antisymmetric (ABM) breathing modes through oscillating external electric or magnetic fields, the time-reversal symmetry of the pair is broken, generating non-reciprocal gyrotopic (Magnus) forces that result in a net displacement of the pair, with the self-propulsion velocity oriented perpendicularly to the bond direction. The Thiele-based model fully describes the symmetric excitation at weak interlayer coupling and allows for the calculation of the self-propulsion velocity by projecting the skyrmion trajectory onto the shape space spanned by the bond length of the skyrmion pair and the dissipation coefficient. The self-propulsion is found to be maximized when the excitation frequency matches the natural frequency of the breathing modes and reaches ultra-high speeds, in the order of hundreds of millions of body lengths per second, more than any known natural or artificial swimmer. Finally, the stochastic analysis further reveals that the thermal noise induces the skyrmion pair to behave as an active Brownian particle (ABP), featuring transition from diffusion-dominated to ballistic motion when the excitation amplitude is increased. The increase of temperature makes the skyrmion pair to exhibit spontaneous reorientations and momentary reversals of the propulsion, behaviors akin to biological swimmers (e.g., flagellated bacteria and microalgae). Although the swimming mechanism for the case of strong coupling is not yet fully understood, our results substantiate the proposal of the skyrmionic active matter based on the motion of skyrmion-skyrmion bound pairs in heterochiral SAFs, opening new perspectives for the design of reconfigurable magnetic metamaterials and neuromorphic computing devices based on these new autonomous topological textures, ultimately bridging the gap between spintronics and active matter physics.

Keywords: Magnetic skyrmions; Synthetic antiferromagnet; Skyrmion Hall Effect; Active matter; Self-propulsion; Spintronics.

RESUMO

Os skyrmions magnéticos são texturas magnéticas semelhantes a redemoinhos de magnetização, estabilizados em materiais ferromagnéticos (FM), despertando crescente interesse para aplicações em spintrônica devido à sua inerente estabilidade topológica, dimensões nanométricas e movimento induzido por baixos valores de correntes elétricas. Esta tese pretende investigar a estabilidade, a dinâmica induzida por corrente elétrica polarizada por spin e a autopropulsão emergente dessas texturas topológicas em antiferromagnetos sintéticos (do inglês, synthetic antiferromagnets ou SAFs) — um sistema de multicamadas FM acopladas de forma antiferromagnética. Com esse intuito, adotamos uma metodologia que combina simulações micromagnéticas da equação semiclássica de Landau-Lifshitz-Gilbert (LLG) estocástica com o desenvolvimento de um modelo analítico baseado na descrição de quase-partícula fornecida pela equação de Thiele. Inicialmente, apresentamos o diagrama de fases magnéticas encontrado para SAFs homoquirais e heteroquirais — compostos, respectivamente, por camadas FM de vetores de Dzyaloshinskii-Moriya (DM) paralelos e antiparalelos — com foco nos pares ligados de skyrmion em arranjos coaxial e não-coaxial obtidos durante nosso estudo. Em particular, verificamos que o aumento do acoplamento Ruderman-Kittel-Kasuya-Yosida (RKKY) intercamada promove o par não-coaxial como o estado ligado preferencial do SAF heteroquiral. Em seguida, investigamos a dinâmica dos pares de skyrmions em SAFs desbalanceados, caracterizados por assimetrias na interação DM (DMI) e na saturação magnética das camadas FM. Já é sabido que os SAFs permitem uma carga topológica líquida nula e, consequentemente, o cancelamento do efeito Hall de Skyrmion (SkHE) — a deflexão transversal de skyrmions em movimento numa camada FM. A eliminação do SkHE é crucial para aplicações em spintrônica que utilizam o skyrmion como o portador de informação, como é o caso das memórias racetrack. Ao investigar a persistência do SkHE quando os skyrmions são colocados em movimento a partir do modelo de Zhang-Li para o efeito de torque de transferência de spin (STT), demonstramos que o cancelamento do SkHE é robusto contra desequilíbrios em diversos parâmetros materiais, tais como as magnitudes relativas das constantes de DMI e da anisotropia, mas é sensível a assimetrias na injeção de correntes polarizadas por spin nas camadas FM e nas magnetizações de saturação. Além disso, revelamos que os SAFs heteroquirais não apenas estabilizam pares ligados em uma configuração não-

coaxial, mas também facilitam sua detecção experimental através de campos dispersos (stray fields). Em uma segunda fase do estudo, demonstramos teoricamente que esses pares não-coaxiais apresentam um comportamento notável, funcionando como "nadadores" autônomos e autopropelidos em escala nanométrica. Ao excitar modos de respiração (breathing modes) simétricos (SBM) ou antissimétricos (ABM) através de campos elétricos ou magnéticos externos e oscilantes, a simetria de reversão temporal do par é quebrada, gerando forças girotópicas (Magnus) não-recíprocas que resultam em um deslocamento líquido do par, com a velocidade de autopropulsão orientada perpendicularmente à direção da ligação. O modelo baseado em Thiele descreve completamente a excitação simétrica em um regime de acoplamento RKKY fraco entre as camadas e permite o cálculo da velocidade de autopropulsão projetando-se a trajetória do skyrmion no espaço de forma (shape space) gerado pelo comprimento da ligação do par de skyrmions e pelo coeficiente de dissipação. Verifica-se que a autopropulsão é maximizada quando a frequência de excitação coincide com a frequência natural dos modos de respiração, atingindo velocidades bastante elevadas, da ordem de centenas de milhões de comprimentos corporais por segundo, superando qualquer nadador natural ou artificial conhecido. Finalmente, a análise com presença de temperatura revela que o ruído térmico induz o par de skyrmions a comportar-se como uma partícula browniana ativa (ABP), apresentando uma transição de movimento dominado por difusão para movimento balístico quando a amplitude de excitação é aumentada. O aumento da temperatura leva o par de skyrmions a exibir reorientações espontâneas e reversões momentâneas da propulsão, comportamentos semelhantes aos de nadadores biológicos (por exemplo, bactérias flageladas e microalgas). Embora o mecanismo de nado para o caso de acoplamento forte ainda não seja totalmente compreendido, nossos resultados fundamentam a proposta de matéria ativa skyrmiônica baseada no movimento de pares ligados de skyrmion-skyrmion em SAFs heteroquiirais, abrindo novas perspectivas para o desenvolvimento de metamateriais magnéticos reconfiguráveis e dispositivos de computação neuromórfica baseados nessas novas texturas topológicas autônomas, estabelecendo uma ponte entre a spintrônica e a física da matéria ativa.

Palavras-chaves: Skyrmions magnéticos; Antiferromagneto sintético; Efeito Hall de Skyrmion; Matéria ativa; Autopropulsão; Spintrônica.

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1 INTRODUCTION

The search for new technologies that can be miniaturized and consume less power is a complex quest involving from fundamental research to the real-world manufacturing in industries. **Magnetic skyrmions**, to be addressed in this thesis, emerge as a viable alternative for future information and processing devices and are a result of many developments in the field of magnetism and spintronics during the 20th century. In the introduction of this thesis, we briefly present the history of magnetism focused on its remarkable applications and techniques ultimately exemplified by the chiral skyrmion texture in thin magnetic films. Finally, we connect this historical background with an overview of our work with skyrmions in synthetic antiferromagnets and present the structure of the thesis for the reader.

1.1 Brief comments on the history of magnetism

The magnetic phenomena are said to be first discussed in the Western tradition by the ancient philosopher Thales of Miletus, which lived between 7th and 6th centuries BCE in the region of Ionia, western coast of contemporary Turkey. Since no writing record of Thales' philosophy is available, we rely on the discussions that several philosophers and historians have made about him afterwards. According to Aristotle, Thales argued that everything in nature has a soul, or "all things are full of gods", including inanimate objects which Thales exemplified by the naturally occurring magnets (ARISTOTLE, 2000). Although the Greek word for soul has a broader meaning that could have been lost in translation (BREMNER, 1983), a mystic aura often surrounded magnetic phenomena in ancient times.

The Chinese, as early as the 3rd century BCE, also provided precise references of the lodestone magnetic properties (NEEDHAM, 1962). Although these sources post-date those of the Greeks, the development of magnetic knowledge in ancient China clearly surpassed their European counterparts: north-south orientation of a lodestone bar as early as 2nd century BCE and the discovery of the magnetic polarity and the Earth's magnetic declination around the 8th and 9th centuries CE (NEEDHAM, 1962). The most practical consequence of the discovery of magnetic declination — a devia-

Figure 1 – Replica of the Chinese south-pointing spoon (*Si Nan* or “south seeker”), an object thought to be a precursor to the magnetic compass approximately 2000 years ago during China’s Han dynasty and, in those days, associated with the divine.



Source: Kingdon (2019).

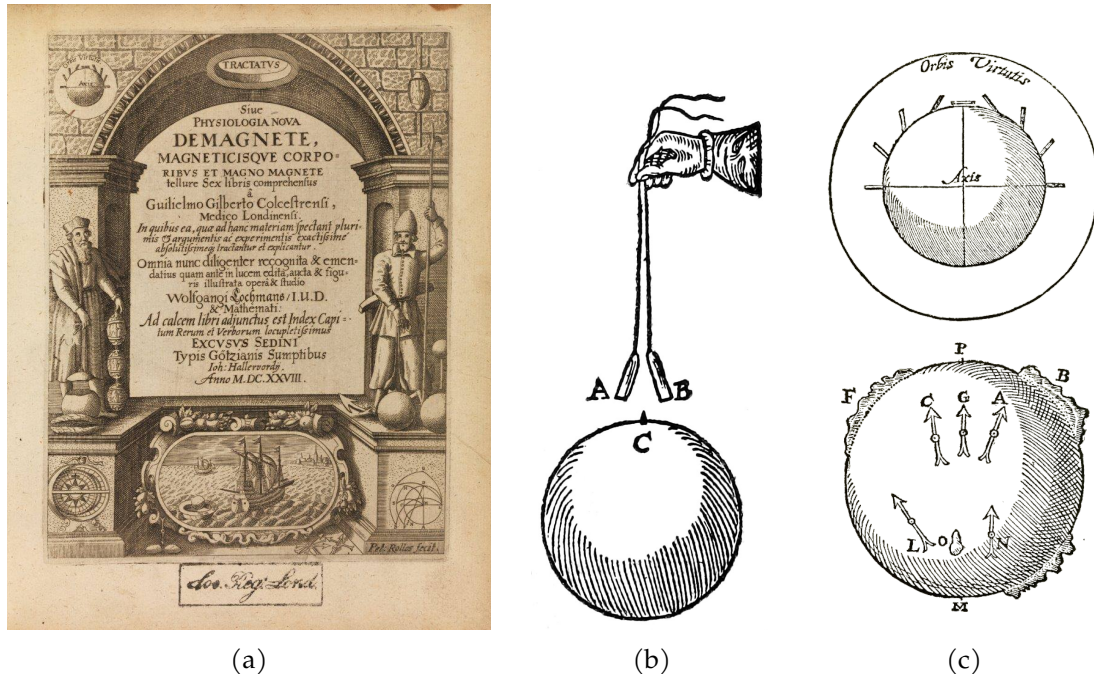
tion between the magnetic north and the Earth’s geographic north — is the **magnetic compass** for navigation. Curiously, however, its first use by the Chinese, represented by the south-pointing spoon (Fig. 1), was associated with “divination” and “geomancy” (NEEDHAM, 1962). Around the 1960s and 1970s, archaeological findings on one of the first Mesoamerican civilizations, the Olmecs, point to many cultural similarities with the Chinese in this matter (CARLSON, 1975).

Although worldwide spread, according to the traditional historiography it was in the region of Magnesia, still in the Anatolian peninsula, that the first magnetic phenomena were recognized and reported. Despite the exact location being still open to debate, thanks to the existence of many ancient “Magnesias”¹, the usual story is that the abundance of a special ore (magnetite or Fe_3O_4) in this venue allowed people in the Antiquity to witness the “magic” attraction and repulsion between naturally magnetized pieces of magnetite (lodestone) and iron (MELFOS; HELLY; VOUDOURIS, 2011). This mystical phenomenon ended up disseminating the unusual and challenging physical idea of the **action at a distance**: an object could instantaneously exert an influence on another body, regardless the nature of space or distance between them (MCMULLIN, 2002).

Many centuries after the Greeks, the scholar and engineer Pierre de Maricourt engaged with this idea during the 13th century CE in his *Epistola de magnete* (1269), considered the first systematic treaty on magnetism and the earliest acknowledge work of

¹ A brief yet elucidating discussion can be found in Melfos, Helly and Voudouris (2011).

Figure 2 – (a) Frontispiece of William Gilbert’s treatise *De magnete* (1628). The nautical illustration indicates how applied magnetism accompanied the first rigorous experiments at that time. (b) Illustration of Gilbert’s model of a magnetized sphere, the *terrella*, was used to explain, for example, the repulsion between the nearby pieces of iron A and B (induced magnetism). (c) Illustrations of *versorium* directions according to the latitude in the *terrella*, showing how the analogues to continental masses, such as B and F, influence local magnetic fields.



Source: The Royal Society of London webpage (a), Gilbert (1900, p. 133) (b), and Gilbert (1900, pp. 157 and 191) (c).

experimental science in Europe. Pierre describes several experiments performed with lodestones, including the construction of magnetic devices such as the pivoted compass and the labeling of a magnet extremity as a “pole”. His conclusions lead to the lodestone possessing a “natural virtue” and “it is clear that the poles of the lodestone derive their virtue from the poles of the heavens” (PEREGRINUS, 1904), enabling the attraction or repulsion at a distance.

In the year 1600, english physicist, physician and natural philosopher William Gilbert² proposed a modern and revolutionary explanation for the compass: the Earth itself is actually a giant “active” magnet which guides the behavior of a magnetic needle. In his thorough treatise *De magnete*, he developed a small model of the planet, the *terrella*, which was a magnetized sphere formed out of lodestone (Fig. 2). Gilbert relied on his own experimental evidence rather than superstition, describing his finds in *De magnete*, which is considered the first modern scientific text (COEY; MAZALEYRAT, 2024).

² Gilbert was the royal physician to Queen Elizabeth I (BBC, 2014).

Around the time of Gilbert, the use of the magnetic compass was crucial to enable the Age of Discovery during the XIV century “leading the Ming admiral Cheng Ho to the coasts of Africa in 1433, (...) Columbus, who rediscovered declination, to America in 1492” (COEY; MAZALEYRAT, 2024), and the Portuguese navigator Pedro Álvares Cabral to the coast of Brazil in 1500. According to Francis Bacon in his *Novum Organum* the magnetic compass was one of the three things — along with printing and gunpowder — that changed history (BACON, 1878).

1.2 Magnetism in contemporaneity

During the 18th century, the professionalization of the scientist, or “natural philosopher” (FARA, 1996 apud COEY, 2010), ultimately lead to the enormous advances witnessed in magnetism during the 19th century, when finally the theory of electricity and magnetism was **unified**. This was enabled by the experiments of Hans Christian Ørsted who in 1820 observed that an electric current could deflect the compass needle, providing the first experimental link between the two areas. Conversely, this discovery inspired André-Marie Ampère to formulate the mathematical description of magnetic forces between current-carrying wires, laying the groundwork for electrodynamics (COEY, 2010). Soon after, Michael Faraday introduced the concept of the **magnetic field** (modern action at a distance) and the description using force lines, and discovered electromagnetic induction — the mechanism behind electric motors and generators. These discoveries culminated in the 1860s with James Clerk Maxwell, who summarized in set of partial differential equations all the previous observations into a single framework (**Maxwell’s equations**) predicting the existence of **electromagnetic waves**, later confirmed experimentally by Heinrich Hertz.

The **telegraph** and the **radio** — which were fundamentally simple electromagnetic solenoids and relays — revolutionized communication, while the distribution and conversion of energy allowed street lighting and first domestic appliances. The excitement arising from scientific progress enjoyed by the European civilization during the *Belle Époque* (1871-1914), was however drastically diminished due to the arise of two world wars. The wonders and idealistic perspectives of science, electromagnetism included, were then replaced by the search and development of technologies contributing to war efforts. Magnetism proved itself to be decisive especially in World War II: the develop-

ment of **magnetic naval mines** and consequent the vessel "degaussing" as a counter-measure (GORDON, 1984); the emergence of **radar** systems via the cavity magnetron (RANDALL; BOOT, 1946); and the **production of fissile uranium** in the context of the Manhattan Project by separating isotopes using the California University Cyclotron (SMYTH, 1945) were crucial for the Allies' victory.

1.3 Spintronics and magnetic skyrmions

The technological advancements propelled by the world wars ultimately laid the foundations for a new approach in magnetism, where research then focused on a purely quantum mechanical property of the electron, the **spin**. After the discovery of the electron and the realization that magnetism is deeply associated with the electronic angular momentum — orbital and, specially, with the intrinsic spin component revealed by the Stern-Gerlach experiment — the fundamental origin of magnetism in matter could be deeply explained after Dirac's equation in 1928.

Two important byproducts of World War II were the Turing machine and the beginning of electronic computation. In the 1950s, the first electronic computers promoted a search for **soft magnetic materials**, whose magnetization could be easily changed for the purpose of information storage (GRÜNBERG, 2001). After the 1970s, novel techniques such as molecular-beam epitaxy and magnetic sputtering allowed the production of **thin films**, layers of a material with thickness from fractions of nanometers to micrometer multilayers. Film technology was crucial to make **integrated circuits** (ICs) possible, as they are composed by multilayers deposited on the top of a wafer. As is commonly known, electronics miniaturization was made possible by ICs, following the exponential Moore's law until quite recently (MOORE, 1965; WALDROP, 2016).

Another development provided by thin magnetic films was **spintronics**. In electronics, electric charges carry information, whereas the spin is ignored as it is not conserved during typical electron scattering due to its small diffusion length (λ_d). λ_d is a characteristic distance that a certain spin population can propagate before spin polarization decays to a fraction of its initial value. Strong spin-orbit coupling materials (e.g., platinum) have λ_d of about a few nm, whereas in weak spin-orbit coupling materials (e.g., copper) λ_d is about hundreds of nm. Therefore, the manufacturing of micrometer-, and after nanometer-, sized films whose thickness is smaller than typical λ_d allowed the ex-

ploration of information transport via spin.

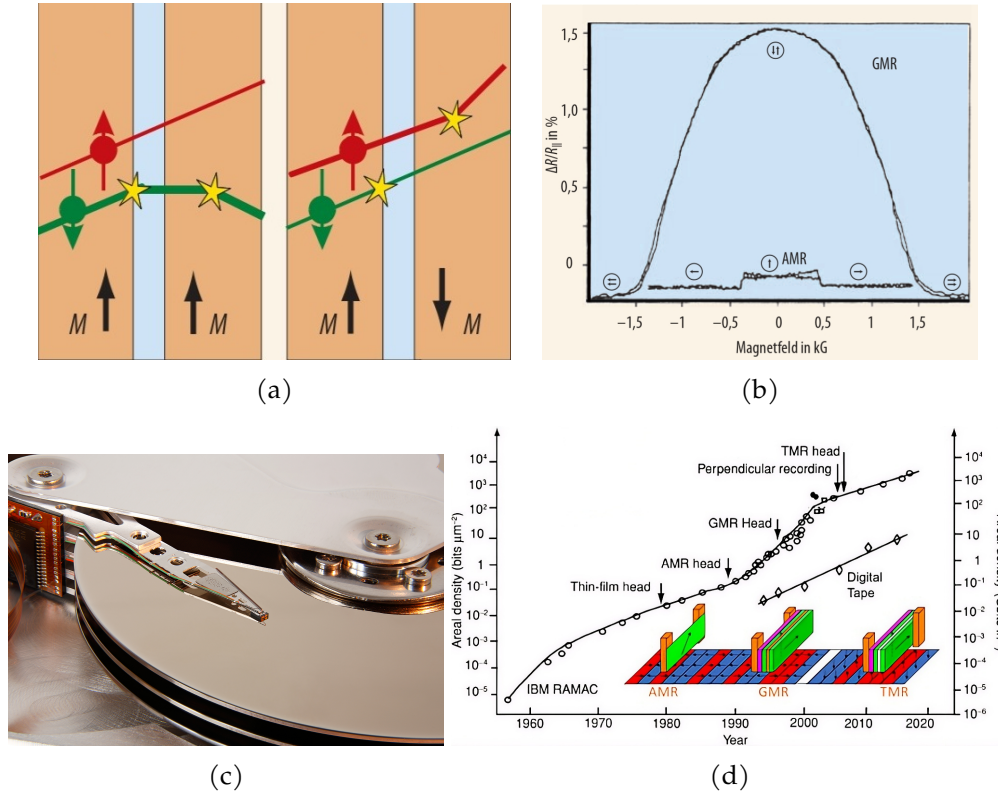
This ultimately led to the discovery of the **giant magnetoresistance (GMR)** in 1988 independently by Albert Fert and Peter Grunberg, which is considered the beginning of **spintronics** (BAIBICH et al., 1988; BINASCH et al., 1989). The idea behind the GMR effect is that, for two ferromagnetic (FM) layers coupled by a spacer (e.g., Copper), the electric resistance of the material is very sensitive upon their relative magnetic alignment [Fig. 3(a),(b)]: parallel alignment provides low resistance, while antiparallel provides higher resistance (SATTler, 2020). Magnetic multilayers would deliver, then, a very sensitive field-sensing device for (very!) small magnetic fields, with much greater sensitivity than conventional FM materials (PARKIN, 1998). An immediate application, a few years later, became commercial in 1997 when the first GMR-based head of hard disk drive was introduced by IBM [Fig. 3(c)]: a magnetic sensor to read orientation of magnetic bits used to store data, boosting the density of information storage (SATTler, 2020). In 2007, the Nobel Prize in Physics was awarded to Albert Fert and Peter Grunberg for discovering the GMR effect.

Although the beginning of the XXI century came with many progress in information storage [Fig. 3(d)], the need for **minimizing Joule heating** and **maximize processing speed** induced the search for alternative solutions regarding information storage and processing. This shift in perspective catalyzed the development of new spintronic technologies utilizing **non-collinear spin configurations** as mobile information carriers.

In this context, the theoretical work of Tony Skyrme in 1961, who proposed topologically stable field configurations as models for particles (SKYRME, 1961), found a surprising resonance in Condensed Matter Physics systems via exotic chiral textures known as **magnetic skyrmions**. Magnetic skyrmions are nanometric, swirling spin textures that are **topologically protected**, meaning that they cannot be easily annihilated by a continuous deformation and making them ideal candidates for carrying information without any loss. In 2009, after a series of theoretical predictions, the magnetic skyrmion was experimentally realized with the skyrmion lattice in the bulk chiral magnet MnSi (MÜHLBAUER et al., 2009), stabilized by the asymmetric **Dzyaloshinskii-Moriya interaction (DMI)** that favors non-collinear alignment of spins.

However, for practical device applications, the focus shifted towards skyrmions in **magnetic thin films** [Fig. 4(a)], which is already available at room temperature (LEGRAND et al., 2020). This is achieved by exploiting the interfacial DMI in **heavy-metal/ferromagnet**

Figure 3 – (a) Spin-dependent scattering in the giant magnetoresistance effect (GMR) in a stack composed of ferromagnetic (orange) and non-magnetic metal spacer (blue) layers. If magnetizations M are parallel (antiparallel), spin-up electrons find a low (high) resistance path. By using small values of magnetic field, we change the magnetic state and, then, the resistance changes dramatically. (b) Relative change in resistance ($\Delta R/R$, vertical axis) due to a change in the magnetic field (horizontal axis, in units of kG = 0.1 T) in the GMR effect and comparison to AMR (anisotropic magnetoresistance). (c) A hard disk drive (HDD) read-head assembly. (d) The impact of GMR on the historical scaling of hard drive areal density.

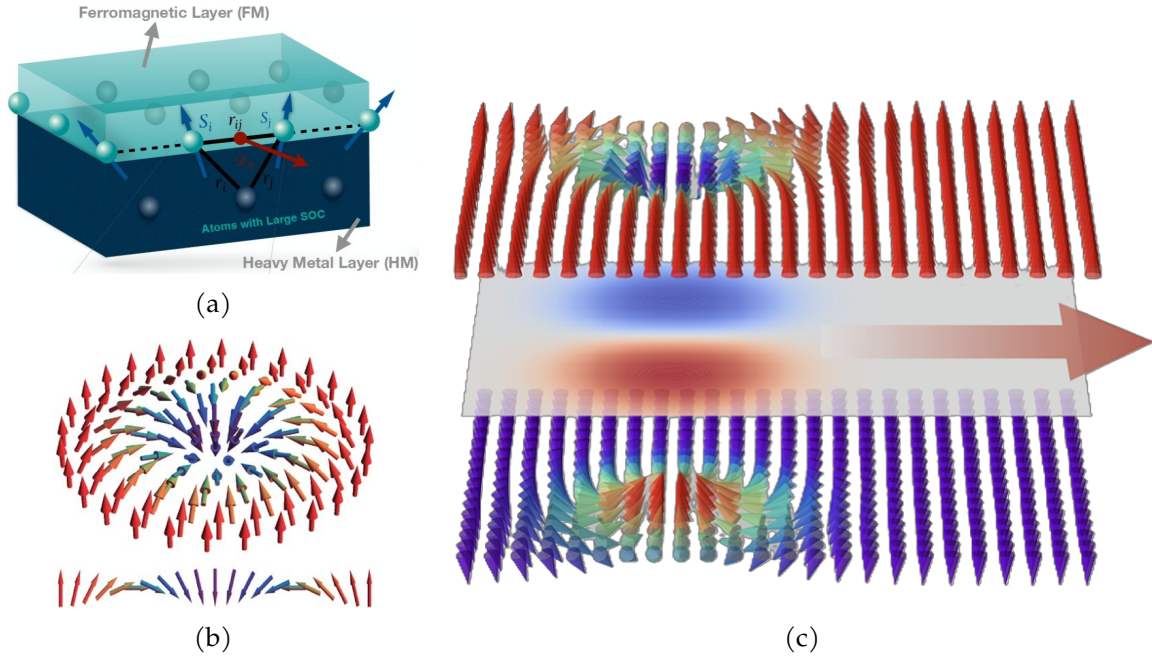


Source: (a)–(b) Grünberg (2007); (c) Wikimedia Commons (2005); and (d) Coey and Mazaleyrat (2024).

magnetic bilayers, such as Pt/Co (TOKURA; KANAZAWA, 2020). In these systems, the strong spin-orbit coupling at the interface favors a chiral twisting of the spins, stabilizing Néel-type skyrmions [Fig. 4(b)] and enabling control of skyrmion motion by many techniques (e.g., electric currents injected into the FM layers).

These interfacial skyrmions are the central subject of the numerical studies summarized in this thesis. We nucleate skyrmions exploring a special configuration of antiferromagnetically coupled ultrathin FM layers known as the **synthetic antiferromagnet** (SAF). SAFs are known to provide many advantages in tuning skyrmion motion, including the suppression of the **skyrmion Hall effect** (SkHE) — an undesirable, topologically based transverse deflection of skyrmion motion. The primary goal of this thesis is to investigate the stability, current-induced dynamics, and emergent self-propulsion

Figure 4 – (a) Schematic mechanism of the interfacial Dzyaloshinskii-Moriya interaction (iDMI). The symmetry breaking at the interface between ferromagnetic (FM) and heavy-metal thin layers generates the iDMI that stabilizes chiral structures. (b) Spin texture of a Néel-type skyrmion, characteristic of iDMI systems, showing the radial rotation of magnetization from the core to the periphery. (c) Illustration of a self-propelled non-coaxial skyrmion pair. Magnetic moments of top and bottom FM layers are represented as cones and the joint out-of-component magnetization is shown in the middle. We found that when internal, skyrmion breathing modes are excited, the pair exhibits autonomous motion perpendicular to the bond.



Source: (a) Mishra et al. (2025); (b) Kézsmárki et al. (2015); and (c) Clécio C.S. Silva in Zhang and Mochizuki (2025).

of magnetic skyrmions in SAFs. To achieve this, the study employs a unified methodology combining micromagnetic simulations of the stochastic Landau-Lifshitz-Gilbert (LLG) equation with analytical models based on the Thiele equation.

Specifically, this work aims to determine the magnetic phase diagram for homochiral and heterochiral SAFs, focusing on the stabilization of skyrmion pairs featuring a vanishing topological charge. Furthermore, it seeks to analyze the dynamics of these pairs under spin-polarized currents, evaluating the robustness of the SkHE cancellation against imbalances in material parameters and current injection, a situation that often happens with real-world samples. Via micromagnetic simulations, we obtain stable **coaxial and non-coaxial bound pairs of skyrmions** resulting from the interlayer coupling and even offering better prospects for experimental probing. We further investigate skyrmion pair dynamics by looking into several scenarios of SkHE in magnetically balanced and unbalanced SAFs. We verify the success of SAFs in suppressing SkHE under interlayer asymmetry in many magnetic parameters such as DMI and

anisotropy, however the Hall angle is found to be sensitive for differences in the **injected current densities and saturation** of the FM layers.

Furthermore, while spintronics regards magnetic skyrmions almost exclusively as **passive entities** whose motion is fully defined by, e.g., external fields or electric currents, in this thesis we aim to present a novel approach that bridges the gap between spintronics and active matter. The manipulation of internal soft modes of a non-coaxial skyrmion pair in a SAF reveals for the first time the phenomenon of **skyrmion self-propulsion**, an autonomous motion akin to active matter systems. Our theoretical model, based on the Thiele equation and backed by numerical simulations, uncovers a self-propelled skyrmion pair behaving as an **Active Brownian Particle**, mimicking **run-and-reverse** behaviors observed in motile bacteria and microalgae, when thermal fluctuations are considered. Finally, the skyrmion pairs reach velocities up to a few meters per second, which corresponds to roughly 10^8 **body lengths per second**, remarkably surpassing all known biological and artificial active swimmers and establishing this new area of “skyrmionic active matter”.

1.4 Thesis outline

This thesis is organized as follows.

- In **Chapter 2**, we provide the theoretical background regarding magnetic phenomena and the theory of magnetic skyrmions. We discuss the role of the magnetic moment and the magnetic order in matter leading to the fundamental magnetic interactions. Chiral magnetic phases are addressed, focusing on the discussion of the solitonic nature of skyrmions, describing its topological properties, induced motion, the Thiele equation, and the skyrmion Hall effect (SkHE).
- In **Chapter 3**, we define the numerical tools regarding the micromagnetic model — with its advantages and limitations — we adopt in this thesis to simulate magnetic skyrmions. Also, we introduce the two distinct synthetic antiferromagnet geometries — homochiral and heterochiral — that we employ in the studies reported in Chapters 4 and 5, discussing the implications for the skyrmion phase.
- In **Chapter 4**, we present our results and discussion on the dynamics of coaxial

and non-coaxial skyrmion pairs in homochiral and heterochiral synthetic antiferromagnets. We include a phase diagram studying the possible magnetic states according to the interlayer coupling and the relative magnitude of DMI between the ferromagnetic layers. We study the skyrmion pair dynamics under the injection of spin-polarized currents, investigating the robustness of the cancellation of the SkHE in scenarios of unbalanced magnetic parameters. We finally address the stability of skyrmion pairs with respect to the injection of spin-polarized currents.

- In **Chapter 5**, we present a review on active matter systems and the results of a novel phenomenon, typical of active matter, regarding non-coaxial skyrmion pairs in a heterochiral SAF, the self-propelled motion. We discuss non-reciprocal swimming in the low Reynolds number regime and the excitation of internal breathing modes of coupled magnetic skyrmions to induce non-reciprocal motion. We exhibit the results for self-propelled pairs in $T = 0$, with the discussion of the swimming mechanism and a theoretical model using Thiele equation to describe self-propulsion in the symmetrical breathing mode (SBM) scenario. We discuss the challenges for the cases of strong coupling and the antisymmetric breathing modes (ABM). We finally consider the case of $T \neq 0$, presenting the skyrmion pairs trajectories, studying its diffusive and ballistic regimes, and debating the similarities between this magnetic system and biological organisms.
- In **Chapter 6**, we summarize our results and present some perspectives regarding self-propelled skyrmions in synthetic antiferromagnets.
- In **Appendix A**, we provide a list of publications associated with this thesis.
- In **Appendix B**, we discuss the origin of the atomic magnetism, addressing the classical and quantum fundamentals for orbital and spin magnetic moments.
- In **Appendix C**, we debate magnetic phenomena in matter, describing the behavior of the magnetization field and the typical magnetic responses of materials, specially ferromagnetism.

In accordance with the research advisor, this thesis was prepared with the partial assistance of a **generative artificial intelligence (AI) model**, namely *Gemini Pro* by

Google, at various stages of the writing process. The tool was used in exploring recent literature, organizing references according to the Brazilian Association of Technical Standards (ABNT) and for stylistic revision (grammatical verification and enhancement of text clarity in English). The author declares that all AI-generated content was cross-checked in order to guarantee the quality of the final version of this text.

2 THEORETICAL BACKGROUND

In this chapter, we discuss the fundamentals of magnetism in matter: the main aspects of classical and quantum magnetism, magnetic order and the interactions that naturally occur in many materials, focusing the ferromagnetic order. Finally, we address the chiral magnetism and the emergence of magnetic skyrmions: topological properties, induced motion, and other related phenomena such as the skyrmion Hall effect (SkHE).

2.1 Magnetic dipole moment

According to Gauss's law for magnetism, which is one of Maxwell's equations for electromagnetism, the magnetic field $\mathbf{B}$ is divergenceless, i.e.,

$$\nabla \cdot \mathbf{B} = 0. \quad (2.1)$$

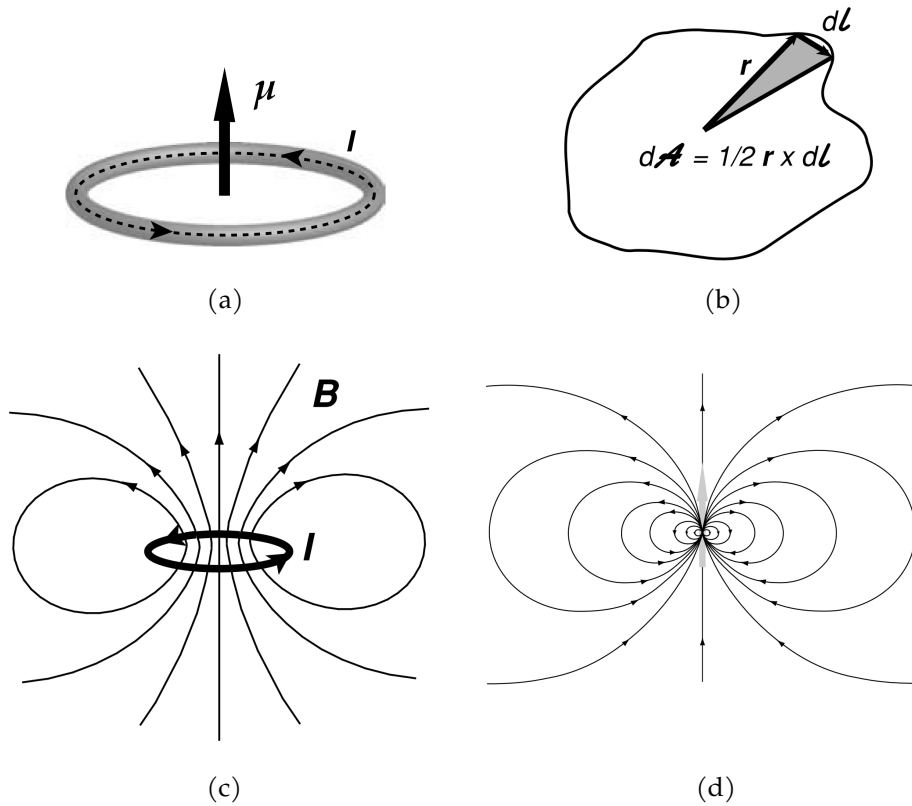
So, unlike electrostatic phenomena, there cannot be any point sources or sinks, where magnetic field lines would begin or terminate similar to electric charges. "Magnetic charges" (or magnetic monopoles), however, were never really observed in nature so far, so we rely on a different basic entity for describing magnetic phenomena: the **magnetic dipole**.

Accordingly, in order to describe strength and direction of magnetic fields in matter we resort to a fundamental quantity, the **magnetic dipole moment** $\boldsymbol{\mu}$. As André-Marie Ampère very cleverly presented in his *mémoire* to the French Academy of Science during the 1820s (AMPÈRE, 1883), we can associate $\boldsymbol{\mu}$ with a very small current loop [Fig. 5(a)], with a current density $\mathbf{j}(\mathbf{r})$, as

$$\boldsymbol{\mu} = \frac{1}{2} \int \mathbf{r} \times \mathbf{j}(\mathbf{r}) d^3r. \quad (2.2)$$

where $\mathbf{j}(\mathbf{r})$ and $\boldsymbol{\mu}$ are measured in SI units of A/m² and Am², respectively, and $\mathbf{r}$ is the distance between the origin and an infinitesimal length element in a current loop. The integration is performed in the whole space considering $\mathbf{j}(\mathbf{r})$ flowing through the cross-section of an infinitesimal volume element d^3r (COEY, 2010). We assume a simple circular planar loop of radius R and constant current I for Eq. 2.2. In this case, $\mathbf{r} \times \mathbf{j}(\mathbf{r}) d^3r = RI \hat{\mathbf{r}} \times d\mathbf{l}$, where $d\mathbf{l}$ is a length element tangent to the loop. Then we solve the

Figure 5 – (a) A current I flows in a planar circular loop, which is equivalent to a magnetic dipole moment μ . (b) Integration of Eq. 2.2 visualized as the sum of oriented area elements $d\mathbf{A}$. Magnetic field lines around a current loop (c) and a magnetic dipole (d).



Source: Adapted from Coey (2010).

resulting contour integral as

$$\boldsymbol{\mu} = \frac{1}{2} \int \mathbf{r} \times \mathbf{j}(\mathbf{r}) d^3r = \frac{1}{2} RI \oint_{\text{Loop}} \hat{\mathbf{r}} \times d\mathbf{l} = (\pi R^2) I \hat{\mathbf{z}}, \quad (2.3)$$

where $A = \pi R^2$ is the area enclosed by the loop and $d\mathbf{A} = \frac{1}{2} \mathbf{r} \times d\mathbf{l}$ is the perpendicular oriented area element [Fig. 5(b)] defining the direction of $\boldsymbol{\mu}$. Equation 2.3, then, gives us a clear interpretation of what $\boldsymbol{\mu}$ means in terms of a current loop.

The magnetic field $\mathbf{B}$ produced by the current loop is obtained using the Biot-Savart law, the magnetostatics equivalent of Coulomb's law in electrostatics — by considering a steady current, we are in the realm of stationary magnetic fields. We obtain the Biot-Savart law as a consequence of two Maxwell's equations of the electromagnetism, the Eq. 2.1 and the Ampère's law for magnetostatics

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}, \quad (2.4)$$

where μ_0 is the magnetic permeability of the vacuum. Consider the infinitesimal volume element δV with cross-section a and current density $\mathbf{j}(\mathbf{r})$. The **Biot-Savart law**

calculates the magnetic field contribution $\delta\mathbf{B}$ of that tiny volume element as (JACKSON, 1999)

$$\delta\mathbf{B} = -\frac{\mu_0}{4\pi} \frac{(\mathbf{r} - \mathbf{r}') \times \mathbf{j}}{|\mathbf{r} - \mathbf{r}'|^3} \delta V, \quad (2.5)$$

where $\mathbf{r} - \mathbf{r}'$ is a vector from the current element at $\mathbf{r}'$ to a point $\mathbf{r}$. Considering again the loop of Fig. 5(a), where I flows in a length element δl such that $\|\mathbf{j}\| = I/a$ and $\mathbf{j} \delta V = I \delta l$, we sum the contribution of $\delta\mathbf{B}$ in multiple locations and obtain the field lines distribution depicted in Fig. 5(c), which is the exact shape of $\mathbf{B}$ for a natural magnetic dipole [Fig. 5(d)]. $\mathbf{B}$ is expressed in spherical coordinates (r, θ, ϕ) , where $\boldsymbol{\mu}$ is aligned with the z axis. The components of $\mathbf{B}$ are finally given by (COEY, 2010)

$$B_r = \frac{\mu_0 \mu}{2\pi r^3} \cos \theta, \quad B_\theta = \frac{\mu_0 \mu}{4\pi r^3} \sin \theta, \quad B_\phi = 0. \quad (2.6)$$

The field has radial and polar components, decaying with r^3 .

The clever suggestion of Ampère comes with no surprise after we discuss the association between a current loop and a dipole. After Ørsted's experiment and Ampère's own results on the forces between currents flowing in conductors, the french physicist conjectured **electric currents** as the ultimate cause for all the magnetic phenomena (COEY; MAZALEYRAT, 2024), rejecting "imponderable fluid" theories, paradigmatic at the beginning of the 1800s (WHITTAKER, 1951, p. 88). Eventually, Ampère's hypothesis that magnetism is nothing more than electricity in motion was not entirely correct, but it was remarkably astute.

To maintain a concise narrative in the main text of this thesis, the **physical origin of the atomic magnetic moment**, alongside the fundamentals of classical and quantum magnetism are discussed, is covered in Appendix B. Furthermore, **magnetic phenomena in matter** are comprehensively addressed in Appendix C, where ferromagnetism and correlated magnetic responses by materials are described.

In the next section, we jump to the investigation of the most relevant magnetic interactions in ferromagnetic materials.

2.2 Fundamental magnetic interactions and chiral magnetism

In this section, we detail the fundamentals of the exchange and how some types of interaction (dipolar, magnetocrystalline anisotropy, and typical or even unusual forms

of exchange) are present in ferromagnetic materials and help us understand how complex non-collinear structures arise in our systems of interest: chiral magnets.

2.2.1 Magnetic dipolar interaction

In Section 2.1 we discussed about the magnetic moment, a fundamental entity of magnetism, and its behavior under an external magnetic field. It is reasonable to consider now, then, how the interaction between magnetic dipoles occurs in ferromagnetic materials. Based on Eq. B.5, we know there is a tendency of $\boldsymbol{\mu}$ to be aligned with $\mathbf{B}$.

What about the interaction between two dipoles? What kind of alignment is energetically favorable when one is subjected to the other's magnetic field? Consider a pair of magnetic dipoles $\boldsymbol{\mu}_i$ and $\boldsymbol{\mu}_j$ at positions $\mathbf{r}_i$ and $\mathbf{r}_j$, respectively, such that $\|\boldsymbol{\mu}_i\| = \|\boldsymbol{\mu}_j\| = \mu$, $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j = r_{ij}\hat{\mathbf{r}}_{ij}$, and $\hat{\mathbf{r}}_{ij}$ is the direction of the line connecting both dipoles. The total energy of the pair (ϵ_{pair}) is given either in terms of $\boldsymbol{\mu}_i$ or of $\boldsymbol{\mu}_j$ experiencing the mutual interaction with the field generated by the other dipole. So we can write a symmetrical expression by adding both as

$$2\epsilon_{\text{pair}} = -\boldsymbol{\mu}_i \cdot \mathbf{B}_{i,j} - \boldsymbol{\mu}_j \cdot \mathbf{B}_{j,i}, \quad (2.7)$$

where $\mathbf{B}_{i,j}$ is the magnetic field experienced by $\boldsymbol{\mu}_i$ due to $\boldsymbol{\mu}_j$.

Besides Eqs. 2.6, $\mathbf{B}_{i,j}$ can also be conveniently expressed (COEY, 2010) with terms parallel to the distance vector $\mathbf{r}_{ij}$ and to the magnetic dipole orientations as

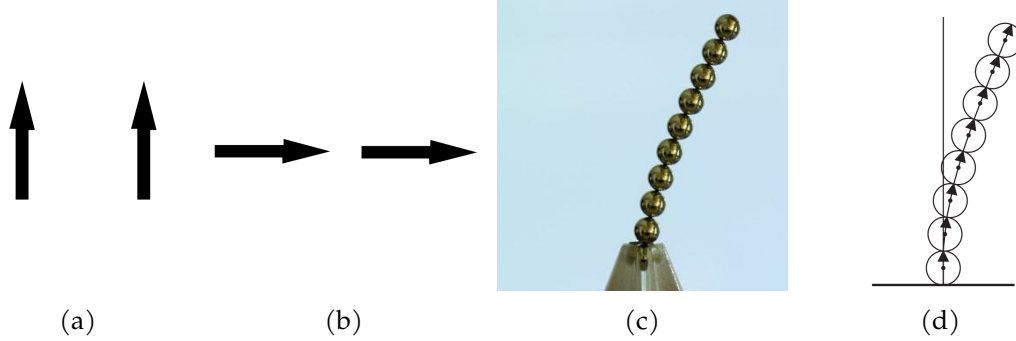
$$\mathbf{B}_{i,j} = \frac{\mu_0}{4\pi r_{ij}^3} [3(\boldsymbol{\mu}_j \cdot \hat{\mathbf{r}}_{ij})\hat{\mathbf{r}}_{ij} - \boldsymbol{\mu}_j]. \quad (2.8)$$

So, the first term in the right side of Eq. 2.7 can be written as

$$-\boldsymbol{\mu}_i \cdot \mathbf{B}_{i,j} = -\frac{\mu_0}{4\pi r_{ij}^3} [3(\boldsymbol{\mu}_i \cdot \hat{\mathbf{r}}_{ij})(\boldsymbol{\mu}_j \cdot \hat{\mathbf{r}}_{ij}) - \boldsymbol{\mu}_i \cdot \boldsymbol{\mu}_j]. \quad (2.9)$$

This allow us to wonder about the possible arrangements for the two interacting magnetic moments. If we consider a parallel (broadside) orientation such as in Fig. 6(a), then $\boldsymbol{\mu}_{i,j} \cdot \hat{\mathbf{r}}_{ij} = 0$ and $\boldsymbol{\mu}_i \cdot \boldsymbol{\mu}_j = \mu^2$. Applying this into Eq. 2.9 and Eq. 2.7, we obtain $\epsilon_{\text{pair}} = -\mu_0\mu^2/4\pi r_{ij}^3$. A second arrangement, however, shown in Fig. 6(b), and known as “nose-to-tail” or “head-to-tail”, gives us the lower energy, as the relative orientation of the two magnetic dipoles leads to $\boldsymbol{\mu}_{i(j)} \cdot \hat{\mathbf{r}}_{ij} = -\mu_{i(j)}$ and $\boldsymbol{\mu}_i \cdot \boldsymbol{\mu}_j = \mu^2$. So we reach $\epsilon_{\text{pair}} = -2\mu_0\mu^2/4\pi r_{ij}^3$, energetically favorable than the parallel arrangement.

Figure 6 – Magnetic moments ordering via dipolar interaction: (a) broadside and (b) head-to-tail arrangements for a pair, the latter with lower energy. The tendency of dipoles to form chains via dipolar interaction is shown via a macroscopic analogue — small magnetic balls in the presence of gravity are analyzed with respect to the stability of the chain they formed — in (c) and (d). In the latter, the head-to-tail pattern forming the chain is illustrated with arrows.



Source: (a)–(b) Coey (2010); (c)–(d) Schönke and Fried (2017).

There is a tendency, thus, for magnetic dipoles to organize themselves in strings or threads. For illustrative purposes, a macroscopic analogue is shown in Fig. 6(c)–(d), where centimeter-sized magnetic balls are subjected to a gravitational field, play the role of magnetic dipoles, and organize themselves in a chain.

At the atomic level, however, the dipolar interaction is typically very small in comparison to the exchange interaction, for example, and can be often neglected. Considering the head-to-tail configuration and typical atomic magnetic moments ($\mu_i = \mu_j = \mu_B$) and distances ($r \approx 1 \text{ \AA}$), then $\epsilon \approx 8.6 \times 10^{-24} \text{ J} \approx 0.05 \text{ meV}$, what corresponds via $E = k_B T$ (k_B is the Boltzmann's constant), to a temperature of 0.62 K. So, typically the order brought by the dipolar interaction is easily broken at room temperature.

For an indefinite large number of dipoles in the material, we generalize Eq. 2.7 and the total dipolar interaction $\mathcal{E}_{\text{dip}}$ should account for

$$\mathcal{E}_{\text{dip}} = -\frac{1}{2} \left(\frac{\mu_0}{4\pi} \right) \sum_{\substack{i,j \\ i \neq j}} \frac{1}{r_{ij}^3} \left[3(\boldsymbol{\mu}_i \cdot \hat{\mathbf{r}}_{ij})(\boldsymbol{\mu}_j \cdot \hat{\mathbf{r}}_{ij}) - \boldsymbol{\mu}_i \cdot \boldsymbol{\mu}_j \right], \quad (2.10)$$

where the sum runs over all the pairs $\boldsymbol{\mu}_i, \boldsymbol{\mu}_j$ in the system. So far, the description of $\mathcal{E}_{\text{dip}}$ via Eq. 2.10 considers a system with discrete components.

Jumping into the continuous limit, we describe this interaction with $\mathbf{M}(\mathbf{r})$. We know, building on Eq. C.1, that $\boldsymbol{\mu}(\mathbf{r}) = (1/V) \int_V \mathbf{M}(\mathbf{r}') d^3\mathbf{r}'$, where V is the volume of the material. So, Eq. 2.8 can be rewritten as we divide by μ_0 to find

$$\mathbf{H}_d(\mathbf{r}) = \frac{1}{4\pi} \int_V \frac{d^3\mathbf{r}'}{\|\mathbf{r} - \mathbf{r}'\|^3} \left\{ \left[3\mathbf{M}(\mathbf{r}') \cdot \hat{\mathbf{r}} \right] \hat{\mathbf{r}} - \mathbf{M}(\mathbf{r}') \right\}, \quad (2.11)$$

where $\hat{\mathbf{r}} = (\mathbf{r} - \mathbf{r}') / \|\mathbf{r} - \mathbf{r}'\|$ and $\mathbf{H}_d$ is typically known as the **demagnetization field**¹.

Replacing the magnetic moment by the magnetization field $\mathbf{M}$ and the discrete sum of dipoles in Eq. 2.10 by a volume integral leads to the continuum limit version, typically solved numerically, of dipole-dipole interaction as a function of $\mathbf{H}_d$,

$$\mathcal{E}_{\text{dip}} = -\frac{\mu_0}{2} \int_V \mathbf{M}(\mathbf{r}) \cdot \mathbf{H}_d(\mathbf{r}) d^3\mathbf{r}. \quad (2.12)$$

2.2.2 Magnetocrystalline anisotropy

This is a form of a structure-related magnetic anisotropy, i.e., the fact that some directions of a crystal are easier to magnetize (easy axes) — requiring a lower amount of energy to do so — than others, where it can be very hard to achieve the saturation (hard axes).

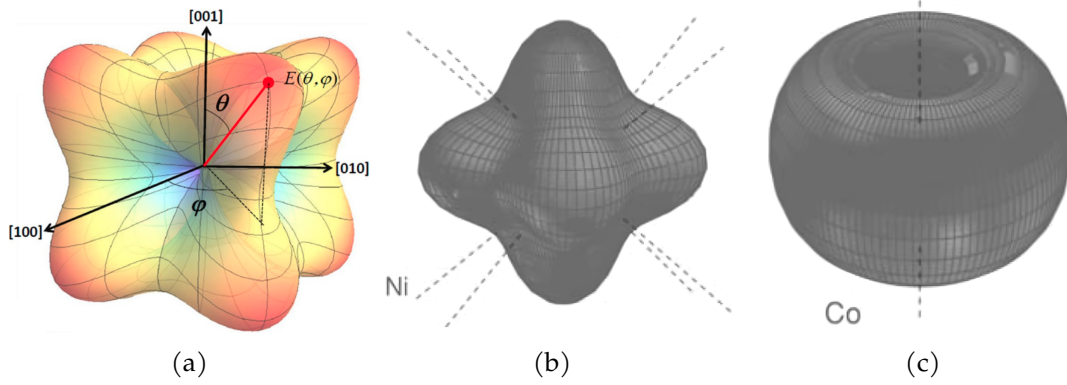
In materials such as iron and cobalt, the **spin-orbit interaction** is behind the magnetocrystalline anisotropy: this is known as **single-ion anisotropy**. In a simple way, the orbits of the electrons are “locked” to the crystal lattice since atoms are surrounded everywhere by their neighbors in a given pattern of that specific material. We know that, for each electron, spin and orbital components of the angular momentum are coupled due to relativistic effects, so the energy required to change the orientation of a spin component is necessarily related to the crystal structure. In less symmetric crystals, the dipole-dipole interaction also has a contribution — sometimes dominantly — to the magnetic anisotropy (**two-ion anisotropy**), since the distribution of magnetic moments is related to a given crystallographic direction (COEY, 2010).

The anisotropy energy $\mathcal{E}_{\text{anis}}$ is defined as the work required to reorient the spin away from an easy direction of magnetization, where a minimum of $\mathcal{E}_{\text{anis}}$ is found. The dependence of $\mathcal{E}_{\text{anis}}$ with respect to polar and azimuthal angles (θ, φ) are shown in Fig. 7 for pure iron, nickel, and cobalt crystals. If multiple easy axes are available due to the unit cell’s geometry, the material possesses **cubic anisotropy** as iron and nickel in Fig. 7(a)–(b) do. For a unique easy axis, such as cobalt in Fig. 7(c), we refer to it as **uniaxial anisotropy**.

Typically, the form of the anisotropy energy is obtained from the quantum, relativis-

¹ This name emerges because this component of the $\mathbf{H}$ -field inside a magnet is directly opposed to $\mathbf{M}$.

Figure 7 – Magnetocrystalline anisotropy energy surfaces. In (a), we show the dependence of $\mathcal{E}_{\text{anis}}$ on the polar and azimuthal angles (θ, φ) for pure cubic iron crystal. It shows the easy axes $[001]$, $[010]$, and $[100]$ at the cubic edges where energy is smaller (blue). Hard axes are located in the diagonals (protuberant in red). In (b), the inverse is found for cubic nickel, where diagonals are the easy axes (dashed lines). In (c), vertical $[001]$ axis (dashed line) is the easy axis for the cobalt hexagonal crystal.



Source: (a) Manh et al. (2020); (b)–(c) Coey (2010, adapted).

tic Hamiltonian (YOSIDA, 1996, p. 34) — which includes a spin-orbit term — via perturbation theory. However, the functional form can be simply deduced by arguments regarding the crystal symmetry (LANDAU; LIFSHITZ, 1960, pp. 149–150). The perturbation theory obtains $\mathcal{E}_{\text{anis}}$ as a power series of the direction cosines of $\mathbf{M}$, associated to the angles between $\mathbf{M}$ and the Cartesian axes. $\mathcal{E}_{\text{anis}}$ should be invariant with respect to the time-reversal transformation², so it should feature only even powers of the cosines.

The uniaxial anisotropy is very common in many materials, so we start by obtaining $\mathcal{E}_{\text{anis}}$ considering the easy axis in the $\hat{\mathbf{z}}$ direction, leading to a power series of $\cos \theta$, where θ is the polar angle in a spherical coordinates system. Therefore, we have

$$\mathcal{E}_{\text{anis}} = \sum_{i=0,1,\dots} K_i (\cos \theta)^{2i} = K_0 + K_1 \cos^2 \theta + K_2 \cos^4 \theta + K_3 \cos^6 \theta + \dots, \quad (2.13)$$

where the K_i constants depend on the material and are given in units of energy per volume (J/m^3). For convenience, we set $\mathcal{E}_{\text{anis}} = 0$ when $\mathbf{M}$ lies in the $\mathbf{z}$ axis and $K_1 > 0$. This is done by transforming the sine into the cosine function and discarding constant isotropic terms and odd powers:

$$\mathcal{E}_{\text{anis}} = \sum_{i=1,2,\dots} K_i (\sin \theta)^{2i} = K_1 \sin^2 \theta + K_2 \sin^4 \theta + K_3 \sin^6 \theta + \dots \quad (2.14)$$

² Otherwise, if we reverse the flow of time, making $t \rightarrow -t$, the sign of $\mathbf{M}$ would change and the crystal energy would depend on the direction of time.

Fortunately, we truncate Eq. 2.14 in its first term because of the **Callen-Callen law** (CALLEN; CALLEN, 1966), which states that as temperature increases the anisotropy weakens, but the decrease occurs at higher rates for the higher-order terms. It reads

$$\frac{K_i(T)}{K_i(T=0)} \approx \frac{1}{M_s} [M(T)]^{\frac{2i(2i+1)}{2}},$$

given that M decreases for increasing T . For example, K_1 weakens as M^3 while K_2 as M^{10} ! So the expression for the uniaxial anisotropy is greatly simplified to

$$\mathcal{E}_{\text{anis}} = K_1 \sin^2 \theta. \quad (2.15)$$

For $K_1 > 0$, the lower energy is found at the z axis. If $K_1 < 0$, we now face an **easy plane** of magnetization because the z axis is now the hard one. For a **hexagonal** crystal such as cobalt, there is only one easy axis, $[001]$ [dashed line in Fig. 7(c)]. See that the protuberances indicate higher energy and rests at a plane perpendicular to $[001]$.

Analogous reasoning is applied to cubic anisotropy, when we expand the anisotropy energy considering three easy axes corresponding to edges of a cube:

$$\mathcal{E}_{\text{anis}} = K_0 + K_1(m_x^2 m_y^2 + m_y^2 m_z^2 + m_z^2 m_x^2) + K_2 m_x^2 m_y^2 m_z^2 + \dots, \quad (2.16)$$

where we discard higher-order terms and find six energy minima when $K_1 < 0$. This corresponds to crystallographic directions $\langle 001 \rangle$, $\langle 010 \rangle$, and $\langle 100 \rangle$. The example of iron is shown in Fig. 7(a). $K_1 > 0$, on the other hand, leads to eight energy minima for nickel, when diagonals such as $[111]$ are the easy axes [Fig. 7(b)].

In the continuum limit, we integrate the uniaxial anisotropy in an easy axis $\hat{\mathbf{n}}$ as

$$\mathcal{E}_{\text{anis}} = -\frac{1}{M_s^2} \int_V K (\mathbf{M} \cdot \hat{\mathbf{n}})^2 d^3 \mathbf{r}, \quad (2.17)$$

where K is known as the **anisotropy constant**, typically given in units of J/m^3 .

2.2.3 Exchange revisited: direct and indirect interactions

Although we begin the discussion regarding the exchange interaction in Appendix C, we now dive into the quantum fundamentals for this interaction and how it is related to the Dzyaloshinskii–Moriya interaction (DMI), to be discussed in the next section and a major source of stability for magnetic skyrmions.

2.2.3.1 Quantum origin of the direct Heisenberg exchange

The exchange interaction is rooted in the indistinguishability of electrons. For a two-electron system at positions $\mathbf{r}_1$ and $\mathbf{r}_2$ occupying states ψ_a and ψ_b — where each ψ_i solves the individual Schrödinger equation (Eq. B.9) — the joint wavefunction Ψ must preserve the electronic density under particle permutation, satisfying $|\Psi(1, 2)|^2 = |\Psi(2, 1)|^2$. Another requirement refers to Ψ itself: the Pauli exclusion principle forbids two (or more) electrons to occupy the same state simultaneously. Therefore, the superposition of the states $\Psi(1, 2) + \Psi(2, 1) = 0$ and thus $\Psi(1, 2) = -\Psi(2, 1)$, so the joint wavefunction is required to be **antisymmetric**.

Whenever the system hamiltonian is allowed to be separated into spatial and spin parts, the solution of Ψ can be decomposed analogously³, reading

$$\Psi(1, 2) = \varphi(\mathbf{r}_1, \mathbf{r}_2) \otimes \chi(s_1, s_2), \quad (2.18)$$

where s_1, s_2 are the spin quantum numbers for each electron. It is required Eq. 2.18 to be antisymmetric and for that we have φ (χ) symmetric (antisymmetric). The overlapping of the individual electron wavefunctions ψ is the mechanism behind the **covalent bonding**, such as the case of H_2 . Since $s_1 = s_2 = 1/2$, the total spin number $S = s_1 - s_2 = 0$ or $S = s_1 + s_2 = 1$.

For $S = 1$, $m_S = -1, 0, 1$, so there are three possible states for the spin wavefunction χ : $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, and $(1/\sqrt{2})[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle]$, where $|\uparrow\downarrow\rangle$ represents the spin function χ for $(m_S)_1 = -(m_S)_2 = 1/2$ for electrons 1 and 2, respectively. In these cases, known as **spin triplet**, χ is symmetric — the exchange of the particles does not change the resulting wavefunction.

For $S = 0$ we find the unique state of $m_S = 0$ (**spin singlet**) by considering that it must be orthogonal to the linear combination showed in the spin triplet, since the eigenvectors of the spin operator⁴ associated to different eigenvalues ($S = 0$ and $S = 1$)

³ This is the case of light atoms, when spin-orbit interaction term mixing $\mathbf{L}$ and $\mathbf{S}$ is small [LS (Russell-Saunders) coupling in Appendix B], or when the system is subjected to an external **uniform** magnetic field.

⁴ In fact, this is true for the discrete spectra of all Hermitian operators (GRIFFITHS; SCHROETER, 2018, p. 128).

are necessarily orthogonal. The simplest combination is, then,

$$\chi_{\text{singlet}} = \frac{1}{\sqrt{2}} [|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle], \quad (2.19)$$

which is antisymmetric. Therefore, the total wavefunction considers the opposite symmetries for the spatial wavefunctions ψ . For the triplet, we adopt the normalized wavefunction

$$\Psi_{\text{triplet}} = \frac{1}{\sqrt{2}} [\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) - \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)] \chi_{\text{triplet}}, \quad (2.20)$$

and for the singlet state,

$$\Psi_{\text{singlet}} = \frac{1}{\sqrt{2}} [\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) + \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)] \chi_{\text{singlet}}. \quad (2.21)$$

Now we are able to obtain the energy E of the states by calculating the expectation value of the Hamiltonian operator $\hat{\mathcal{H}}$, i.e., $E = \int \Psi^* \hat{\mathcal{H}} \Psi d^3\mathbf{r}_1 d^3\mathbf{r}_2$. Applying it for the triplet and singlet states, we calculate their energy difference as

$$E_{\text{triplet}} - E_{\text{singlet}} = 2 \int \psi_a^*(\mathbf{r}_1)\psi_b^*(\mathbf{r}_2) \hat{\mathcal{H}} \psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1) d^3\mathbf{r}_1 d^3\mathbf{r}_2. \quad (2.22)$$

From the considerations of spin alignment we observed that $S = 1$ triplet and $S = 0$ singlet favor **parallel** and **antiparallel** order, respectively. We build the exchange energy term $\mathcal{E}_{\text{ex}}$ by assuming that its magnitude — the cost to flip a given spin and change the magnetic order — is given by the **exchange constant** $J_{\text{ex}} = J_{12} = E_{\text{singlet}} - E_{\text{triplet}}$ as

$$\mathcal{E}_{\text{ex}} = -J_{12} \hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2, \quad (2.23)$$

where the minus sign is a convention. If $J_{\text{ex}} > 0$ ($J_{\text{ex}} < 0$), energy is minimized when the spins are parallel (antiparallel) to each other. This is how we obtain the Heisenberg hamiltonian in Eq. C.6, when many electrons are interacting.

Moreover, in the continuous limit we can start precisely from Eq. C.6. Firstly, we assume that the exchange constant J is uniform and we change the index of j to n to account for the nearest neighbors of the i -th spin. So the exchange energy associated with that spin is

$$\mathcal{E}_i = -J \sum_n \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_n. \quad (2.24)$$

We change the spin moment $\mathbf{S}_i$ in a site i of a cubic lattice with parameter a by the normalized magnetization field $\mathbf{m}(\mathbf{r})$. We assume that the spin is positioned at $\mathbf{r}$ and

its nearest neighbors are at $\mathbf{r} + \boldsymbol{\delta}$, where $\boldsymbol{\delta} = \pm a\hat{x}, \pm a\hat{y}, \pm a\hat{z}$. So the sum index is now over $\boldsymbol{\delta}$:

$$\mathcal{E}_i = -JS^2 \sum_{\boldsymbol{\delta}} \mathbf{m}(\mathbf{r}) \cdot \mathbf{m}(\mathbf{r} + \boldsymbol{\delta}) \quad (2.25)$$

Since the variation of $\mathbf{m}$ is small between $\mathbf{r}$ and $\mathbf{r} + \boldsymbol{\delta}$, we use Taylor expansion to find $\mathbf{m}(\mathbf{r} + \boldsymbol{\delta})$ and the dot product $\mathbf{m}(\mathbf{r}) \cdot \mathbf{m}(\mathbf{r} + \boldsymbol{\delta})$ becomes

$$\mathbf{m}(\mathbf{r}) \cdot \mathbf{m}(\mathbf{r} + \boldsymbol{\delta}) \approx 1 + \mathbf{m} \cdot [(\boldsymbol{\delta} \cdot \nabla) \mathbf{m}] + \frac{1}{2} \mathbf{m} \cdot [(\boldsymbol{\delta} \cdot \nabla)^2 \mathbf{m}]. \quad (2.26)$$

Inserting the last expression and neglecting the constant term,

$$\sum_{\boldsymbol{\delta}} \mathbf{m}(\mathbf{r}) \cdot \mathbf{m}(\mathbf{r} + \boldsymbol{\delta}) \approx \mathbf{m} \cdot \left[\sum_{\boldsymbol{\delta}} (\boldsymbol{\delta} \cdot \nabla) \mathbf{m} \right] + \frac{1}{2} \mathbf{m} \cdot \left[\sum_{\boldsymbol{\delta}} (\boldsymbol{\delta} \cdot \nabla)^2 \mathbf{m} \right]. \quad (2.27)$$

The first sum over the odd terms cancels out because of positive and negative axis neighbors' position, while $\sum_{\boldsymbol{\delta}} (\boldsymbol{\delta} \cdot \nabla)^2 = 2a^2 (\partial_x^2 + \partial_y^2 + \partial_z^2) = 2a^2 \nabla^2$. The summation over the neighbors becomes

$$\sum_{\boldsymbol{\delta}} \mathbf{m}(\mathbf{r}) \cdot \mathbf{m}(\mathbf{r} + \boldsymbol{\delta}) \approx a^2 (\mathbf{m} \cdot \nabla^2 \mathbf{m}). \quad (2.28)$$

Since $\nabla^2 (\mathbf{m} \cdot \mathbf{m}) = 2(\nabla \mathbf{m})^2 + 2\mathbf{m} \cdot \nabla^2 \mathbf{m} = 0$, then $\mathbf{m} \cdot \nabla^2 \mathbf{m} = -(\nabla \mathbf{m})^2$. The final expression for the energy of a single site i is then

$$\mathcal{E}_i = a^2 JS^2 (\nabla \mathbf{m})^2 \quad (2.29)$$

Transforming Eq. 2.29 in an energy density, we divide by the volume of the site (a^3) and by two to avoid counting the interaction twice. Integrating in the whole material yields

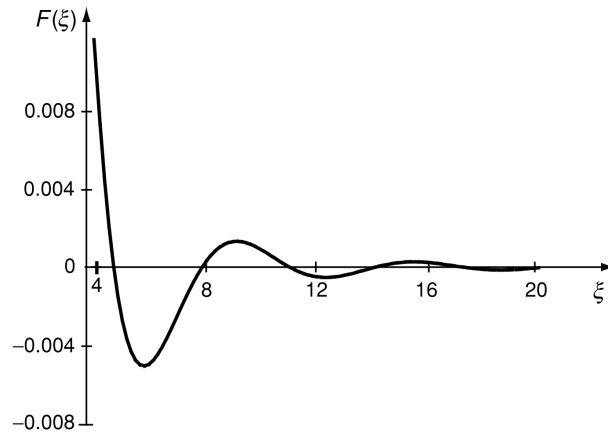
$$\mathcal{E}_{\text{ex}} = A_{\text{ex}} \int_V [(\nabla m_x)^2 + (\nabla m_y)^2 + (\nabla m_z)^2] d^3 \mathbf{r}, \quad (2.30)$$

where $A_{\text{ex}} = JS^2/2a$ is the exchange stiffness constant, given in units of J/m³. A_{ex} was derived for a cubic lattice, however the procedure is analogous for others. By checking Eq. 2.30, we confirm the energy is reduced for $\nabla \mathbf{m} = 0$, when spins are aligned.

2.2.3.2 RKKY coupling

Equation 2.23 is a result of the direct overlap of the electron's wavefunctions, so it is a case of **direct exchange**. There are, however, many cases when an **indirect ex-**

Figure 8 – The magnetization generated by the long-range, oscillatory spatial behavior of RKKY interaction is proportional to the homonym function $F(\xi) = (\sin \xi - \xi \cos \xi) / \xi^4$, where $\xi = 2k_F r$ and k_F is the Fermi wave vector.



Source: Coey (2010, p. 142).

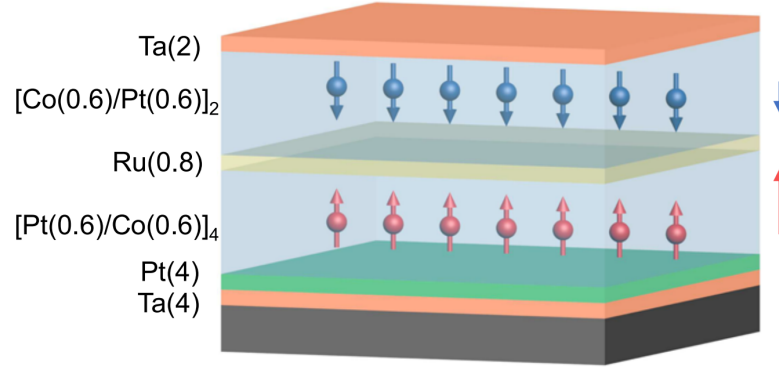
change is mediated via an additional, middle agent. One example, detected in many rare-earth⁵ compounds, is the Ruderman-Kittel-Kasuya-Yosida (**RKKY**) **interaction**. This is an **itinerant** exchange initially proposed for localized nuclear magnetic moments. In RKKY, the conduction electrons of a metal mediate the exchange between magnetic ions. The RKKY interaction is a long range interaction which have a spatial oscillatory dependence (Fig. 8): according to the distance between magnetic moments, it changes its sign and the behavior may be either ferromagnetic or antiferromagnetic.

While the RKKY interaction was originally formulated to describe the coupling between localized moments in a bulk non-magnetic material, the same physical mechanism governs the **indirect exchange coupling of magnetic multilayers**, discovered in 1986 (GRÜNBERG et al., 1986). In these magnetic heterostructures, two nanometric ferromagnetic (FM) layers are separated by a non-magnetic metallic spacer, typically Ru, Ir, or Cu. Here, the spacer is the medium for the conduction electrons, playing the analogous role of the “host metal” in the bulk theory. The spin polarization of these conduction electrons, induced by the moments at the interface of one FM layer, **propagates across the spacer** to interact with the moments of the second layer, effectively coupling to them.

Unlike bulk systems, where the impurities are randomly distributed so atoms couple

⁵ Rare-earth metals are very important elements that receive this denomination because they are often difficult to isolate and purify. Presently, they are in increasing demand for use in advanced electronics and renewable energy technologies, thus presenting a geopolitical challenge in obtaining such elements from the leading reserve regions, such as China and Brazil (BURTON; TEIXEIRA, 2024).

Figure 9 – Illustration of a synthetic antiferromagnet (SAF) stabilized by interlayer coupling. The thin 0.8 nm ruthenium (Ru) spacer layer mediates a strong antiferromagnetic Ruderman–Kittel–Kasuya–Yosida (RKKY) interaction between the two ferromagnetic Co/Pt superlattices. This indirect exchange coupling forces the magnetization of the top $[\text{Co}(0.6)/\text{Pt}(0.6)]_2$ stack (blue arrows, pointing down) to align antiparallel to the magnetization of the bottom $[\text{Pt}(0.6)/\text{Co}(0.6)]_4$ stack (red arrows, pointing up).



Source: Ma et al. (2025).

ferro and antiferromagnetically in an uncontrolled manner, in magnetic multilayers **the coupling strength is tuned by the spacer thickness**. This was demonstrated in early experiments by Parkin, More and Roche (1990) in Fe/Cr/Fe and Co/Ru/Co multilayers, where the coupling sign is inverted simply by varying the spacer thickness by a few Å, allowing the two magnetic layers to spontaneously align parallel or antiparallel to one another, even in the absence of any external field.

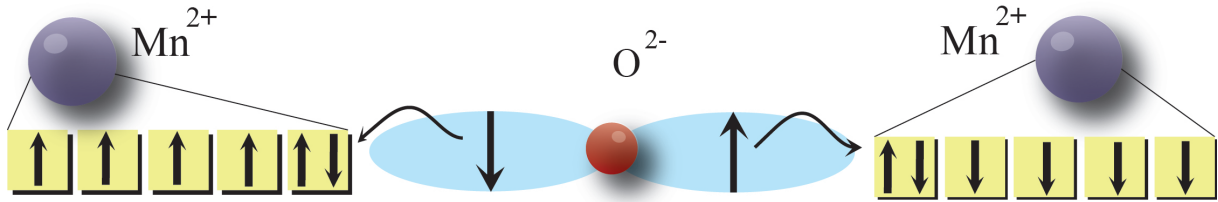
In 1995, Patrick Bruno gave the interlayer coupling a theoretical treatment using the RKKY theory (BRUNO, 1995). The coupling mechanism is described as interferences of the conduction electron waves, creating quantum well states depending on the spacer thickness and the topology of the Fermi surface. This theory provides the justification for treating the interface coupling as an energy density

$$\mathcal{E}_{\text{RKKY}} = -A_i(\mathbf{m}_1 \cdot \mathbf{m}_2), \quad (2.31)$$

where $\mathbf{m}_1$ and $\mathbf{m}_2$ are the magnetization vectors of the FM layers and the sign of A_i is determined by the spacer.

This ability to tune A_i to a negative value leads to the celebrated **synthetic antiferromagnets** (SAFs) (Fig. 9). By selecting a spacer thickness corresponding to an antiferromagnetic coupling peak (such as the first peak of Ruthenium at ≈ 0.7 nm), the two FM layers are locked in the antiparallel configuration. This structure mimics the magnetic order of a bulk intrinsic antiferromagnet offering a significant advantage for

Figure 10 – Superexchange in MnO: electrons from the mediating ion O^{2-} become delocalized and transiently hop from the O^{2-} p -orbital to the manganese d -orbitals in order to lower their kinetic energy. Each Mn^{2+} shares one delocalized electron with O^{2-} and, due to the Pauli exclusion principle, the energy is only reduced if the Mn^{2+} ions are aligned antiferromagnetically.



Source: Wikimedia Commons (2015).

a spintronic device: **the net magnetic moment of the multilayer is zero**, so we eliminate long-range dipolar stray fields that typically destabilize small magnetic textures (e.g., skyrmions) in single layers.

2.2.3.3 Superexchange

Another indirect exchange we mentioned is the **superexchange**, the standard exchange mechanism in transition metal compounds (ionic solids) such as MnO and MnF_2 . It is the interaction between two magnetic ions mediated by a non-magnetic ion in between and it is based on the virtual electronic “hopping” between the magnetic ions and the mediator — typically oxygen (O), fluorine (F) or chlorine (Cl). We call it “super” because typically the exchange interaction is short-ranged, however magnetic ground states indeed reveals the superexchange as a strong, long-ranged interaction.

The superexchange in MnO is illustrated in Fig. 10. The virtual hopping occurs because of the electron confinement: due to the uncertainty principle, electrons lower kinetic energy by “spreading out”. However, when electrons delocalize they should satisfy the Pauli exclusion principle upon interacting with the metal orbitals. Since the two electrons hopping from the filled oxygen p -shell have opposite spins, they can only be accommodated if the neighboring Mn^{2+} ions have antiparallel spins. Consequently, superexchange favors antiferromagnetic ordering.

2.2.3.4 Anisotropic exchange

In general, the superexchange features an isotropic, Heisenberg-like hamiltonian and the energy depends only on the relative angles between the magnetic moments. However, when spin-orbit interaction is also taken into account, the magnetic moments **orientation with respect to the crystal lattice** also becomes important, so the exchange may be **anisotropic**.

In order to account for anisotropy, we write the Heisenberg hamiltonian as

$$\mathcal{H}_{\text{ex}} = -\frac{1}{2} \sum_{i,j} \hat{\mathbf{n}}_i \mathcal{J}_{ij} \hat{\mathbf{n}}_j, \quad (2.32)$$

where $\mathcal{J}_{ij}$ is a tensor given by

$$\mathcal{J}_{ij} = \begin{bmatrix} J_{ij}^{xx} & J_{ij}^{xy} & J_{ij}^{xz} \\ J_{ij}^{yx} & J_{ij}^{yy} & J_{ij}^{yz} \\ J_{ij}^{zx} & J_{ij}^{zy} & J_{ij}^{zz} \end{bmatrix}. \quad (2.33)$$

The tensorial notation is useful for many magnetically frustrated systems and in spintronics, where anisotropy plays a major role in the exchange. The general form of Eq. 2.33 can be decomposed into three different components $\mathcal{J} = \mathcal{J}^{\text{iso}} + \mathcal{J}^{\text{sym}} + \mathcal{J}^{\text{anti}}$, where

- $\mathcal{J}^{\text{iso}} = J_{ij} \mathbf{I}$ is the isotropic, symmetric part, and $\mathbf{I}$ the identity matrix, resulting in the standard Heisenberg hamiltonian (Eq. C.6);
- $\mathcal{J}^{\text{sym}}$ is the anisotropic, symmetric part, with elements $\mathcal{J}_{ij}^{\text{sym}} = \frac{1}{2}(\mathcal{J}_{ij} + \mathcal{J}_{ji}) - J_{ij}\delta_{ij}$, including terms such as Kitaev or pseudo-dipolar interactions (MORIYA, 1960); and
- $\mathcal{J}^{\text{anti}}$ is the anisotropic, antisymmetric component, with elements $\mathcal{J}_{ij}^{\text{anti}} = \frac{1}{2}(\mathcal{J}_{ij} - \mathcal{J}_{ji})$, resulting in the asymmetric exchange we discuss in the next section: the Dzyaloshinskii-Moriya interaction (DMI).

2.2.4 Dzyaloshinskii–Moriya interaction

The decomposition of Eq. 2.33 in the previous section was performed by Japanese physicist Tôru Moriya (MORIYA, 1960). Moriya expanded the Anderson superexchange

theory on electron hopping to account for the spin-orbit interaction as a perturbation. Moriya was able to explain microscopically the weak ferromagnetism in compounds such as $\alpha\text{-Fe}_2\text{O}_3$ [hematite, Fig. 11(a)] and other Mn and Co carbonates.

Theoretically, $\alpha\text{-Fe}_2\text{O}_3$ is an antiferromagnetic insulator, so its spins should produce a zero net magnetization [Fig. 11(b)]. However, experimental data confused physicists at the time because it exhibited a small ferromagnetic response above the so-called Morin temperature $T_M \approx 250$ K. For $T < T_M$ magnetic moments comfortably lie in an antiferromagnetic pattern along the trigonal axis [black arrows along vertical direction in Fig. 11(b)]. However, at $T = T_M$ the sign of the magnetic anisotropy changes: instead of favoring the trigonal axis, the basal plane (001) is now a better deal for magnetic moments to lower the system energy, which results in a 90° spin rotation. Igor Dzyaloshinskii showed by symmetry arguments the existence of an interaction which favored the **spin canting** — when the magnetic moments are tilted by a an angle about their axis — for $T > T_M$ at the basal plane, producing a finite net magnetization in a state that was, after all, an intrinsic property of the material.

In the words of Moriya addressing the problem in his 1960 paper,

among the terms of the form allowed under this crystal symmetry **there was a term which favors the canted spin arrangement rather than the antiferromagnetic one** and therefore is essential to the weak ferromagnetism. (MORIYA, 1960, p. 92, emphasis added).

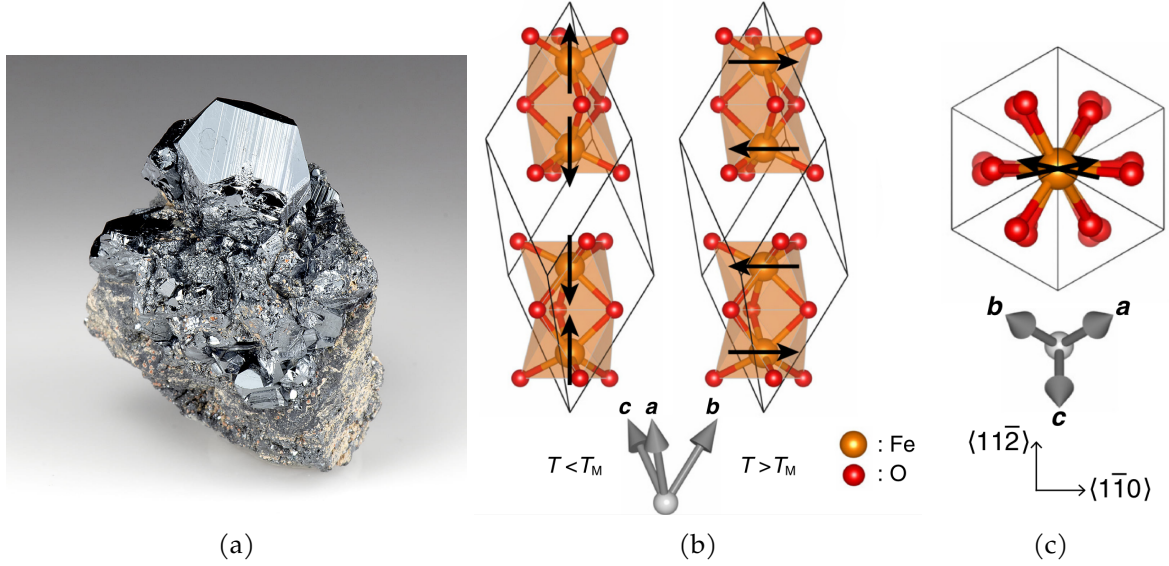
The phenomenological term Moriya is referring to is given by

$$\mathcal{E}_{\text{DMI}} = \mathbf{D} \cdot (\mathbf{S}_1 \times \mathbf{S}_2), \quad (2.34)$$

where $\mathbf{D}$ is now called the Dzyaloshinskii-Moriya (DM) vector. In $\alpha\text{-Fe}_2\text{O}_3$, $\mathbf{D}$ is found to be parallel to the trigonal axis, so what happens when $T > T_M$ is that spins are lying in the basal plane making a finite $\mathcal{E}_{\text{DMI}}$ to appear. Because $\mathbf{D}$ points along the trigonal axis ($\hat{z}$), it forces the “twist” to occur in the plane perpendicular to it. Note that $\mathcal{E}_{\text{DMI}} = 0$ for $T < T_M$, since the magnetic moments are parallel to the trigonal axis. This extra interaction is small, but makes the magnetic moments to have a tiny net component along the $\hat{z}$ direction — now becoming uncompensated — and then to produce a weak ferromagnetic response in the material [Fig. 11(c)].

$\mathcal{E}_{\text{DMI}}$ is the Dzyaloshinskii-Moriya interaction (DMI), obtaining such a form from the anisotropic, antisymmetric exchange component $\mathcal{J}^{\text{anti}}$ we discussed earlier and it

Figure 11 – α -Fe₂O₃ structure and Morin transition: (a) The hematite crystal. (b) α -Fe₂O₃ orthorhombic cell unit and magnetic ground states for temperatures below and above T_M , where black arrows indicate the direction of magnetic moments. For $T < T_M$, the material is a pure antiferromagnet. For $T > T_M$, spins rotate by 90° and, thanks to the Dzyaloshinskii-Moriya interaction (DMI), do not compensate each other but are slightly canted producing weak ferromagnetism. (c) Projection of the crystal structure onto the (111) plane. Here, the canting angle is very small: about 0.065° .



Source: Weinrich Minerals, Inc. (2025) (a) and Miyawaki et al. (2017) (b–c).

is given by

$$\mathcal{E}_{\text{DMI}} = -\frac{1}{2} \sum_{i,j} \hat{\mathbf{n}}_i \mathcal{J}_{ij}^{\text{anti}} \hat{\mathbf{n}}_j = -\frac{1}{2} \sum_{i,j} \mathbf{D}_{ij} \cdot (\hat{\mathbf{n}}_i \times \hat{\mathbf{n}}_j) \quad (2.35)$$

when the DM vector is written as

$$\mathbf{D}_{ij} = \frac{1}{2} \left(J_{ij}^{yz} - J_{ij}^{zy}, J_{ij}^{xz} - J_{ij}^{zx}, J_{ij}^{xy} - J_{ij}^{yx} \right). \quad (2.36)$$

In the continuous limit, we obtain an expression for Eq. 2.34 assuming the effect of DMI in a pair of magnetic moments for simplicity, as did Camley and Livesey (2023). We position the DM vector along the $\hat{x}$ direction and the pair of magnetic moments are joined together by a distance vector $\mathbf{r}_{ij}$ in the $\hat{z}$ direction. Therefore,

$$\mathcal{E}_{\text{DMI}} = D_{12} \hat{x} \cdot (\mathbf{S}_1 \times \mathbf{S}_2) = D_{12} (\mathbf{S}_1 \times \mathbf{S}_2)_x = D_{12} (S_1^y S_2^z - S_1^z S_2^y) \quad (2.37)$$

Let us assume a chain of magnetic moments along $\hat{z}$ direction, instead of the initial pair. The separation between moments is the lattice parameter a and a spin is labeled by i ,

with two neighbors $i - 1$ and $i + 1$. The i -th spin then interacts with both neighbors via

$$\mathcal{E}_{\text{DMI}} = D_{ij}(S_{i-1}^y S_i^z - S_{i-1}^z S_i^y) + D_{ij}(S_i^y S_{i+1}^z - S_i^z S_{i+1}^y) \quad (2.38)$$

$$= 2aD_{ij} \left[\frac{(S_{i-1}^y S_i^z - S_{i-1}^z S_i^y)}{2a} + \frac{(S_i^y S_{i+1}^z - S_i^z S_{i+1}^y)}{2a} \right] \quad (2.39)$$

$$= 2aD_{ij} \left[S_i^y \left(\frac{S_{i+1}^z - S_{i-1}^z}{2a} \right) - S_i^z \left(\frac{S_{i+1}^y - S_{i-1}^y}{2a} \right) \right]. \quad (2.40)$$

Using the finite-difference approximation and considering a smoothly varying spin field S , we transform the terms in parentheses into derivatives along the chain sites:

$$\mathcal{E}_{\text{DMI}} \approx 2aD_{ij} \left[S_i^y \frac{\partial S^z}{\partial z_i} - S_i^z \frac{\partial S^y}{\partial z_i} \right], \quad (2.41)$$

where we assume DM vector to be spatially homogeneous. Replacing the spin moment S by $\mathbf{m} = \mathbf{M}/M_s$ in the site i (with a volume a^3), then

$$\mathcal{E}_{\text{DMI}} \approx D \left[m_y \frac{\partial m_z}{\partial z} - m_z \frac{\partial m_y}{\partial z} \right], \quad (2.42)$$

where $\mathbf{D} = \mathbf{D}_{ij}/a^2$ is the **micromagnetic DM vector** given in J/m² and the energy density $\mathcal{E}_{\text{DMI}}$ in J/m³. Observing Eq. 2.42, we see that a gradient of $\mathbf{m}$ produces a difference in the energy according to the direction. Therefore, DMI establishes a “**distinct handedness**” favored by the system.

Symmetry is, then, very important to determine whether a given material features DMI or not. Incorporating superexchange theory in his 1960 paper, Moriya showed that the DM vector is proportional to the change of $\mathbf{L}$ during the electron hopping (MORIYA, 1960, p. 93). The addition of a perturbative spin-orbit interaction affects the trajectory of hopping electrons, tending to “twist” it and disturbing orbital motion. This is energetically favorable if the inversion symmetry is (locally) broken in the bonding site of the magnetic moment, since DM vector is not canceled out and phenomena like the spin canting can be appreciable.

This scenario is connected to a major consequence of DMI. Besides explaining weak ferromagnetism, the distinct character exhibited by Eq. 2.42 shows that DMI gives rise to important **non-collinear** magnetic structures when competing with other interactions such as the direct exchange, establishing **chiral magnetism**. This occurs since energy is minimized for perpendicular moments in Eq. 2.34, instead of collinear ferro or antiferromagnetic arrangements of Heisenberg energy. Chirality is the property that a given

object has a non superimposable mirror image, so **it lacks inversion symmetry**, which is indeed the case of the $\alpha\text{-Fe}_2\text{O}_3$ bonding of magnetic moments.

In bulk magnetic materials, though, DMI is small — typically below 1% of direct Heisenberg exchange magnitude (CAMLEY; LIVESEY, 2023). However this is not the case for **magnetic thin films**, where a strong DMI plays a significant role in defining magnetic structures. Noncollinear magnetic textures such as helicoids, conics, and **magnetic skyrmions** then arise in the presence of DMI. As you are aware, the latter is the main subject of our work and is to be deeply covered in the next chapter.

2.3 Magnetic Skyrmions

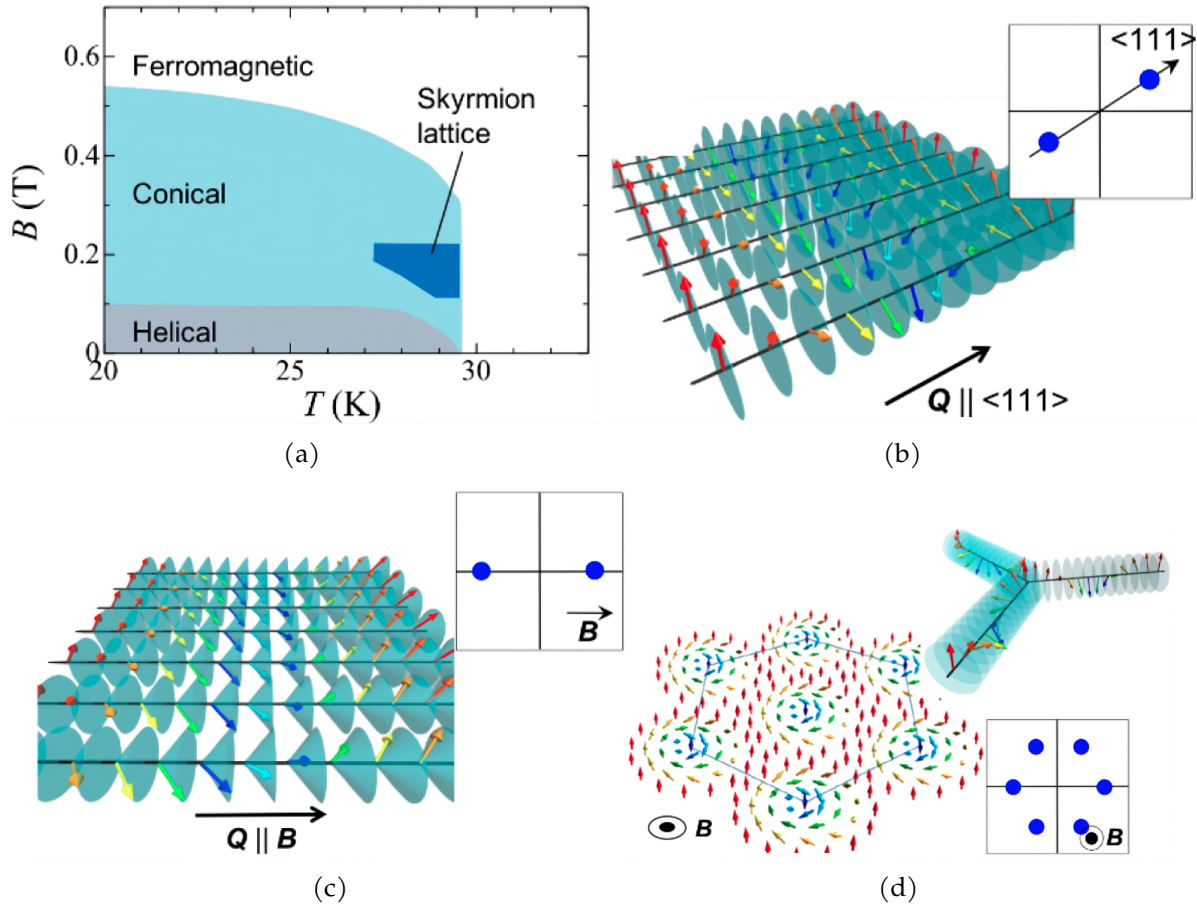
In this section, we address the theory of **magnetic skyrmions**, beginning with the phenomenon of chiral magnetism and non-collinear magnetic structures. We illustrate how skyrmions emerge in bulk materials and ferromagnetic thin films, how they are stabilized, and their major properties regarding the induced-motion by spin-polarized currents and the skyrmion Hall effect (SkHE).

2.3.1 Chiral magnetic phases

As we discuss in Sec. 2.2.4, the tendency of spins to twist under Dzyaloshinskii-Moriya interaction (DMI) gives us exotic magnetic textures characterized by non-trivial winding of magnetization, with interesting properties for spintronic applications. A remarkable, textbook example of a chiral magnet is the **MnSi**, a weak itinerant ferromagnet. A representation of the MnSi phase diagram in Fig. 12 exhibits distinct magnetic phases according to temperature (T) and magnitude of an applied magnetic field (B). If the magnitude of B is strong enough ($B \approx 0.6$ T), the material is field-polarized with spins aligned just as an ordinary ferromagnet. Paramagnetic behavior is observed as T increases above $T_c \approx 30$ K and thermal energy breaks the magnetic order. Moreover, in the center of the diagram we note three different phases: helical, conical, and the skyrmion lattice.

For a small magnetic field, the **helical phase** (in gray) appears illustrated in Fig. 12(b). This magnetic structure is a combination of rotation and spatial translation of the magnetization vector along a “screw” axis (or **helical axis**) $\hat{h}$, forming a helix structure

Figure 12 – (a) The magnetic phase diagram of MnSi in the temperature (T) vs. magnetic field (B) plane, delineating helical, conical, and skyrmion lattice phases. (b) Representation of the helical spin structure observed at $B = 0$, with the propagation vector Q parallel to the $\langle 111 \rangle$ crystal axis. (c) The conical spin structure induced by B , where Q aligns parallel to B . (d) The skyrmion lattice phase, illustrating the hexagonal arrangement of particle-like skyrmion tubes (top-down and perspective views). Insets in panels (b)–(d) show blue spots representing the small-angle neutron scattering (SANS) detector's image.



Source: Tokura and Kanazawa (2020) (adapted). Original data by Mühlbauer et al. (2009).

along the $\langle 111 \rangle$ direction with a spatial period of about 18 nm at 4.2 K (ISHIKAWA et al., 1976). Experiments using small-angle neutron scattering (SANS) report that the spiral period (“pitch”) in MnSi shows no dependence on B or T within helical phase (GRIGORIEV et al., 2006), what is expected since the pitch scales with A_{ex}/D , which remains roughly constant. Varying T and B also does not affect the single-handedness of the spiral, i.e., the magnetic moments rotate along $\hat{h}$ in a unique sense [clockwise in Fig. 12(b)]. Eventually, the helical order is destroyed by thermal fluctuations for $T \approx 28$ K.

Further increase of B leads to a rotation of the helix wave vector toward the magnetic field, establishing the **conical phase**. In this **field-polarized** state, spins rotate

around a cone whose defining axis lies in the direction of $\mathbf{B}$ [Fig. 12(c)]. It represents a compromise between the DMI (which favors twisting) and the Zeeman energy (which favors alignment with $\mathbf{B}$). As B increases even further, the spins tilt towards the field direction. The cone angle closes completely when we increase B up to the critical field B_{c2} , and the system enters the ferromagnetic state (ISHIKAWA et al., 1977).

In a small sector of the phase diagram of Fig. 12(a) (in dark blue) we finally encounter the **skyrmion lattice phase**, just below the magnetic order temperature T_c . The skyrmions are **localized swirls of magnetization** forming a triangular lattice immersed in a ferromagnetic background and oriented perpendicularly to $\mathbf{B}$ [Fig. 12(d)]. In the detail of panel (d), magnetic moments twist along the edge of the lattice's hexagon ring going from spin up (red arrow) to spin down (blue arrow) and then back to spin up again. This kind of spatial reversal in magnetization is a common feature of all skyrmions.

Inspired by the Abrikosov vortex in type-II superconductors, Alexei Bogdanov and Dmitrii Yablonskii predicted that skyrmions in magnetic systems could be a thermodynamically stable state in crystals with broken inversion symmetry (BOGDANOV; YABLONSKII, 1989) — rather than just metastable excitations. Almost two decades later, Roessler, Bogdanov and Pfleiderer (2006) developed a phenomenological continuum theory for cubic helimagnets such as the MnSi, correctly predicting **spontaneous** — i.e., without the help of external magnetic fields — skyrmionic ground states as a result of competing exchange and DMI. This was on the brink of experimental discovery: the skyrmion triangular lattice was first detected in 2009 by Mühlbauer et al. (2009)⁶ using low-temperature neutron scattering, where the incident magnetic field was applied parallel to the neutron beam. The result was the phase diagram of Fig. 12 we discussed.

We provide a depiction of the detector image in the square panels of Fig. 12(b)–(d). In experiments using the small-angle neutron scattering (SANS) technique, we essentially obtain the reciprocal space map (Fourier transform) of the magnetic periodicity inside the crystal. The incident neutrons carry magnetic moment and interact with the periodic magnetic structure. Because of spatial periodicity, they act as a magnetic diffraction grating, scattering neutrons into the Bragg peaks [circular blue spots

⁶ The discovery was made almost by accident by then doctoral student Stephen Mühlbauer on April 1 at the Technical University of Munich, for which his professor (coincidentally, Christian Pfleiderer!) initially thought it was an April Fool's joke on him (Technische Universität München, 2009).

of Fig. 12(b)–(d)]. The radial position of the spots reveals the magnitude of the wave vector $\|\mathbf{Q}\| = 2\pi/\lambda_{mag}$, where λ_{mag} is the “pitch” of the spiral. Crucially, the angular position of the spots gives us the orientation of the magnetic state. In Fig. 12(b), $\mathbf{Q}$ lies in the $\langle 111 \rangle$, pinned to the crystal lattice diagonals. Conical and skyrmion phases decouple from the lattice and aligns to $\mathbf{B}$, however in a different manner. In panel (c), two blue spots of conical phase indicate $\mathbf{Q} \parallel \mathbf{B}$. Conversely, in panel (d) the skyrmion lattice is defined by a six-fold orientation in a plane perpendicular to $\mathbf{B}$, so $\mathbf{Q} \perp \mathbf{B}$.

What these three chiral phases observed in MnSi share in common is the **lack of inversion symmetry** in the B20 crystal structure. The geometry allows DMI to favor the perpendicular rotation of spins, producing a **fixed chirality** (or **handedness**): the textures twists in a specific preferred direction (to the left or to the right), according to the sign of the DM vector that is, in turn, defined by the crystallographic structure. The length scale of the magnetic phase is then modulated by the ratio between competing exchange and DMI, though the wave vector $\mathbf{Q}$ orientation differs dramatically.

Despite the common origin of chiral states, the **non-trivial** topological properties distinguishes the skyrmion. Different from the helical and conical phases, we cannot continuously deform skyrmions into the topologically trivial ferromagnetic state. This is of great importance for spintronics applications since skyrmion topology enables a protection against deformations, defects and thermal fluctuations, making it more stable, as we shall discuss in the next section.

2.3.2 Topological properties of a skyrmion

In the framework of skyrmions, an important property arises: **topological protection**. In the field of topology, the properties of physical objects are not defined by specific geometric lengths or angles, but rather by the global connectivity and the **shape** of a configuration associated to a given physical field. As described in Lancaster (2019),

the topologist imagines all objects as made from a moldable clay that can be **stretched and shrunk**, subject to the rules that holes cannot be punched in the clay, and any existing holes cannot be healed up. Shapes that can be transformed into each other following the rules of this game are said to be **homeomorphic** to each other. (emphasis added).

Consequently, we conclude that two non-homeomorphic field configurations cannot be transformed into the other through continuous deformation — as one does in the famous example of the donut/coffee mug homeomorphism.

At first glance, the stretching and bending of fields by topologists seems inappropriate for describing magnetic phenomena, given that magnetic materials are ultimately constructed from discrete, discontinuous ordinary matter as we observe in a crystal lattice. However, the exchange interaction favors **continuity** (FINOCCHIO; PANAGOPOULOS, 2021), which culminates in a fair description of magnetism via the spin averaging provided by the slowly varying, **continuous** field $\mathbf{m}(\mathbf{r})$. Besides proving itself useful in the micromagnetic simulations adopted in this thesis, the continuum approximation justifies the association of topology to magnetism. The topological protection, for example, can be translated into magnetic systems as an **energy barrier** between two distinct magnetic states, e.g., the skyrmion and the ferromagnetic phases. The connection between topology and magnetism is further illustrated by the first proposal of skyrmions by Tony Skyrme⁷, which predates the work of Bogdanov and colleagues in 1989, and was applied in a drastically different context: nuclear and high-energy physics.

2.3.2.1 Solitons and the Skyrme model

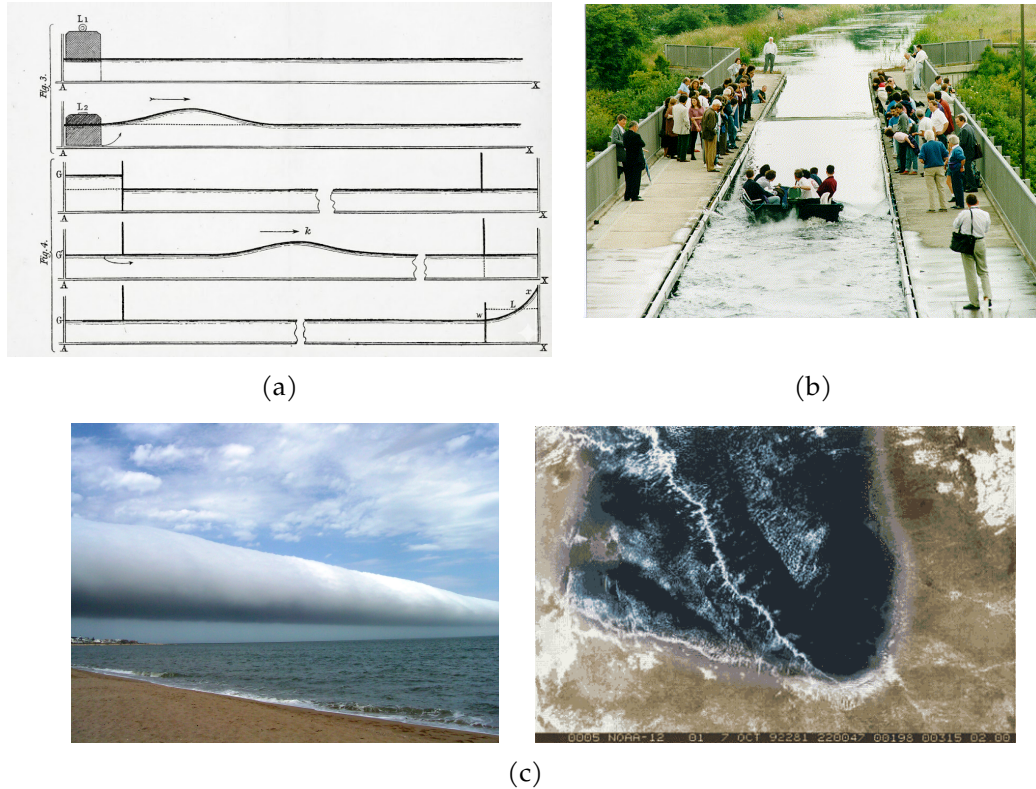
In the 1960s, Skyrme (1961) describes how baryons — such as protons and neutrons (nucleons) composing the atomic nuclei — can emerge with stability via force-mediating fields of lighter particles known as pions (π^0 or $\pi^\pm$). Back then, the nucleons were thought to be fundamental point particles — the model for quarks would arise little after Skyrme work (ZWEIG, 1964; GELL-MANN, 1964). The idea of Skyrme was that baryons emerged as **topological solitons**⁸ (or **topological defects**) formed by “twists” in the mediating pion field.

Solitons are stable, nonlinear, localized wave solutions first documented in 1834 by the naval architect John Scott Russell, who observed a “a large solitary elevation, a rounded, smooth and well-defined heap of water” (KASMAN, 2018) generated by a stopping canal boat [Fig. 13(b)–(c)]. There are many models in which solitons appear as a solution, from cosmic strings in cosmology (GARRIGA; VERDAGUER, 1987) to the dynamics of exquisite roll clouds [Fig. 13(d)]. In the 1960s, Zabusky and Kruskal studied one of them: the **Korteweg–de Vries (KdV) equation**, which models waves of shallow

⁷ The skyrmion derives its name from this British physicist — who, in addition to his relevant theoretical contributions, arguably possesses one of coolest names in the history of physics.

⁸ This terminology appeared only after the work of Skyrme.

Figure 13 – Historical and natural observations of the solitary wave or solitons. (a) Diagrams from Russell’s 1844 Report on Waves, illustrating the generation and propagation of a solitary wave in a water tank located in his back garden. (b) A modern recreation of a solitary wave traveling through the Scott Russell aqueduct at the same Union Canal in 1995, mimicking Russell’s original observation. (c) An atmospheric soliton, specifically a Morning Glory cloud (*volatus*) in Uruguay (left) and the satellite image of such type of cloud in Australia (right).



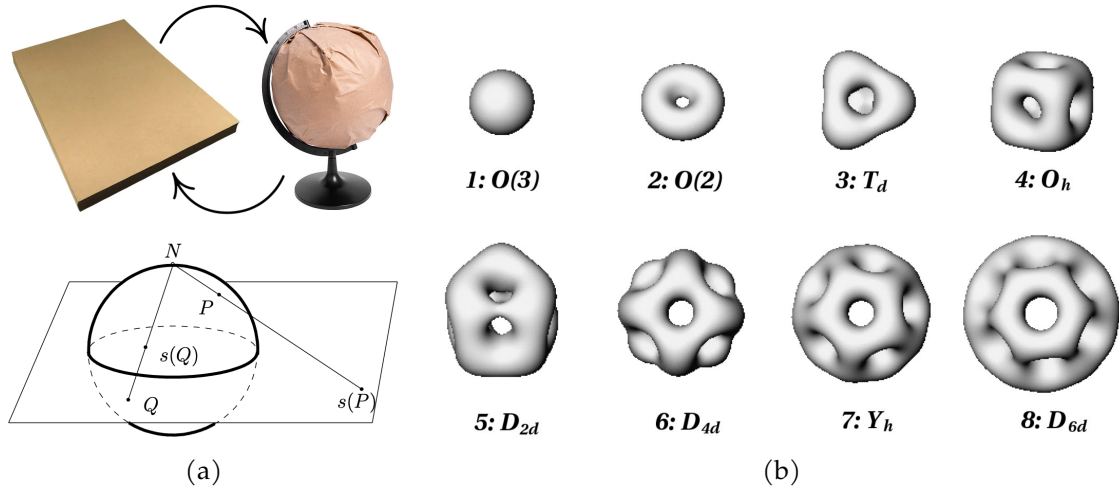
Source: Russell (1844) (a); Technische Universität Wien (1995) (b); Eberl (2009) and National Oceanic and Atmospheric Administration (1998) (c).

water surfaces (KASMAN, 2018). They observed that the Russell’s solitary wave, when subjected to perturbation, broke apart into a train of localized pulses at different speeds. Instead of merging or dispersing as ordinary waves, they exhibit a **particle-like behavior** (ZABUSKY; KRUSKAL, 1965).

In the case of Skyrme, he analogously describes protons and neutrons as solitons via the **non-linear σ model** of quantum field theory, interpreting particles as arising from “knots” in the pionic mediating field $U(\mathbf{r})$. However, the **Derrick’s theorem** asserts that localized solutions of a wide class of non-linear wave equations were possible, but unstable (DERRICK, 1964), thus particles were expected to collapse under the standard σ model in (3+1) dimensions. Nevertheless, Skyrme added an ad-hoc⁹ short-range repulsion term in the Lagrangian, preventing soliton instability in a modified σ model

⁹ This term was later interpreted as the exchange of an excited state of the pion, the heavy ρ meson (ZAHED; BROWN, 1986, p. 49).

Figure 14 – The Skyrme model for baryons. (a) Illustration of the stereographic projection. In the context of the model, the wrapping of the 2D plane into a usual sphere is a lower-dimensional analogue of mapping the entire physical space ($\mathbb{R}^3$) onto the field manifold (S^3). In the bottom diagram, a point P in the sphere is taken to $s(P)$ in the plane. (b) Skyrmion configurations obtained by numerical simulations for several baryon numbers B . $B = 1$ corresponds to the Skyrme nucleon, while $B > 1$ represent other atomic nuclei.



Source: (a) Walmart Canada (2026), iStock (2025), and Lyons (2021); (b) Manton (2022).

(SKYRME, 1961).

Crucially, Skyrme provides a topological interpretation of the **baryon number** (B) **conservation**. He regards a stable baryon as a given field configuration $U_i(\mathbf{r})$, a map from the physical space S^3_{space} (“domain”) to the target space S^3_{target} (“image”) where we find the values of U . Since domain and image spaces have the same geometry, we can assign an integer Q , known as the **topological invariant** (or **winding number**), to describe how many times $U_i(\mathbf{r})$ “wraps” around the target space. This is analogous to the wrapping of a typical sphere (S^2) by a 2D surface (e.g., a sheet of paper) [Fig. 14(b)]. In the Skyrme model, Q is interpreted as the baryon number B , where some field configurations are shown in Fig. 14(c): the trivial vacuum solution ($B = 0$), a single baryon ($B = 1$), and the atomic nuclei ($B > 1$). If two configurations have different Q , then one cannot be morphed smoothly and continuously into the other, so they feature **topological protection**, i.e., a state with $Q = B_i$ cannot change into $Q = B_j$ by continuous deformation, since the process would require overcoming an infinite energy barrier.

2.3.2.2 Magnetic skyrmion as a 2D topological soliton

By considering a two-dimensional magnetic skyrmion, we obtain an analogous behavior with respect to topology as it is also regarded as a topological soliton. In Condensed Matter systems, skyrmions are often realized in **ultrathin magnetic films**, where the main features occur in the plane. Therefore the physical space is effectively two-dimensional, as long as the thickness of the sample is not much larger than the exchange interaction characteristic length $\Lambda = \sqrt{2A_{ex}/\mu_0 M_s^2}$ (FINOCCHIO; PANAGOPOULOS, 2021).

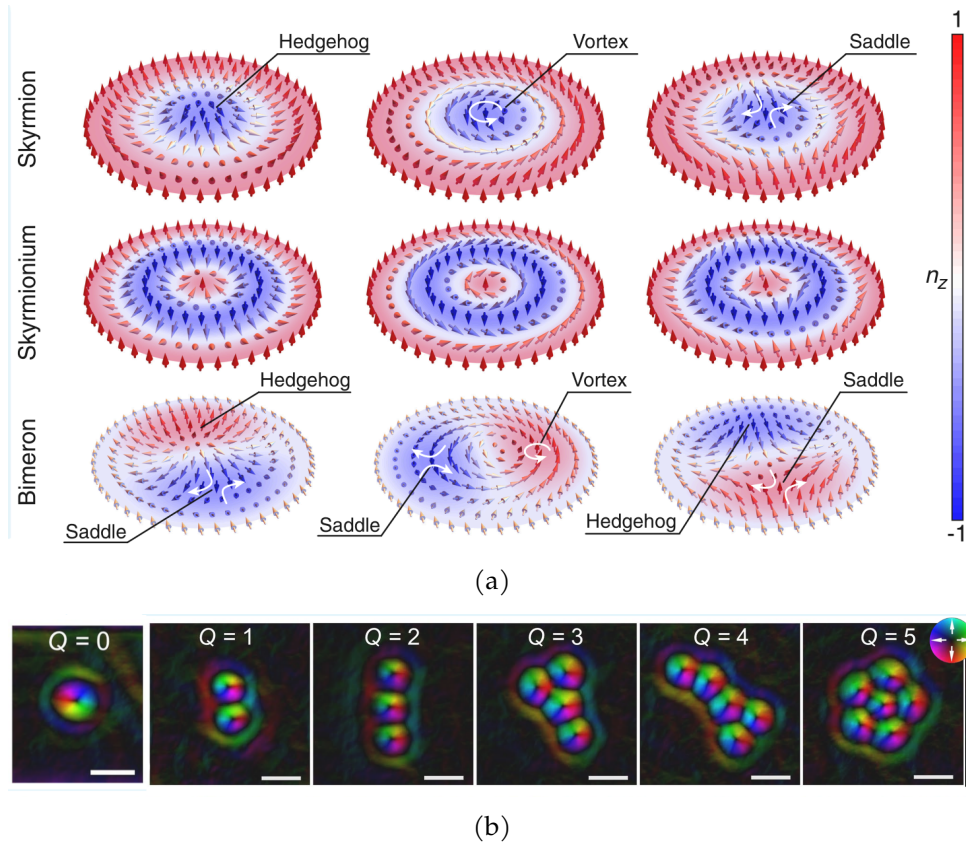
Instead of Skyrme's scalar field, the magnetization vector field $\mathbf{m}(\mathbf{r}) = \mathbf{m}(x, y)$ is wrapped around the unit Riemann sphere [Fig. 14(b)]. In this procedure, we obtain a topological invariant called the **skyrmion number** or the skyrmion **topological charge** Q . We calculate Q considering the wrapping of $\mathbf{m}(\mathbf{r})$ as a **stereographic projection** by using

$$Q = \frac{1}{4\pi} \int \mathbf{m} \cdot \left(\frac{\partial \mathbf{m}}{\partial x} \times \frac{\partial \mathbf{m}}{\partial y} \right) d^2 \mathbf{r}, \quad (2.43)$$

where the integration is performed across the 2D plane. For 2D standard skyrmions, $|Q| = 1$, meaning that the magnetization field covers all possible orientations on the unit sphere exactly once.

Other two-dimensional field configurations of $\mathbf{m}(\mathbf{r})$ with alternative values of Q are possible, just as Skyrme's model. Figure 15 exhibits a few cases. In the first row, typical hedgehog (left) and vortex (center) patterns are shown in Néel and Bloch skyrmions with $Q = -1$, while a **antiskyrmion** characterized by opposite $Q = +1$ appears in the left with a saddle pattern. The **skyrmionium** (second row) is a structure where a central skyrmion is embedded within a concentric ring of opposite magnetization, effectively constituting a skyrmion-antiskyrmion pair with $Q = 0$. In contrast, the **bimeron** (third row) is the in-plane topological counterpart of a skyrmion (GÖBEL et al., 2019) and it is composed of a meron-antimeron pair preserving $|Q| = 1$ [see distinct pairing of vortex/hedgehog core with a saddle in third row of Fig. 15]. Finally, in panel (b), we exhibit the **skyrmion bags**, a class of composite magnetic solitons with arbitrary integer Q . It consists of a domain wall boundary (the bag) encapsulating N smaller skyrmions of opposite polarity. Q is determined by the sum of the winding numbers ($Q = N - 1$). In this framework, $Q = 0$ ($N = 1$) corresponds to the skyrmionium.

Figure 15 – Overview of non-trivial textures with varying Q . In (a), representation of the out-of-plane component of magnetization for the standard skyrmion (top row), the skyrmionium (middle row), and the bimeron (bottom row). (b) Lorentz Transmission Electron Microscopy (LTEM) color-map reconstructions of skyrmion bags carrying integer topological charges ranging from $Q = 0$ to $Q = 5$ and consisting of a domain wall boundary encapsulating N nested skyrmions, where $Q = N - 1$. The $Q = 0$ bag (far left) is topologically equivalent to the skyrmionium.

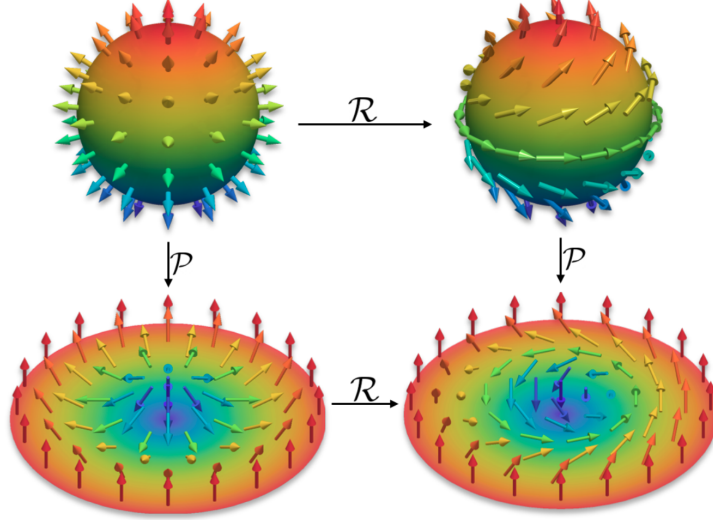


Source: Adapted from Shen (2021) (a) and Tang et al. (2021) (b).

The ferromagnetic phase, on the contrary, is a **topologically trivial** one. Since spins point only in one direction, they cannot “wrap” the Riemann sphere not even once, meaning $Q = 0$. Consequently, a skyrmion with $|Q| = 1$ is **topologically distinct** of the ferromagnetic phase and cannot be taken to the (topologically trivial) FM phase through a smooth, continuous deformation. This ensures a **topological protection** against deformations for the skyrmion in a film with ferromagnetic background.

Figure 16 exhibits a **Néel** skyrmion ($Q = -1$) in the bottom left corner. The magnetization swirls in the radial direction from $+\hat{z}$ in the periphery to $-\hat{z}$ in the center. By applying a rotation $\mathcal{R}$ of $\pi/2$ about the axis perpendicular to the skyrmion plane, we still obtain a skyrmion with $Q = -1$ but with the magnetization “laying down” on the plane, a **Bloch** skyrmion. The operation is correspondingly performed in the mag-

Figure 16 – Stereographic projection and rotation for magnetic skyrmions: the hedgehog pattern (top left) of magnetization field $\mathbf{m}(\mathbf{r})$ (shown as color-coded arrows) is projected via $\mathcal{P}$ as the Néel skyrmion onto the plane (bottom left) whereas the twisted pattern (top right) is projected as a Bloch skyrmion (bottom right). The two skyrmions are topologically equivalent since both are connected by a continuous global rotation $\mathcal{R}$ of $\mathbf{m}$ by 90° about the perpendicular axis.



Source: Lancaster (2019).

netic moments lying in the Riemann sphere (top), where the stereographic projection $\mathcal{P}$ connects the magnetization vectors from the sphere to the plane. Note that the rotation $\mathcal{R}$ is a **continuous** operation, so Néel and Bloch skyrmions are topologically equivalent, with similar Q . Therefore, for describing the transformation of a Néel to a Bloch skyrmion we should provide **other variables** characterizing skyrmions, to be discussed in the next section.

2.3.3 Skyrmion classification

Q is the fundamental physical quantity characterizing skyrmion topology. However, solving the integral of Eq. 2.43 reveals other two variables of interest, the vorticity and the helicity.

The following parametrization exploits the skyrmion circular symmetry. We recall that to describe the skyrmion texture, we consider the magnetization vector $\mathbf{m}(\mathbf{r})$ to be normalized by the saturation magnetization M_s , so $\|\mathbf{m}\| = 1$. If $\mathbf{r} = (r, \phi)$ contains the spatial coordinates in the polar form, $r = \sqrt{x^2 + y^2}$ is the radial distance and $\phi = \tan^{-1}(y/x)$ is the polar angle. The three component magnetization $\mathbf{m}(x, y) = \mathbf{m}(r, \phi)$

can be expressed using spherical coordinates as

$$\mathbf{m}(r, \phi) = [\sin \Theta(r) \cos \Phi(\phi), \sin \Theta(r) \sin \Phi(\phi), \cos \Theta(r)], \quad (2.44)$$

where $\Theta = \Theta(r)$ is the polar angle and $\Phi = \Phi(\phi)$ is the azimuthal angle of the magnetization.

Using Eq. 2.44, we calculate Q by the definition of Eq. 2.43, yielding (KOSHIBA; NAGAOSA, 2017)

$$\begin{aligned} Q &= \frac{1}{4\pi} \int_0^\infty dr \int_0^{2\pi} d\phi \frac{d\Theta(r)}{dr} \frac{d\Phi(\phi)}{d\phi} \sin \Theta(r) \\ &= [\cos \Theta(r)]_{r=0}^{r=\infty} [\Phi(\phi)]_{\phi=0}^{\phi=2\pi}, \end{aligned} \quad (2.45)$$

where we classify skyrmion structures based on the boundary conditions to be imposed. For a standard skyrmion, the spin at the core ($r = 0$) is characterized to be in the **opposite direction** of the ferromagnetic background ($r \rightarrow \infty$). For example, $\Theta(0) = \pi$ (spin down) and $\Theta(\infty) = 0$ (spin up). So, the first factor becomes $[\cos \Theta(r)]_{r=0}^{r=\infty} = 2$. The second factor depends on the relationship between the magnetization azimuthal angle Φ and the spatial angle ϕ , given by $\Phi(\phi) = \nu\phi + h$ and introducing: (i) the **vorticity** $\nu = [\Phi(\phi)]_{\phi=0}^{\phi=2\pi} / 2\pi$, the integer determining how many times the magnetization winds around the unit circle as one traverses skyrmion perimeter ($0 \leq \phi \leq 2\pi$), and (ii) the phase shift h known as the **helicity**, indicating the rotational sense of the spin swirl. Once boundary conditions are defined for r and we integrate in ϕ , we obtain $Q = -\nu$.

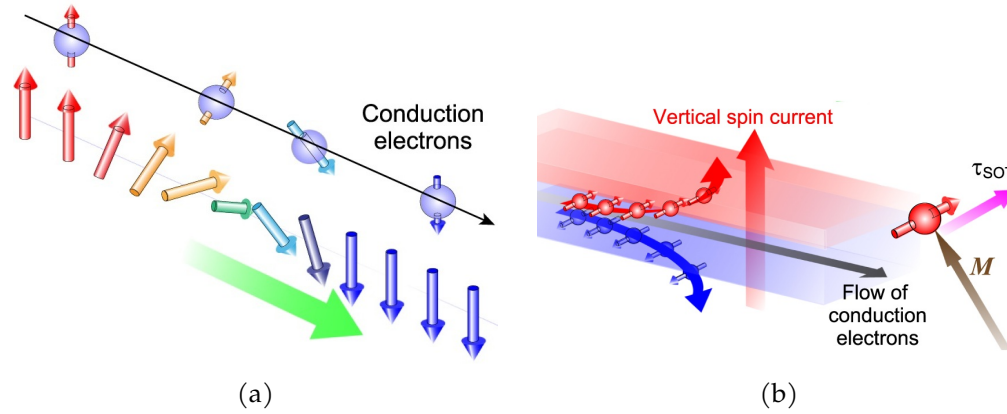
This allows for the following classification of axially symmetric skyrmions:

- **Néel-type** ($\nu = 1, h = 0$ or π), where magnetization rotates radially, similar to a hedgehog;
- **Bloch-type** ($\nu = 1, h = \pm\pi/2$), where magnetization rotates tangentially, forming a vortex-like structure; and
- **antiskyrmions** ($\nu = -1$).

2.3.4 Setting skyrmions in motion

For spintronic applications, the ability to transport skyrmions efficiently is as critical as their stability. Recent literature has demonstrated diverse strategies to manipulate

Figure 17 – (a) Spin-Transfer Torque (STT): conduction electrons flowing through a non-collinear magnetic texture (such as a domain wall or a skyrmion) align their spins to the local magnetization. The resulting exchange of angular momentum exerts a torque driving the texture in the direction of electron flow (green arrow). (b) Spin-Orbit Torque (SOT): An in-plane charge current (grey arrow) in a heavy-metal layer generates a vertical spin current via the Spin Hall Effect (SHE) (red vertical arrow), exerting a torque (τ_{SOT}) on the magnetization M .



Source: Adapted from Ohki and Mochizuki (2025).

skyrmion, including in-plane magnetic field gradients (MOON et al., 2016), temperature gradients (QIN et al., 2022), optical manipulation via a high-frequency laser (HIROSAWA et al., 2022), and even the use of acoustic waves (NEPAL; GÜNGÖRDÜ; KOVALEV, 2018).

Electrical manipulation, however, remains as the most technologically relevant approach for integrated devices, specially via the interaction of skyrmion textures with **spin-polarized currents**. This interaction can be roughly categorized into two effects: **Spin Transfer Torque (STT)** and **Spin-Orbit Torque (SOT)**. In the STT regime, illustrated in Fig. 17(a), a charge current flowing through the FM material becomes spin-polarized. As these conduction electrons traverse the non-collinear magnetic texture of the skyrmion, their spins adiabatically follow the local magnetization. By conservation of angular momentum, a back-reaction torque is exerted on the local magnetic moments, effectively pushing the magnetic texture in the direction of the electron flow. A model for the STT mechanism by Zhang-Li is described in Chapter 3. Conversely, in the case of SOT, an in-plane charge current in the heavy-metal layer generates a pure transverse spin current via the Spin Hall Effect (SHE). This spin current diffuses into the adjacent ferromagnetic layer, exerting a torque on the skyrmion texture [Fig. 17(b)].

A defining feature of the current-driven skyrmion motion is the **low critical current density** (j_c) required to initiate motion. Experimental observations in materials like MnSi reported j_c on the order of 10^6 A/m² (JONIETZ et al., 2010), standing in contrast to

the required for ferromagnetic domain walls (DW), typically exceeding 10^{11} A/m² (HU et al., 2013). DW are spatially extended objects easily pinned by impurities and grain boundaries. Skyrmions, however, act as particle-like entities. When they encounter a pinning center (an impurity or defect), they can slightly deform and move around the obstacle (IWASAKI; MOCHIZUKI; NAGAOSA, 2013).

The quantitative description of this motion, including the deflection arising from the skyrmion Hall effect, is captured by the **rigid-body approximation**. This allows us to map the Landau-Lifshitz-Gilbert (LLG) dynamics onto a simpler equation of motion for the skyrmion center of mass, known as the Thiele equation.

2.3.5 Thiele equation

The quasiparticle description of magnetic skyrmions leads to the Thiele equation (THIELE, 1973; NAGAOSA; TOKURA, 2013). In this approach, we consider the dynamics of the center of topological charge (CTC) of a skyrmion within a thin magnetic film and assuming that it maintains circular shape. The CTC is located at $\mathbf{R} \equiv (x, y)$ with a velocity vector $\dot{\mathbf{R}} \equiv (\dot{x}, \dot{y})$. The Thiele equation is obtained from time evolution of the magnetization field $\mathbf{m}(\mathbf{r}, t)$, described by the Landau-Lifshitz-Gilbert (LLG) equation. The LLG equation is projected onto the translational modes of skyrmion, so Thiele equation is a balanced force equation reading

$$\mathbf{G} \times \dot{\mathbf{r}} + \alpha \mathcal{D} \dot{\mathbf{r}} + \nabla V(\mathbf{r}) = 0. \quad (2.46)$$

The first term is a gyrotropic (Magnus) force arising from the curvature of the magnetization in space. $\mathbf{G} = G \hat{z} = 4\pi Q(M_s d / \gamma) \hat{z}$ is known as the **gyromagnetic coupling vector**, depending on the saturation magnetization M_s , the film thickness d , and the electron gyromagnetic ratio γ . Crucially, G is a function of $Q = \pm 1$ making this the explicit topological term in Thiele equation, not depending on the details of skyrmion profile.

The second term is the viscous drag, where dissipation is proportional to α , the material-dependent, dimensionless Gilbert parameter. $\mathcal{D}$ is known as the **dissipation**

tensor (or dyadic) with components $\mathcal{D}_{ij}$ given by

$$\mathcal{D}_{ij} = \int (\partial_i \mathbf{m} \cdot \partial_j \mathbf{m}) d^2r \quad (2.47)$$

where i and j are the x or y coordinates. Different from the case of the gyrovector $\mathbf{G}$, the dissipation tensor is not topological. Instead, it depends heavily on the shape and size of the skyrmion, sensitive to vibrational excitation or the skyrmion interaction with edges. A perfectly circular skyrmion yields a diagonal $\mathcal{D}$, then $\mathcal{D}_{ij} = \mathcal{D} \delta_{ij}$. It means $\mathcal{D} = (dM_s/\gamma) \pi \mathcal{I}$, where

$$\mathcal{I} = \int_0^\infty \rho d\rho \left[\left(\frac{d\Theta}{d\rho} \right)^2 + \frac{\sin^2 \Theta}{\rho^2} \right], \quad (2.48)$$

is the dimensionless dissipation integral, with $\Theta(\rho)$ the polar angle of the spins positioned at a distance ρ from skyrmion center. If the skyrmion is deformed, $\mathcal{D}$ becomes anisotropic, so the drag force becomes dependent on the direction of motion.

The third term is the force exerted on the skyrmion center-of-mass due to any conservative potentials. For example, the injection of a in-plane spin-polarized current into a two-dimensional FM film via STT modifies Eq. 2.46 to

$$\mathbf{G} \times (\mathbf{u} - \dot{\mathbf{r}}) + \mathcal{D}(\beta \mathbf{u} - \alpha \dot{\mathbf{r}}) - \nabla V(\mathbf{r}) = 0, \quad (2.49)$$

where $\mathbf{u} = u\hat{x}$ is the drift velocity of conduction electrons and β is the non-adiabaticity parameter [See Zhang and Li (2004)]. If the current is applied along the x direction ($u_y = 0$), the Thiele equation can be separated into its two cartesian components. In the absence of other forces ($\nabla V = 0$), we obtain the velocity vector $\dot{\mathbf{R}}$ components as (IWASAKI; MOCHIZUKI; NAGAOSA, 2013)

$$\dot{x} = \left(\frac{G^2 + \mathcal{D}^2 \alpha \beta}{G^2 + \alpha^2 \mathcal{D}^2} \right) \nu_x, \quad \dot{y} = \left[G \mathcal{D} \frac{(\alpha - \beta)}{G^2 + \alpha^2 \mathcal{D}^2} \right] \nu_x. \quad (2.50)$$

Equations 2.50 describe the velocity of the skyrmion due to an applied in-plane spin-polarized current in the absence of external forces and impurities, where the skyrmion reaches a constant (terminal) velocity under an overdamped dynamics.

2.3.6 Generalized Thiele equation

When extra soft modes other than translation are present in the skyrmion dynamics, we obtain a generalized description of a **semi-rigid** skyrmion also via the conversion of a field equation (LLG equation) into an equation of motion (Thiele). Stating explicitly, the dynamics of the magnetization field $\mathbf{m}(\mathbf{r}, t)$ is classically provided by the **Landau-Lifshitz-Gilbert (LLG) equation** given by

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times (\mathbf{B}_{\text{eff}} + \mathbf{B}_{\text{th}}) + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} \quad (2.51)$$

where $\mathbf{B}_{\text{eff}} = M_s^{-1}(\delta \mathcal{H} / \delta \mathbf{m})$ is the **effective magnetic field** that considers all the interactions in the Hamiltonian $\mathcal{H}$ (a functional), and $\mathbf{B}_{\text{th}}$ is the stochastic field representing thermal fluctuations, whose properties are discussed in Chapter 3.

Our first step in the derivation is to cross multiply the LLG equation by $\mathbf{m}$:

$$\mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times [\mathbf{m} \times (\mathbf{B}_{\text{eff}} + \mathbf{B}_{\text{th}})] + \alpha \mathbf{m} \times \left(\mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} \right). \quad (2.52)$$

Here, $\partial \mathbf{m} / \partial x$ and $\partial \mathbf{m} / \partial y$ emerge as natural modes describing translational motion. Projecting LLG equation into those modes and integrating in the whole sample — assuming uniform magnetic behavior in the out-of-plane z direction of a thin film with thickness d — provide us with the global equations of motion for $\mathbf{R}(t) = [x(t), y(t)]$. This is the usual procedure in obtaining Thiele equation in the form of Eq. 2.46 or Eq. 2.49.

To obtain the desired generalized form of Thiele equation, we consider a set of extra soft modes $\boldsymbol{\xi} = \{\xi_1, \dots, \xi_N\}$ coupling with the translation and defining the generalized coordinates $\mathbf{X} = \{x, y, \xi_1, \dots, \xi_N\}$. These extra soft modes can be, for example, the domain wall width w or the skyrmion radius $\mathcal{R}$. With N extra (soft modes) coordinates the magnetization is parameterized as $\mathbf{m} = \mathbf{m}[x(t), y(t), \xi_1(t), \dots, \xi_N(t)]$ with the time derivative given by the chain rule as

$$\dot{\mathbf{m}} = \sum_j \dot{X}_j \frac{\partial \mathbf{m}}{\partial X_j}, \quad j = 1, 2, \dots, N + 2. \quad (2.53)$$

The deformation of magnetic textures as skyrmions is then interpreted as a generalized displacement along the ξ_i axis (LIU; CHEN; ZHENG, 2023). The approach to be

reproduced here is adopted, for example, in describing the motion of domain walls dominated by a few soft modes in FM nanowires (TRETIAKOV et al., 2008) and to model current-driven dynamics of deformable skyrmions in the presence of line defects (LIU; CHEN; ZHENG, 2023).

Now, the equations of motion for $\mathbf{X}$ are obtained via projecting LLG equation onto the tangent space of the manifold spanned by $\partial\mathbf{m}/\partial\mathbf{X}$. By taking the dot product of Eq. 2.52 with the components $\partial\mathbf{m}/\partial X_i$ and integrating over the volume V of the sample,

$$\int_V (\mathbf{m} \times \dot{\mathbf{m}}) \cdot \left(\frac{\partial\mathbf{m}}{\partial X_i} \right) d^3r = \int_V \left\{ -\gamma \mathbf{m} \times [\mathbf{m} \times \mathbf{B}'] + \alpha \mathbf{m} \times (\mathbf{m} \times \dot{\mathbf{m}}) \right\} \cdot \left(\frac{\partial\mathbf{m}}{\partial X_i} \right) d^3r, \quad (2.54)$$

where $\mathbf{B}' = \mathbf{B}_{\text{eff}} + \mathbf{B}_{\text{th}}$. Using the vector identity $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$, as well as the relations for the magnetization unit vector ($\mathbf{m} \cdot \mathbf{m} = 1$ and $\mathbf{m} \cdot \dot{\mathbf{m}} = 0$), we simplify the triple products as $\mathbf{m} \times (\mathbf{m} \times \mathbf{B}') = \mathbf{B}' - (\mathbf{B}' \cdot \mathbf{m})\mathbf{m}$ and $\mathbf{m} \times (\mathbf{m} \times \dot{\mathbf{m}}) = \dot{\mathbf{m}}$. So, the integrands are simplified to

$$\int_V (\mathbf{m} \times \dot{\mathbf{m}}) \cdot \left(\frac{\partial\mathbf{m}}{\partial X_i} \right) d^3r = \int_V \left\{ -\gamma [\mathbf{B}' - \mathbf{m}(\mathbf{B}' \cdot \mathbf{m})] + \alpha \dot{\mathbf{m}} \right\} \cdot \left(\frac{\partial\mathbf{m}}{\partial X_i} \right) d^3r. \quad (2.55)$$

We now replace the time derivative of $\mathbf{m}$ by the chain rule result and remove the magnetic field $\mathbf{B}' \cdot \mathbf{m}$ parallel to $\mathbf{m}$ as it does not contribute to the skyrmion motion (since $\mathcal{E}_z = -\mathbf{m} \cdot \mathbf{B}$ is uniform in this case),

$$\int_V \mathbf{m} \times \left(\sum_j \dot{X}_j \frac{\partial\mathbf{m}}{\partial X_j} \right) \cdot \left(\frac{\partial\mathbf{m}}{\partial X_i} \right) d^3r = \int_V \left[-\gamma \mathbf{B}' + \alpha \left(\sum_j \dot{X}_j \frac{\partial\mathbf{m}}{\partial X_j} \right) \right] \cdot \left(\frac{\partial\mathbf{m}}{\partial X_i} \right) d^3r \quad (2.56)$$

Reorganizing the triple products, multiplying the equation by (M_s/γ) , and integrating in the uniform z axis, we obtain

$$\begin{aligned} \frac{dM_s}{\gamma} \sum_j \dot{X}_j \int_S \mathbf{m} \cdot \left(\frac{\partial\mathbf{m}}{\partial X_i} \times \frac{\partial\mathbf{m}}{\partial X_j} \right) d^2r &= dM_s \sum_j \int_S \left(\mathbf{B}' \frac{\partial\mathbf{m}}{\partial X_j} \right) d^2r \\ &\quad - \alpha \frac{dM_s}{\gamma} \sum_j \dot{X}_j \int_S \left(\frac{\partial\mathbf{m}}{\partial X_i} \cdot \frac{\partial\mathbf{m}}{\partial X_j} \right) d^2r. \end{aligned} \quad (2.57)$$

We are now in the position to define the **generalized gyrotropic tensor** $\mathcal{G}$ with components given by

$$\mathcal{G}_{ij} = \frac{dM_s}{\gamma} \int_S \mathbf{m} \cdot \left[\left(\frac{\partial\mathbf{m}}{\partial X_i} \right) \times \left(\frac{\partial\mathbf{m}}{\partial X_j} \right) \right] d^2r. \quad (2.58)$$

Note that we previously introduced the gyrotropic vector $\mathbf{G}$ for Eq. 2.46. Although formally defined as a tensor, due to the antisymmetric property of $\mathcal{G}_{ij}$, $\mathcal{G}$ acting on the velocity vector $\dot{\mathbf{R}}$ for $(i, j) = (x, y)$ is operationally equivalent to a vector cross product. Specifically, the force term $\mathcal{G}_{ij}\dot{R}_j$ can be rewritten as $\mathbf{G} \times \dot{\mathbf{R}}$.

For the moment, we set $\mathbf{B}_{th} = 0$ ($T = 0$) leading to $\mathbf{B}' = \mathbf{B}_{eff}$. Recalling that $\mathbf{B}_{eff} = -(1/M_s)(\delta E/\delta \mathbf{m})$, the first term in the right side of Eq. 2.57 can be described as a generalized force:

$$\begin{aligned} M_s \sum_j \int_V \left(-\frac{1}{M_s} \frac{\delta E}{\delta \mathbf{m}} \right) \cdot \left(\frac{\partial \mathbf{m}}{\partial X_j} \right) d^3r &= - \sum_j \int_V \left(\frac{\delta E}{\delta \mathbf{m}} \cdot \frac{\partial \mathbf{m}}{\partial X_j} \right) d^3r \\ &= - \sum_j \frac{\delta E}{\delta X_j} = \mathbf{F}. \end{aligned} \quad (2.59)$$

An analogous term would be obtained replacing $\mathbf{B}_{eff}$ by $\mathbf{B}_{th}$ in Eq. 2.59, but containing a stochastic, Langevin-type force instead of the deterministic $\mathbf{F}$ we obtained.

Also, we define the **generalized dissipation tensor** $\mathcal{D}$ with components given by

$$\mathcal{D}_{ij} = \frac{dM_s}{\gamma} \int_S \left(\frac{\partial \mathbf{m}}{\partial X_i} \right) \cdot \left(\frac{\partial \mathbf{m}}{\partial X_j} \right) d^2r. \quad (2.60)$$

The equation is finally simplified to

$$\sum_j \mathcal{G}_{ij} \dot{X}_j = \mathbf{F} - \alpha \sum_j \mathcal{D}_{ij} \dot{X}_j \quad (2.61)$$

Let us separate the translation coordinates of the CTC, $\mathbf{R} = (x, y)$, in Eq. 2.61 to obtain a separate term for the extra soft modes ξ as

$$-\mathcal{G}_{ix}\dot{x} - \mathcal{G}_{iy}\dot{y} - \sum_j \mathcal{G}_{ij}\dot{\xi}_j = \mathbf{F} - \alpha \left(\mathcal{D}_{ix}\dot{x} + \mathcal{D}_{iy}\dot{y} + \sum_j \mathcal{D}_{ij}\dot{\xi}_j \right), \quad (2.62)$$

Obtaining

$$-(\mathcal{G}_{ix} - \alpha \mathcal{D}_{ix})\dot{x} - (\mathcal{G}_{iy} - \alpha \mathcal{D}_{iy})\dot{y} = \mathbf{F} + \sum_j (\mathcal{G}_{ij} - \alpha \mathcal{D}_{ij})\dot{\xi}_j, \quad (2.63)$$

where the last term accounts for the extra soft modes. According to the component assigned to $i = \{x, y\}$ we have the two following coupled equations:

$$\begin{pmatrix} \alpha \mathcal{D}_{xx} & -\mathcal{G}_{xy} + \alpha \mathcal{D}_{xy} \\ -\mathcal{G}_{yx} + \alpha \mathcal{D}_{yx} & \alpha \mathcal{D}_{yy} \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \mathbf{F} + \sum_j \begin{pmatrix} \mathcal{G}_{xj} - \alpha \mathcal{D}_{xj} \\ \mathcal{G}_{yj} - \alpha \mathcal{D}_{yj} \end{pmatrix} \dot{\xi}_j. \quad (2.64)$$

Finally, we can combine Eqs. 2.64 to obtain

$$\mathbf{G} \times \dot{\mathbf{R}} + \alpha \mathcal{D} \dot{\mathbf{R}} = \mathbf{F} + \sum_j \begin{pmatrix} \mathcal{G}_{xj} - \alpha \mathcal{D}_{xj} \\ \mathcal{G}_{yj} - \alpha \mathcal{D}_{yj} \end{pmatrix} \dot{\xi}_j, \quad (2.65)$$

where $\dot{\mathbf{R}} = (\dot{x}, \dot{y})$ is the velocity of the center of topological charge of the skyrmion and

$$\mathcal{D} = \begin{pmatrix} \mathcal{D}_{xx} & \mathcal{D}_{xy} \\ \mathcal{D}_{yx} & \mathcal{D}_{yy} \end{pmatrix}.$$

2.3.7 Skyrmion Hall effect

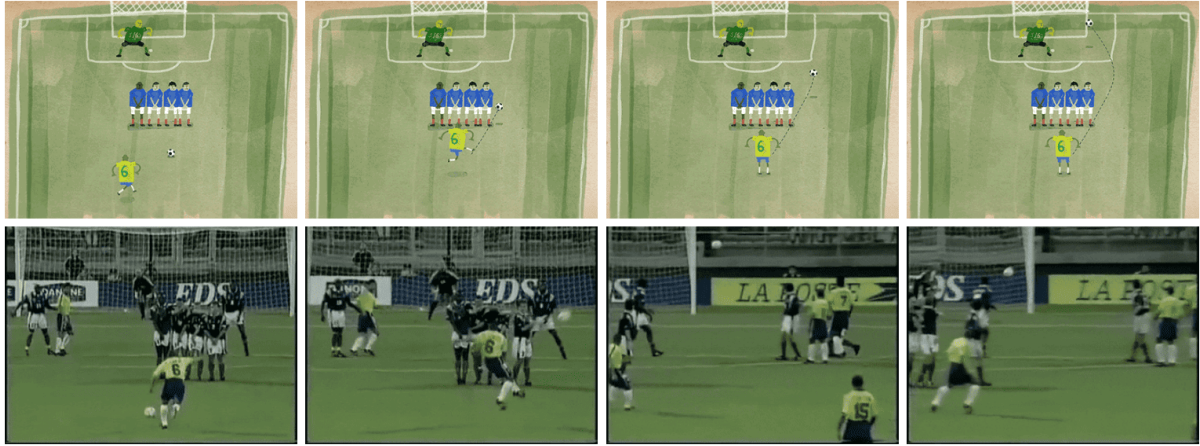
The fact that skyrmions are subjected to a gyrotropic/Magnus force implies that they suffer an often undesirable transverse deviation from a straight line [$\dot{y} \neq 0$ in Eq. 2.50] in a phenomenon known as the **skyrmion Hall effect**. By solving the Thiele equation for a skyrmion in a single ferromagnetic layer, we obtain the measure of such deviation, given by the **Hall angle**

$$\Theta_{\text{SkH}} \equiv \arctan \left(\frac{\dot{y}}{\dot{x}} \right) = \arctan \left[\frac{\mathcal{G}\mathcal{D}(\alpha - \beta)}{\mathcal{G}^2 + \alpha\beta\mathcal{D}^2} \right]. \quad (2.66)$$

Since Θ_{SkH} is proportional to $\mathcal{G}$, the deflection depends on the sign of $Q = \pm 1$, providing a (topological) charge separation just like an ordinary Hall effect.

The effect is analogous to the **classical Magnus effect**, in which the rotation of an object changes its trajectory when moving in a fluid. A famous example of classical Magnus effect is the “impossible” free kick of Brazilian soccer player Roberto Carlos when playing against France in 1997, illustrated in Fig. 18 (first row). To this day the player is still confused and baffled by his accomplishment, where he managed to shoot the ball making it spin, remarkably changing its trajectory during the motion by a transverse deflection to the left, ultimately reaching the goal. The camera operator was probably not expecting such strong curvature, so the frames from television broadcast are blurred as the camera searches for the ball [Fig. 18 (second row)]. In the magnetic counterpart, the magnetic moments are the ones to rotate as the skyrmion moves throughout the film.

Figure 18 – Classical Magnus effect: illustration (top row) and broadcast footage frames (bottom row) of Roberto Carlos's famous free kick against France in 1997. The four panels of top row depicts the player approach and the curved trajectory followed by the spinning ball around the defensive wall and into the goal. In the bottom, the corresponding sequence from the actual broadcast footage of the event, with a confused camera operator changing the camera orientation during the ball's curved flight.



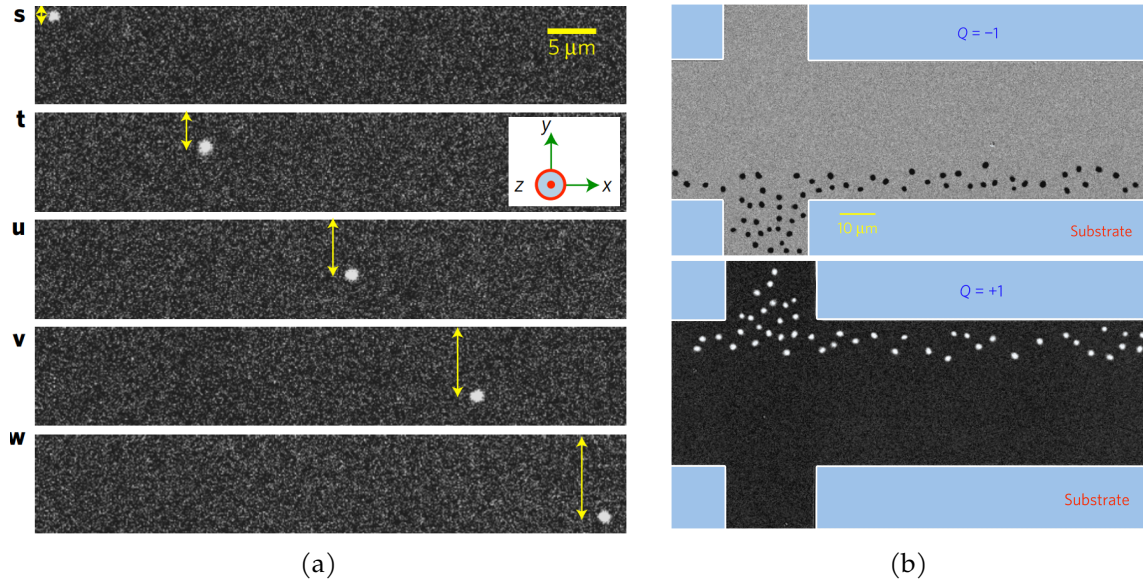
Source: TED-Ed (2016) (top row) and TF1 (1997) (bottom row).

Twenty years later, the Skyrmion Hall effect is observed experimentally for the first time by Jiang et al. (2017) using the Kerr microscopy technique in a magnetic multilayer of Ta/CoFeB(1.1nm)/TaO_x (Fig. 19), when electric currents are injected in the longitudinal x direction, pushing the skyrmion in this very direction but with a transverse motion indicated by the yellow arrows [Fig. 19(a)]. The characteristic dependence on the charge Q is visualized by the skyrmion accumulation depicted in panel (b).

The development of spintronic devices, such as the non-volatile **racetrack memories** conceptually illustrated in Fig. 20, faces a critical challenge after SkHE. These devices are based in the controlled current-based motion of skyrmions that are manipulated (reading/writing) to record and process information. Racetrack memories were first proposed by IBM using domain walls and are promised to offer higher storage density than actual solid state memories. However the ubiquitous, transverse Magnus force causes skyrmions to deviate from the direction of the current, leading to a curved trajectory and eventually driving the skyrmion toward the track edge where it can be annihilated, leading to data loss.

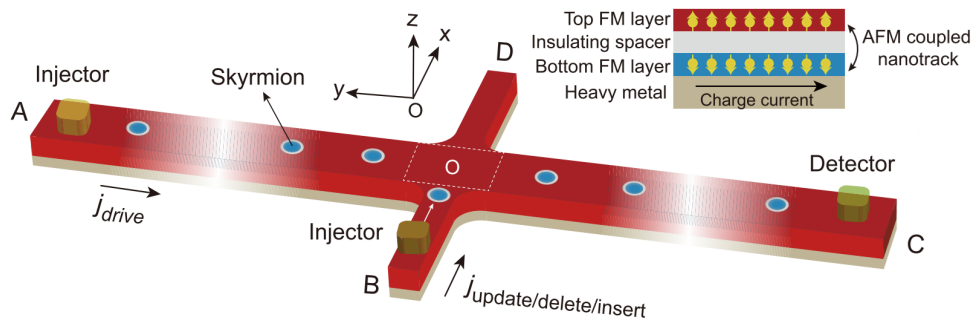
Several mitigation strategies have been proposed to suppress SkHE, ranging from the use of antiferromagnetic (AFM) skyrmions (BARKER; TRETIAKOV, 2016) to geometric confinement. One effective physical method is the utilization of a **narrow channel** (TOSCANO et al., 2020). By reducing the width of the nanotrack, the repulsive potential

Figure 19 – The skyrmion Hall effect (SkHE). (a) Sequential imaging (panels s–w) tracking the trajectory of a single magnetic skyrmion. As the skyrmion moves horizontally, propelled by a current density in the x direction, it exhibits a transverse deflection indicated by the yellow arrows. (b) Comparison of skyrmion motion based on the topological charge Q . The top panel shows skyrmions with $Q = -1$ accumulating at the bottom edge of the track, while the bottom panel shows skyrmions with $Q = +1$ deflected toward the top edge, demonstrating that the direction of the transverse force depends on the sign of Q , just as classical Hall effect depends on the electric charge's.



Source: Adapted from Jiang et al. (2017).

Figure 20 – (a) Schematic representation of a racetrack memory device where skyrmions are driven by a current J_{drive} . The device includes injectors (A, B) for writing and a detector (C) for reading magnetic bits.



Source: Zhu et al. (2018).

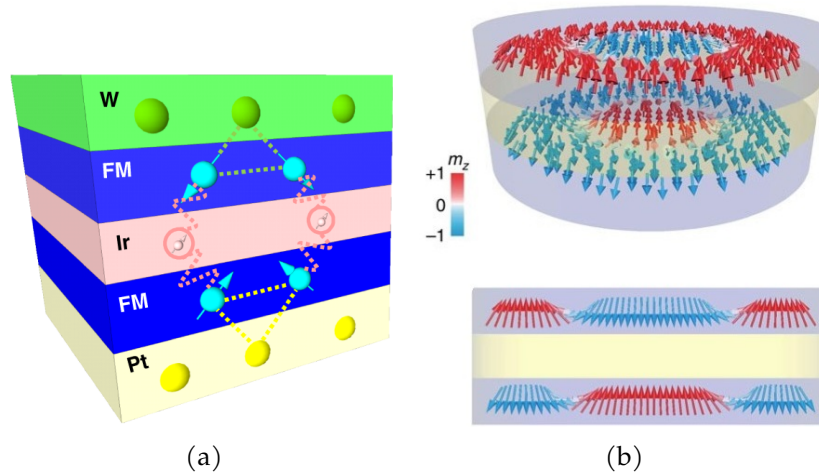
from the track edges acts against the Magnus force, ultimately balancing the forces. Other geometric solutions include the engineering of “attractive” magnetic strips or potential wells along the track center to further pin the skyrmion to a straight path (TOSCANO et al., 2020; SONG et al., 2017).

A more fundamental solution, though, inspired by the AFM skyrmions however offering several advantages such as the tunability of magnetic parameters, is that of

the **synthetic antiferromagnets** (SAFs) (Sec. 2.2.3.2). As depicted in Fig. 21(a), a SAF structure is composed of two FM layers coupled antiferromagnetically through a non-magnetic spacer such as Iridium.

In this configuration, skyrmions nucleate as **coupled pairs** located in the top and bottom layers. Crucially, as the magnetization of the two layers is antiparallel, the topological charges of the paired skyrmions are opposite ($Q_{\text{Top}} = -Q_{\text{Bottom}}$) [Fig. 21(b)]. When a current drives this coupled system, the Magnus forces acting on each layer are **equal in magnitude but opposite in direction**, leading to a perfect cancellation of the transverse forces and the suppression of SkHE.

Figure 21 – (a) Illustration of the heavy-metal/ferromagnet heterostructure (e.g., W/FM/Ir/FM/Pt). The central Iridium (Ir) spacer mediates the antiferromagnetic RKKY coupling between the two ferromagnetic (FM) layers, as indicated by the dashed interaction paths. The external heavy-metal layers (W and Pt) induce the necessary interfacial DMI. (b) Three-dimensional and cross-sectional views of the AFM coupled skyrmion pair. The color map represents the out-of-plane magnetization (m_z), demonstrating the antiparallel alignment. Due to this configuration, the topological gyrotropic forces acting on each layer effectively cancel each other out, leading to the suppression of the skyrmion Hall effect.



Source: (a) Dohi et al. (2019) and (b) Zhang, Zhou and Ezawa (2016).

2.3.8 Magnetic skyrmions in synthetic antiferromagnets (SAFs)

While the theoretical advantages of SAFs for magnetic skyrmion stabilization are well-established, practical implementations face challenges regarding efficient **nucleation** and **observation**. Recent experimental advancements have demonstrated that magnetic skyrmions in SAFs can be nucleated not only via standard electrical current pulses (SAMPAIO et al., 2013; YU et al., 2017) but also through ultra-fast laser illumina-

tion, providing a pathway for ultra-fast writing processes in devices (JUGE et al., 2022). These methods overcome the energy barriers associated with the vanishing total magnetization and the net $Q = 0$ of SAF skyrmions, facilitating their stabilization, with the benefit of also **avoiding parasitic Joule heating** – in the optical case — that can sometimes destabilize the very texture we are trying to write.

Once nucleated, the dynamical properties of fully compensated SAFs offer significant advantages, which have recently been confirmed in experimental studies. Crucially, Pham et al. (2024b) demonstrated that magnetic skyrmions can be driven by electrical currents to **velocities reaching up to 900 m/s** in these architectures. This unprecedented speed, which is nearly an order of magnitude faster than what is typically observed in conventional ferromagnetic thin films (WOO et al., 2016; LITZIUS et al., 2017), is achieved by exploiting the exact cancellation of the net topological charge, leading to a vanishing Hall effect. This scenario provides a critical experimental foundation for the high-velocity dynamics modeled in Chapter 4.

To further tune these translational properties in SAFs, recent theoretical efforts have explored **synthetic ferrimagnets (SFi)**, where an **imbalance in the saturation magnetization** of the coupled FM layers is observed. It has been shown that introducing a **spatial gradient** in M_s can effectively modulate the Hall angle, leading to a robust suppression of the transverse deflection even in the presence of varying magnetic environments (BO et al., 2024). This approach offers an alternative to control the trajectory of magnetic skyrmions in racetrack geometries without requiring perfectly compensated AFM coupling, which can be difficult to achieve in real samples due to manufacturing limitations (e.g., in magnetic sputtering). Additionally, SFi structures can be also **intentionally** prepared in order to enhance skyrmion detection via, for example, the magneto-optical Kerr effect (MOKE) (DOHI et al., 2019).

Beyond translational motion, the internal dynamics of magnetic skyrmions in SAFs exhibit richer behaviors compared to their ferromagnetic counterparts. Specifically, the interlayer coupling leads to the **splitting of the characteristic radial breathing modes** into distinct in-phase (**symmetric**) and out-of-phase (**antisymmetric**) resonance frequencies as observed in a SAF trilayer (LONSKY; HOFFMANN, 2020) and in a four repetition SAF multilayer (BARKER et al., 2023). The micromagnetic simulations by Lonsky and Hoffmann (2020) have mapped how the splitting of the breathing mode frequencies occurs within both synthetic ferri- and antiferromagnetic trilayers. Their findings

focus on the excitation of these coupled modes **via out-of-plane AC magnetic fields** or **spin torques**, primarily framing coaxial pairs of skyrmions as **localized resonators**. This is highly relevant for experimental investigations, since **in the out-of-phase mode the magnetic compensation is circumvented** — the skyrmions in different layers respond oppositely to the excitation — and a detectable signature remains in the form of a stray field that does not cancel out.

In the next chapter, we discuss the numerical methods we adopt for numerically simulate skyrmions in synthetic antiferromagnets, presenting the possibilities for the nucleation of bound skyrmion pairs and studying the suppression of the SkHE in several scenarios.

3 NUMERICAL METHODS

In this chapter we describe and discuss the micromagnetic model we use to simulate the nucleation and motion of magnetic skyrmions within ultrathin coupled ferromagnetic layers. Here, we describe the magnetization dynamics, the numerical methods, the geometry of our magnetic multilayer and the parameters we use throughout our numerical studies.

3.1 The Landau-Lifshitz-Gilbert equation and micromagnetics

The fundamental dynamics of the magnetization field $\mathbf{m}(\mathbf{r}, t)$ in ferromagnetic materials are governed by the conservation of angular momentum, where the time-evolution of the magnetic moment is dictated by interaction torques. As we discussed in Sec. 2.1, in the discrete atomic scale an individual magnetic moment $\boldsymbol{\mu}_i$ experiences a torque arising from an external magnetic field $\mathbf{B}$ (Eq. B.6). When $\boldsymbol{\mu}_i$ is, however, within a ferromagnetic material, multiple interactions have to be considered when analyzing the time evolution of $\boldsymbol{\mu}_i$.

Strictly speaking, matter is made of atoms and, therefore possesses a **discrete** structure of many magnetic moments $\boldsymbol{\mu}_i$. So we should describe magnetic systems via a discrete atomistic model in order to obtain all the available physical information. However, to guarantee a computationally feasible way of implementing our numerical study on a mesoscopic system, we transition — via a coarse-graining procedure — from a discrete model to a continuum approximation known as the **micromagnetic model**. In this approximation, as we occasionally discussed earlier, the discrete moments $\boldsymbol{\mu}_i$ are averaged over a small volume V of the material (Fig. 23) *sufficiently large to contain many atoms, but sufficiently small to assume the magnetization is uniform inside V* , yielding a description via the continuous vector field $\mathbf{M}(\mathbf{r})$ instead.

This hypothesis relies on the assumption that the orientation of $\mathbf{M}(\mathbf{r})$ varies slowly relative to the inter-atomic spacing. Consequently, the initial discrete version of multiple interactions are replaced by the continuous expressions we obtained in Chapter 2. For example, the Heisenberg exchange energy (Eq. C.6) is replaced by a spatial gradient term involving the exchange stiffness constant A_{ex} (Eq. 2.30).

In order to “encapsulate” the effect of multiple magnetic interactions, we define a new quantity, the **effective magnetic field** $\mathbf{B}_{\text{eff}}$, holding the information of all the interactions in the material. Fundamentally, $\mathbf{B}_{\text{eff}}$ determines the time evolution of the local magnetization $\mathbf{M}(\mathbf{r}, t)$ via the previously introduced cornerstone of micromagnetics, the **Landau-Lifshitz-Gilbert (LLG) equation**

$$\frac{\partial \mathbf{m}}{\partial t} = -\frac{\gamma}{1 + \alpha^2} (\mathbf{m} \times \mathbf{H}_{\text{eff}}) - \frac{\gamma\alpha}{1 + \alpha^2} [\mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}})], \quad (3.1)$$

where γ is the electron gyromagnetic ratio, α is the phenomenological dimensionless Gilbert damping parameter, and $\mathbf{m}(\mathbf{r}, t) = \mathbf{M}(\mathbf{r}, t)/M_s$ is normalized by the saturation magnetization M_s of the material. The effective field $\mathbf{B}_{\text{eff}}$ in the continuum limit is derived from the variational derivative of the total magnetic energy density functional (the micromagnetic Hamiltonian) $\mathcal{H}[\mathbf{m}]$ — which we shall discriminate further — such that

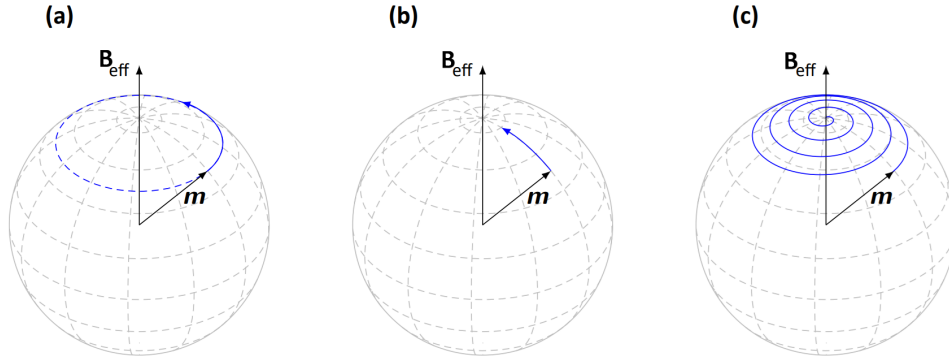
$$\mathbf{B}_{\text{eff}} = -\frac{1}{M_s} \frac{\delta \mathcal{H}}{\delta \mathbf{m}}. \quad (3.2)$$

In this way, each magnetic interaction contained in $\mathcal{H}$ contributes to $\mathbf{B}_{\text{eff}}$. The first term of Eq. 3.1 reflects the classical Larmor precession around the field axis [Fig. 22(a)], whereas the second one is a damping term¹ which reflects energy dissipation [Fig. 22(b)] and consequently relaxation of physical systems into the equilibrium [Fig. 22(c)].

The spatial and time derivatives are calculated by a **finite-difference method**, using the discretization of space and time domains — which is typical in a numerical integration — to convert the LLG equation into an algebraic equation. The latter can be finally solved using differences of function values of nearby points. Ironically, thus, after taking a problem of discrete spins and assuming a continuous magnetization field, we go back to space discretization within the finite-difference approach. We assume that the small volume V corresponds to a cubic **micromagnetic cell** with sides a [Fig. 23(a)], the smallest unit in the model, but covering several inter-atomic distances. The micromagnetic cell size should be smaller than the exchange characteristic length $l_{\text{ex}} = \sqrt{2A_{\text{ex}}/\mu_0 M_s^2}$, otherwise calculating spatial derivatives would not capture the details of entire magnetic structures, such the skyrmion nucleus (WANG; YUAN; WANG,

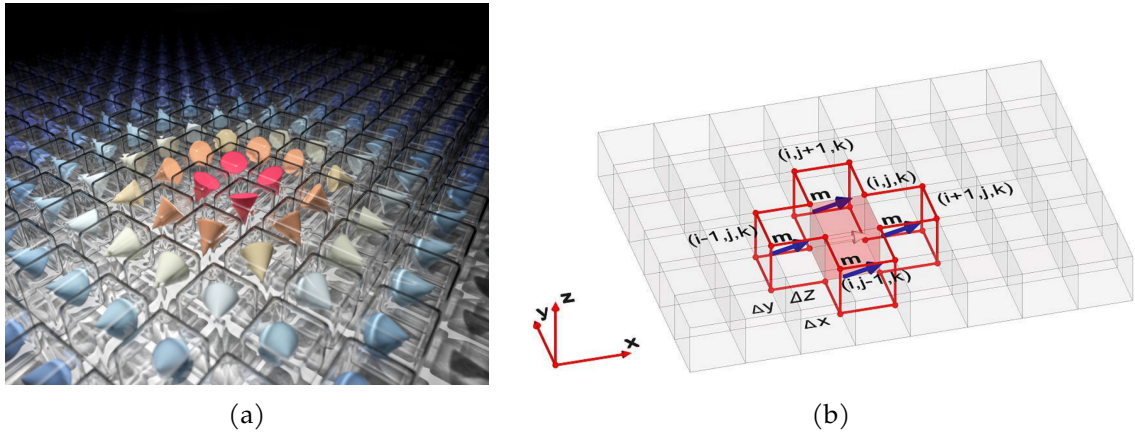
¹ This term, in the form shown in Eq. 3.1, was originally proposed by Landau and Lifshitz (1965), however leading to physical inconsistencies for high α . Gilbert later in 1955 proposed a damping term proportional to $d\mathbf{M}/dt$, correcting the issue and obtaining the same term of Landau and Lifshitz, but with the factor $1/(1 + \alpha^2)$ (GILBERT, 2004).

Figure 22 – Illustration of the magnetization dynamics terms in the Landau-Lifshitz-Gilbert (LLG) equation. (a) The precessional torque ($\sim \mathbf{m} \times \mathbf{H}_{\text{eff}}$) causes the magnetization vector $\mathbf{m}$ to rotate around the effective field $\mathbf{B}_{\text{eff}}$ on a constant-energy orbit. (b) The damping torque drives the magnetization towards the effective field, representing the energy dissipation. (c) The superposition of both terms results in the full LLG dynamics, where the magnetization follows a spiraling trajectory until it aligns with $\mathbf{B}_{\text{eff}}$.



Source: Adapted from Abert (2013).

Figure 23 – Panel (a) illustrates a cubic lattice of micromagnetic cells with volume V , where the average local normalized magnetization $\mathbf{m}(\mathbf{r})$ (oriented cones) results from the average of discrete spins inside the volume V and with orientation represented by colors [from red ($-\hat{z}$) to blue ($+\hat{z}$)]. In panel (b), we represent a finite-difference method, where spatial (and time) derivatives of $\mathbf{m}$ can be obtained by calculating value differences at neighboring sites.



Source: Fangohr (2015) (a) and Makarov (2014) (b).

2018). Finally, the combination of the local magnetization for every cell (or site) provides us the spatial profile of $\mathbf{M}(\mathbf{r})$ of the whole lattice, where derivatives are calculated via a finite-difference method [Fig. 23(b)].

The micromagnetic model is, thus, a powerful tool for mesoscopic scale (1 to 1000 nm) though it naturally breaks down at length scales comparable or smaller than the lattice constant a , since we cannot define $\mathbf{M}(\mathbf{r}, t)$ properly in this situation. It means that very small skyrmions (with only a few nm in diameter) cannot have its internal details well described for typical values adopted for a . Also, some physical processes such as

the skyrmion collapse necessarily implies changes of $\mathbf{M}(\mathbf{r}, t)$ in a scale comparable to interatomic distances, thus atomistic simulations are indeed necessary.

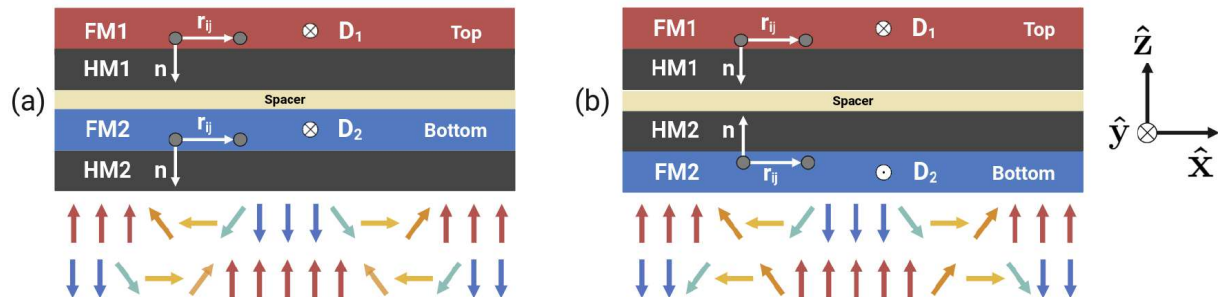
Nevertheless, from a computational physics perspective, the preference for micromagnetics over an atomistic model is justified by the **computational feasibility** of device-scale simulations, extending from hundred of nm to several μm . At this scale, atomistic simulations become prohibitive due to the vast number of degrees of freedom involved in resolving individual atomic moments (ALLEN; TILDESLEY, 2017). Also, the micromagnetic model remains **physically reasonable** for this work as $\mathbf{M}(\mathbf{r}, t)$ associated with skyrmions in synthetic antiferromagnets varies on a length scale significantly larger than the interatomic spacing. Therefore, implementing the micromagnetic model is well suited for the purpose of this thesis.

3.2 The multilayer system: synthetic antiferromagnet (SAF)

We investigate magnetic multilayers comprising two ferromagnet–heavy-metal (FM-HM) bilayers separated by a non-magnetic spacer. We assume that both FM layers are **coupled** antiferromagnetically via the RKKY coupling discussed in Sec. 2.2.3.2. Our multilayer structure is illustrated in Fig. 24.

We aim to understand what magnetic phases appear at the ferromagnetic (FM) coupled layers by assuming different interfaces between FM and heavy-metal (HM) films, those of (a) and (b) in Fig. 24. Recalling that in ultrathin films the DMI is pretty much

Figure 24 – The synthetic antiferromagnet (SAF). In the top, two stack structures are illustrated for parallel (a) and anti-parallel (b) Dzyaloshinskii–Moriya (DM) vectors $\mathbf{D}_k$ of the k -th ferromagnetic (FM) layer, according to the interface with the heavy metal. $\mathbf{D}_k$ is perpendicular both to the distance between the i -th and j -th lattice sites, $\mathbf{r}_{ij}$, and to $\hat{\mathbf{n}}$, a vector perpendicular to the interface. Changing the order of HM and FM in (b) modifies the orientation of $\mathbf{D}_2$ and then the skyrmion chirality in FM2. In the bottom, we show the lateral view of magnetic moments in each FM layer across a skyrmion pair for each case. In both cases, $Q_1 = -Q_2 = -1$.



Source: Correia, Velásquez and Silva (2024).

an interfacial effect arising from the HM layer's spin-orbit interaction, changing the interface directly impacts the DM vectors in each FM layer since $\mathbf{D}_{1,2} \propto \hat{\mathbf{n}} \times \mathbf{r}_{ij}$, where $\hat{\mathbf{n}}$ is the unit vector normal to the FM/HM interface, pointing to the HM layer, and $\mathbf{r}_{ij}$ is the distance vector between spins i and j . Also, varying the DMI constants implies changes in the skyrmion size, since the energy cost of the nucleus becomes different with respect to the opposite ferromagnetic background.

The first configuration, (a), illustrates an arrangement where the DM vectors $\mathbf{D}_1$ and $\mathbf{D}_2$ have the **same orientation**, parallel and pointing to $+\hat{\mathbf{y}}$, but different magnitudes in general since DMI constants $D_1 \neq D_2$. Below the multilayer, we depict the lateral view of the magnetic moments within FM layers, illustrating the profile of top and bottom skyrmions nucleated in the case of (a). Here, similar to situation in (b), the skyrmions have opposite topological charges ($Q_1 = -Q_2 = -1$) and different radii. Another property, the helicity h , defined in the azimuthal component of $\mathbf{m}$ given by $\Phi(\phi) = \phi + h$, is also found to differ among the layers. In the top skyrmion, as we can visualize next its domain wall in Fig. 24, the magnetic moments points outwards for $\phi = 0$ (right to the skyrmion nucleus): $\Phi_1(\phi) = 0 + h_1 = 0$. So, $h_1 = 0$. Conversely in the bottom skyrmion, for $\phi = 0$: $\Phi_1(\phi) = 0 + h_2 = \pi$. So, $h_2 = \pi$. The physical chirality (rotational sense), however, are **similar** since magnetic moments of both rotate counterclockwise, so the case (a) is known as the **homochiral SAF**.

We make a small but crucial change in the layer ordering of (b): the HM2 layer is now located above the FM2, while maintaining the FM1/HM1 interface fixed. This makes $\mathbf{D}_2$ to become antiparallel with respect to $\mathbf{D}_1$, reversing the chirality of the bottom skyrmion — the magnetic moments of FM2 now rotates in the clockwise direction. So the skyrmion helicities become the same ($h_1 = h_2 = 0$), with both pointing radially outwards, but their **chiralities are opposed**, making situation (b) to be known as the **heterochiral SAF**.

Moreover, we also considered the case of a **synthetic ferrimagnet** (SF_i) configuration, where the saturation magnetization of both bilayers, $M_{s(1)}$ and $M_{s(2)}$, are chosen to be different, i.e., the case of an **unbalanced multilayer**.

Accordingly, the effective field for each layer, $\mathbf{B}_{\text{eff},i}(t) = -M_s^{-1}(\delta\mathcal{H}/\delta\mathbf{m}_i)$ is calcu-

lated via the functional derivative of the total micromagnetic Hamiltonian

$$\mathcal{H} = \int dS [\mathcal{E}_1 d + \mathcal{E}_2 d + A_{\text{int}} \mathbf{m}_1 \cdot \mathbf{m}_2], \quad (3.3)$$

where $A_{\text{int}} > 0$ is the RKKY **interlayer** coupling stiffness, which favors antiferromagnetic alignment between spins of different layers, and $\mathcal{E}_i$ is the **intralayer** energy density of the i -th layer. $\mathcal{E}_i$ includes exchange, uniaxial anisotropy, Zeeman, and Dzyaloshinskii–Moriya (DM) contributions as we discussed earlier in Sec. 2.2 yielding

$$\mathcal{E}_i = A_i [(\partial_x \mathbf{m}_i)^2 + (\partial_y \mathbf{m}_i)^2] - K m_{z(i)}^2 - M_s \mathbf{B} \cdot \mathbf{m}_i + D_i [m_{z(i)} (\nabla \cdot \mathbf{m}_i) - \mathbf{m}_i \cdot (\nabla m_{z(i)})]. \quad (3.4)$$

A_i is the exchange stiffness and D_i is the DM constant for the i th ferromagnetic layer, with D_i possessing a lower and a upper limit allowing skyrmion phase stabilization in the sample.

The micromagnetic simulation is based on solving the LLG equation on a lattice of $L \times L \text{ nm}^2$ grid per layer, with $L = 100 - 250$, discretized into cells of size (a_x, a_y, a_z) , with periodic boundary conditions using a second order stochastic Runge-Kutta method and a custom finite-difference code developed in-house to be further described. To find the possible **equilibrium configurations** of the skyrmion pair, we initialize the FM layers with opposite magnetization background ($m_z = \hat{z}$ in the top and $m_z = -\hat{z}$ in the bottom). The magnetization vector in the circularly symmetric skyrmion texture is described by

$$\mathbf{m} = (\sin \Theta \cos \Phi, \sin \Theta \sin \Phi, \cos \Theta) \quad (3.5)$$

with $\Theta = \Theta(r)$ and $\Phi(\phi) = \nu \phi + h$, where ν is the vorticity and h is the helicity. We introduce a prototype skyrmion in each layer using a 360° domain-wall ansatz (WANG; YUAN; WANG, 2018)

$$\Theta(r) = 2 \arctan \left[\frac{\sinh(R/w)}{\sinh(r/w)} \right], \quad (3.6)$$

where R is the skyrmion radius, w is the domain wall width, and skyrmions are initially separated by a typical distance of 10 nm. Skyrmions of both layers have $\nu = 1$ and

- $h_1 = 0$ and $h_2 = \pi$, in the homochiral SAF; or
- $h_1 = h_2 = 0$, in the heterochiral SAF.

Finally, we choose between homo and heterochiral scenarios by changing the values of the DMI constants $\mathcal{D}_i$ in the simulation.

3.3 Validation and error analysis

The time integration of the LLG equation is performed using an explicit second-order **Predictor-Corrector method** (Midpoint method) (FIDLER; SCHREFFL, 2000). The predictor step estimates the local effective field at the half-step ($t + \Delta t/2$) using a forward Euler approximation. The corrector step then uses this intermediate field to advance the magnetization to $t + \Delta t$, when we manually insert the Stratonovich geometric constraint ($\|\mathbf{m}\| = 1$) at the end of each time integration step to ensure physical validity (GARCÍA-PALACIOS; LÁZARO, 1998). This scheme guarantees a global **time truncation error** of $\mathcal{O}(\Delta t^2)$.

To ensure **numerical stability** and properly resolve the high-frequency precessional modes of the magnetic skyrmions, the dimensionless time step is set to $\Delta t = 0.001$ in units of $(\mu_0 M_s \gamma)^{-1}$. Using the common material parameter $M_s = 0.58$ MA/m and $\gamma = 1.761 \times 10^{11}$ rad/(T · s), this corresponds to a physical time step of approximately 7.8 fs. Because this time step is several orders of magnitude smaller than the characteristic timescales of the Larmor precession, the temporal integration is within the stability region for a proper numerical integration. For the same reason, the time step is also more than suitable when we excite skyrmion breathing modes via cardinal sine signal with frequencies up to 100 GHz (Chap. 5), when the Predictor-Corrector code executes over a thousand integration steps per period. Therefore, we expect no numerical artifacts whatsoever arising in the simulation.

With respect to the **finite-difference method**, the spatial derivatives for the exchange and DMI are discretized using square grid with $a_x = a_y = 1$ nm. A five-point central difference stencil is utilized to compute the discrete Laplacian for the exchange field, yielding a spatial truncation error of $\mathcal{O}(a_x^2)$. To strictly validate the precision of this cell size, the grid resolution must be compared to the **exchange length**, defined as $l_{ex} = \sqrt{2A/\mu_0 M_s^2}$ (VANSTEENKISTE et al., 2014). Given the exchange stiffness $A = 15$ pJ/m and the saturation $M_s = 0.58$ MA/m, the exchange length is $l_{ex} \approx 8.42$ nm. Thus, the selected grid size strictly satisfies the condition $a_x = a_y \ll l_{ex}$, ensuring that the continuous rotation of $\mathbf{m}$ within the skyrmion domain walls is captured without

artificial lattice-pinning effects.

For the simulations of spin relaxation toward an equilibrium configuration, the system is considered to have converged to an equilibrium configuration once the absolute variation in the magnetization vectors over a fixed number of integration steps (i.e., the torque) drops below a predefined threshold (VANSTEENKISTE et al., 2014). Mathematically, **the relaxation is halted** when the difference of the components of $\mathbf{m}$ for both the top ($\mathbf{m}_1$) and bottom ($\mathbf{m}_2$) FM layers satisfies

$$\max_{i,j} \left(\sum_{\alpha=x,y} |m_{1,\alpha}^t - m_{1,\alpha}^{t-n}| + |m_{2,\alpha}^t - m_{2,\alpha}^{t-n}| \right) < \epsilon_m,$$

where t is the current number of time steps in the simulation, $n = 500$ the fixed testing interval, and $\epsilon_m = 10^{-8}$ is the tolerance threshold. This ensures the configuration is relaxed before any excitations are applied, although it does not guarantee we found a global minimum (ground state).

3.4 Stochastic dynamics and Spin-transfer torque

The dynamics of the normalized magnetization $\mathbf{m}_i(t) = \mathbf{M}_i(t)/M_{s(i)}$ ($i = 1$ for top and $i = 2$ for bottom) of each FM layer is described by the **stochastic** version of the LLG equation (Eq. 3.1),

$$\frac{\partial \mathbf{m}_i}{\partial t} = -\frac{\gamma}{1+\alpha^2} \left\{ \mathbf{m}_i \times (\mathbf{B}_{\text{eff},i} + \mathbf{B}_{\text{th}}) + \alpha \mathbf{m}_i \times [\mathbf{m}_i \times (\mathbf{B}_{\text{eff},i} + \mathbf{B}_{\text{th}})] \right\}. \quad (3.7)$$

where thermal fluctuations are modeled by a **stochastic field**, $\mathbf{B}^{\text{th}}(\mathbf{r}, t)$, as in the seminal work of Brown Jr. (1963). At temperature T , $\mathbf{B}^{\text{th}}(\mathbf{r}, t)$ satisfies the following properties, which are very similar to those of random forces in Brownian motion:

- vanishing average, i.e., $\langle B_j^{\text{th}}(\mathbf{r}, t) \rangle = 0$;
- spatio-temporal white noise: $\langle B_k^{\text{th}}(\mathbf{r}, t) B_l^{\text{th}}(\mathbf{r}', t') \rangle = 2\lambda_{kl}\delta(\mathbf{r} - \mathbf{r}')\delta(t - t')$, where k, l are the cartesian components, $\lambda_{kl} = (\alpha k_B T / \gamma M_s) \delta_{k,l}$ is the strength of thermal fluctuations, k_B is the Boltzmann's constant, and $\langle \rangle$ represents the ensemble average (BROWN JR., 1963).

The introduction of thermal fluctuations transforms the LLG equation into a stochastic differential equation (Eq. 3.7), requiring **highly efficient random number genera-**

tion due to the extended time scales of the simulation, so the GNU Scientific Library (GSL) for C/C++ is adopted to this end (GALASSI et al., 2009). We also emphasize that macroscopic parameters such as M_s , A and D are treated as temperature-independent constants.

In short, we obtain equilibrium configurations making $\mathbf{B}^{\text{th}} = 0$ and the system is relaxed at $T = 0$. This allows skyrmions to reach their natural shape and equilibrium separation. In some cases, skyrmion undergo collapse or explosion, and the magnetic moments rearrange themselves into a different magnetic phase. In the cases where $T > 0$ ($\mathbf{B}^{\text{th}} \neq 0$), the system is initiated from a equilibrium condition and, after a transient, it reaches steady-state conditions.

The **current-driven motion** is based on the Spin-transfer torque mechanism (Sec. 2.3.4). When we inject an electrical current into the FM layers at $T = 0$, we need to add a new torque term into the LLG equation (Eq. 3.7). We model the effect of a **spin-polarized current**, $\mathbf{j}_i = -j_i \hat{\mathbf{x}}$, applied along the x -direction into the i th FM layer via the Zhang-Li model for the **spin-transfer torque** (ZHANG; LI, 2004):

$$\boldsymbol{\tau}_i = u_i \left[\frac{\beta - \alpha}{1 + \alpha^2} \mathbf{m}_i \times \frac{\partial \mathbf{m}_i}{\partial x} - \frac{1 + \alpha\beta}{1 + \alpha^2} \frac{\partial \mathbf{m}_i}{\partial x} \right], \quad (3.8)$$

where

$$u_i = \frac{P\mu_B}{eM_{s(i)}(1 + \beta^2)} j_i \quad (3.9)$$

is the drift velocity of electrons in the spin-polarized current applied into the i th layer, β is the non-adiabaticity factor, P is the spin polarization, $\mu_B = 9.274 \times 10^{-24} \text{ J T}^{-1}$ is the Bohr magneton, and e is the electron charge.

4 EQUILIBRIUM AND CURRENT-INDUCED MOTION OF SKYRMIONS IN SYNTHETIC ANTIFERROMAGNETS (SAFS)

In this chapter, we describe the magnetic phases and the motion of skyrmion pairs in the synthetic antiferromagnet (SAF) according to several parameters such as the Dzyaloshinskii–Moriya interaction (DMI), the layer chirality, and the RKKY interlayer coupling. We obtain the formation of a bound skyrmion pair that can move with constant velocity in the longitudinal direction of the sample when a spin-polarized current is applied via spin-transfer torque (STT) effect. The current-induced motion is found to be affected by the Skyrmion Hall effect (SkHE) for different combinations of parameters — such as current amplitude, chirality, and material properties. Crucially for skyrmionic devices, we report that the SkHE persists in the presence of any asymmetries in the current and material parameters (e.g., M_s). Finally, the heterochiral SAF exhibits interesting properties both in equilibrium — a change in the skyrmion–skyrmion equilibrium distance due to RKKY coupling — and dynamics — enhanced experimental visualization of skyrmions via stray fields. Here, we adopt a micromagnetic cell of sides $a_x = a_y = 1$ nm and $a_z = 0.4$ nm, as well as $A_1 = A_2 = 15$ pJ/m.

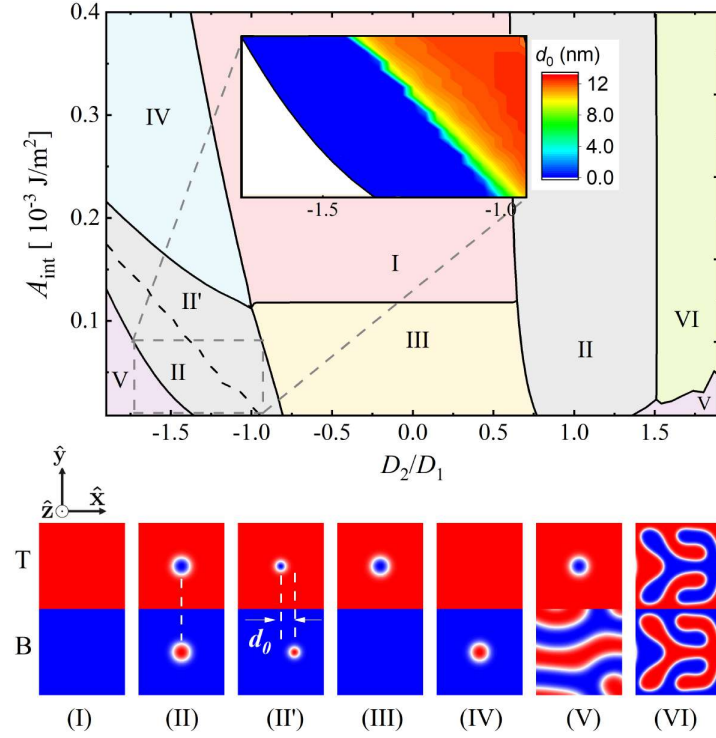
The results of this chapter are based on a study published in Correia, Velásquez and Silva (2024), where my main contributions were the following: (i) calculations regarding Hall angle via Thiele equation formalism; (ii) micromagnetic simulations for skyrmion dynamics in balanced and unbalanced SAFs; and (iii) the study of skyrmion stability via critical current analysis.

4.1 Stability of magnetic phases in a SAF

In this section, we describe the equilibrium magnetic phases obtained for a synthetic antiferromagnet, which include the case of skyrmion pairs in two different arrangements: (i) coaxial and (ii) non-coaxial. The energetics at equilibrium and the properties arising from the transition from (i) to (ii) is discussed in detail to justify the transition and to address improvements in experimental skyrmion tracking in SAFs systems.

The phase diagram of the multilayer arrangements (a) and (b) of Fig. 24 is shown in Fig. 25, where regions I–VI at the top indicate the magnetic phases obtained by micromagnetic simulations by changing both the relative DMI strength D_2/D_1 and the AFM

Figure 25 – Phase diagram for the multilayer in Fig. 24. Top panel shows phases (Roman numerals I–VI) as a function of the RKKY coupling A_{int} (vertical axis) and the relative amplitude of DMI D_2/D_1 (horizontal axis). For $D_2/D_1 < 0$ FM layers have opposite chiralities. Bottom panels (I–VI) depicts the phases by displaying the z component of magnetization for top (T) and bottom (B) layers with red (blue) indicating $m_z = +1$ (-1). Phases II and II' are separated by a dashed line (top panel) and correspond to coaxial and non-coaxial bound skyrmion pairs, respectively. The inset maps skyrmion–skyrmion separation d_0 for II and II'.



Source: Correia, Velásquez and Silva (2024).

coupling A_{int} . Below the diagram, these regions can be correspondingly visualized in panels I–VI, which display $m_z(x, y)$ for both layers. The diagram is obtained by initializing all the spins in the top (bottom) layer with $\mathbf{m}_i = \pm \hat{z}$, except for a circular region at the center of the sample where we flip the spins to $\mp \hat{z}$. We tested different initial separations between skyrmion nuclei (0, 10, and 18 nm) to assess potential metastability and then allowed the system to relax to equilibrium at $T = 0$. Finally, the spins either arrange themselves in the skyrmions' shape and mutual distance or produce a different final configuration (FM saturation, spin spiral or labyrinthian) after a longer transient.

For small $|D_2/D_1|$, the system is in saturation of both layers (I) or only at the bottom (III). In this situation, and when the RKKY coupling is small, $|D_1|$ is large enough to stabilize a skyrmion (III), which is not the case for $|D_2|$. If A_{int} increases above some threshold, the fully polarized bottom layer, plus the AFM coupling, acts as an effective

magnetic field $\mathbf{B}_{\text{Top}}$ given by

$$\mathbf{B}_{\text{Top}} = -\frac{1}{M_s} \frac{\delta}{\delta \mathbf{m}_1} \int dS (A_{\text{int}} \mathbf{m}_1 \cdot \mathbf{m}_2) = -\frac{A_{\text{int}}}{M_s} \mathbf{m}_2 = \frac{A_{\text{int}}}{M_s} \hat{z}. \quad (4.1)$$

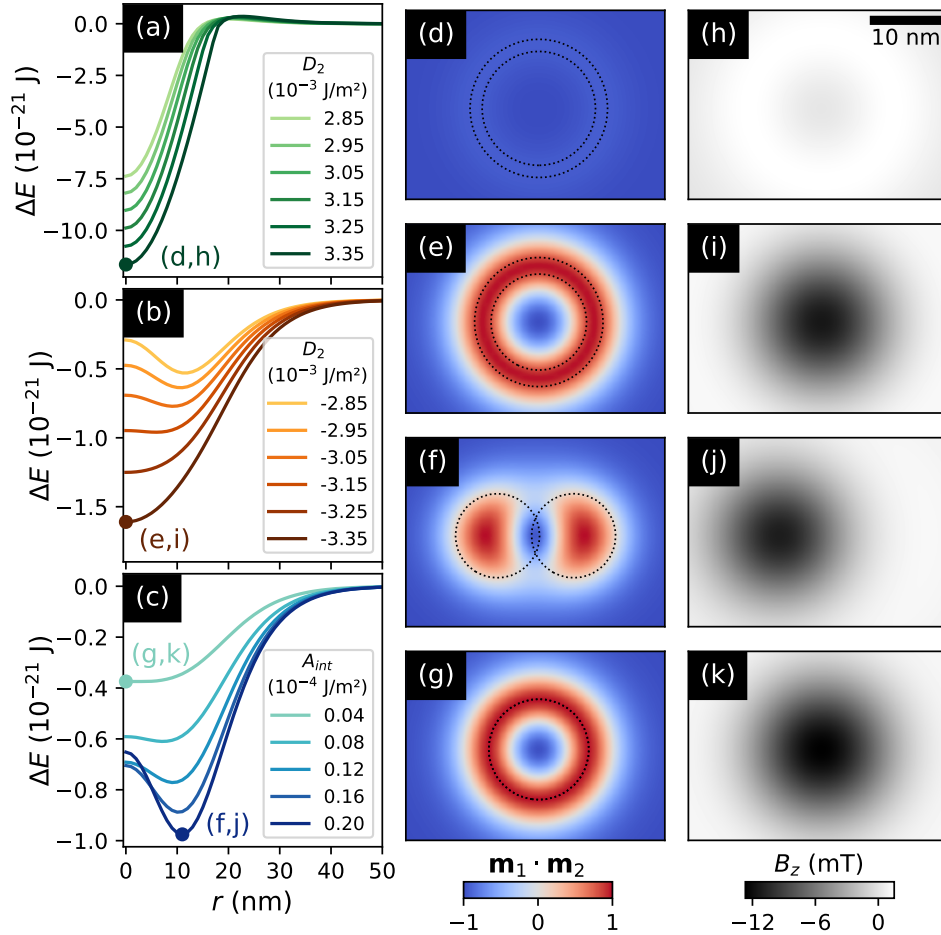
$\mathbf{B}_{\text{Top}}$ is homogeneous and increases with A_{int} , so above $A_{\text{int}} \approx 0.12 \text{ mJ/m}^2$ the top layer ultimately decays into the ferromagnetic phase (region I). So far, the picture is the same for homochiral and heterochiral SAFs, respectively in the right and left halves of the phase diagram in Fig. 25.

For small coupling and $D_2/D_1 \leq -1$, the stabilization of bound skyrmion pairs is observed. Skyrmions are laterally separated by $\mathbf{d}_0 = d_0 \hat{x}$ in **coaxial** (II, for $d_0 = 0$) and **non-coaxial** (II', for $d_0 > 0$) alignments of the heterochiral SAF. The inset in the top panel of Fig. 25 shows the skyrmion-skyrmion distance d_0 in the narrow vicinity of II and II' phases, where a coaxial to non-coaxial transition is observed as d_0 rapidly increases up to 12 nm. The skyrmion pair is such that the top layer features a Néel skyrmion with a central spin $m_z = -1$ ($Q = -1$), whereas the opposite $m_z = 1$ ($Q = +1$) occurs at the bottom's. Within either phases II or II', increasing interlayer coupling A_{int} makes the skyrmion radius to decrease as a result of enhancing the background spins effective field (Eq. 4.1). Ultimately, starting from the II' configuration and increasing A_{int} long enough, we reach phase IV: the top skyrmion collapses, as it cannot be further stabilized, and magnetic saturation is achieved in the top layer.

Homochiral SAFs, however, are restricted to exhibit coaxial pairs for the entire range of A_{int} analyzed and for approximately $0.75 \leq |D_2/D_1| \leq 1.5$. As long as DMI is within this stabilizing range, changing A_{int} does not affect the pair stability, since the energy cost of maintaining concentric nuclei and domain wall is not affected by the simultaneous expansion or reduction of skyrmions at different layers.

The energy analysis exhibited in Fig. 26 helps us to understand how II and II' phase's pairs behave. In panels (a)–(c) we show the profiles obtained numerically for the skyrmion-skyrmion interaction according to their separation r . The profiles are built by the **frozen-core method**: the central spin of the skyrmions are fixed in simulation, so they are positioned at a distance r and the energy difference $\Delta E(r) = E(r) - E_\infty$ is calculated, where $E(r)$ is the energy of freezing skyrmions and E_∞ is the energy of infinitely separated skyrmions ($r \rightarrow \infty$). In (a), we show the homochiral case (II) for D_1 and D_2 with same sign, but different magnitudes. Clearly, the energy minima locates at $r = 0$,

Figure 26 – (a)–(c) Panels show skyrmion-skyrmion interaction energy profiles ΔE as a function of skyrmions' distance r in three situations: (a) D_1 and D_2 with same sign, (b) D_1 and D_2 opposite signs, and (c) $D_1 = -D_2$ and different values of AFM coupling A_{int} . For (a),(b), $A_{\text{int}} = 12 \times 10^{-3} \text{ mJ/m}^2$ is fixed. In all cases, $D_1 = 3.05 \times 10^{-3} \text{ mJ/m}^2$. (d)–(g) Contour plots of normalized RKKY coupling energy density $\mathbf{m}_1 \cdot \mathbf{m}_2$ for the correspondent solid circles of the energy panels (a)–(c). Circular dotted lines indicate the domain wall for the skyrmions. (h)–(k) Correspondent contour plots for the out-of-plane component of the stray field as calculated 10 nm above the top layer.



Source: Correia, Velásquez and Silva (2024).

so the pair is indeed coaxial. In (b), the heterochiral case (II') with DMI of opposite sign and different magnitudes leads to a change in the energy minimum location for decreasing bottom DMI magnitude: from $r = 0$ at $D_2 = -3.35 \text{ mJ/m}^2$ to $r \approx 11 \text{ nm}$ at $D_2 = -3.05 \text{ mJ/m}^2$. This is equivalent to follow a horizontal line to the right when crossing from II to II' phases. A similar minimum transition is observed in the profiles of (c), when both DMI constants are fixed with opposite sign ($D_1 = -D_2 = 3.05 \text{ mJ/m}^2$) and the interlayer coupling is increased from 0.004 (coaxial) to 0.02 mJ/m^2 (non-coaxial).

Additionally, in order to visualize the skyrmion-skyrmion interaction in detail, four solid points indicated in panels (a)–(c), are selected. The corresponding heat maps

of the interlayer coupling energy density $\mathcal{E}_{\text{RKKY}}/A_{\text{int}} = \mathbf{m}_1 \cdot \mathbf{m}_2$ are shown in panels (d)–(g). Dotted circular lines indicate the boundary for skyrmions nuclei ($m_{i,z} = 0$) with different radii — since $D_1 \neq D_2$. Blue regions depict lower energy for the system (spins are antiparallel), while reddish ones indicate higher energy cost. In (d), a coaxial pair is shown almost completely in blue — some white-ish background is still observed — indicating an imperfect match of spin orientation between top and bottom layers' magnetization due to different DMI constants. By inverting the sign of D_2 , we obtain the situation of panel (e): the change of bottom skyrmion's chirality (now spins rotate in opposite sense) creates a **mismatch** between the two skyrmions in the pair. The result is a reddish circular region (annulus), which is energetically demanding for a system which favors antiparallel spins of different layers. The mismatch is critical along the skyrmion's domain wall, as anticipated in the bottom illustration of Fig. 24. Nevertheless, the system still minimizes energy in this situation by maintaining the coaxial arrangement.

The magnetization mismatch is not sustainable if the interlayer coupling largely increases in the heterochiral case. In panel (g), the red annulus appears in a coaxial skyrmion pair of $D_1 = -D_2 = 3.05 \text{ mJ/m}^2$ for $A_{\text{int}} = 4 \times 10^{-3} \text{ mJ/m}^2$. However at $A_{\text{int}} = 20 \times 10^{-3} \text{ mJ/m}^2$ [panel (f)] the situation changes dramatically: skyrmions separate themselves along the x axis, becoming a non-coaxial pair, even if a large spatial part of a single skyrmion's magnetization is not matching the characteristic antiparallel alignment with the other layer. Therefore, the interlayer RKKY coupling induces a change in the equilibrium pair as it was previously observed in Koshibae and Nagaosa (2017).

The inversion of DMI sign, transitioning from a homochiral to an heterochiral SAF, also suggests crucial improvement in the visualization of skyrmions during experiments. Alongside the $\mathbf{m}_1 \cdot \mathbf{m}_2$ heatmaps, panels (h)–(k) exhibit the maps for the out-of-plane component of the stray field for the multilayer in the correspondent situations depicted in panels (d)–(g). The field is calculated in the plane located 10 nm above the top layer. From the homochiral to heterochiral cases, the contrast in the maps between (h) and (i)–(k) is observed to dramatically increase, allowing for better tracking of skyrmion's trajectories via techniques such as Magnetic Force Microscopy (MFM) (DOHI et al., 2019; LEGRAND et al., 2020; PHAM et al., 2024a) — which is based on the scanning of stray field's landscape above the multilayer and often faces a challenge in

determining the dynamics of (topologically neutral) coaxial skyrmion pairs.

Lastly, in the phase diagram a spin spiral (panel V) appears for $|D_2/D_1| > 1.5$ in the bottom layer, while for homochiral alone both skyrmions explode ($A_{\text{int}} \geq 0.05$) into a coupled labyrinthine configuration (panel VI). This establishes the parameter limits of coaxial and non-coaxial skyrmion pairs at equilibrium in the SAF. In the next section, we shall discuss our findings for the skyrmion dynamics when induced by spin-polarized currents.

4.2 Dynamics of coaxial and non-coaxial skyrmion pairs in synthetic antiferromagnets (SAF) and ferrimagnets (SFi)

In this section we discuss the current-induced motion — via the spin torque transfer effect — of the skyrmion pairs discussed in the previous section. Firstly, we present the quasi-particle description of the problem provided by the Thiele equation, addressing the Skyrmion Hall Effect regarding the possibilities of mitigating or eliminating it in the dynamics. For that matter, we also discuss unbalanced multilayers which we refer as synthetic ferrimagnets (SFi), where saturation magnetization is different for each layer. Moreover, the results obtained from micromagnetics are discussed and compared to the theoretical prediction by the Thiele equation, with a final analysis on the stability of skyrmion pairs during the injection of spin-polarized currents.

4.2.1 Dynamics of skyrmion pairs in SAF/SFi: Thiele equation and micromagnetics

4.2.1.1 Spin transfer torque (STT) scenario

The description of a skyrmion as a quasi-particle is valid as long as its circular shape is considered to be intact, i.e., when skyrmions are not deformed in any form. Under this regime, we proceed by describing the skyrmion pair motion by the injection of in-plane spin-polarized current $\mathbf{j}_i = j_i \hat{\mathbf{x}}$ using the Thiele equation. We adopt $\beta = 0.6$ as the non-adiabaticity factor and $P = 0.4$ as the spin polarization in the Zhang-Li torque (Eqs. 3.8 and 3.9). Here, the author was in charge of the complete set of micromagnetic simulations and Thiele equation calculations. In our case, the skyrmion at the i th FM

layer ($i = 1, 2$) is described by an equation in the form

$$\mathbf{F}_{ij} + \mathbf{G}_i \times (\mathbf{u}_i - \dot{\mathbf{r}}_i) + \mathcal{D}_i (\beta_i \mathbf{u}_i - \alpha_i \dot{\mathbf{r}}_i) = 0. \quad (4.2)$$

Unraveling Eq. 4.2, $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$ is the force on skyrmion i imparted by skyrmion j in the other layer, i.e., the binding force, $\mathbf{G}_i = \mathcal{G}_i \hat{\mathbf{z}} = 4\pi Q_i (d_i M_{s(i)} / \gamma) \hat{\mathbf{z}}$ is the gyromagnetic coupling vector, $Q_1 = -Q_2 = -1$ is the skyrmion topological charge, $\mathbf{u}_i$ is the drift velocity of the electrons in the current, $\dot{\mathbf{r}}_i$ is the velocity of the skyrmion's center-of-mass. The dissipation tensor of the skyrmion, $\mathcal{D}_i = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathcal{D}_i$, is assumed to be diagonal, reproducing the assumption that skyrmions do not deform. Considering the azimuthal symmetry of the skyrmions, we write the dissipation tensor as

$$\mathcal{D}_i = (d_i M_{s(i)} / \gamma) \pi \mathcal{I}_i, \quad (4.3)$$

where

$$\mathcal{I}_i = \int_0^\infty \rho d\rho \left[\left(\frac{d\Theta}{d\rho} \right)^2 + \frac{\sin^2 \Theta}{\rho^2} \right] \quad (4.4)$$

is the dimensionless dissipation integral, with $\Theta(\rho)$ as the polar angle of the spins positioned at a distance ρ from the skyrmion center. Equation 4.2 can be viewed as a force balance condition for each skyrmion, where the force exerted by the other skyrmion (first term) is balanced by both the Magnus force (second term) and the downstream drag force (third term). This approach allows for the derivation of analytical estimates for skyrmion trajectories, which is compared with micromagnetic simulations.

By assuming $\mathbf{u}_i = u_i \hat{x}$ and separating Eq. 4.2 into its cartesian components

$$\hat{x}: \quad \mathcal{G}_i \dot{y}_i - \alpha_i \mathcal{D}_i \dot{x}_i + \beta_i \mathcal{D}_i u_i + F_{ij}^{(x)} = 0, \quad (4.5a)$$

$$\hat{y}: \quad \mathcal{G}_i (u_i - \dot{x}_i) - \alpha_i \mathcal{D}_i \dot{y}_i + F_{ij}^{(y)} = 0, \quad (4.5b)$$

we immediately recover the Hall angle $\Theta_{\text{SkH}}^{(i)}$ for a single isolated skyrmion (IWASAKI; MOCHIZUKI; NAGAOSA, 2013) by making $\mathbf{F}_{ij} = 0$, i.e., in the absence of coupling:

$$\tan \Theta_{\text{SkH}}^{(i)} = \frac{\dot{y}_i}{\dot{x}_i} = \frac{(\alpha_i - \beta_i) \mathcal{G}_i \mathcal{D}_i}{\mathcal{G}_i^2 + \alpha_i \beta_i \mathcal{D}_i^2}. \quad (4.6)$$

By considering a strong coupling, the skyrmions are assumed to move together with the same velocity in steady state, i.e., $\dot{\mathbf{r}}_1 = \dot{\mathbf{r}}_2 = \mathbf{v}$. For symmetric FM layers — with the

same magnetic parameters such as α , M_s and D , summing Eq. 4.5b for skyrmion 1 and 2 leads to $v_y = 0$, thus eliminating the SkHE as there is no transverse motion, what is expected for a coaxial pair within a synthetic antiferromagnet.

However, when the layers are asymmetric, the same procedure leads to the following general relation

$$v_y = 4 \frac{(\mathcal{M}_1 - \mathcal{M}_2)v_x - (\mathcal{M}_1 u_1 - \mathcal{M}_2 u_2)}{(\alpha_1 \mathcal{M}_1 \mathcal{I}_1 + \alpha_2 \mathcal{M}_2 \mathcal{I}_2)}, \quad (4.7)$$

where $\mathcal{M}_i = d_i M_{s(i)}$ is the magnetic moment of the layer per unit area. This formula exhibits the situations in which the SkHE is eliminated in the sample: if the speeds of the spin polarized electrons, u_i , and the magnetic moments, $\mathcal{M}_i$, are perfectly balanced, then the numerator of Eq. 4.7 vanishes and the SkHE is removed. This happens irrespective of any other imbalance in the Gilbert damping parameters, α_i , non-adiabaticity factors, β_i , and in the dissipation integrals, $\mathcal{I}_i$. Therefore, **the cancellation of the SkHE in SAF bilayers is robust with respect to asymmetries in several magnetic parameters**, such as A , D , and K . These parameters determine the size of the skyrmions and thereby result in asymmetric dissipation integrals. In this way, we can host skyrmions with different sizes and, even so, the SkHE is absent. However, the elimination of the Hall effect is not robust against asymmetries in u_i or $\mathcal{M}_i$, which we address below.

Firstly, we analyze the sensitivity of the Hall angle Θ_{SkH} with respect to an **asymmetry in the current density**, i.e., when $j_1 \neq j_2$. Assuming equal magnetic moments for both layers ($\mathcal{M}_1 = \mathcal{M}_2 = \mathcal{M}$), the tranverse velocity for the pair (Eq. 4.7) becomes

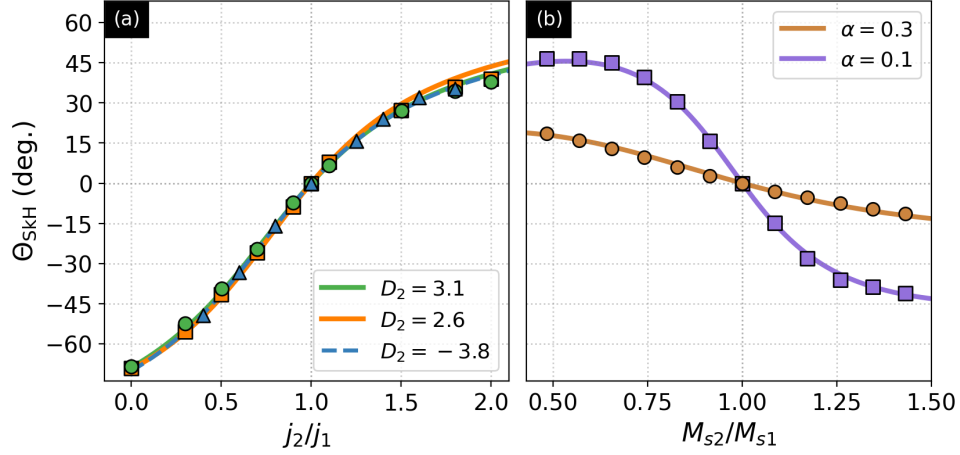
$$v_y = -4 \frac{(u_1 - u_2)}{\alpha_1 \mathcal{I}_1 + \alpha_2 \mathcal{I}_2}, \quad (4.8)$$

and combining with the original Eqs. 4.5a, the Hall angle for the coupled skyrmion pair reads

$$\Theta_{\text{SkH}} = \arctan \left[-\frac{4(u_1 - u_2)}{\beta_1 \mathcal{I}_1 u_1 + \beta_2 \mathcal{I}_2 u_2} \right]. \quad (4.9)$$

Fig. 27(a) shows Θ_{SkH} calculated analytically via Eq. 4.9 (solid lines) and from micromagnetic simulations (symbols) as a function of j_2/j_1 for fixed $D_1 = 3.1 \text{ mJ/m}^2$. We recall that u_i and j_i are related by Eq. 3.9. During simulations, we keep $j_1 = 9.45 \times 10^{11} \text{ A m}^{-2}$ applied to the top layer and vary j_2 within the range $0 < j_2 < 2j_1$. This choice of parameters guarantees the integrity of the pair, so the skyrmions remain coupled to each other. For each j_2 , Θ_{SkH} is calculated from the steady state velocity

Figure 27 – Hall angle Θ_{SkH} of the skyrmion pair in a SAF subjected to unbalanced currents (a) and a SFi subjected to uniform current (b) for analytical solutions of the Thiele equation (lines) and micromagnetic data (symbols). $M_{s1} = 0.58 \times 10^6$ A/m and $D_1 = 3.1$ are fixed. In (a), j_2 is varied and $\beta_1 = \beta_2 = 0.3$ for $D_2 = 3.1$ (green) and $D_2 = 2.6$ (orange) when $j_1 = 9.45$ and $A_{\text{int}} = 0.012$. For $D_2 = -3.8$ (dotted blue), $j_1 = 4.72$ and $A_{\text{int}} = 0.06$. In (b), we vary M_{s2} for $j_1 = 9.45$ and $j_2 = (M_{s2}/M_{s1})j_1$ for $\alpha = 0.3$, $A_{\text{int}} = 0.012$ (beige) and $\alpha = 0.1$, $A_{\text{int}} = 0.024$ (purple). Here, A_{int} , D_1 , and D_2 are in units of 10^{-3} J/m², whereas j_i are in units of 10^{11} A/m².



Source: Correia, Velásquez and Silva (2024).

components of the skyrmion pair, after some initial transient. We assume symmetric non-adiabaticity factors $\beta_1 = \beta_2$ and saturation magnetizations $M_{s(1)} = M_{s(2)}$, so that $j_2/j_1 = u_2/u_1$ (see Eq. 3.9). Additionally, we consider two different values of the DMI constant for the bottom layer, $D_2 = 2.6$ mJ/m² and $D_2 = -3.1$ mJ/m² each one resulting in different shapes of both skyrmions and, thereby, different values of the dissipation integrals $\mathcal{I}_i$, calculated numerically using Eq. 4.4.

As observed by the overlapping of symbols and solid lines in Fig. 27, the analytical estimates obtained from Eq. 4.9 are found to be in excellent agreement with the simulations. The case for $j_1 = j_2$ ($u_1 = u_2$), with no Hall effect, is reproduced in the center. The curves and points do not depend on the magnitude and sign of bottom DMI. Interestingly, for $j_2 = 0$ ($u_2 = 0$), where only the top skyrmion is pushed by the current while the bottom one is dragged along because of the coupling, the Hall angle reduces to a minimum of $\Theta_{\text{SkH}} = \arctan[-4/(\beta_1 \mathcal{I}_1)] = -68.4^\circ$. This value is in stark contrast with the Hall angle of an isolated (uncoupled) skyrmion in the top layer, which from Eq. 4.6 is 16° , and is due precisely to the larger transverse force originated by dragging the bottom skyrmion, making Θ_{SkH} to drastically swing via an “anchor” effect.

Now we discuss **asymmetries in saturation magnetizations** M_s , which results in

an uncompensated SAF stack, creating a **synthetic ferrimagnet (SFi)**. Changes in M_s have a direct impact on the drift velocity u , as $u \propto M_s^{-1}$ according to the Zhang Li model (Eq. 3.9). In particular, if the same current density is applied to both FM layers ($j_1 = j_2$) and further assuming that P and β are the same for both layers, we obtain the condition $M_{s(1)}u_1 = M_{s(2)}u_2$ and, from Eq. 4.7, the Hall angle Θ_{SkH} is given by

$$\Theta_{\text{SkH}} = \arctan \left[\frac{4(\mathcal{M}_1 - \mathcal{M}_2)}{(\alpha_1 \mathcal{M}_1 \mathcal{I}_1 + \alpha_2 \mathcal{M}_2 \mathcal{I}_2)} \right]. \quad (4.10)$$

However, it is instructive to evaluate the **isolated effect** of unbalanced magnetizations. For that, we assume all magnetic parameters are the same for both layers, except $\mathcal{M}$ and $\mathcal{I}$, and that the spin polarized currents are such that $u_1 = u_2$. Combining Eqs. 4.5 for both skyrmions in this situation, we obtain

$$\Theta_{\text{SkH}} = \arctan \frac{4(\beta - \alpha)\Delta\mathcal{M}(\mathcal{M}_1\mathcal{I}_1 + \mathcal{M}_2\mathcal{I}_2)}{\alpha\beta(\mathcal{M}_1\mathcal{I}_1 + \mathcal{M}_2\mathcal{I}_2)^2 + (4\Delta\mathcal{M})^2}, \quad (4.11)$$

where $\Delta\mathcal{M} = \mathcal{M}_1 - \mathcal{M}_2$.

In Fig. 27(b) we present the Hall angle of a skyrmion pair driven in a SFi as a function of the ratio M_{s2}/M_{s1} for fixed $M_{s1} = 0.58 \times 10^6$ A/m. The symbols of micromagnetic simulations correspond to two different values of D_2/D_1 and the lines were obtained from Eq. 4.11. Once again, the analytical Thiele approach shows excellent agreement with the micromagnetics. Similar to the case of uneven values of u , the imbalance in magnetic moments results in a non-zero Hall angle and is rather insensitive to the DMI ratio (as long as it falls within the range of the skyrmion pair phase).

Both analysis — imbalance of applied current and of saturation magnetization of the FM layers — show, therefore, that the Hall angle Θ_{SkH} of a SAF skyrmion pair is **highly sensitive** to unevenness in any of the two physical quantities.

Moreover, since such asymmetries are essentially unavoidable under typical experimental conditions — for example due to the irregularities during sample manufacturing — it is important to investigate parameter choices that **minimize this sensitivity** in the limit of small asymmetries. Defining the relative deviations in u , $\epsilon_u = (u_2 - u_1)/u_1$, and in M_s , $\epsilon_M = (M_{s2} - M_{s1})/M_{s1}$, and assuming these deviations to be small, Θ_{SkH} satisfies a linear relation

$$\Theta_{\text{SkH}} = \gamma_\mu \epsilon_\mu \quad (4.12)$$

for each case, with $\mu = u, M$, where the linear coefficients,

$$\gamma_u = \left[\frac{\partial \Theta_{\text{SkH}}}{\partial \epsilon_u} \right]_0 = -\frac{4}{\beta(\mathcal{I}_1 + \mathcal{I}_2)}, \text{ for } \mathcal{M}_1 = \mathcal{M}_2, \quad (4.13a)$$

$$\gamma_M = \left[\frac{\partial \Theta_{\text{SkH}}}{\partial \epsilon_M} \right]_0 = \frac{4}{\alpha_1 \mathcal{I}_1 + \alpha_2 \mathcal{I}_2}, \text{ for } j_1 = j_2, \text{ and} \quad (4.13b)$$

$$\gamma_M = \left[\frac{\partial \Theta_{\text{SkH}}}{\partial \epsilon_M} \right]_0 = \frac{4(\beta - \alpha)}{\alpha\beta(\mathcal{I}_1 + \mathcal{I}_2)}, \text{ for } u_1 = u_2, \quad (4.13c)$$

were obtained from Eqs. 4.9, 4.10, and 4.11, respectively.

Therefore, in both cases of SAFs and SFIs, the route to lower sensitivity to sample inhomogeneities is to minimize the linear coefficients by maximizing the dissipation integrals $\mathcal{I}_i$ in both layers. Additionally, for the cases of magnetic asymmetries, it is important to avoid materials with small values of the Gilbert parameter α , since the responsiveness arising from such asymmetries can be enlarged at low damping. For example, in the case considered in Fig. 27(b), where $\alpha = 0.3$, a 10% mismatch in magnetizations leads to a small Hall angle, $\Theta_{\text{SkH}} \simeq 3^\circ$. By assuming materials with $\alpha = 0.1$ [Fig. 27(b), in purple], the Hall angle for this small mismatch jumps to 16° ! The downside of adopting materials with larger α , however, is the consequently decrease of the skyrmion pair velocity.

4.2.1.2 Hall-angle under the spin-orbit torque (SOT) scenario

In the case where the skyrmions are driven by the spin-orbit torque mechanism (Sec. 2.3.4), the current $\mathbf{j}^{hm}$ is now injected into the heavy-metal layer and the Thiele equation can be expressed as (JIANG et al., 2017)

$$\mathbf{F}_{ij} - \mathbf{G}_i \times \dot{\mathbf{r}}_i - \alpha_i \mathcal{D}_i \dot{\mathbf{r}}_i + \mathbf{F}_{D,i} = 0, \quad (4.14)$$

where $\mathbf{F}_{D,i} = 4\pi \mathcal{B}_i \mathbf{j}_i^{hm}$ is the skyrmion driving force, $\mathbf{j}_i^{hm}$ is applied to the i -th HM layer and $\mathcal{B}_i$ quantifies the efficiency of the spin Hall effect.

This equation is similar to the STT case, Eq. 4.2, but with an important difference: in the SOT case, the Magnus forces [second term in 4.14] **does not depend on the driving forces**. Therefore, asymmetries in the currents applied to the HM layers cannot per se induce the Skyrmion Hall effect. Indeed, repeating the procedure we adopt in the STT case, we sum Eq. 4.14 for both skyrmions under the coupling condition where

$\dot{\mathbf{r}}_1 = \dot{\mathbf{r}}_2 = \mathbf{v}$). This leads to the following general equation for the SOT-driven skyrmion Hall angle:

$$\Theta_{\text{SkH}}^{\text{SOT}} = \arctan \left[\frac{4(\mathcal{M}_1 - \mathcal{M}_2)}{\alpha_1 \mathcal{M}_1 \mathcal{I}_1 + \alpha_2 \mathcal{M}_2 \mathcal{I}_2} \right]. \quad (4.15)$$

This formula coincides with Eq. 4.10, which is the case of STT when identical current densities are applied to the FM layers. Here, we should emphasize, Eq. 4.15 applies to any combination of $\mathbf{j}_1^{hm}$ and $\mathbf{j}_2^{hm}$, even in the case where the current is applied to only one of the HM layers! Therefore, within the SOT scenario, balancing the magnetic moments of the layers is a condition enough to eliminate the SkHE.

4.3 The breaking of the skyrmion pair

In this section, we analyze the limits of the skyrmion pair regarding its stability when we inject spin-polarized currents. We want to determine the maximum in-plane current density j_c we can apply without breaking the pair, as well as the maximum longitudinal speed v_x it reaches before decoupling.

We consider two scenarios:

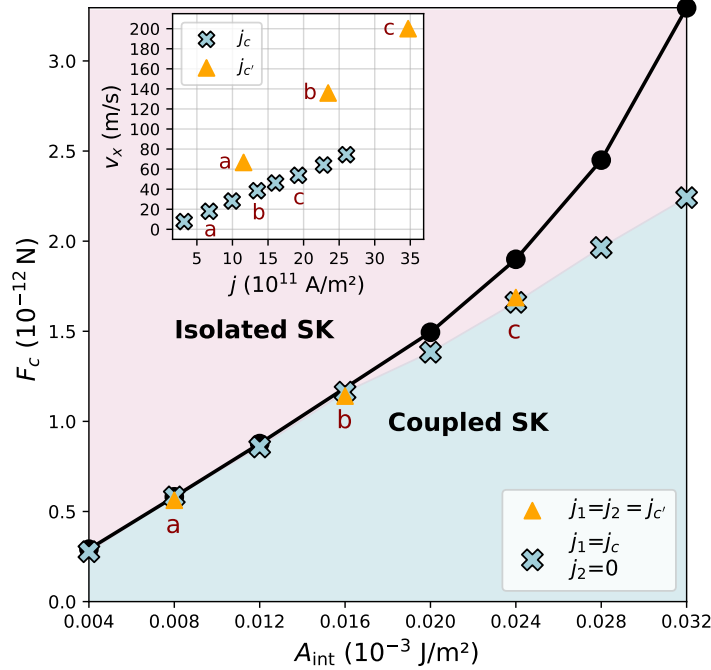
- (i) applying the same spin-polarized current to both layers ($j_1 = j_2$); and
- (ii) applying a spin-polarized current only to the top layer ($j_2 = 0$).

For the sake of simplicity during the analysis, we assume a perfectly symmetrical SAF stack in both scenarios. In scenario (i), since the Hall angle is zero according to Eq. 4.9, the pair splitting is provoked only by the **oppositely oriented Magnus forces** acting on the skyrmions in the transverse y-direction, as follows from inspection of the Thiele Eq. 4.2. In scenario (ii), not only the Magnus force but also the difference in viscous drag between the top skyrmion — which moves downstream — and the bottom skyrmion — which moves in “still water” — contributes to the splitting.

For each scenario, we performed micromagnetic simulations for different values of the RKKY interaction A_{int} , starting from a coupled pair of identical skyrmions and then slowly changing the current density until the pair breaks. We calculate the critical binding force, F_c , the skyrmions exert on each just below the critical decoupling current, j_c , using Eq. 4.2 to solve for F_{ij} and replacing the skyrmion pair velocity and dissipation

tensor by the values obtained numerically from the micromagnetics data. The results of this semi-analytical approach are shown in Fig. 28.

Figure 28 – Critical decoupling force F_c for the skyrmion pair in a symmetric SAF ($D_1 = D_2 = 3.05 \times 10^{-3} \text{ J m}^{-2}$). Black line with dots refers to F_c as obtained by the frozen-core method. Symbols indicate micromagnetic results for two current configurations: $j_1 = j_c$ and $j_2 = 0$ [scenario (i), blue], and $j_1 = j_2 = j_c'$ [scenario (ii), orange], where j_c and j_c' are the critical decoupling currents of the respective case. The inset shows the maximum skyrmion pair velocity component parallel to $\mathbf{j}$. Letter labels identify A_{int} in each case.



Source: Correia, Velásquez and Silva (2024).

We observe a linear dependence of F_c on A_{int} for $0.004 \leq A_{\text{int}} \leq 0.032$ in units of mJ/m^2 . Moreover, both scenarios lead to essentially the same binding force (see the overlap of orange over blue symbols in a, b and c), thus supporting our analysis via the force balance (Thiele) equations. Regarding the corresponding skyrmion maximum speed before decoupling, the inset depicts its value according to the critical current densities j_c' [scenario (i)] or j_c [scenario (ii)]. Notice that, for a given A_{int} , the $j_1 = j_2$ (or $u_1 = u_2$) scenario leads to considerably higher top speeds of the pair — about four times, by observing points a, b, and c.

Additionally, we compare the binding force estimated from the micromagnetic simulations with that obtained by the frozen-core method. In this alternative approach, we obtain the maximum derivative of the skyrmion-skyrmion pair potential, which corresponds to the maximum force in which the pair remains bounded. As seen in Fig. 28, the

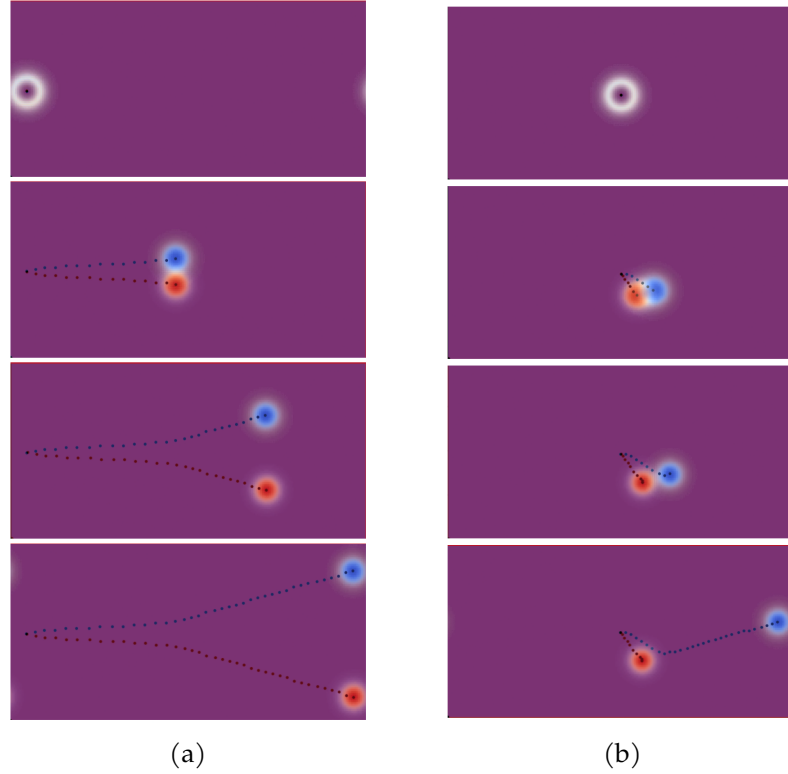
frozen-core method successfully anticipates the maximum binding force for the case of small RKKY coupling, while for $A_{\text{int}} \geq 0.02 \times 10^{-3} \text{ J m}^{-2}$ the two methods diverge. The issue is that the frozen-core technique constrains the spins at the skyrmion core possibly leading to spurious deformations in the rest of the skyrmion body. This effect is more pronounced for large A_{int} , which might explain the poor agreement observed in this range. On the other hand, estimating pair potentials can be very useful for obtaining analytical results or for investigating the many-skyrmions dynamics via **molecular dynamics simulations** — which exploits the quasi-particle nature of skyrmion textures and are much less computationally demanding than micromagnetics.

Finally, illustrations of the decoupling scenarios are presented by the superimposed heatmaps of the z component of $\mathbf{m}_1$ and of $\mathbf{m}_2$ obtained from micromagnetic simulations in Fig. 29. Blue (red) colors indicate top (bottom) skyrmion nucleus and time runs forward from top to bottom frames. In (a), for the scenario (i) for $j_1 = j_2 > j_c$, skyrmions begin the simulation as a bound pair with an approximately longitudinal trajectory. However, they gradually split in opposite transverse directions because of opposite unbalanced Magnus force. This is much more clear in the third frame, when the splitting is complete and each skyrmions follow symmetrical paths of $\Theta_{\text{SKH}} = \pm 16^\circ$. In (b), the scenario (ii) for $j_2 = 0$ (or $u_2 = 0$), the pair initially follows an oblique trajectory with a Hall angle $\Theta_{\text{SKH}} = \arctan(-4/\beta I) = -53^\circ$, as follows from Eq. 4.9, just before decoupling in the third frame. Upon decoupling, the Hall angle of the top skyrmion, which is now isolated, suddenly changes sign, becoming $\Theta_{\text{SKH}} = +16^\circ$, while the bottom skyrmion is left behind.

4.3.1 Envisioning experimental realization

Regarding real-world systems, different DMI strengths can be achieved by choosing different materials for the adjacent heavy metal layers HM1 and HM2, whereas the orientation of DM vectors can be explored by the relative positions of ferromagnetic and heavy-metal layers, as depicted in Fig. 24. With respect to the **investigation of the skyrmion Hall angle Θ_{SKH} sensitivity** to any imbalances in the magnetic properties and in-plane current densities, a synthetic ferrimagnet (SFi) stack with different FM materials such that $M_{s(1)} = M_{s(2)}$ at a given temperature would allow such an investigation by using independent application of current densities in each FM layer.

Figure 29 – Snapshots of the skyrmion pair decoupling scenarios, with time running forward from top to bottom. In (a), the scenario (i), with $j_1 = j_2 > j_c$, shows the vertical/transverse separation of skyrmions due to Magnus force alone. Here, skyrmions ultimately follow symmetrical paths of opposite Θ_{SkH} separated from each other. In (b), the scenario (ii), with $j_1 > j'_c$ and $j_2 = 0$, illustrates the decoupling via the abrupt change of Θ_{SkH} for the top skyrmion (from $\approx -53^\circ$ to $+16^\circ$), behaving as an isolated skyrmion at last, whereas the bottom skyrmion is left behind.



Source: The author (2025).

Our results also suggest that the **experimental investigation of SAF skyrmions**, which faces the challenge of the topological neutrality of antiferromagnetically coupled FM layers, can be addressed in a distinct way. Instead of deliberately unbalance the multilayer to create a synthetic ferrimagnet (SFi), which comes at the cost of an inevitable Skyrmion Hall effect, we can engineer interfacial DMIs with opposite signs for a magnetically balanced stack ($M_{s(1)} = M_{s(2)}$). The skyrmion pairs are then formed with magnetically frustrated regions where adjacent spins in different layers are parallel, irrespective of the AFM coupling, inducing detectable magnetic moments with much better contrast in magnetic imaging.

Finally, it remains still unknown whether the cancellation of the SkHE would be observed in the **presence of disorder effects**, which are present in real materials and provides a path for future studies. Even for nominally symmetric stacks [e.g., Pd/Co/Pd

multilayer in Carvalho et al. (2023)] interfacial disorder due to atomic intermixing via random local defects — which is deeply related to the manufacturing process of sputtered samples and to the interface quality — can unintentionally lead to an asymmetric multilayer (BRANDÃO et al., 2019; CARVALHO et al., 2023).

Moreover, the role of intralayer inhomogeneities in the magnetic parameters, such as those induced by **strain gradients** applied to the stack, is still unclear as it would potentially lead to a **geometric skyrmion Hall effect** (LIU et al., 2022) originated from a geometric asymmetry of the skyrmion. In this chapter, the work focuses on the SkHE caused by asymmetries at the interlayer level only, assuming magnetic parameters such as DMI and exchange homogeneous within each FM layer. In the next chapter, however, skyrmion shape deformations arising from alternating external magnetic and electric fields comes into play inducing novel features in the dynamics of coupled SAF skyrmions.

5 SELF-PROPULSION OF MAGNETIC SKYRMIONS IN HETEROCHIRAL SYNTHETIC ANTIFERROMAGNETS

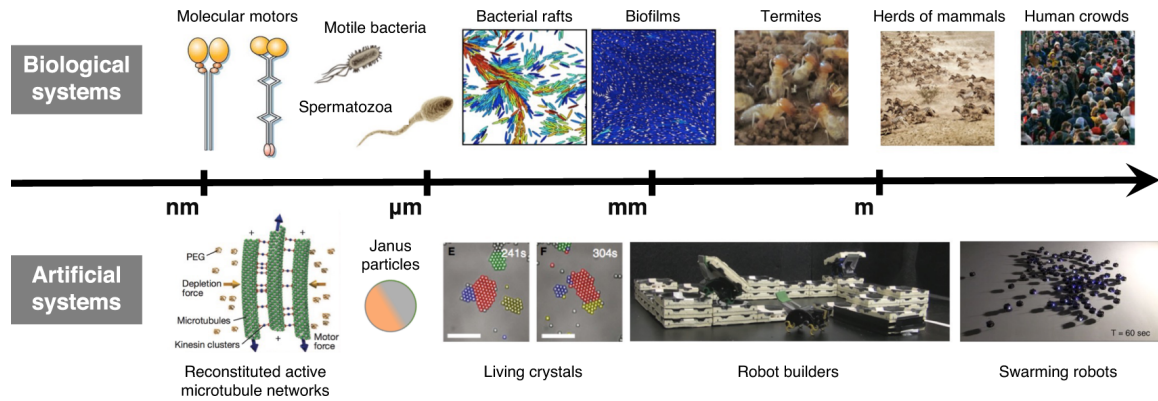
In the first part of our work (Chapter 4), as well as in the majority of the spintronics literature, we describe magnetic skyrmions as **passive** quasi-particles, i.e., an entity which has no means of self-propulsion. In this chapter, we discuss what active (self-propelled) particles are and how we characterize them in an important regime of motion with no inertia. Notably, we address the situation in which, by using an oscillating magnetic or electric field to excite specific internal dynamic modes, the non-coaxial skyrmion pair in a synthetic antiferromagnet exhibits autonomous motion. Thus, we describe the skyrmion pair as an autonomous magnetic swimmer behaving as an active brownian particle and mimicking strategies found in several microorganisms. Finally, preliminary results on the interaction between many skyrmion pairs and the proposal of “skyrmionic” metamaterials are discussed.

With the exception of Sec. 5.5, the results of this chapter are based on a study published in Silva, Correia and Velásquez (2025), where my main contributions were the following: (i) collaborate with the energy analysis of equilibrium configurations; (ii) perform the spectral analysis for skyrmion’s breathing modes in a synthetic antiferromagnet; (iii) run micromagnetic simulations of symmetric breathing mode (SBM) $T = 0$ (to help characterize the phenomenon) and at $T > 0$ (to obtain trajectories under low and high excitation). Here, we adopt a micromagnetic cell of sides $a_x = a_y = 1$ nm and $a_z = 0.4$ nm, as well as $A_1 = A_2 = 15$ pJ/m for the exchange interaction.

5.1 Active matter systems

Active particles are physical objects which are capable of obtaining energy from the environment and convert it into autonomous motion, i.e., endowing the constituents with **self-propulsion**. During this process, the constant flow of energy qualifies them as belonging to nonequilibrium systems that are summarized and named as **active matter**. Recent interest in active matter is due to two main causes: (i) the diversity of natural systems featuring active particles and relevant collective behavior, such as human crowds, flocks of birds, and motile micro-organisms (e.g., flagellated bacteria) (GOMPPER et al., 2025); and (ii) the development of artificial active matter, inspired by locomotion mech-

Figure 30 – Active matter systems come in different shapes and sizes, as exhibited by the diagram. The arrow indicate the length scale involved and the whole group is divided into natural or artificial active matter.



Source: Volpe et al. (2022).

anisms found in biological systems, including the development of self-organized active colloids (MORAN; POSNER, 2020), external field-propelled¹ micro/nanobots for complex medical tasks such as targeted drug delivery (YANG et al., 2022), and reconfigurable entities where the structure and function of the material can be changed on the fly (ZHANG et al., 2021).

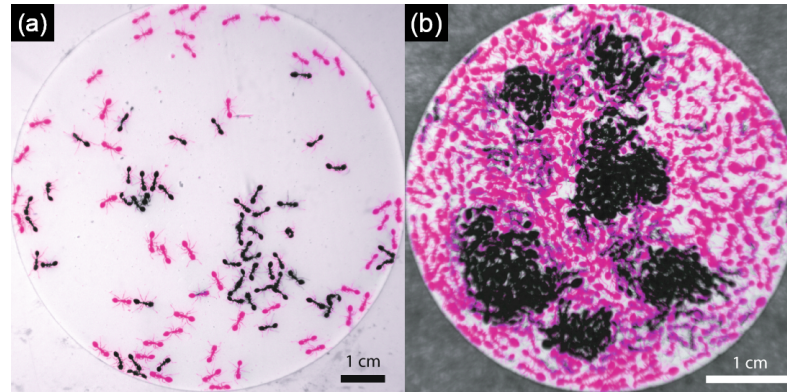
5.1.1 General features

An overview of natural and artificial examples of active matter is shown in Fig. 30, where the characteristic size is indicated by the central arrow. We note that active matter systems appear in natural and artificial systems at very different scales. Also, we refer to living organisms that are typically present in very different, sometimes complex environments, such as soil, oceans, rivers, blood, and tissues (GOMPPER et al., 2025). Therefore, it is fundamental to understand the individual and collective behavior of such systems in order to provide real-world synthetic applications of active matter.

Unlike passive systems, which obey detailed balance at thermal equilibrium, active matter is intrinsically driven far from equilibrium by energy injection at the single-particle level. Because of this continuous energy input, the dynamics become irreversible and, consequently, **time-reversal symmetry is fundamentally broken at the microscopic scale** (PELITI; PIGOLOTTI, 2021). This symmetry breaking gives rise to exotic collective behaviors, including anomalous self-assembly and clustering. While passive phase

¹ Including light, electrical, acoustics and magnetic fields.

Figure 31 – Visualization of Motility-Induced Phase Separation (MIPS) in a population of fire ants. This is a false color figure where ants that remain stationary over thirty seconds are marked in black, otherwise they are marked in magenta. (a) At lower density, ants are largely dispersed. (b) At higher density, the ants aggregate into large, dense clusters, demonstrating the coexistence of dense and dilute phases typical of MIPS.



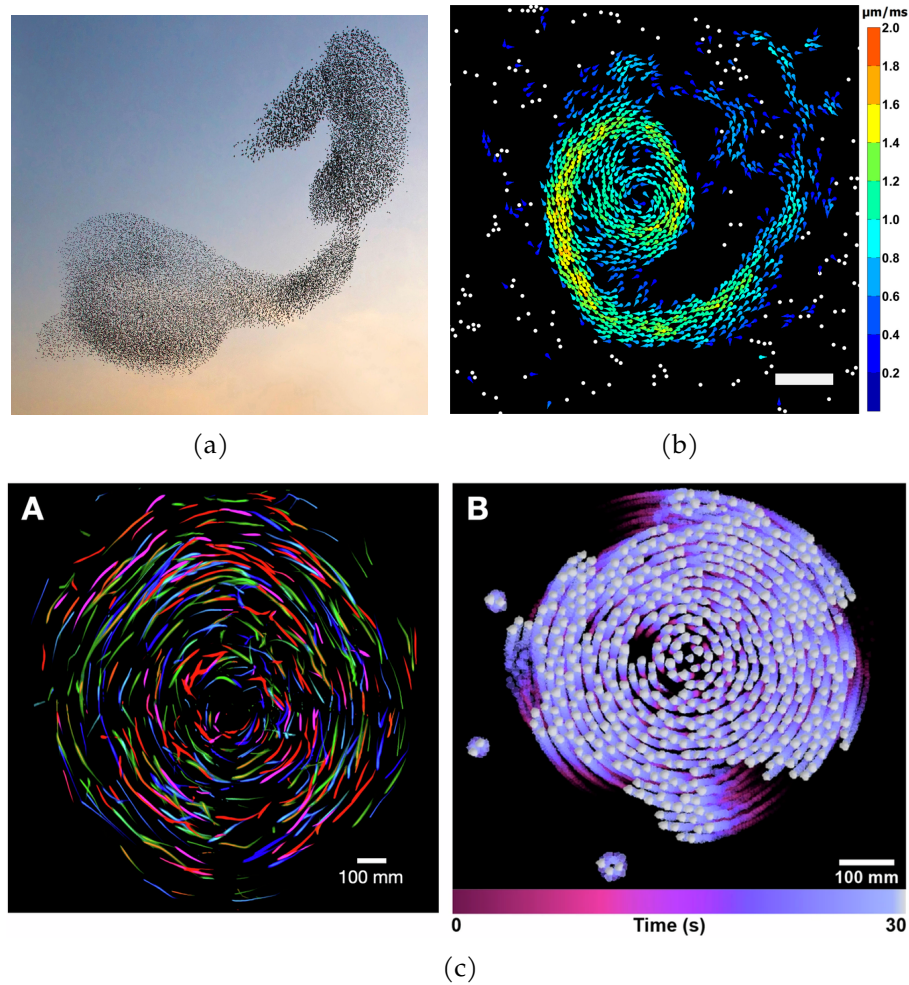
Source: Anderson and Fernandez-Nieves (2022).

transitions are typically dictated by temperature or attractive potentials, active matter can undergo transitions driven purely by the interplay between **motility and local density**.

A paradigmatic example of this non-equilibrium physics is the formation of dense clusters even in the absence of cohesive forces, where the slowing down of particles due to collisions leads to spatial accumulation. This phenomenon is known as the **Motility-Induced Phase Separation (MIPS)**, describing the transition of an initially uniform dispersion of active particles spontaneously separating into (i) coexisting dense, solid-like phases and (ii) dilute, gas-like phases when a critical density is reached. This behavior is visualized in Fig. 31, when fire ants act as macroscopic active agents. As shown in panel (a), at lower populations the ants are relatively dispersed. However, as the number of agents grows [panel (b)], crowding effects cause them to aggregate into large, dense clusters, illustrating the phase separation.

Overall, collective cohesion in nature arises by multiple reasons, including directional sensing (e.g. visual perception guiding the motion of individuals), non-reciprocal interactions, and self-steering in response to information acquired in the environment (VICSEK; ZAFEIRIS, 2012; SHANKAR et al., 2022; BECHINGER et al., 2016). The interest in active matter spikes as these mechanisms can be relevant not only for the self-organization of living systems, but also for the design of robust and scalable autonomous systems.

Figure 32 – Natural and artificial active matter organizing patterns. Phenomenon of *murmuration*, a flock of starlings (*Sturnidae*) in (a) and the spiral flock of active spherical colloids (“Quincke rollers”) in (b) — color bar indicates speed, where white particles are at rest, and the white scale bar is set for 100 μm . Clockwise rotating sunbleak fish (*Leucaspius delineatus*) school and a swarm of reconfigurable micro-robots (“magbots”) are shown in (c).

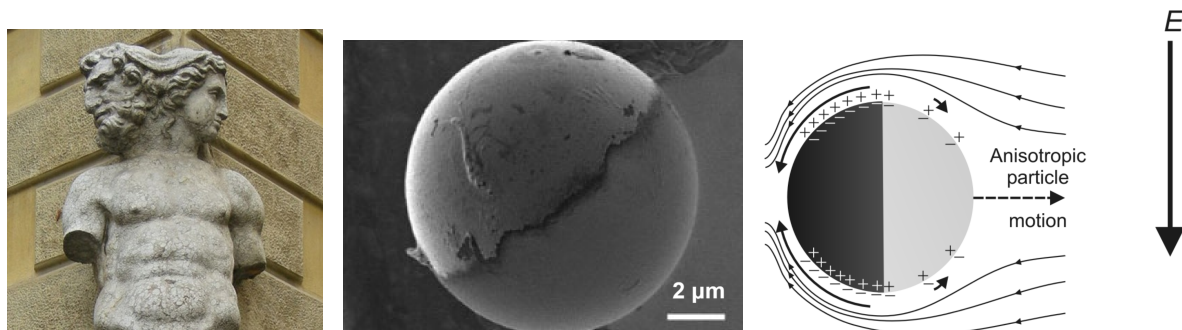


Source: Greif (2015) (a); (b) is adapted from Liu et al. (2021); (c) was obtained in Couzin (2025) with micro-robots by Wang et al. (2024).

5.1.2 Active matter at different scales

Macroscopic active matter (MaAM) allows complex interactions, such as the case of animal groups and human crowds. A fundamental feature of MaAM systems is that we can artificially tailor simple interaction rules and strategies in order to understand how very complex behavior emerges. Through millions or even billions of years, natural selection guided and tuned animal collectives to increase the protection of the individuals (like fish schools or flocking birds) and the search for nutrients, and now works as inspiration for many synthetic systems (Fig. 32).

Figure 33 – In the left, sculpture of the bifaced *Janus*, the Roman god of changes. In the center, scanning electron micrograph reveals a spherical Janus colloid and its two hemispheres with distinct material properties (dark and light halves). In the right, the propulsion mechanism of a Janus particle under an applied electric field E . The field induces asymmetric surface charge distributions, electro-osmotic fluid flows (curved arrows) and net directed motion.



Source: Sogari (1576) (left); Erb, Jenness and Clark (2009) (center); and Shen et al. (2020) (right).

Because of typical nanoscale size of the skyrmion texture, we highlight the case of microscopic active matter (MiAM). This group is greatly exemplified by biology: spermatozoa, motile bacteria and algae, phytoplankton and protozoa feature different mechanisms of **self-propulsion** in order to gather nutrients and avoid dangerous locations. Similarly as their living counterparts, it is possible to synthesize active particles that can convert energy from their environment into directed motion and self-organize in complex collective states such as swarms or flocks.

The benchmark example of artificial MiAM research are the **Janus particles**: synthetic micro or nanoparticles, typically spheres (Fig. 33, center), where the two hemispheres possess distinct physico-chemical properties on their surface. For example, one hemisphere is modified (e.g., coated with a thin metal layer) while the other remains bare silica or polystyrene. The inspiration for naming these particle comes from *Janus*, the Roman god of transitions and changes, which is typically represented by a bifaced head (Fig. 33, left). This illustrates the main feature of Janus particles, which is the broken spatial symmetry, crucial for transforming them from passive into active agents capable of converting energy into autonomous motion. The self-propulsion mechanism is based on **phoretic phenomena**, where local gradients generate fluid flows along the particle surface. As illustrated in Fig. 33 (right), external energy sources — such as chemical fuels, light, or, in this case, a uniform electric field (E) — interact differently with the two contrasting faces. Under the applied electric field, induced charges accumulate asymmetrically on the different hemispheres, driving unbalanced fluid flows around the particle. By conservation of momentum, this net fluid flow exerts a thrust

on the particle, propelling it in a given direction.

The importance of Janus particles extends from fundamental physics to engineering applications. As **controllable synthetic microswimmers**, since we can prepare the surface and control the external excitation, they serve as excellent model systems for understanding non-equilibrium statistical physics, allowing researchers to study collective behaviors like swarming and self-assembly in a simplified environment analogous to biological systems.

5.1.3 Reynolds and Péclet numbers

The behavior of active matter systems can be rooted on how the self-propelled (directed) motion is comparable to the diffusion occurring due to thermal fluctuations, which deeply affects microscopic entities. This is formalized as follows by using two dimensionless quantities in the model: the **Reynolds number** and the **Péclet number**.

We can study MiAM by modeling a solvent surrounding a self-propelled object (microscopic swimmer) as an incompressible Newtonian fluid governed by the Navier-Stokes equation (LANDAU; LIFSHITZ, 1987). This equation describes the conservation of momentum in the medium as

$$\underbrace{\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right)}_{\text{Inertial}} = -\nabla p + \underbrace{\eta \nabla^2 \mathbf{u}}_{\text{Viscous}} + \mathbf{f}_{\text{active}} \quad (5.1)$$

where ρ is the fluid density, $\mathbf{u}$ is the flow velocity, η is the viscosity, and $\nabla \cdot \mathbf{u} = 0$ — since the fluid is incompressible. The left-hand side of Eq. 5.1 represents **inertial forces**, including the non-linear term $\mathbf{u} \cdot \nabla \mathbf{u}$ that typically drives turbulence. The right-hand side balances these against the pressure gradient ∇p , the **viscous forces** $\eta \nabla^2 \mathbf{u}$, and the active force density $\mathbf{f}_{\text{active}}$ produced by the active particles swimming in the fluid by using different mechanisms. Here $\mathbf{f}_{\text{active}}$ is critical as it represents the local forces exerted by the micro or nanoswimmers on the fluid, creating the flow fields that mediate hydrodynamic interactions.

Using the characteristic length a — for example, the body size of some bacterium — and a typical swimming speed v , we see that the inertial term scales as $\rho v^2 / a$, while

the viscous term scales as $\eta v/a^2$. Their ratio is defined as the **Reynolds number**:

$$\mathcal{Re} = \frac{|\text{Inertial forces}|}{|\text{Viscous forces}|} = \frac{\rho v^2/a}{\eta v/a^2} = \frac{\rho v a}{\eta}. \quad (5.2)$$

Typical values of $\mathcal{Re}$ are in the order of 10^4 for a human being swimming in the water, whereas a goldfish possess 10^2 in the same conditions. As the length scales become very small, such as for spermatozoa or bacteria, then $\mathcal{Re} \ll 1$ for MiAM systems — typically 10^{-4} or 10^{-5} (PURCELL, 1977).

The second quantity, the **Péclet number** $\mathcal{Pe}$, instead characterizes the relative importance of directed motion (provided by the self-propulsion mechanism) versus the diffusion (due to the Brownian motion) for an active particle, yielding $\mathcal{Pe} = va/D$, where D is the translational diffusion coefficient for the particle. The higher the Péclet number, the less relevant is Brownian motion in determining the dynamics of an active particle. In the limit of $\mathcal{Pe} \rightarrow 0$, diffusion completely dominates over directed motion, $\mathcal{Pe} = 1$ means equal characteristic times for diffusion and advection, and $\mathcal{Pe} \rightarrow \infty$ the particle translates with no diffusion.

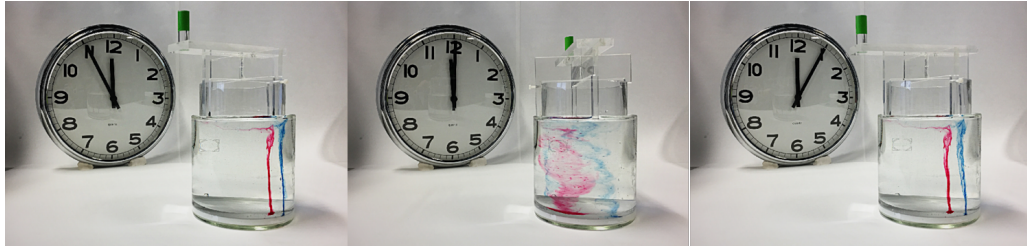
Let us calculate $\mathcal{Pe}$ for a bacterium such as *E. coli*, with typical size about 1 or 2 μm , immersed in the water and moving with an average speed of 20 $\mu\text{m/s}$. The D coefficient is calculated via the Stokes-Einstein equation $D = \frac{k_B T}{6\pi\eta R} \approx 10^{-13} \text{ m s}^{-2}$. Then, $\mathcal{Pe} = \frac{(20\mu\text{m/s})(1\mu\text{m})}{10^{-13}\text{m}^2/\text{s}} = 200$. Thus, although water molecules are continuously colliding with it, since $\mathcal{Pe} \gg 1$ the bacterium can swim directly for considerable distances.

5.1.4 Low Reynolds numbers: the life of tiny swimmers

However, life as a microscopic swimmer is not the same as in the human length scale. In the range of $\mathcal{Re} \ll 1$, also known as the **Stokes regime**, inertial forces become almost negligible. This can be seen in Eq. 5.1 if we rewrite it in a dimensionless version by replacing v (u) by $v\tilde{v}$ ($u\tilde{u}$), a given length x by $x\tilde{x}$, pressure p by $(\eta v/a)\tilde{p}$ and t by $(a/v)\tilde{t}$ (consequently, $\partial/\partial t = (v/a)(\partial/\partial\tilde{t})$, where tilde goes over dimensionless variables. This leads to

$$\frac{\rho a v}{\eta} \left[\frac{\partial \tilde{\mathbf{u}}}{\partial \tilde{t}} + (\tilde{\mathbf{u}} \cdot \tilde{\nabla}) \tilde{\mathbf{u}} \right] = -\tilde{\nabla} \tilde{p} + \tilde{\nabla}^2 \tilde{\mathbf{u}}. \quad (5.3)$$

Figure 34 – Time-reversibility in the fluid flow at low Reynolds number: as we try to mix two dyes with different colors by a mechanical system of rotating cylinders, we got frustrated given that the laminar flow is such that the dyes revert essentially to their original state after rotation.



Source: Adapted from Wikimedia Commons (2018).

The term multiplying the inertial left side is precisely the Reynolds number. Thus, in the low Re regime, we obtain the approximation known as the **Stokes equation** given by

$$-\nabla p + \eta \nabla^2 \mathbf{u} + \mathbf{f}_{\text{active}} \approx 0. \quad (5.4)$$

Equation 5.4 is linear and brings a key feature for microswimmers: the fluid flow — here also known as “creeping” flow — is **instantaneous** and **time-reversible**. This has profound consequences on the swimming mechanisms at microscopic scale.

First, the fluid flow responds instantaneously to any disturbances in pressure, force or boundary conditions. The dynamics is now **overdamped**, so we have a comeback of the Aristotelian physics with no inertia: the applied forces are proportional to the flow velocity and not to acceleration. So, if there is no force, there is no movement for any swimmer. The second consequence, the time-reversibility, is classically visualized as the difficulty of mixing liquids in the low Re regime (Fig. 34).

As we try to mix two dyes of different colors by rotating the inner cylinder of the so-called Taylor-Couette cell, we observe a mixing and subsequent “unmixing” of the material, making the dyes return to their original state when rotation is reversed. This is also known as kinematic reversibility. The time-reversibility indicates that other kinds of non-reciprocity are needed for objects to swim in the low Re regime.

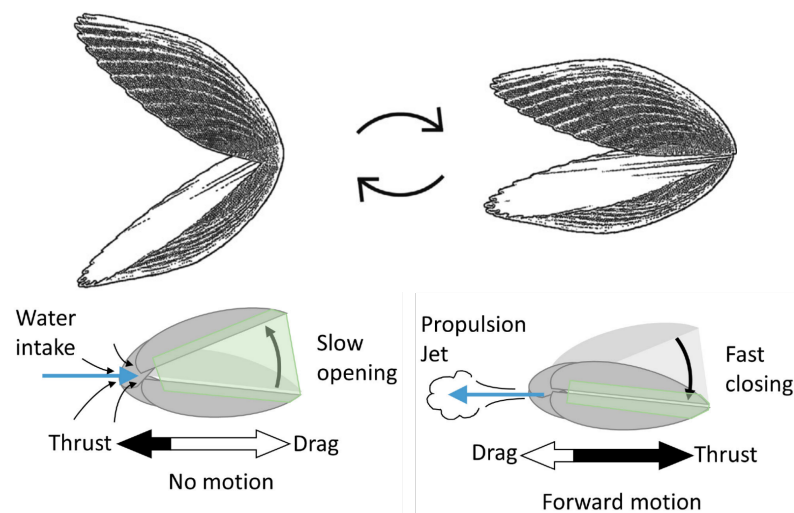
In general, to propel themselves within a fluid, macroscopic animals and micro-organisms use some kind of periodic deformation of their own bodies. For example, when addressing the subject in his celebrated 1977 paper *Life at low Reynolds number*, physicist Edward Purcell comments on the motion of the **scallop** (PURCELL, 1977) shown in Fig. 35. The scallop periodically opens their shell slowly and closes it very fast in order to produce motion, as we visualize in the bottom of Fig. 35 the fluid movement

along the swimming cycle. The new thrust is such that the scallop can produce forward motion. This is an example of **reciprocal motion**, since the sequence of deformations is such that opening and closing geometry is perfectly similar, so it is time-reversible. Nonetheless, this works very well for the scallop, since inertia is present and allows for a net displacement.

However, Purcell raises a warning for microscopic swimmers: reciprocal motion produces no net displacement at low Re regime, just as we could not mix the dyes by performing a rotation followed by a counter-rotating motion (Fig. 34). Due to the absence of inertia and the instantaneous nature of the viscous fluid dynamics, the forward motion is overridden by the subsequent backwards motion. The scallop would only oscillate back and forth if it was supposed to swim in a very viscous fluid. For that reason, only **non-reciprocal** deformation strategies allows for any net displacement. This is summarized as the **Purcell's scallop theorem** (PURCELL, 1977): due to time-reversibility, the reciprocal motion of a scallop cannot produce net motion in the low Reynolds number regime.

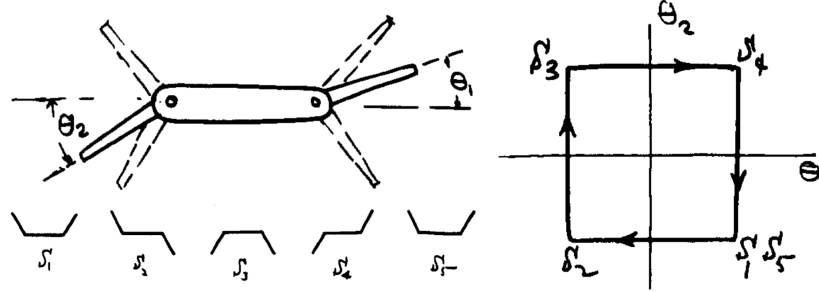
For that reason, organisms develop complex ways to swim, such as the case of the corkscrew rotation provided by the beating of cilia or the corkscrew rotation of helical flagella (NAMBA; VONDERVISZT, 1997, pp. 6-7). Moreover, in order to swim in the Stokes

Figure 35 – Scallop bivalve swimming cycle. The diagram demonstrates the relationship between shell velocity and net motion for the animal for a periodic opening and closing of the double shell. During the slow opening, the reaction forces are negligible and there is basically no motion for small water intake. During the fast closing, the rapid expulsion of fluid creates a propulsion jet where the thrust vector significantly exceeds the drag vector, finally propelling the scallop forward.



Source: Adapted from Qiu et al. (2014) (top) and Robertson, Efremov and Paik (2024) (bottom).

Figure 36 – Purcell’s non-reciprocal swimmer. In the top left, the swimmer features two degrees of freedom, θ_1 and θ_2 . This allows the time sequence of body deformations from S_1 to S_5 , which is non-reciprocal — if the reverse time, going from S_5 to S_1 instead, the deformations are not geometrically identical. In the right, S_1 to S_5 sequence produces a loop in the (θ_1, θ_2) configuration space, which indicates a net displacement for the swimmer within the fluid.



Source: Purcell (1977).

regime, Purcell proposes a second swimmer, with the appearance of a Swiss army knife, illustrated in Fig. 36. The idea is to have at least two degrees of freedom allowing the swimmer to draw a non-trivial loop in the configuration space during the periodic motion (Fig. 36, right). For the scallop, there is only one degree of freedom, whereas the Purcell’s swimmer has two (θ_1 and θ_2). Since the time sequence of configurations from S_1 to S_5 ($S_5 \equiv S_1$) is not symmetric when we time-reverse, the swimmer motion is said to be **non-reciprocal**. So Purcell’s swimmer is able to navigate in the viscous fluid, producing net motion which is proportional to the area inside the square loop of Fig. 36.

Before mapping the non-reciprocal situation to that of a magnetic skyrmion pair in the synthetic antiferromagnet, we shall summarize in the next section how we can manipulate the internal **vibrational modes** of skyrmions.

5.2 Skyrmion dynamic modes

The magnetic skyrmion pair is a deformable texture navigating within an overdamped environment, as indicated by the damping term of Thiele equation (Eq. 4.2, whose amplitude is controlled by the Gilbert parameter α). Also, the non-coaxial configuration we obtained in Sec. 4.1 breaks rotational symmetry. Given these two conditions, we wonder if we could energetically supply the pair to produce **self-propulsion**, using its own internal dynamics, regardless of external drives, i.e., if we could build a magnetic correspondent of the non-reciprocal swimmer in our synthetic antiferromagnet. One way of providing energy and activate a non-reciprocal deformation sequence is to

excite internal vibrational modes of the skyrmions.

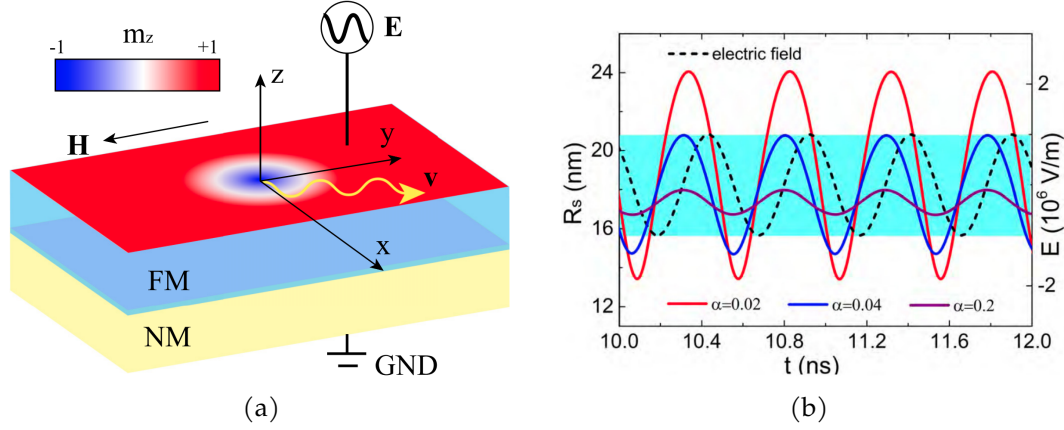
Skyrmions are a result of a delicate balance of competing interactions, resulting in a state of an energy minimum. When an external excitation acts on the elastic skyrmion texture, it exhibits different modes of vibration. One example is the gyrotropic rotational mode, when the skyrmion rotates around an equilibrium position. This is typically excited by an in-plane magnetic field (WANG et al., 2015).

Another vibrational mode, which we focus here due to the possibility of setting non-reciprocal swimming cycle for the skyrmion pair, is the radially symmetric **breathing mode**. In this case, the skyrmions texture expands and contracts periodically as a response to a periodic external excitation, maintaining its rotational symmetry. As we anticipated in the equilibrium properties of the non-coaxial skyrmion pair, this mode can be excited either by changing the anisotropy via an alternating electric field or by applying a perpendicular magnetic field.

An alternating electric field can be applied across the interface of ultrathin magnetic films in order to manipulate several magnetic properties such as exchange, DMI, and the uniaxial anisotropy. The control of the perpendicular magnetic anisotropy using $\mathbf{E}$ has been investigated in numerical studies and experimental works (MARUYAMA et al., 2009; UPADHYAYA et al., 2015; YUAN et al., 2019). Typically, the influence of these voltage-induce methods is very restricted to the surface, so the use of ultrathin ferromagnetic layers is a requirement — for example, the Fe layer in Maruyama et al. (2009) has only a few monoatomic layers within the Fe/MgO junction. Applying $\mathbf{E}$ promotes significant changes in the equilibrium and dynamical properties of a skyrmion. Small electric fields as small as 100 mV/nm causes up to 40% change in the anisotropy field (MARUYAMA et al., 2009). The introduction of an alternating out-of-plane $\mathbf{E}$ modulates an effective magnetic anisotropy $K_{\text{eff}}(t) = K_0 + \Delta K \sin(\omega t)$.

The $\mathcal{E}_{\text{anis}}$ term of $\mathcal{H}$ favors the uniform ferromagnetic alignment for spins. Since the skyrmion radius R is a result of a balance between – among others — competing anisotropy and DMI forces, a change in K_0 modifies the energy cost of the skyrmion core with inverted spins. For $K_{\text{eff}} > K_0$, **the out-of-plane component is energetically favored** for spins, so the number of in-plane spins should decrease, i.e., the number of spins in the skyrmion domain wall should also decrease, leading to the contraction of the skyrmion. For $K_{\text{eff}} < K_0$ we have analogous reasoning, but for a radially expanding skyrmion.

Figure 37 – Voltage-controlled magnetic anisotropy. (a) Illustration of a system of magnetic layers where an oscillating out-of-plane electric field $\mathbf{E}$ is applied in the z direction in the top of ferromagnetic layer, where skyrmions reside. (b) Skyrmion radius R (color solid lines) as a function of time when an alternating $\mathbf{E}$ with a frequency $f \approx 13$ GHz (dashed line) is applied for different Gilbert damping coefficients: $\alpha = 0.02$ (red), $\alpha = 0.04$ (blue), and $\alpha = 0.20$ (purple).



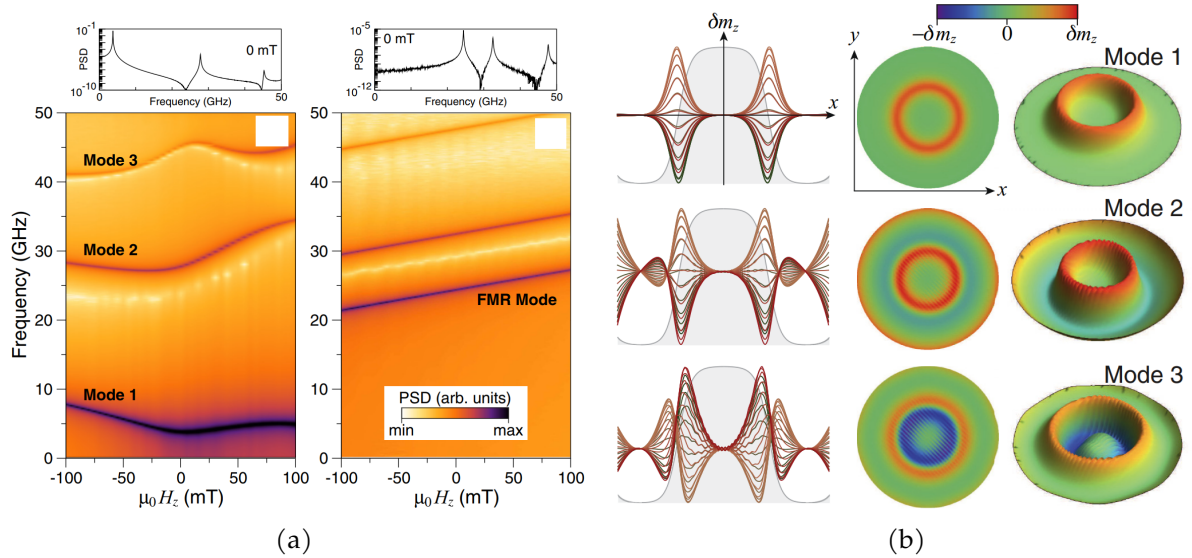
Source: Yuan et al. (2019).

The change of skyrmion radius induced by a periodic electric field is verified in Yuan et al. (2019) as illustrated in Fig. 37. The skyrmion is observed to periodically increase and decrease its radius following the behavior of $\mathbf{E}$, except for a phase shift, maintaining its circular shape. Finally, the changes on R are larger for low Gilbert damping, since when α is small the magnetic moments can precess further away from equilibrium before the damping torque arrests their motion. Consequently, the skyrmion can expand and contract more dramatically as we minimize the energy loss per oscillation cycle.

To prevent electric current from flowing through the stack, we can introduce an insulating spacer, as demonstrated experimentally in Zhang et al. (2020), where NiO is used to provide the RKKY coupling. This offers a significant advantage for overall energy consumption, since the system effectively avoids the accompanying Joule heating losses during the modulation (QIU et al., 2021; TAKEUCHI; MIZUSHIMA; MOCHIZUKI, 2019). Finally, we stress that while the changing electric field theoretically modifies both the local anisotropy and the DMI, the dynamic response of the system is dominated by the changes in the magnetic anisotropy (RODRIGUES et al., 2023).

A second way of exciting skyrmion breathing modes is the application of a **spatially uniform, oscillating magnetic field** $\mathbf{B}$, such as $\mathbf{B}(t) = B_z \hat{z} = B_0 \sin(\omega t) \hat{z}$. The field is assumed to be perpendicular to the film plane. This immediately affects the Zeeman energy of the system (Eq. B.5) and modifies the skyrmion radius $R(t)$ according to the instantaneous sign of B_z and the orientation of magnetic moments at the skyrmion

Figure 38 – Excitation spectrum and spatial profiles of breathing modes in a confined magnetic skyrmion. (a) Maps of the power spectral density (PSD) showing the magnetic excitation frequencies as a function of the external perpendicular magnetic field ($\mu_0 H_z$). Left panel displays the spectrum for the skyrmion, revealing distinct resonant branches (Modes 1, 2, and 3) with non-monotonic dependence on H_z . The right panel shows the spectrum for the ferromagnetic state. (b) Spatial profiles of the out-of-plane magnetization deviation ($\delta m_z(t) = m_z(t) - \langle m_z \rangle$) with respect to equilibrium ($\langle m_z \rangle$) for the three modes in (a).



Source: Adapted from Kim et al. (2014).

core. Consider, for instance, a skyrmion with central spin oriented along $+\hat{z}$ and a surrounding background magnetization of $-\hat{z}$ (down). When $B_z > 0$, it aligns parallel to the skyrmion core, so an outward pressure is exerted on the domain wall, causing the core to expand in order to maximize the area of aligned spins, minimizing Zeeman energy. Conversely, when $B_z < 0$, it aligns with the background and opposes the core orientation, increasing the energy cost of the core and exerting an inward pressure that forces the skyrmion to contract.

If the frequency of either $\mathbf{E}(t)$ or $\mathbf{B}(t)$ matches the natural frequency of the breathing modes, typically located in the GHz range, skyrmion undergoes **resonant expansion and contraction** — when $|\dot{R}(t)|$ is maximum. A theoretical study using micro-magnetic simulations of individual magnetic skyrmions confined in ultrathin film dots under oscillating out-of-plane magnetic field is done by Kim et al. (2014). The power spectrum density obtained by the authors is shown in Fig. 38(a), where the skyrmion panel (left) is markedly different from the spectrum corresponding to the ferromagnetic phase (right). Skyrmion exhibits three special modes, whose resonance peaks are exemplified in the top inset of panel (a) and spatial profiles are depicted in Fig. 38(b). The static field dependence of the breathing modes is non-monotonically and is unique

for each mode, as can be visualized in panel (a), offering an extra possibility of experimental detection of the skyrmion phase utilizing ferromagnetic resonance (FMR) as it distinguished from the linear dependence of ferromagnetic phase [panel (a), right].

5.3 Self-propulsion of a non-coaxial skyrmion pair

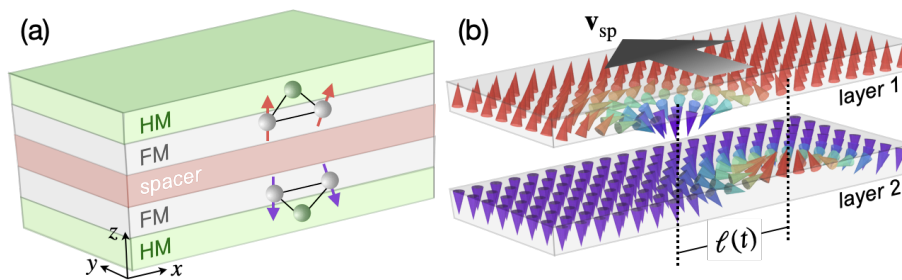
5.3.1 Geometry and equilibrium properties

Our system is the same synthetic antiferromagnet of Chapter 4, setting magnetic parameters to obtain a **non-coaxial** skyrmion pair [Fig. 39(a)]. Recalling Purcell (1977), a non-reciprocal cycle demands at least two degrees of freedom in order to obtain autonomous motion for low Re number swimmer. This non-coaxial configuration is of interest because it breaks rotational symmetry producing another relevant degree of freedom, the bond length (the distance between the skyrmions) $\ell(t)$ [Fig. 39(b)].

For the micromagnetic simulations, we fixed the thicknesses of the FM layers $d_1 = d_2 = 0.4$ nm, Dzyaloshinskii-Moriya constants $D_1 = -D_2 = 3.05$ mJ/m², and saturation magnetizations $M_{s(1)} = M_{s(2)} = 0.58$ MA/m.

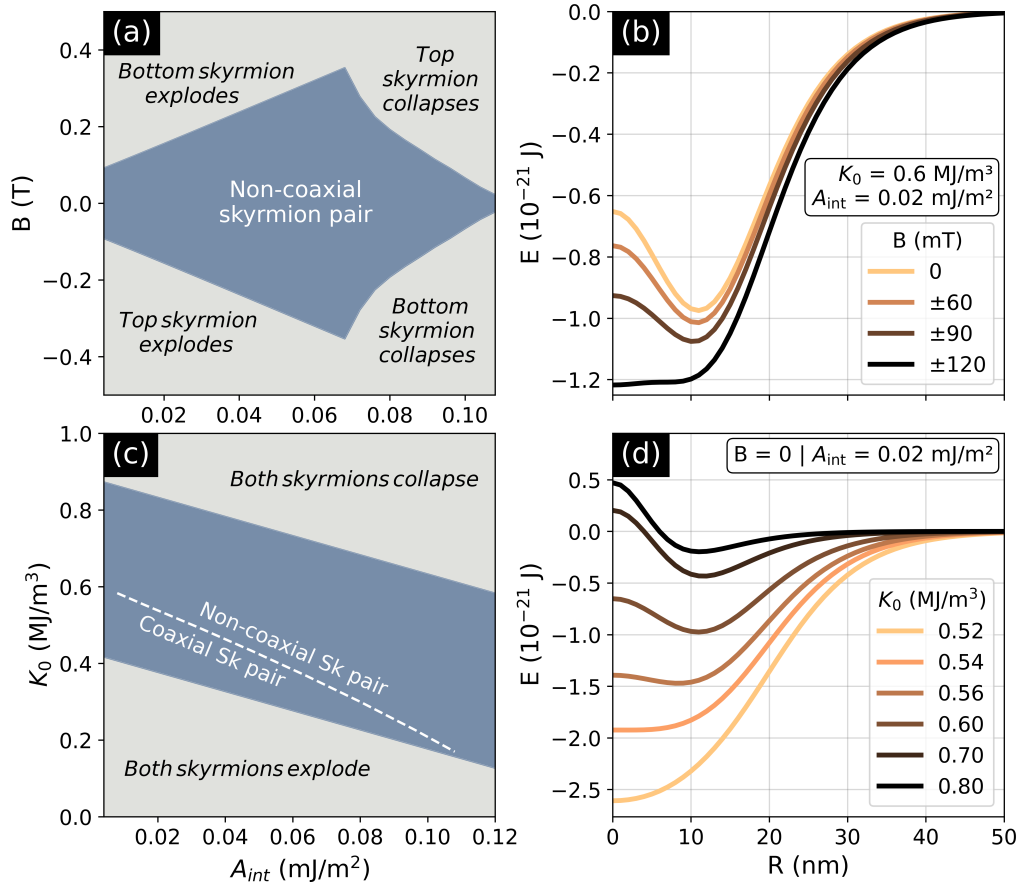
In order to define the stability region for the non-coaxial pair when we vary our parameters of interest, the uniaxial anisotropy K_0 , the out-of-plane magnetic field B , and the interlayer coupling A_{int} , we prepare the diagrams visualized in Fig. 40 at $T = 0$. In (a), B is varied for fixed $K_0 = 0.6$ MJ/m³, whereas in (b) K_0 is varied for $B = 0$. Also, in panels (c) and (d) we select several values of B and K_0 to calculate the skyrmion-

Figure 39 – (a) Synthetic antiferromagnet (SAF) stack showing ferromagnetic (FM) and heavy metal (HM) layers with the non-magnetic spacer. The Dzyaloshinskii-Moriya constants have opposite signs, as indicated by the diagram in the FM/HM interfaces. (b) Color-coded representation of magnetization vector in the FM layers for a non-coaxial skyrmion pair. A purple (red) cone stands for $\mathbf{m} = -\hat{z}$ ($\mathbf{m} = +\hat{z}$) and the skyrmions are separated by a distance $\ell(t)$. The pair exhibits self-propulsion with a velocity v_{sp} perpendicular to the bond.



Source: Silva, Correia and Velásquez (2025).

Figure 40 – Phase diagram for the skyrmion pair according to A_{int} and to a constant magnetic field B (a) or to the anisotropy constant K_0 (b). Grey areas show regions where the skyrmion pair is not stable, either collapsing or exploding. Regions of stability (in blue) are subdivided by white dashed lines separating coaxial and non-coaxial pairs. Energy profiles of selected values of diagram from (a) and (b) are shown in (b) and (d), respectively, where the interaction potential energy is plotted against skyrmion separation R . B , K_0 , and A_{int} are in units of T, MJ m^{-3} , and mJ m^{-2} , respectively. The equilibrium bond length of the pair can be identified as the position R of minimum E .



Source: Silva, Correia and Velásquez (2025).

skyrmion interaction energy E versus the skyrmion separation R at weak coupling ($A_{\text{int}} = 0.02 \text{ mJ/m}^2$).

In order to track the motion of the skyrmion pair during the excitation cycle, we use the coordinates of the **center of topological charge** (CTC), given by

$$\mathbf{R}_i = \frac{1}{Q_i} \int \mathbf{r} \rho_i(\mathbf{r}) d^2\mathbf{r} = (X_{\text{CTC}}, Y_{\text{CTC}}). \quad (5.5)$$

for the i -th skyrmion, where $\varrho_i = (1/4\pi) \mathbf{m}_i \cdot (\partial_x \mathbf{m}_i \times \partial_y \mathbf{m}_i)$ is the topological charge density in layer i , $\mathbf{m}_i(\mathbf{r})$ its normalized magnetization, and $Q_i = \int d^2r \varrho_i$ its total topological charge. The **bond length** ℓ is then defined as the distance between the topological centers of the two skyrmions: $\ell = |\mathbf{R}_1 - \mathbf{R}_2|$. Our analysis indicates that, in the grey

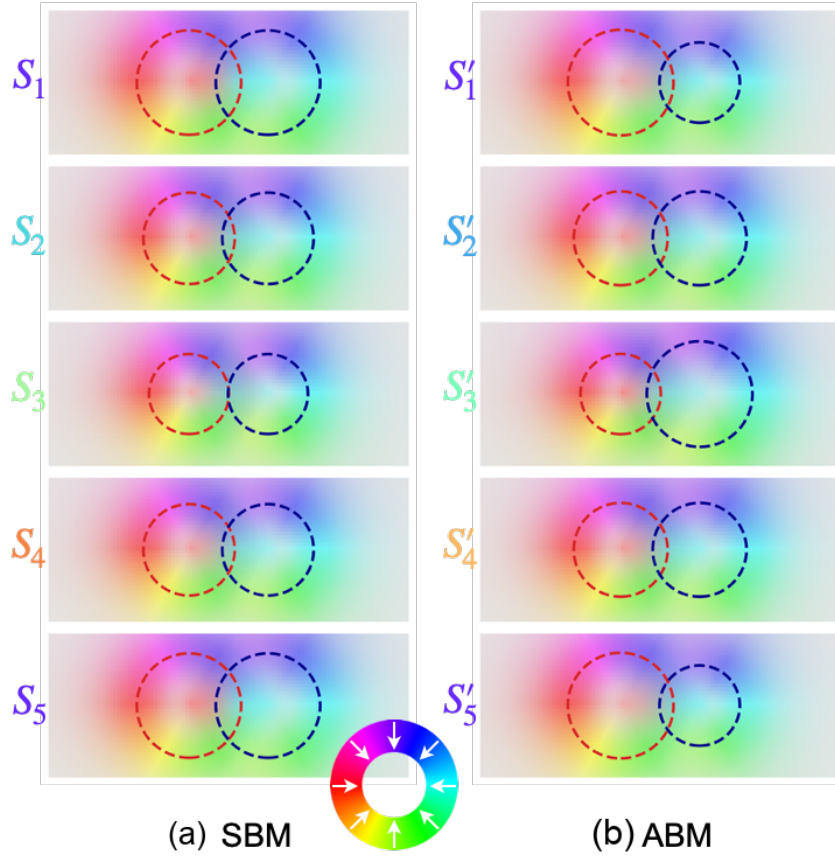
regions of Fig. 40, skyrmions experience an “explosion” when the parameters cause them to grow excessively, reaching a size comparable to the simulation box length, or a “collapse” when their size becomes smaller than or similar to the micromagnetic cell. However, we find blue regions of stability in (a),(c) for the non-coaxial bonding and the change of the bond length ℓ as the parameters B or K_0 are varied in (b),(d). This is a clear indication that varying B or K_0 — ultimately, applying alternating magnetic or electric fields in the sample — should excite the longitudinal vibrational mode associated with the bond and induce relative motion among skyrmions, providing $\ell = \ell(t)$.

5.3.2 Exciting symmetric and antisymmetric breathing modes

In the synthetic antiferromagnet, the skyrmions are coupled and we verify that the resulting pair responds to external periodic excitations of the vibrational modes in a symmetric or antisymmetric way. The author was responsible for conducting the complete spectral analysis in this study. In the SBM scenario, while $B = 0$, we excite the sample changing the effective uniaxial anisotropy as $K(t) = K_0 + \Delta K \sin \omega t$, where K_0 is the value defined at equilibrium. This excitation makes both skyrmions to suffer periodic, in-phase contraction and expansion, what we refer to as the excitation of the **symmetric breathing mode (SBM)**. In Fig. 41(a), we show the sequence of synchronous contraction and expansion of the skyrmions along a period $T = 2\pi/\omega$ (an excitation cycle), where the skyrmions radii ($m_z = 0$) are indicated as the circular dotted lines for top (red) and bottom (blue) layer. The simulation snapshots from S_1 to S_5 correspond to a full excitation cycle where skyrmions are contracted in S_3 and expanded in $S_1 = S_5$. This synchronization emerges as the uniaxial anisotropy is invariant under mirror reflection: since top and bottom skyrmions are separated by the transformation $m_z \rightarrow -m_z$ and $\mathcal{E}_{\text{anis}} \propto (m_z \sin \theta)^2$ (Eq. 2.15), then the skyrmions respond similarly to the excitation of the anisotropy field.

In contrast, applying an out-of-plane magnetic field $\mathbf{B} = \hat{z}B_z = \hat{z}B_0 \sin \omega t$ breaks the inversion symmetry since Zeeman interaction $\mathcal{E}_{\text{Zeeman}} \propto -\mathbf{m} \cdot \mathbf{B} = -m_z B_z$ inverts sign upon the transformation $m_z \rightarrow -m_z$. This makes the top skyrmion to contract while the bottom skyrmion is expanding while $B_z > 0$ (and vice-versa for $B_z < 0$), which we refer to as the excitation of the **antisymmetric breathing mode (ABM)**. The ABM excitation cycle is shown in Fig. 41(b), where the snapshots S'_1 to S'_5 show the

Figure 41 – Close-up visualization of the skyrmion pair at exciting stages of the (a) SBM ($\Delta K/K_0 = 0.005$ and $f = 18$ GHz) and (b) ABM ($B_0 = 4$ mT and $f = 19.24$ GHz) modes. The dashed circles depict the domain wall ($m_z = 0$) of the top (red) and bottom (blue) skyrmions. The in-plane magnetization of both layers is shown by overlaying their maps with partial transparency in the background. The directions of in-plane $\mathbf{m}$ is given by the color wheel. Here, $A_{\text{int}} = 0.02$ mJ/m² (weak coupling), $K_0 = 0.6$ MJ/m³, and $\alpha = 0.02$.



Source: Adapted from Silva, Correia and Velásquez (2025).

out-of-phase evolution of skyrmions sizes.

Although not immediately apparent in Fig. 41, both breathing modes are accompanied by the longitudinal oscillation of the skyrmion bond $\ell(t)$. SBM and ABM scenarios, however, feature different behaviors for $\ell(t)$: in SBM, $\ell(t)$ oscillates with a frequency $\omega_\ell = \omega$ that matches the external excitation's, whereas in ABM it oscillates with $\omega_\ell = 2\omega$. The latter can be visualized in the sequence of Fig. 41. For example, the bond length $\ell(t)$ is similar in the frames S'_1 and S'_3 (or S'_2 and S'_4), as well as the magnitude of B , despite its sign being opposite. The key is that $\ell(t)$ is the same whether we change positions of top and bottom skyrmions during the excitation, i.e., it is invariant under field inversion and determined solely by $\|\mathbf{B}\|$, which results in the observed frequency doubling.

The description of breathing modes is complemented by the **excitation spectra** we obtain for both ABM and SBM in a similar procedure as described in Kim et al. (2014). At $T=0$, we compute the spatially averaged magnetization $\langle m_z \rangle = (1/V) \int m_z(\mathbf{r}) dV$ for each FM layer at equilibrium, denoted as $\langle m_z \rangle_{\text{eq}}$. In order to excite the system we apply either

- a time-varying magnetic field $\mathbf{B}_{\text{ext}}(t) = \hat{\mathbf{z}}B_0\text{sinc}(f_c t)$ for ABM, where $B_0 = 4$ mT and we fix $K = K_0$, or
- an oscillating anisotropy parameter $K(t) = K_0 + \Delta K\text{sinc}(f_c t)$ for SBM, where $\Delta K / K_0 = 0.004$ and we set $B_{\text{ext}} = 0$,

both cases featuring a cutoff frequency $f_c = 100$ GHz. The choice of the cardinal sine shape of the excitations, $\text{sinc}(x) = \sin(\pi x)/\pi x$, guarantees uniform excitation across frequencies up to f_c as the Fourier transform of $\text{sinc}(x)$ is a rectangular function in the frequency domain. During excitation, we record the deviation of the mean magnetization from equilibrium, $\delta m_z(t) = \langle m_z(t) \rangle - \langle m_z \rangle_{\text{eq}}$, every 2 ps throughout $\Delta t = 10$ ns. The excited modes are, then, finally identified from the power spectrum of the magnetization response

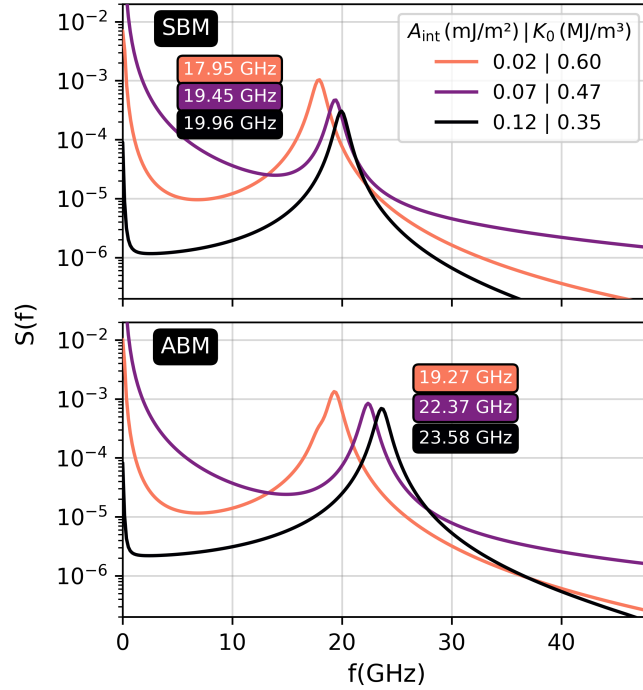
$$S(f) = \left| \int_0^{\Delta t} dt e^{-i2\pi f t} \delta m_z(t) \right|^2. \quad (5.6)$$

The power spectra $S(f)$ is shown in Fig. 42 for selected values of A_{int} and K_0 from Fig. 40, covering weak to strong coupling. The top panel corresponds to excitations of $K(t)$, which selectively excite SBM, while the bottom panel corresponds to ABM excitations via $\mathbf{B}_{\text{ext}}(t)$. The frequency $f = f_R$ of the resonance peaks is found to increase with the AFM coupling A_{int} , as also indicated by the numerical values next to the curves, and the ABM spectra exhibit slightly higher f_R values.

5.3.3 Skyrmion pair swimming at $T=0$

Now we focus on the central results of this chapter, that the excitation of skyrmion breathing modes results in a directed, autonomous motion of the non-coaxial skyrmion pair, i.e., the pair swims autonomously.

Figure 42 – Power spectra of the magnetization response when exciting symmetric (SBM, top) and anti-symmetric (ABM, bottom) breathing modes of the skyrmion pair ($\alpha = 0.02$). The resonance frequencies (f_R) in the GHz range are indicated in rectangles and are shown to increase with A_{int} . Weak ($A_{\text{int}} = 0.02$), intermediate ($A_{\text{int}} = 0.07$), and strong ($A_{\text{int}} = 0.12$) AFM coupling regimes are represented. K_0 is decreased to ensure the stability of a non-coaxial pair (Fig. 40). For SBM, $\Delta K / K_0 = 0.004$ and $B = 0$, whereas for ABM $B_0 = 4$ mT and $K = K_0$.



Source: Silva, Correia and Velásquez (2025).

Firstly, we characterize the internal configuration of the skyrmion pair under weak coupling regime, assuming that the skyrmion maintains its circular shape. We, then, adopt two degrees of freedom:

- the **bond length** $\ell = |\mathbf{R}_1 - \mathbf{R}_2|$, where $\mathbf{R}_i$ is the center of topological charge (CTC) (Eq. 5.5) of skyrmions featuring $Q_2 = -Q_1 \equiv Q = 1$; and
- the **effective dissipation coefficient** of the skyrmions $\mathcal{D}_i(t)$ given by

$$\mathcal{D}_i \equiv \sqrt{\det \mathcal{D}_i}, \quad [\mathcal{D}_i]_{\mu\nu} = \frac{M_s d}{\gamma} \int d^2 r \partial_\mu \mathbf{m}_i \cdot \partial_\nu \mathbf{m}_i, \quad (5.7)$$

where $\mu, \nu = x, y$ and γ is the gyromagnetic factor.

Together, ℓ and $\mathcal{D}$ are the main variables determining the internal configuration and dynamics of the pair. They form the shape manifold we use to illustrate and characterize the swimming mechanism — just as Purcell (1977) did with his two-armed swimmer.

Since the dissipation tensor $\mathcal{D}_i$ depends only on the skyrmion shape and size, and both skyrmions are identical under SBM, $\mathcal{D}_1 = \mathcal{D}_2$ at all times. Under weak interlayer coupling, we assume the same conditions adopted in the first SAF study (CORREIA; VELÁSQUEZ; SILVA, 2024): $\mathcal{D}_{xx} \simeq \mathcal{D}_{yy} \simeq \mathcal{D}$ and $\mathcal{D}_{xy} \simeq 0$, so the dissipation tensor is diagonal, indicating an essentially circular skyrmion profile. In this regime, $\mathcal{D}$ scales directly with the skyrmion radius.

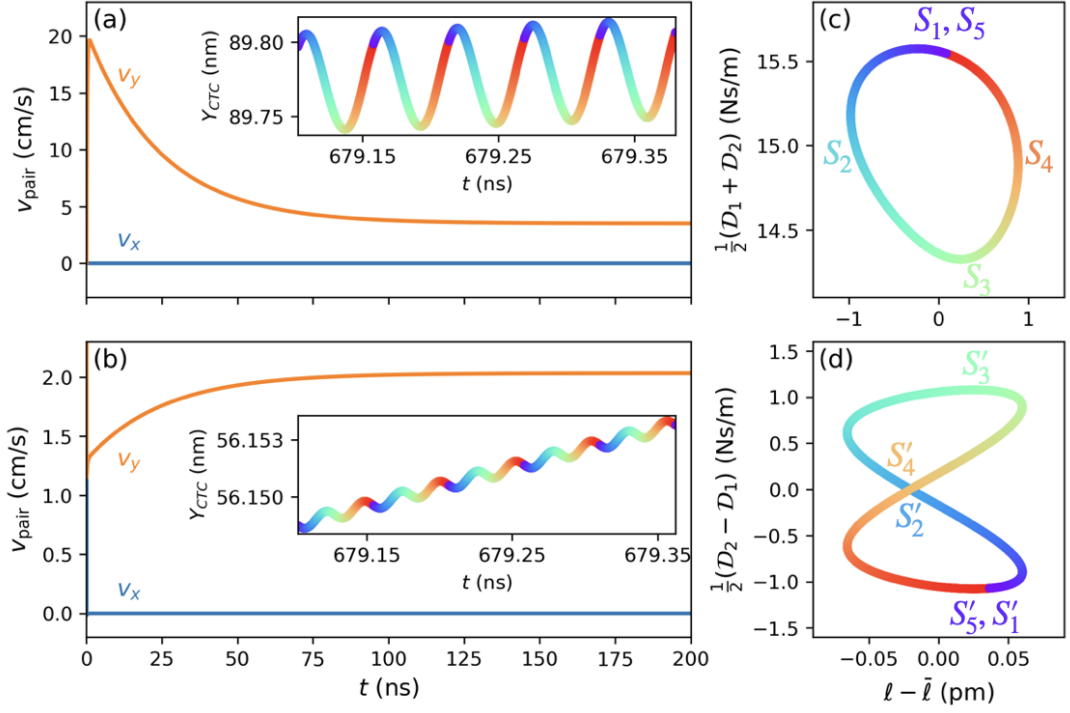
We proceed to discuss the main finding obtained at $T = 0$: the skyrmion pair is observed to have autonomous, self-propelled motion upon periodic excitations of SBM and ABM. The velocity v_{sp} of the pair is oriented along the direction perpendicular to the bond and depends only on the internal skyrmion pair dynamics, thus characterizing it as a self-propulsion velocity. Moreover, the magnitude of v_{sp} varies with ω and shows resonance peaks at breathing modes' natural frequencies. Here, the author performed preliminary micromagnetic simulations and analysis for the SBM scenario.

The time series for the velocity $\mathbf{v}_{\text{pair}} = (v_x, v_y)$ of the skyrmion pair is shown in Fig. 43(a)–(b) for weak coupling ($A_{\text{int}} = 0.02 \text{ mJ/m}^2$) and weak damping ($\alpha = 0.02$). Skyrmions are initialized at equilibrium positions in the middle of the sample with the ansatz of Eq. 3.6. The time series for $\mathbf{v}_{\text{pair}}$ during both ABM and SBM excitations show that after a transient of approximately 100 ns, the steady state results indicate that the perpendicular component v_y saturates at a few cm/s and $v_x \approx 0$. Therefore, at $T = 0$ the pair indeed moves in a path perpendicular to the skyrmion bond, in the y direction.

The insets in panels (a)–(b) of Fig. 43 show the evolution of the perpendicular component of the CTC, Y_{CTC} . An oscillatory behavior arises from the breathing modes excitation and, ultimately, a net displacement towards the positive y direction is observed. The main and inset plots of (a)–(b) correspond exactly to the situation of Fig. 41, so they are color-coded similarly.

Crucially, a period of the skyrmion pair motion defines a **closed loop in the shape plane** defined by the deviation of equilibrium bond length $\ell - \bar{\ell}$, where $\bar{\ell}$ is the bond length obtained in equilibrium, and one of the linear combinations of the dissipation coefficients given by $\frac{1}{2} [\mathcal{D}_2(t) \pm \mathcal{D}_1(t)]$. In Fig. 43(c), an ellipse is drawn under SBM excitation, whereas in (d) we have a vertical lemniscate curve for ABM. These are known as **Lissajous curves** (THORNTON; MARION, 2003), resulting from the superposition of two oscillating quantities — here, the bond length and the skyrmion size. The ellipse in (c) is an 1:1 Lissajous curve indicating a phase shift, as $\mathcal{D}$ responds to the breath-

Figure 43 – Self-propulsion of non-coaxial skyrmion pair for A_{int} , K_0 , and α similar to Fig. 41. In (a),(b) we show the time series of v_{pair} components v_x and v_y , parallel and perpendicular to the bond. Excitations of SBM (a), with $\Delta K/K_0 = 0.005$ and $f = 18$ GHz, and ABM (b), with $B_0 = 4$ mT and $f = 19.24$ GHz, are represented. After transient, v_y saturates. Evolution of perpendicular component of the CTC, Y_{CTC} , is shown in the insets during five cycles. Curves of insets in (a),(b) and of main plots in (c),(d) are color-coded to match stages S_i and S'_i of Fig. 41. In (c),(d), the Lissajous figures drawn during the swimming cycle for SBM (c) and ABM (d). Configuration plane is defined by bond ($\ell - \bar{\ell}$) and shape $[\frac{1}{2}(\mathcal{D}_1 \pm \mathcal{D}_2)]$ deformations. $\mathcal{D}_1$ and $\mathcal{D}_2$ are calculated from micromagnetics for $\bar{\ell}_{\text{SBM}} = 10.98$ nm and $\bar{\ell}_{\text{ABM}} = 10.39$ nm.



Source: Adapted from Silva, Correia and Velásquez (2025).

ing mode while ℓ is dominated by the pair binding force, between skyrmion size and bond length dynamics during SBM excitation, characterized by $\omega_\ell = \omega$. The lemniscate, in turn, represents a 1:2 Lissajous curve, illustrating the mentioned double frequency $\omega_\ell = 2\omega$ arising from ABM excitation.

5.3.4 Generalized Thiele equation

We invoke the generalized Thiele equation (Eq. 2.65) to model the isotropic breathing excitation, so skyrmions do not undergo any other deformation besides radial expansion and contraction. In this simplified model, the corresponding soft mode is the

skyrmion radius $\mathcal{R}(t)$ such that $\xi_j = \mathcal{R}$ in Eq. 2.65, yielding

$$\mathbf{G} \times \dot{\mathbf{R}} + \alpha \mathcal{D} \dot{\mathbf{R}} = \mathbf{F} + \begin{pmatrix} \mathcal{G}_x \mathcal{R} - \alpha \mathcal{D}_x \mathcal{R} \\ \mathcal{G}_y \mathcal{R} - \alpha \mathcal{D}_y \mathcal{R} \end{pmatrix} \dot{\mathcal{R}}, \quad (5.8)$$

where $\mathbf{R}$ is the position of the skyrmion's center of topological charge. The dissipation tensor components scales with

$$\mathcal{D}_{\mu\mathcal{R}} \propto \int_0^\infty r dr \int_0^{2\pi} d\phi \left(\frac{\partial \mathbf{m}}{\partial \mu} \cdot \frac{\partial \mathbf{m}}{\partial \mathcal{R}} \right), \quad (5.9)$$

where $\mu = \{x, y\}$ and the $\partial \mathbf{m} / \partial \mu$ introduce a dependence on $\sin \phi$ or $\cos \phi$ for the integrand, whereas $\partial \mathbf{m} / \partial \mathcal{R}$ is isotropic. Therefore, $\mathcal{D}_{\mu\mathcal{R}}$ vanishes as we integrate in the symmetric domain of ϕ (0 to 2π). Because of an analogous odd integrand of $\mathcal{G}_{\mu\mathcal{R}}$ also vanishes. Therefore, by symmetry, isotropic modes such as the skyrmion radius $\mathcal{R}(t)$ do not contribute explicitly to the right-hand side of Eq. 5.8. However, the dynamics of $\mathcal{R}(t)$ still implicitly provides the modulation of the dissipation tensor $\mathcal{D}(t)$. In this case, Eq. 5.8 is labeled to describe the breathing of the i -th skyrmion and simplified to

$$\mathbf{F}_i = \mathbf{G}_i \times \dot{\mathbf{R}}_i + \alpha \mathcal{D}_i \dot{\mathbf{R}}_i, \quad (5.10)$$

featuring a time-dependent $\mathcal{D}$, $\mathbf{F}_i$ is the binding force of the skyrmion pair, $\mathbf{G}_i = \mathcal{G}_i \hat{\mathbf{z}} = 4\pi(M_s d / \gamma) Q_i \hat{\mathbf{z}}$ is the gyrocoupling vector, $\mathbf{v}_i = d\mathbf{R}_i / dt$ is the skyrmion velocity, and α is the Gilbert damping parameter for both layers.

Analogously to the procedure in the last chapter (Sec. 4.2.1), we break Eq. 5.10 into two components, but now in the directions parallel ($\hat{n}_{\parallel}$) and perpendicular ($\hat{n}_{\perp}$) to the skyrmion bond as

$$\hat{n}_{\parallel} : F_{i\parallel} = \mathcal{G}_i v_{i\perp} + \alpha \mathcal{D}_i v_{i\parallel}, \quad (5.11a)$$

$$\hat{n}_{\perp} : F_{i\perp} = -\mathcal{G}_i v_{i\parallel} + \alpha \mathcal{D}_i v_{i\perp}. \quad (5.11b)$$

Equations 5.11 can be solved yielding $v_{i\perp} / v_{i\parallel} = -\mathcal{G}_i / \alpha \mathcal{D}_i$ for each skyrmion. In the SBM regime, the skyrmions mirror each other's dynamics so

$$v_{1\perp} = v_{2\perp} \equiv v_{\perp}, \quad (5.12a)$$

$$v_{1\parallel} = -v_{2\parallel} \equiv v_{\parallel}. \quad (5.12b)$$

Then, the parallel component of the pair velocity vanishes and the perpendicular one provides a strictly perpendicular motion to the bond, since

$$\mathbf{v}_{\text{pair}} = \frac{1}{2}(\mathbf{v}_1 + \mathbf{v}_2) = v_{\perp} \hat{n}_{\perp}, \quad (5.13)$$

while $d\ell/dt = -2v_{\parallel}$. Now, let us evaluate the self-propulsion velocity in the SBM scenario. In this case, $\mathcal{D}_1 = \mathcal{D}_2 = \mathcal{D}$ and $\mathcal{G}_1 = -\mathcal{G}_2 = G$. v_{sp} can be calculated as the time average of $v_{\text{pair}} = v_{\perp}$, i.e., as the integral of $v_{\perp}$ over one excitation cycle (period T):

$$v_{\text{sp}} = \bar{v}_{\perp} = \frac{1}{T} \int v_{\perp} dt = \frac{\omega}{2\pi} \int \left(-\frac{G}{\alpha \mathcal{D}} v_{\parallel} \right) dt \quad (5.14)$$

$$= \frac{\omega}{2\pi} \int \left[-\frac{G}{\alpha \mathcal{D}} \left(-\frac{1}{2} \frac{d\ell}{dt} \right) \right] dt. \quad (5.15)$$

So the self-propulsion velocity can be finally written as

$$v_{\text{sp}} = \bar{v}_{\perp} = \frac{\omega}{4\pi} \oint \frac{G}{\alpha \mathcal{D}} d\ell, \quad (5.16)$$

revealing that v_{sp} is proportional to the geometric area enclosed by the trajectory in $\ell, \mathcal{D}$ space (the ellipse). Also, it addresses two fundamental elements for the skyrmion self-propulsion:

- the coupled magnetic textures must be **topologically nontrivial** ($Q \neq 0$) in order to obtain a non-zero value of G and, consequently, obtain a nonvanishing integral for v_{sp} ; and
- the swimming cycle needs to be **nonreciprocal**. The nonreciprocity emerges at the SBM scenario as $\ell(t)$ and $\mathcal{D}(t)$ oscillate at the same frequency ω but — decisively — with different phase lags², resulting in the nonreciprocal closed cycle in the ℓ - $\mathcal{D}$ space.

For the ABM case, the situation is rather complex due to the breaking of mirror symmetry as skyrmions oscillate in antiphase. In this way, ℓ , $\mathcal{D}_1$, and $\mathcal{D}_2$ must be treated as independent geometric parameters. It means that, unlike the case of SBM, it is not possible to obtain, from a unique value of ℓ , another unique value of $\mathcal{D}$, which is actually two-valued. In order to illustrate this we compare the frames S'_1 and S'_3 of Fig. 41: the same ℓ corresponds to different cases of $\mathcal{D}_1$ and $\mathcal{D}_2$. Alternatively, we can check this

² For an ellipse as Lissajous curve, the phase shift is valued anywhere between 0 and $\pi/2$.

via the lemniscate curve the system still traces when projected onto the plane defined by ℓ and $\Delta\mathcal{D} \equiv \frac{1}{2}(\mathcal{D}_2 - \mathcal{D}_1)$ [Fig. 43(d)]. Moreover, as discussed above, ℓ oscillates at twice the excitation frequency due to symmetry constraints introducing **inherent nonlinearity** even for small amplitudes. For example, attempting to use a line integral similar to the SBM case would result in zero net velocity ($v_{\text{sp}} = 0$), as the two lobes of the lemniscate cancel each other out. Therefore, the ABM excitation implies that a simple analytical treatment, as in the SBM case, is more challenging.

To obtain v_{sp} in the presence of **anisotropic modes**, we need to apply the Eq. 2.65 to the SBM dynamics of a SAF skyrmion pair assuming perfect mirror symmetry³ and solve it for the self-propulsion velocity yielding

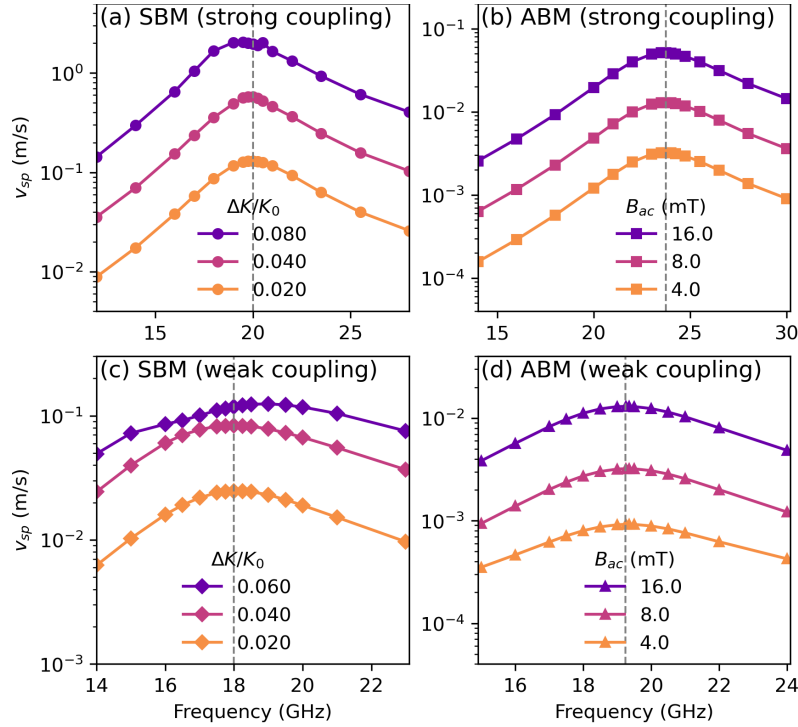
$$v_{\perp} = -\frac{G - \alpha\mathcal{D}_{xy}}{2\alpha\mathcal{D}_{yy}}\dot{\ell} + \sum_j \frac{\mathcal{G}_{yj} - \alpha\mathcal{D}_{yj}}{\alpha\mathcal{D}_{yy}}\dot{\xi}_j, \quad (5.17)$$

where x and y axes are defined respectively parallel and perpendicular to the skyrmion bond length ℓ . This equation allows to **express the self-propulsion speed as a closed-path integral in an extended shape space**: $\{\ell, \xi_1, \xi_2, \dots\}$. In the approximation of circular skyrmions ($\mathcal{D}_{xx} = \mathcal{D}_{yy} = \mathcal{D}$ and $\mathcal{D}_{xy} = 0$) undergoing isotropic breathing dynamics, the second term in Eq. 5.17 vanishes and we recover the case of Eq. 5.16. In contrast, when we excite other shape modes that couple non-reciprocally to translation, Eq. 5.16 becomes insufficient (the case of ABM), thus the full dynamics of Eq. 5.17 should be considered in future studies.

Regarding both SBM and ABM scenarios, the behavior of the self-propelled velocity v_{sp} with respect to the excitation frequency is summarized in Fig. 44 for $\alpha = 0.1$ and for both weak and strong couplings. Panels show that the skyrmion pair velocity reaches up to 0.1 m/s at weak coupling and 1 m/s at strong coupling. In each case, v_{sp} peaks exactly at the resonance frequencies of the excited breathing modes obtained in Fig. 42. Fig. 45 reveals that, for both weak and strong interlayer couplings, the propulsion speed initially scales quadratically with the gyrocoupling-to-friction ratio $\Gamma = G/\alpha\bar{\mathcal{D}}$. At high Γ , however, skyrmions grow too large in one of the half-cycles of excitation due to the small damping, temporarily favoring the coaxial state in the dynamics. This leads to the saturation of v_{sp} as can be visualized in Fig. 45 or, alternatively, in the flattened peaks of panels (a) and (c) in Fig. 44 for large $\Delta K/K_0$. For that reason, we cannot increase

³ This condition is robust against increase in the RKKY coupling, as shown in Fig. 47.

Figure 44 – Self-propulsion speed of the skyrmion pair as a function of excitation frequency ($\alpha = 0.1$). At strong coupling, panel (a) shows v_{sp} for SBM for different $\Delta K/K_0$, whereas (b) shows ABM with different magnetic field amplitudes B_{ac} . Dashed lines indicate the resonance frequency of the corresponding mode. (c, d) Same as (a, b) for weak coupling.

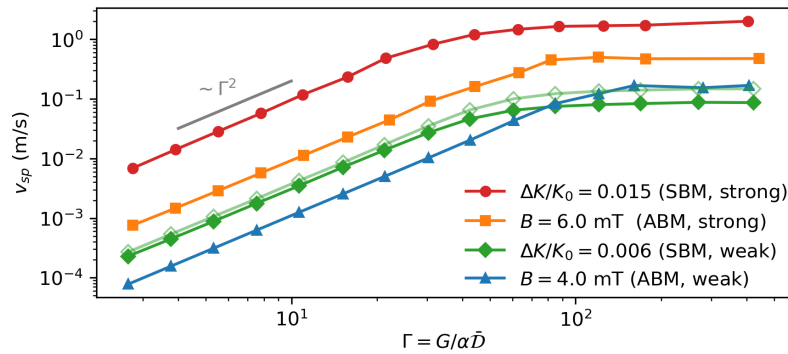


Source: Adapted from Silva, Correia and Velásquez (2025).

self-propulsion velocity indefinitely via larger excitation amplitudes.

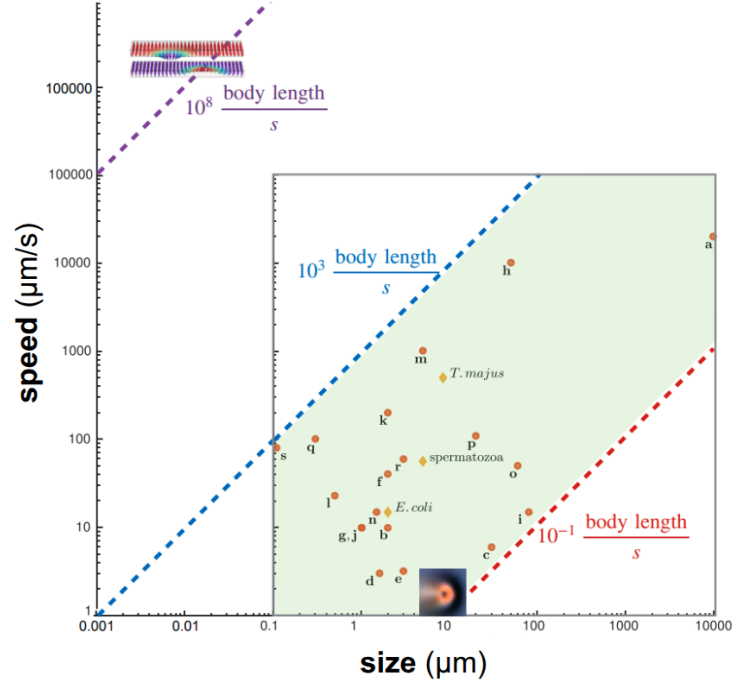
Finally, we compare the velocities observed with that of other natural and artificial swimmers in Fig. 46. Assuming the skyrmion pair to have roughly a nanometer scale ($\ell \approx 10$ nm) moving up to 1 m/s ($10^6 \mu\text{m/s}$), it reaches speeds in the order of 10^8 body

Figure 45 – The self-propulsion velocity v_{sp} at resonance as a function of the gyrocoupling-to-friction ratio $\Gamma = G/\alpha\bar{D}$ for ABM and SBM scenarios at weak and strong coupling regimes. The gray line indicates Γ^2 behavior. The open green symbols in (e) exhibit estimated v_{sp} under weak-coupling SBM using the ℓ - $\bar{D}$ integral, Eq. 5.16.



Source: Adapted from Silva, Correia and Velásquez (2025).

Figure 46 – The SAF skyrmion pair located in the upper left is compared with natural and artificial swimmers depicted in the diagram according to their typical sizes and speeds. The green band indicates that the majority of known swimmers (biological or artificial) perform at a propulsion speed between 10^{-1} and 10^3 body lengths/s, whereas our estimates for the self-propelled skyrmions suggest speeds around 10^8 body lengths/s.



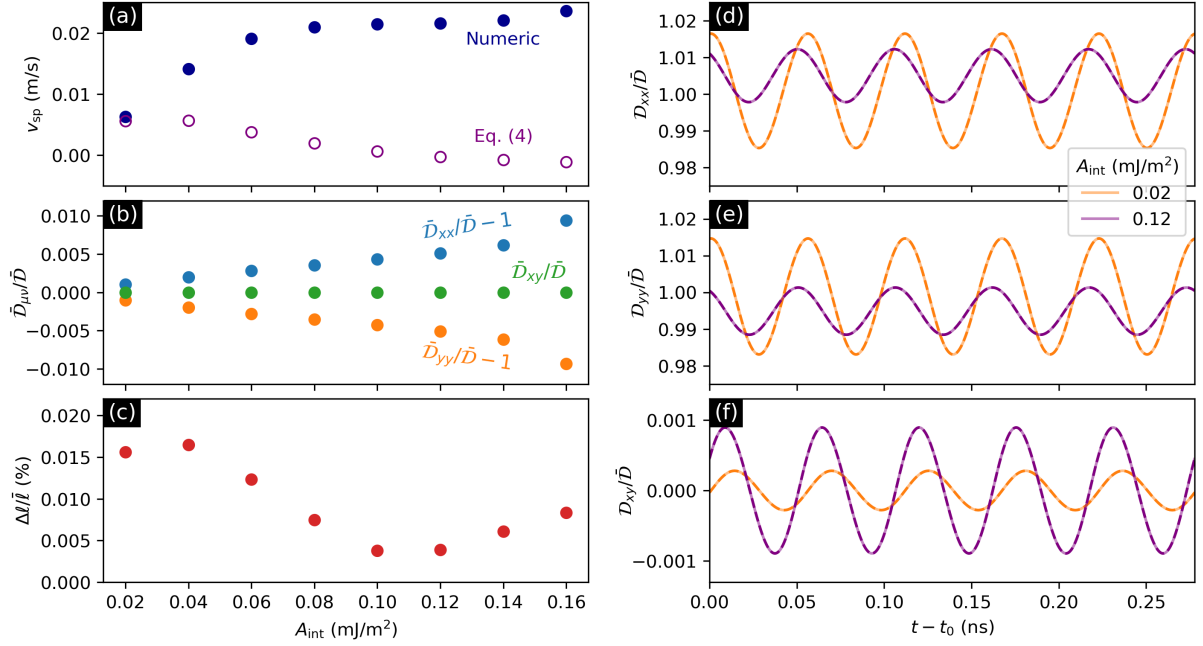
Source: Adapted by Clécio C. de Souza Silva from Bechinger et al. (2016).

lengths per second. This immediately places our self-propelled skyrmion pair in the distinguished upper left sector of the diagram (purple dotted line), indicating that it is much faster than any other natural or artificial swimmer indicated in the diagram.

5.3.5 The case of strong coupling

The Thiele equation approach (Eq. 5.16) works incredibly well near the resonance frequency for weak coupling SBM regime ($f \approx 18$ GHz) in comparison with the data obtained via micromagnetic simulations from the coordinates of the CTC, as illustrated in the open green symbols of Fig. 44(e). This agreement reflects the assumption of the model that the skyrmion motion is dominated by isotropic shape deformations, which is crucial for validating the use of ℓ and $\mathcal{D}$ as effective shape variables in this regime. But for strong coupling the simplified “isotropic” Thiele equation (Eq. 5.10) we obtained fails to capture the full dynamics — as illustrated in panel (a) of Fig. 47 —, indicating that **extra soft modes** become relevant for dynamics and couple to the translation of

Figure 47 – (a) Self-propulsion speed, v_{sp} , (b) normalized time-averaged dissipation tensor components of the top skyrmion, $\bar{\mathcal{D}}_{\mu\nu}/\bar{\mathcal{D}}$, relative to their isotropic limit values, and (c) relative amplitude of the bond length, $\Delta\ell/\bar{\ell}$, as a function of interlayer coupling A_{int} . In (a), closed symbols represent the propulsion speed calculated directly from the simulation, while open dots correspond to the weak-coupling estimate, Eq. 5.16. (d)–(f) Steady-state time evolution of the dissipation tensor components of the top (semi-transparent solid lines) and bottom (dashes) skyrmion, $\mathcal{D}_{xx}$ (d), $\mathcal{D}_{yy}$ (e), and $\mathcal{D}_{xy}$ (f), for weak ($A_{int} = 0.02 \text{ mJ/m}^2$, $K_0 = 0.6 \text{ MJ/m}^3$) and strong ($A_{int} = 0.12 \text{ mJ/m}^2$, $K_0 = 0.35 \text{ MJ/m}^3$) couplings.



the CTC, leading to a shape manifold more complex than ℓ - $\mathcal{D}$.

Anisotropic deformations can be captured in part by the components of the dissipation tensor. To assess such deviations with respect to the isotropic condition, where the dissipation tensor is such that $\mathcal{D}_{xx} = \mathcal{D}_{yy} = \mathcal{D}$ and $\mathcal{D}_{xy} = 0$, we calculated the steady state time evolution of all the dissipation tensor components given by

$$[\mathcal{D}_i]_{\mu\nu} = \frac{M_s d}{\gamma} \int d^2r \partial_\mu \mathbf{m}_i \cdot \partial_\nu \mathbf{m}_i \quad (5.18)$$

under different coupling regimes and varying A_{int} and K_0 . The relative ratio of dissipation tensor components with their isotropic values $\bar{\mathcal{D}}_{\mu,\nu}/\bar{\mathcal{D}}$ (μ, ν are the cartesian components) is exhibited in Fig. 47(b). The ratios are shown as a function of A_{int} for SBM excitation at excitation frequency of 18 GHz with $\Delta K = 0.01K_0$ and $\alpha = 0.1$. Under weak interlayer coupling, the violation of the isotropic condition is small, so the isotropic model provides good agreement with numerical results. But the increase of $\bar{\mathcal{D}}_{xx}$, decrease of $\bar{\mathcal{D}}_{yy}$ and the practically vanishing non-diagonal component $\bar{\mathcal{D}}_{xy}$ as the coupling increases by eight times (from 0.02 to 0.16 mJ/m²) indicate an average elonga-

tion of skyrmions in the $\hat{y}$ direction (perpendicular to the bond and parallel to $\mathbf{v}_{\text{sp}}$, enabling anisotropic dynamics. Combined with the fact that a nonmonotonic dependence is observed in the bond length deformation $\Delta\ell/\bar{\ell}$ with respect to RKKY coupling, we should expect that the skyrmion elongation is a major drive of the propulsion mechanism.

However, the dynamics is actually more intricate than this, as revealed by the behavior of the $\mathcal{D}$ components for weak ($A_{\text{int}} = 0.02$, in orange) and strong ($A_{\text{int}} = 0.12$, in purple) coupling in panels (d)–(f) of Fig. 47. The mirror-symmetry of the heterochiral SAF is found to be preserved from weak to strong coupling since the diagonals components calculated for the two skyrmions are similar in each regime, as the solid (top skyrmion) and dashed (bottom skyrmion) lines overlap. Although smaller, we observe that $\mathcal{D}_{xy}(t)$ [panel (f)] generally increases with A_{int} , while the diagonal components [panels (d) and (e)] decrease. The oscillation of $\mathcal{D}_{xy}$ can be interpreted as wobbling or “trembling” of the main anisotropy axis around the direction perpendicular to the bond, as it induces a oscillating drag term $\sim \mathcal{D}_{xy}(t)$ in the direction parallel to the bond (Eq. 5.10) when we assume a non-vanishing off-diagonal component of $\mathcal{D}$. However, it remains still unclear whether this wobbling couples to the translational mode of the skyrmion pair and accounts for the self-propulsion beyond the weak coupling regime.

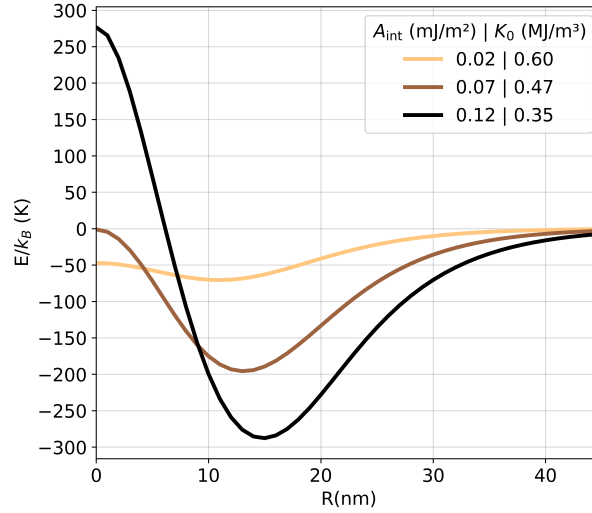
In summary, extra anisotropic soft modes should also be considered as ingredients of Eq. 2.65 and, consequently, as a major contribution for a full description of the dynamics of skyrmion’s CTC under strong coupling in both ABM and SBM scenarios.

5.4 Thermal stochastic dynamics

Finally, we address more realistic conditions by considering the effect of a finite temperature T . The skyrmion pair is found to behave as an active Brownian particle (ABP), transitioning from a diffusion-dominated into a ballistic motion when the amplitude of excitation increases for a given T . The author was responsible for preparing and running the simulations illustrating the trajectories under temperature effects. Remarkably, the pair mimics natural run-and-reverse behavior during its self-propulsion, akin to swimming strategies adopted by micro-organisms such as flagellated bacteria and microalgae.

Firstly, we analyze the pair stability with respect to thermal fluctuations, in order to

Figure 48 – Thermal stability of the non-coaxial pair of skyrmions: skyrmion-skyrmion interaction potential E/k_B in units of K (vertical axis) is shown against the separation R (in nm) of the joint skyrmions for weak ($A_{\text{int}} = 0.02$ and $K_0 = 0.60$) and strong ($A_{\text{int}} = 0.12$ and $K_0 = 0.35$) coupling, chosen with the guidance of diagram of Fig. 40.



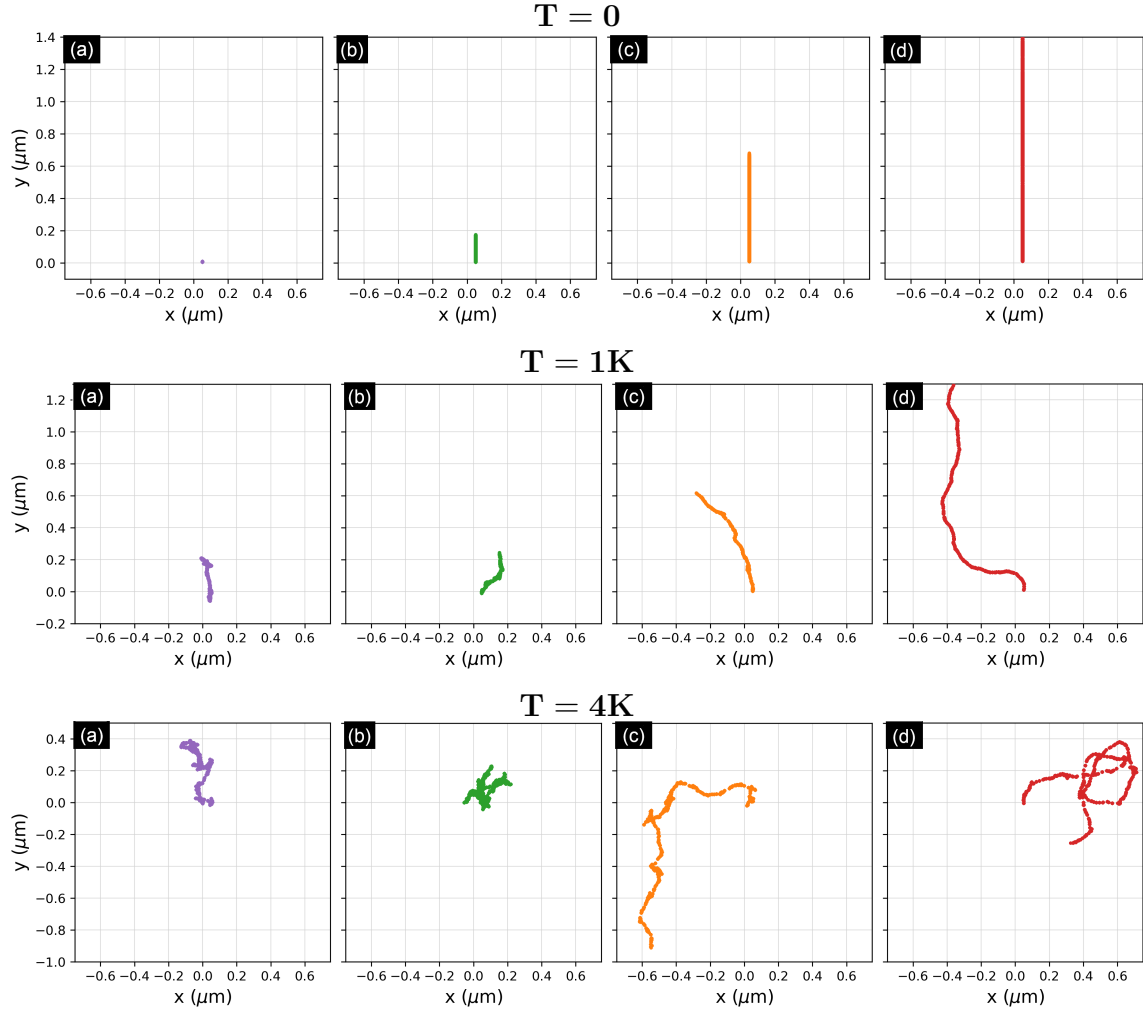
Source: Silva, Correia and Velásquez (2025).

avoid the transition of the non-coaxial state into the metastable coaxial alignment and also the breaking of the bond. Figure 48 shows the bonding energy for selected cases of weak (yellow), intermediate (brown) and strong (black) RKKY coupling. For the weak coupling case, we verify that the energy barriers for coaxial alignment and pair dissociation are approximately 20 K and 70 K, respectively. In this way, the skyrmion pair is expected to be highly sensitive to thermal noise even at a few Kelvin. In contrast, for stronger coupling both the energy barriers become very large, resulting in much greater thermal resilience. Therefore, we choose to verify the self-propulsion robustness by running stochastic LLG simulations in the range of 0.5 to 7.0 K.

The resulting trajectories of the skyrmion pair in several scenarios for the SBM excitation by following CTC coordinates are exhibited in Fig. 49. Each row represents a simulation of a given T for the same time interval. The first row illustrates the previously discussed situation of $T = 0$, when the self-propulsion velocity is perpendicular to the bond length and the skyrmion pair follows a straight trajectory in the y direction. From (a) to (d), the excitation amplitude $\Delta K/K_0$ is increased and so is the speed of the bound pair in the 2-d plane.

At $T \neq 0$, for low or zero anisotropy amplitudes $\Delta K/K_0$ [see panels (e), (f), (i), and (j) in Fig. 49 for $T = 1$ K and $T = 4$ K] the pair jiggles back and forth and changes orientation randomly. So, the trajectory points describes an area very close to the initial

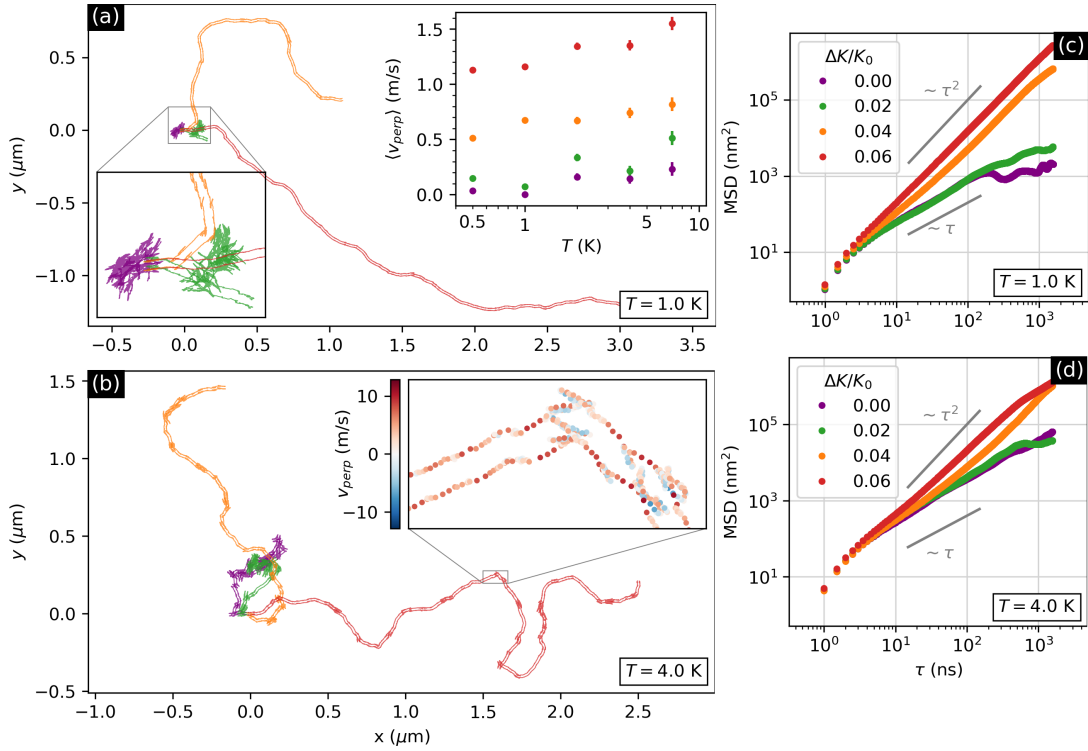
Figure 49 – Trajectories obtained from the center of topological charge (CTC) at $T = 0$ (first row), $T = 1K$ (second row) and $T = 4K$ (third row) and for SBM excitation in a $100 \times 100 \text{ nm}^2$ simulation box. The parameters are $A_{\text{int}} = 0.12 \text{ mJ}^2 \text{ m}^{-1}$, $K_0 = 0.35 \text{ MJ m}^{-3}$, and $\alpha = 0.1$ at $f = 20 \text{ GHz}$ for different excitation amplitudes $\Delta K / K_0$: 0 (a), 0.02 (b), 0.04 (c), and 0.06 (d).



Source: Adapted from Silva, Correia and Velásquez (2025).

position in the origin at $(0, 0)$, which characterizes the pair basically as an **anisotropic Brownian particle** where the motion is dominated by **diffusion** (VOLPE; GIGAN; VOLPE, 2014). While temperature is still $T = 1 \text{ K}$ and we increase $\Delta K / K_0$ to 0.04 [panel (g)], the skyrmion pair departs from the diffusion dominated regime, trying to maintain a straight line path induced by the self-propelled motion, however thermal fluctuations prevent this from happening by a net deviation to the left. At $T = 4 \text{ K}$ and for high enough $\Delta K / K_0$ [see panel (k)], we observe a radically different dynamics of the pair: now it can sustain directed motion over larger distances before reorientation occurs, similar to active Brownian entities such as Janus particles or motile micro-organisms (BECHINGER et al., 2016).

Figure 50 – Self-propelled skyrmion trajectories under SBM excitation for $f = 20$ GHz, $A_{\text{int}} = 0.02$ $\text{mJ}^2 \text{m}^{-1}$, $K_0 = 0.35$ MJ m^{-3} , and $\alpha = 0.1$. Trajectories points (recorded at every 0.5 ns) started at the origin, lasted for ≈ 3 μs and are exhibited in panels (a) and (b) for $T = 1\text{K}$ and $T = 4\text{K}$, respectively. Color indicates the excitation amplitude $\Delta K/K_0$: 0 (purple), 0.02 (green), 0.04 (orange), and 0.06 (red). Insets in (a),(b) exhibit detailed zoomed-in trajectories: small diffusion in (a) for $\Delta K/K_0 = 0, 0.02$ and run-and-reverse pattern for $\Delta K/K_0 = 0.06$ via the orientation of v_{perp} in (b). The mean squared displacement MSD (Eq. 5.19) for trajectories in (a) and (b) is shown in (c) and (d), respectively. Gray lines are references for diffusive ($\sim \tau$) and ballistic ($\sim \tau^2$) behaviors.



Source: Adapted from Silva, Correia and Velásquez (2025).

We proceed to closely inspect the trajectories for $T = 1\text{ K}$ and $T = 4\text{ K}$ in panels (a) and (b) of Fig. 50. Purple ($\Delta K/K_0 = 0$) and green ($\Delta K/K_0 = 0.02$) trajectories in panel (a) confirm the diffusion-dominated motion for vanishing or low SBM excitation, while orange ($\Delta K/K_0 = 0.04$) and red ($\Delta K/K_0 = 0.06$) trajectories correspond to the directed motion sustained by stronger self-propulsion, although disturbed by thermal fluctuations characterizing a slow orientational diffusion. The inset of panel (a) reveals that the averaged self-propulsion velocity slightly increases with temperature, with values close to zero for the diffusion-dominated cases of low excitation amplitude.

We can better characterize the trajectories with respect to diffusion and directed motion by calculating the **mean squared displacement (MSD)** for the skyrmion pair

(PATHRIA; BEALE, 2011), which reads

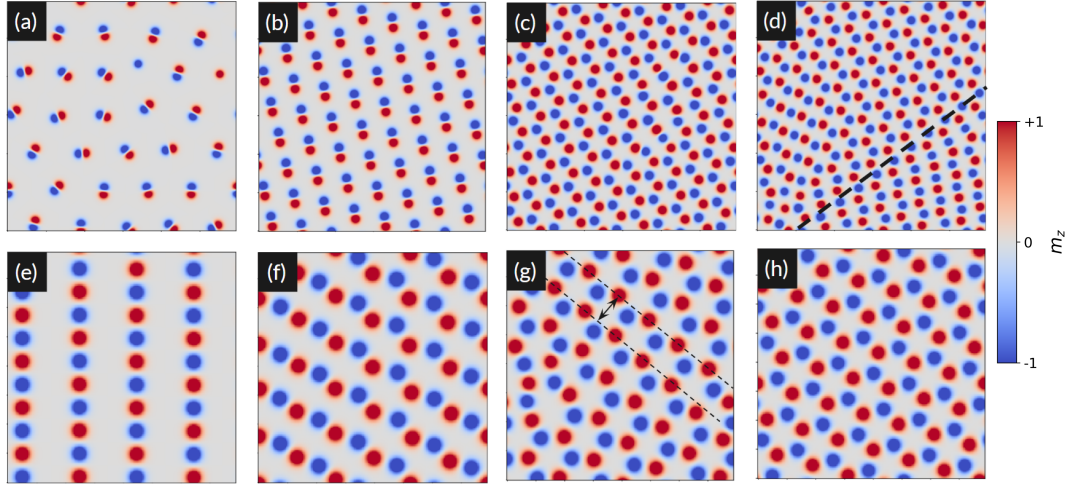
$$\text{MSD}(\tau) = \frac{2}{\Delta t} \int_0^{\Delta t/2} |\mathbf{R}(t + \tau) - \mathbf{R}(t)|^2 dt \quad (5.19)$$

where $\mathbf{R}(t)$ is the vector position of the center of topological charge. Diffusive and ballistic behaviors are known to scale respectively with τ and τ^2 . The panel (c) in Fig. 50 shows the MSD for $T = 1\text{K}$, confirming that a diffusion-dominated trajectory ($\sim \tau$) is found for vanishing or low excitation amplitude (in purple and green) where the net motion is effectively suppressed. A transition in MSD to ballistic motion ($\sim \tau^2$) is verified for increasing $\Delta K/K_0$ (in orange and red), where the directed autonomous motion is persistent. A similar plot is drawn for $T = 4\text{ K}$ with similar results.

However, at $T = 4\text{ K}$ the trajectories for $\Delta K/K_0 = 0.04$ and $\Delta K/K_0 = 0.06$ become more sinuous and a remarkable emergent behavior is observed: during motion, the pair occasionally halts, undergoes a spontaneous 180° reversal in its propulsion direction briefly (only for a few nanoseconds), and then resumes forward motion, as illustrated in the inset of Fig. 50(b). We color code the forward (backward) pair velocity by reddish (blueish) points. In some cases, the pair performs two or more back-and-forth jitters before restoring the persistent forward motion, often emerging along a new direction. Moreover, this reassures that the direction of the autonomous motion is indeed dependent on the internal dynamics of the pair, rather than previously defined by external oscillating fields, which is a main feature of self-propulsion.

Notably, all abrupt changes in orientation coincide with such reversal events which appear to be triggered by small fluctuations that momentarily push skyrmions too close to one another. Interestingly, this emergent **run-and-reverse** behavior is a well-documented strategy employed by a large class of motile bacteria to explore complex environments more efficiently (SON; GUASTO; STOCKER, 2013). This is the case of roughly almost 70% of marine bacteria (MITCHELL; PEARSON; DILLON, 1995), such as *P. haloplanktis*, which uses reversals to escape dead ends, respond to nutrient patches, and climb gradients more rapidly, thus increasing exposure to larger nutrient concentrations (XIE et al., 2011).

Figure 51 – Maps of $m_z = m_{z,1} + m_{z,2}$ for weak (top row, $A_{\text{int}} = 0.02$) and strong (bottom row, $A_{\text{int}} = 0.12$) coupling showing steady-state phases obtained for many skyrmion pairs under SBM excitation ($\Delta K/K_0 = 0.05$) as a function of N prototype skyrmion pairs. In top row [(a)–(d)], simulation box of $L^2 = 300 \times 300 \text{ nm}^2$, $f = 17.75 \text{ GHz}$, and, from (a) to (d), $N = 30, 60, 100$, and 120. In (d), dashed line separates lattice and strip configurations. In bottom row [(e)–(h)], $L^2 = 200 \times 200 \text{ nm}^2$, $f = 20.00 \text{ GHz}$, and, from (e) to (h), $N = 20, 30, 40$, and 45. Color bar indexes indicate $m_z = +1$ (red), $m_z = 0$ (grey), and $m_z = -1$ (blue).



Source: The author (2025).

5.5 Envisioning spin-based active matter and metamaterials

In addition to simulations of a single skyrmion pair, we now present some preliminary results on the interaction of many pairs, where we obtain a rich stationary-state phase diagrams at $T = 0$ for the SBM scenario. Moreover, inspired by colloidal meta-machines, we exemplify how to control rotational and translational motion of magnetic structures formed by a few skyrmion pairs, conceiving “skyrmionic” metamaterials.

5.5.1 Density—Interlayer-coupling phase diagram

We start by exploring the collective phases of many skyrmions pairs in several situations in the SBM scenario. The results are presented in Fig. 51 as maps of $m_z = m_{z,1} + m_{z,2}$, providing a condensed and clear presentation of the magnetic phases in the steady state. The excitation frequency is $f = 20 \text{ GHz}$ and the excitation amplitude is $\Delta K/K_0 = 0.05$. In an extended simulation box with area L^2 , we initialize N prototype skyrmion pairs with randomly distributed orientation and spatial coordinates, avoiding overlap of skyrmions within the same layer, and, after an equilibration time, we apply the oscillating field. We emphasize that the configuration obtained under the

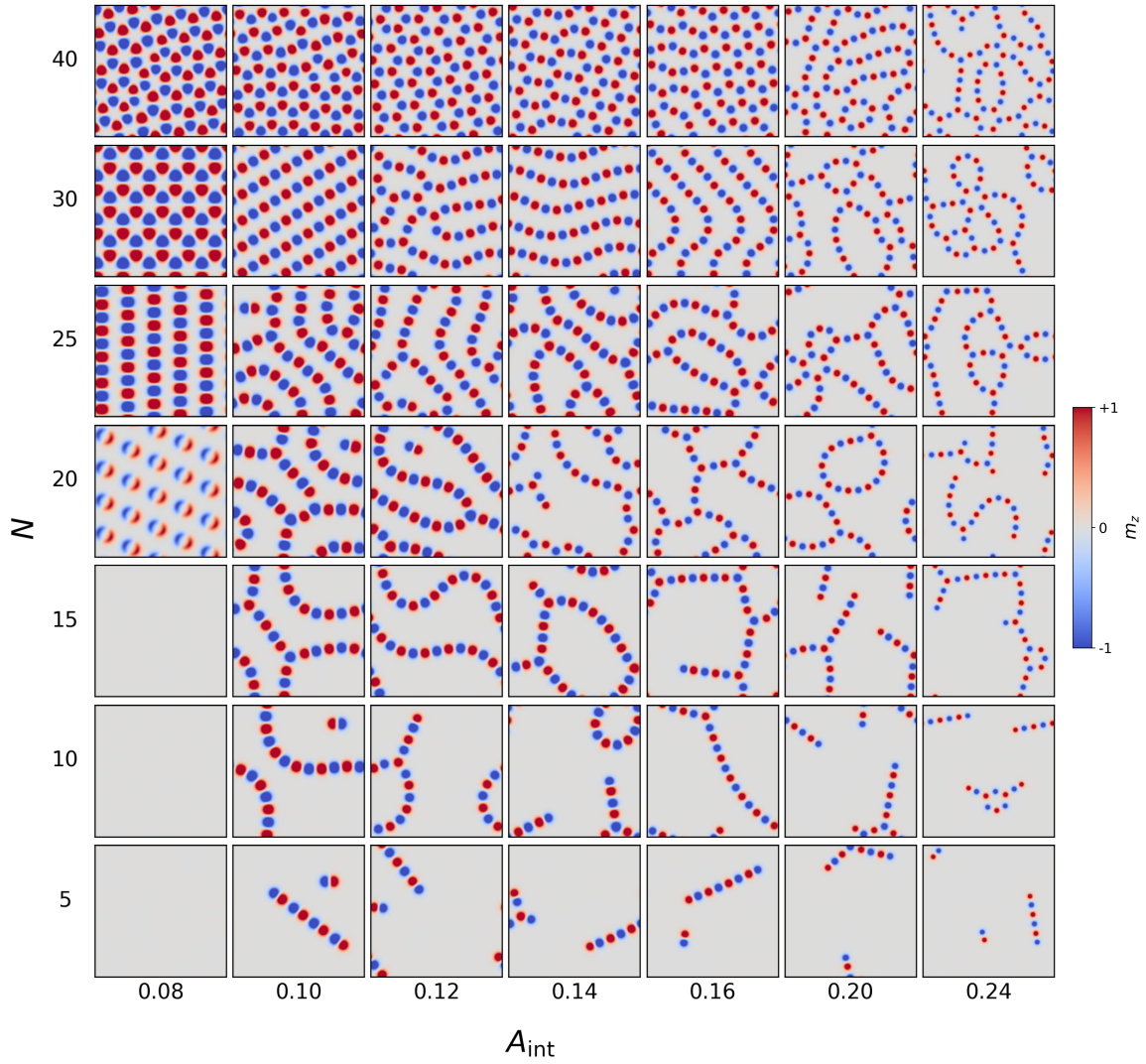
external excitation is modified with respect to the equilibrium configuration. For each row, we fix interlayer coupling A_{int} while increase N and, then, the number density $n_{\text{sk}} = 2N/L^2$.

In the first row, at weak coupling, panels (a)–(d) exhibit the increase of n_{sk} from 0.66 to 2.67 in units of $\times 10^3 \mu\text{m}^{-2}$. In (a), a dilute phase of skyrmions pairs exhibits a few unpaired skyrmions. In (b), the increase of N leads to skyrmions' reorganization into diagonal strips formed by skyrmions pairs equally distanced along each strip, with no skyrmion left unpaired. Further increase of N in panels (c) and (d) illustrates smaller separation between skyrmions, with the formation of defected triangular lattices of top and bottom skyrmions shifted from each other (non-coaxial), sometimes in coexistence with strips [dotted line is a separatrix in panel (d)]. For strong pinning [panels (e)–(h)], n_{sk} ranges from 1.00 to 2.25 in units of $\times 10^3 \mu\text{m}^{-2}$. At low density, the strip phase dominates [panels (e)–(f)], with smaller spatial period for increasing N and with some defects pointing to the formation of triangular lattices [panel (f)]. Non-coaxial lattices are finally formed in (g) and (h), with no defects visualized in the latter. For reference, the distance between two lines of skyrmions in (g) is roughly 40 nm.

After the initial inspection provided by Fig. 51, we visualize a comprehensive dependence of the magnetic phases with respect to both density and interlayer coupling in the preliminary phase diagram depicted in Fig. 52. The diagram reveals a strong density-dependent transition in the collective behavior. In the high-density regime ($N \geq 30$), the system is dominated by **crowding effects**, initially forming an hexagonal lattice at lower coupling strengths ($A_{\text{int}} \leq 0.10 \text{ mJ/m}^2$). As the interlayer coupling A_{int} increases, this ordered lattice destabilizes into a labyrinthine state. This suggests that a stronger RKKY coupling alters the effective inter-particle potential, introducing **competing interactions** that frustrate the triangular symmetry, since the attractive interlayer RKKY coupling competes with the repulsion of skyrmions within the same layer.

The role of the RKKY coupling A_{int} appears to be the critical control parameter for the “stiffness” of the skyrmion pairs. Moving horizontally across the diagram from $A_{\text{int}} = 0.08$ to 0.24 mJ/m^2 , we observe a morphological transition from repulsive, space-filling distributions to attractive, clustering behaviors. This is visible for the cases of $N = 20 - 40$ and $A_{\text{int}} = 0.08$, where we find skyrmions organized in a distinct hexagonal lattice of pairs or vertical strips. At stronger coupling strengths, however, the skyrmions aggregate into chains rather than maintaining uniform separation.

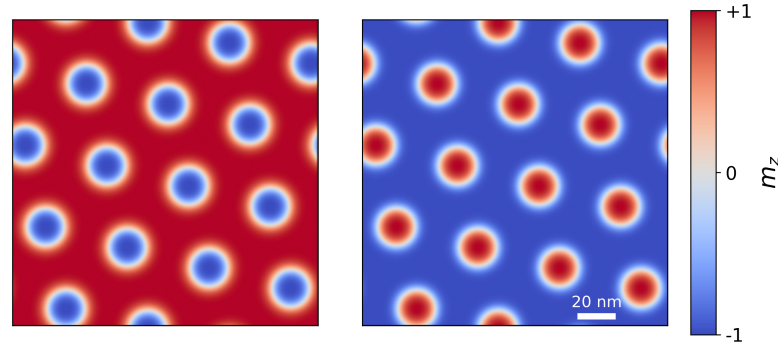
Figure 52 – Preliminary phase diagram of steady-state configurations in the SBM scenario at $f = 20$ GHz and $\Delta K/K_0 = 0.05$ for approximately 150 ns and simulation box of $200 \times 200 \text{ nm}^2$. The snapshots display the net out-of-plane magnetization $m_z = m_{z,1} + m_{z,2}$ for a range of the initial number of skyrmion pairs ($N \in [5, 40]$) and $A_{\text{int}} \in [0.08, 0.24] \text{ mJ/m}^2$. Uniform grey regions (low N and low A_{int}) indicate perfectly compensated top and bottom skyrmion lattices. Color bar indicate $m_z = +1$ (red), $m_z = 0$ (grey), and $m_z = -1$ (blue).



Source: The author (2025).

Finally, in the low-density regime ($N \leq 15$), we visualize entirely grey panels for $A_{\text{int}} = 0.08$. This corresponds to perfectly matching triangular skyrmion lattices of top and bottom layers (multiple coaxial pairs), as revealed in detail by Fig. 53. However, as A_{int} increases, symmetry is broken and for roughly $A_{\text{int}} \geq 0.12$ the system relaxes into discrete, stable “molecules”: dimers, trimers, small chains and even a funny horned structure ($N = 10$ and $A_{\text{int}} = 0.24$). These discrete structures at small N may serve as a realization of “skyrmionic metamaterials” analogous to colloidal active matter systems.

Figure 53 – Steady-state magnetic configurations of top (left) and bottom (right) layers, where red (blue) stands for $m_z = +1(-1)\hat{z}$. The panels illustrate the spatial alignment of the skyrmion cores between layers, characteristic of the coaxial state. The lattice is stabilized for $N = 15$ and an interlayer coupling of $A_{\text{int}} = 0.08 \text{ mJ/m}^2$.



Source: The author (2025).

5.5.2 Skyrmionic metamachines

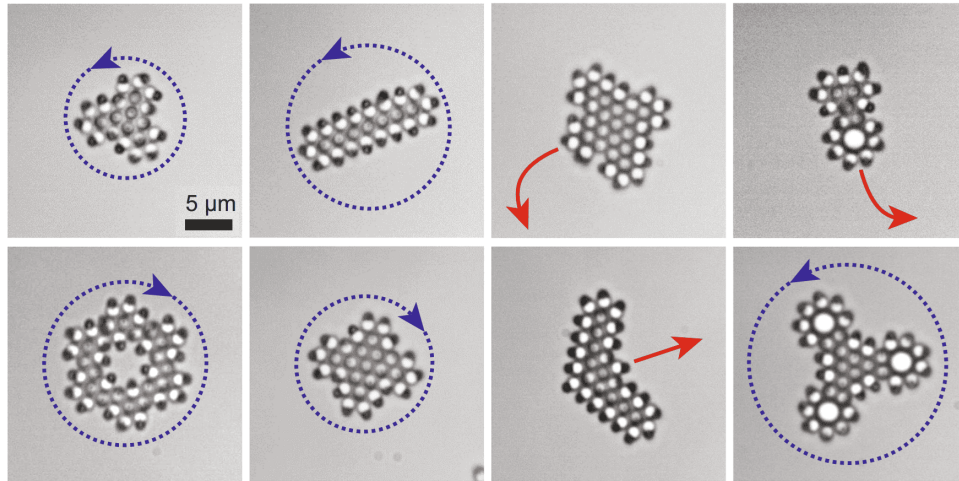
In colloidal physics, the macroscopic response of a cluster — whether it exhibits pure translation, rotation, or chiral motion — can be dictated by the number and geometric symmetry of its constituents (AUBRET; MARTINET; PALACCI, 2021), giving rise to “colloidal metamachines”. Several realizations of these colloidal structures can be seen in Fig. 54, where multiple combinations of active colloids — constructed via optical traps provides — a rich dynamics involving translation, rotation and hybrid states including the transport of passive objects.

By tuning N and the initial geometry for arranging top and bottom skyrmions, this inspires us to artificially engineer specific configurations exploring symmetry and promote dynamical responses of interest to external excitations. The $(n_T + n_B)$ configuration, with n_T (n_B) the number of skyrmions initially nucleated at top (bottom) layers, may respond to external fields with pure rotations, pure translations, or a combination of both by changing the geometry on-the-fly.

In the last chapter we discussed the basic (1+1) arrangement, the skyrmion pair, which responds with a pure (self-propelled) translation. The same is achieved for a linear (2+2) configuration, as exhibited in Fig. 55. The initial “template” in panel (a) accommodates and reaches equilibrium in the linear chain in (b). Then, we apply the SBM excitation in (c) and find that the structure responds with a pure translation perpendicular to the chain, similar as the skyrmion pair mechanism.

However, by preparing a different template of an asymmetric chain of (4+3), the gy-

Figure 54 – Colloidal metamachines from active heterodimers. Microscopy images show architectures formed using optical traps. Anisotropic forces are controlled to program distinct functions, so the specific geometry of each cluster dictates its autonomous dynamics: chiral structures rotating in a deterministic direction (first column); centrosymmetric structures rotating due to spontaneous symmetry breaking (second column); axisymmetric structures exhibiting translational motion (third column, red arrows); and hybrid structures composed of active heterodimers and passive spheres (fourth column).

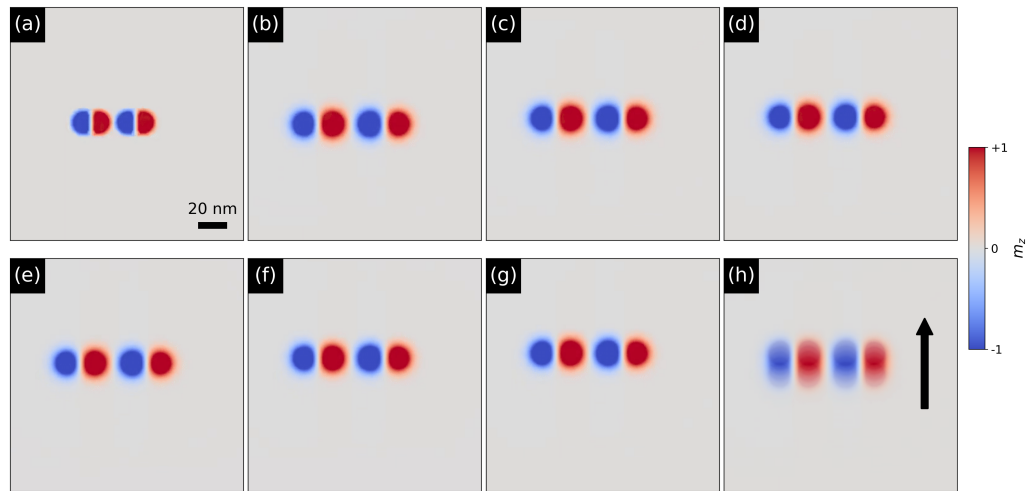


Source: Adapted from Aubret, Martinet and Palacci (2021).

rotropic (Magnus) forces between the top and bottom layers are not balanced anymore, due to the unpaired top skyrmion (Fig. 56). This promotes a torque in the chain, that remains unbroken, ultimately producing a clockwise rotation as a rigid body around the axis perpendicular to the central (bottom) skyrmion [panels (c)–(g)]. In panel (h) we superimpose the snapshots for better visualizing the rotation.

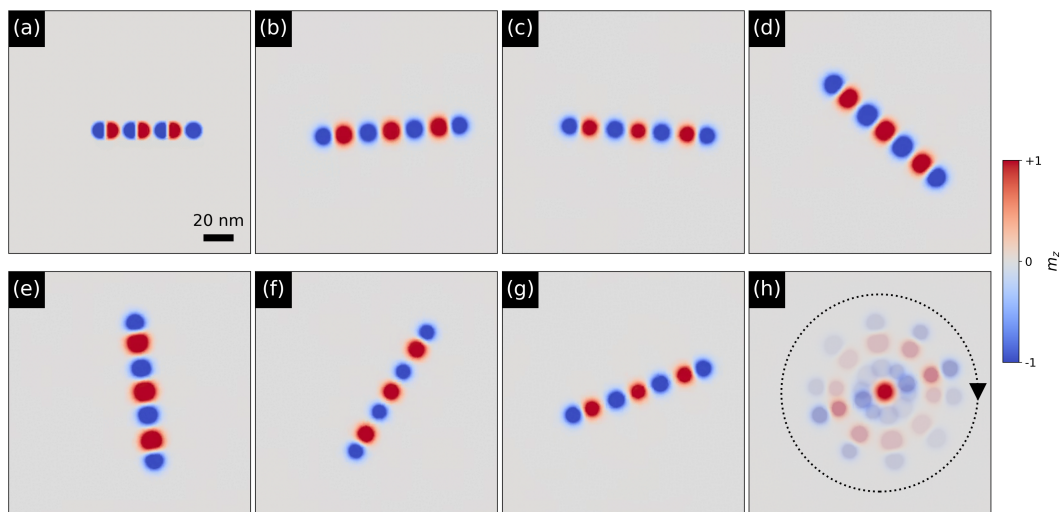
With those preliminary results in mind, we open possibilities for investigating in detail the phase diagram of Fig. 52 in other scenarios and to engineer complex structures at small N , tuning the behavior of these skyrmionic systems envisioning spintronic applications.

Figure 55 – Pure translation in a skyrmionic metamaterial. Panel (a) shows symmetric (2+2) configuration constructed from initial prototype skyrmion pairs: two top and two bottom skyrmions. (b) The system relaxes into a stable linear chain with alternating polarities. (c)–(g) Time-resolved snapshots illustrating the system's evolution under SBM excitation. The collective dynamics drive the center of mass upward while strictly maintaining the horizontal orientation of the chain. (h) Superimposed visualization of the trajectory from (c) to (g).



Source: The author (2026).

Figure 56 – Rotational dynamics of a skyrmionic metamaterial. Panel (a) displays the asymmetric (3+4) configuration constructed from initial prototype skyrmions: three top and four bottom skyrmions. The system relaxes into the stable linear chain shown in (b). Panels (c)–(g) illustrate the time-evolution under SBM excitation. (h) Superimposed visualization of the dynamics from (c) to (g).



Source: The author (2026).

6 CONCLUSIONS

The exploration for next-generation information technologies has encountered in magnetic skyrmions a promising candidate for high-density and energy-efficient spintronics. Their inherent topological protection, nanoscale dimensions, and the ability to be manipulated by low-current densities make them very suitable for applications ranging from racetrack memories to neuromorphic computing. However, the common Skyrmion Hall Effect (SkHE) in ferromagnetic monolayers remains a significant obstacle for their controllable motion. In this thesis, we established that synthetic antiferromagnets (SAFs) not only offer a robust solution to the SkHE, but also provide a rich playing field for exploring novel physical phenomena, from tunable magnetic phases to the emergence of spin-based active matter behavior.

In the first part, based on our first work published in *Physical Review B* (CORREIA; VELÁSQUEZ; SILVA, 2024), we provided a comprehensive analysis on the stability and current-induced dynamics of skyrmion pairs in SAFs. We constructed the equilibrium phase diagram for both homochiral and heterochiral stacks, revealing that the interplay between the Dzyaloshinskii-Moriya interaction (DMI) and the interlayer RKKY coupling dictates the formation of coaxial or non-coaxial bound pairs. A key finding is the demonstration that the suppression of the SkHE in SAFs is remarkably robust against asymmetries in material parameters, such as magnetic anisotropy and DMI constants. However, we identified that this cancellation is sensitive to imbalances in the saturation magnetization or the spin-polarized currents injected into the ferromagnetic layers. Furthermore, we showed that the non-coaxial configuration in heterochiral SAFs significantly enhances the experimental visibility of these textures via stray fields, facilitating their detection in Magnetic Force Microscopy (MFM) apparatus.

In the second part, based on our work published in *Physical Review Letters* (SILVA; CORREIA; VELÁSQUEZ, 2025), we shifted our perspective from traditional “passive” spintronics to the physics of active matter. We theoretically demonstrated that the non-coaxial skyrmion pairs we found in heterochiral SAFs can function as autonomous, self-propelled “swimmers”. By exciting internal breathing modes — specifically symmetric (SBM) and antisymmetric (ABM) modes — through oscillating external fields, we showed that the time-reversal symmetry of the pair is broken. This process generates

non-reciprocal gyrotropic forces resulting in a net directed motion, effectively turning the skyrmion pair into a nanoscale active Brownian particle. Our stochastic analysis also revealed that thermal fluctuations induce a “run-and-reverse” behavior similar to motility strategies observed in biological micro-organisms like marine bacteria, thus connecting magnetic topology and biological active systems.

The results presented in this thesis open several pathways for future research. A natural extension of our work involves detailed investigation of extra dynamic modes, other than breathing, to fully explain the self-propelled motion. Also, as the preliminary results in the last section of Chapter 5 suggest, the study of collective dynamics in systems with higher skyrmion density, when interacting self-propelled pairs can form complex steady-state phases, offers a route to design reconfigurable magnetic meta-materials inspired by colloids, where the macroscopic properties can be tuned by the skyrmion pair activity. This effectively unlocks a new area of research, of **active spintronics systems**, where we may either evoke active matter techniques to implement novel functionalities in future spintronic devices or using active skyrmionic matter to investigate out-of-equilibrium phenomena.

REFERENCES

- ABERT, C. W. *Discrete Mathematical Concepts in Micromagnetic Computations*. Phd Thesis (PhD Thesis) — University of Hamburg, Hamburg, 2013. Available at: <<https://ediss.sub.uni-hamburg.de/handle/ediss/5161>>. Accessed on: 08 jan. 2026.
- ALLEN, M. P.; TILDESLEY, D. J. *Computer simulation of liquids*. 2nd. ed. Oxford: Oxford University Press, 2017. Available at: <<https://doi.org/10.1093/oso/9780198803195.001.0001>>. Accessed on: 17 jan. 2026.
- AMPÈRE, A. h.-M. *Théorie mathématique des phénomènes électro-dynamiques: uniquement déduite de l'expérience*. Paris: A. Hermann, 1883. Available at: <<https://archive.org/details/thoriemathmatiq00ampgoog>>. Accessed on: 21 Mar. 2026.
- ANDERSON, C.; FERNANDEZ-NIEVES, A. Social interactions lead to motility-induced phase separation in fire ants. *Nature Communications*, London, v. 13, n. 1, p. 6710, 2022. Available at: <<https://doi.org/10.1038/s41467-022-34181-0>>. Accessed on: 21 mar. 2026.
- ARFKEN, G. B.; WEBER, H. J. *Mathematical Methods for Physicists*. 6th. ed. Burlington, MA: Elsevier Academic Press, 2005.
- ARISTOTLE. *On the soul*. South Bend: Alex Catalogue, 2000. Available at: <<https://archive.org/details/in.ernet.dli.2015.183331>>. Accessed on: 21 mar. 2026.
- AUBRET, A.; MARTINET, Q.; PALACCI, J. Metamachines of pluripotent colloids. *Nature Communications*, London, v. 12, n. 1, p. 6398, 2021. Available at: <<https://doi.org/10.1038/s41467-021-26699-6>>. Accessed on: 21 mar. 2026.
- BACON, F. *Novum Organum*. Oxford: Clarendon Press, 1878. Available at: <https://books.google.com.br/books?id=tH4_AAAAYAAJ>. Accessed on: 21 Mar. 2026.
- BAIBICH, M. N. et al. Giant magnetoresistance of (001)fe/(001)cr magnetic superlattices. *Physical Review Letters*, College Park, v. 61, n. 21, p. 2472–2475, 1988. Available at: <<https://doi.org/10.1103/PhysRevLett.61.2472>>. Accessed on: 21 Mar. 2026.
- BARKER, C. E. A.; HALTZ, E.; MOORE, T. A.; MARROWS, C. H. Breathing modes of skyrmion strings in a synthetic antiferromagnet multilayer. *Journal of Applied Physics*, v. 133, n. 11, p. 113901, 2023. Available at: <<https://doi.org/10.1063/5.0142772>>. Accessed on: 24 Mar. 2026.
- BARKER, J.; TRETIKOV, O. A. Static and dynamical properties of antiferromagnetic skyrmions in the presence of applied current and temperature. *Physical Review Letters*, College Park, v. 116, n. 14, p. 147203, 2016. Available at: <<https://doi.org/10.1103/PhysRevLett.116.147203>>. Accessed on: 21 Mar. 2026.
- BARNETT, S. J. Magnetization by rotation. *Physical Review*, College Park, v. 6, n. 4, p. 239–270, 1915. Available at: <<https://doi.org/10.1103/PhysRev.6.239>>. Accessed on: 21 Mar. 2026.

BAUER, M. *The Stern-Gerlach Experiment*, Translation of: "Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld". Ithaca: [s.n.], 2023. ArXiv:2301.11343. Available at: <<https://arxiv.org/abs/2301.11343>>. Accessed on: 21 Mar. 2026.

BBC. *William Gilbert (1544 - 1603)*. 2014. Available at: <https://www.bbc.co.uk/history/historic_figures/gilbert_william.shtml>. Accessed on: 29 Jun. 2025.

BECHINGER, C.; LEONARDO, R. D.; LÖWEN, H.; REICHHARDT, C.; VOLPE, G.; VOLPE, G. Active particles in complex and crowded environments. *Reviews of Modern Physics*, College Park, v. 88, n. 4, p. 045006, 2016. Available at: <<https://doi.org/10.1103/RevModPhys.88.045006>>. Accessed on: 21 mar. 2026.

BINASCH, G.; GRÜNBERG, P.; SAURENBACH, F.; ZINN, W. Enhanced magnetoresistance in layered magnetic structures with antiferromagnetic interlayer exchange. *Physical Review B*, College Park, v. 39, n. 7, p. 4828–4830, 1989. Available at: <<https://doi.org/10.1103/PhysRevB.39.4828>>. Accessed on: 21 Mar. 2026.

BO, L.; ZHANG, X.; MOCHIZUKI, M.; ZHANG, X. Suppression of the skyrmion hall effect in synthetic ferrimagnets with gradient magnetization. *Physical Review Research*, v. 6, n. 2, p. 023199, 2024. Available at: <<https://doi.org/10.1103/PhysRevResearch.6.023199>>. Accessed on: 24 Mar. 2026.

BOGDANOV, A. N.; YABLONSKII, D. A. Thermodynamically stable “vortices” in magnetically ordered crystals. the mixed state of magnets. *Soviet Physics JETP*, Moscow, v. 68, n. 1, p. 101–103, 1989. Available at: <<http://jetp.ras.ru/cgi-bin/e/index/e/68/1/p101?a=list>>. Accessed on: 21 Mar. 2026.

BOHR, N. On the constitution of atoms and molecules: Part I. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, London, v. 26, n. 151, p. 1–25, 1913. Available at: <<https://doi.org/10.1080/14786441308634955>>. Accessed on: 21 Mar. 2026.

BOHR, N. On the constitution of atoms and molecules: Part II. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, London, v. 26, n. 153, p. 476–502, 1913. Available at: <<https://doi.org/10.1080/14786441308634993>>. Accessed on: 21 Mar. 2026.

BOHR, N. On the constitution of atoms and molecules: Part III. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, London, v. 26, n. 155, p. 857–875, 1913. Available at: <<https://doi.org/10.1080/14786441308635031>>. Accessed on: 21 Mar. 2026.

BOHR, N. *The structure of the atom*. 1922. Nobel Lecture, December 11, 1922. Nobel Foundation. Available at: <<https://www.nobelprize.org/uploads/2018/06/bohr-lecture.pdf>>. Accessed on: 13 Nov. 2025.

BOHR, N. The doctor's dissertation (text and translation). In: ROSENFELD, L.; NIELSEN, J. R. (Ed.). *Early Work (1905–1911)*. Amsterdam: Elsevier, 1972, (Niels Bohr Collected Works, v. 1). p. 163–393. Available at: <[https://doi.org/10.1016/S1876-0503\(08\)70015-X](https://doi.org/10.1016/S1876-0503(08)70015-X)>. Accessed on: 21 Mar. 2026.

BRANDÃO, J. et al. Observation of magnetic skyrmions in unpatterned symmetric multilayers at room temperature and zero magnetic field. *Scientific Reports*, London, v. 9, n. 1, p. 4144, 2019. Available at: <<https://doi.org/10.1038/s41598-019-40705-4>>. Accessed on: 21 Mar. 2026.

BREMMER, J. N. The early greek concept of the soul. In: *The early Greek concept of the soul*. Princeton: Princeton University Press, 1983. p. 11–69. Available at: <<https://pure.rug.nl/ws/files/3416652/c2.pdf>>. Accessed on: 21 mar. 2026.

BROGLIE, L. D. *Recherches sur la théorie des quanta*. Phd Thesis (PhD Thesis) — University of Paris, Paris, 1924. Available at: <<https://tel.archives-ouvertes.fr/tel-00006807>>. Accessed on: 21 Mar. 2026.

BROWN JR., W. F. Thermal fluctuations of a single-domain particle. *Physical Review*, College Park, v. 130, n. 4, p. 1677–1686, 1963. Available at: <<https://doi.org/10.1103/PhysRev.130.1677>>. Accessed on: 17 jan. 2026.

BRUNO, P. Theory of interlayer magnetic coupling. *Physical Review B*, College Park, v. 52, n. 1, p. 411–439, 1995. Available at: <<https://doi.org/10.1103/PhysRevB.52.411>>. Accessed on: 21 mar. 2026.

BURTON, M.; TEIXEIRA, F. *Brazil joins race to loosen China's grip on rare earths industry*. London: [s.n.], 2024. Available at: <<https://www.reuters.com/markets/commodities/brazil-joins-race-loosen-chinas-grip-rare-earths-industry-2024-06-17/>>. Accessed on: 19 dez. 2025.

CALLEN, H. B.; CALLEN, E. The present status of the temperature dependence of magnetocrystalline anisotropy, and the $l(l+1)/2$ power law. *Journal of Physics and Chemistry of Solids*, Amsterdam, v. 27, n. 8, p. 1271–1285, 1966. Available at: <[https://doi.org/10.1016/0022-3697\(66\)90012-6](https://doi.org/10.1016/0022-3697(66)90012-6)>. Accessed on: 21 Mar. 2026.

CAMLEY, R. E.; LIVESEY, K. L. Consequences of the dzyaloshinskii-moriya interaction. *Surface Science Reports*, Amsterdam, v. 78, n. 3, p. 100605, 2023. Available at: <<https://doi.org/10.1016/j.surfrep.2023.100605>>. Accessed on: 21 Mar. 2026.

CARLSON, J. B. Lodestone compass: Chinese or olmec primacy? multidisciplinary analysis of an olmec hematite artifact from san lorenzo, veracruz, mexico. *Science*, Washington, DC, v. 189, n. 4205, p. 753–760, 1975. Available at: <<https://doi.org/10.1126/science.189.4205.753>>. Accessed on: 21 Mar. 2026.

CARVALHO, P. C. et al. Correlation of interface interdiffusion and skyrmionic phases. *Nano Letters*, Washington, DC, v. 23, n. 11, p. 4854–4861, 2023. Available at: <<https://doi.org/10.1021/acs.nanolett.3c00428>>. Accessed on: 21 Mar. 2026.

COEY, J. M. D. *Magnetism and magnetic materials*. Cambridge: Cambridge University Press, 2010. Available at: <<https://doi.org/10.1017/CBO9780511845000>>. Accessed on: 21 mar. 2026.

COEY, J. M. D.; MAZALEYRAT, F. History of magnetism. In: *Encyclopedia of Condensed Matter Physics*. 2nd. ed. Amsterdam: Elsevier, 2024. v. 2, p. 1–17. Available at: <<https://doi.org/10.1016/B978-0-323-90800-9.00155-4>>. Accessed on: 21 Mar. 2026.

CORREIA, M. V.; VELÁSQUEZ, J. C. P. n.; SILVA, C. C. de S. Stability and dynamics of synthetic antiferromagnetic skyrmions in asymmetric multilayers. *Phys. Rev. B*, College Park, v. 110, n. 9, p. 094430, 2024. Available at: <<https://doi.org/10.1103/PhysRevB.110.094430>>. Accessed on: 21 mar. 2026.

COUZIN, I. D. Collective intelligence in animals and robots. *Nature Communications*, London, v. 16, n. 1, p. 9574, 2025. Available at: <<https://doi.org/10.1038/s41467-025-65814-9>>. Accessed on: 21 mar. 2026.

DERRICK, G. H. Comments on nonlinear wave equations as models for elementary particles. *Journal of Mathematical Physics*, New York, v. 5, n. 9, p. 1252–1254, 1964. Available at: <<https://doi.org/10.1063/1.1704233>>. Accessed on: 21 mar. 2026.

DIRAC, P. A. M. The quantum theory of the electron. *Proceedings of the Royal Society of London. Series A*, London, v. 117, n. 778, p. 610–624, 1928. Available at: <<https://doi.org/10.1098/rspa.1928.0023>>. Accessed on: 21 Mar. 2026.

DOHI, T.; DUTTAGUPTA, S.; FUKAMI, S.; OHNO, H. Formation and current-induced motion of synthetic antiferromagnetic skyrmion bubbles. *Nature Communications*, London, v. 10, n. 1, p. 5153, 2019. Available at: <<https://doi.org/10.1038/s41467-019-13182-6>>. Accessed on: 21 mar. 2026.

EBERL, D. M. Roll Cloud seen on January 25th 2009, in "Las Olas Beach" located in "Punta del Este". San Francisco: [s.n.], 2009. Available at: <<https://commons.wikimedia.org/wiki/File:Roll-cloud.JPG>>. Accessed on: 25 dez. 2025.

EINSTEIN, A.; HAAS, W. J. de. Experimental proof of the existence of ampère's molecular currents. *Proceedings of the Royal Netherlands Academy of Arts and Sciences*, Amsterdam, v. 18, p. 696–711, 1915. Available at: <<https://dwc.knaw.nl/DL/publications/PU00012546.pdf>>. Accessed on: 21 Mar. 2026.

EISBERG, R.; RESNICK, R. *Quantum physics of atoms, molecules, solids, nuclei, and particles*. 2nd. ed. New York: John Wiley & Sons, 1985.

Encyclopedia Magnetica. *Orbital spin magnetic moment*. Warsaw: [s.n.], 2023. Available at: <http://www.e-magnetica.pl/doku.php/file/orbital_spin_magnetic_moment_em_png>. Accessed on: 6 nov. 2025.

ERB, R. M.; JENNESS, N. J.; CLARK, R. L. Towards holonomic control of janus particles in optomagnetic traps. *Advanced Materials*, Hoboken, v. 21, n. 47, p. 4825–4829, 2009. Available at: <<https://doi.org/10.1002/adma.200900892>>. Accessed on: 21 Mar. 2026.

FANGOHR, H. *PhD project on High Performance Computing for Computational Micromagnetics*. 2015. Available at: <<https://fangohr.github.io/vacancies/hpc-micromagnetics.html>>. Accessed on: 20 Jan. 2026.

FARA, P. *Sympathetic Attractions: Magnetic Practices, Beliefs, and Symbolism in Eighteenth-Century England*. Princeton: Princeton University Press, 1996.

FIDLER, J.; SCHREFFL, T. Micromagnetic modelling - the current state of the art. *Journal of Physics D: Applied Physics*, Bristol, v. 33, n. 15, p. R135, 2000. Available at: <<https://doi.org/10.1088/0022-3727/33/15/201>>. Accessed on: 23 Mar. 2026.

FINOCCHIO, G.; PANAGOPOULOS, C. (Ed.). *Magnetic Skyrmions and Their Applications*. Duxford: Woodhead Publishing, 2021. Available at: <<https://www.sciencedirect.com/book/edited-volume/9780128208151/>>. Accessed on: 21 mar. 2026.

FRIEDRICH, B.; HERSCHBACH, D. Stern and gerlach: How a bad cigar helped reorient atomic physics. *Physics Today*, College Park, v. 56, n. 12, p. 53–59, 2003. Available at: <<https://doi.org/10.1063/1.1650229>>. Accessed on: 21 Mar. 2026.

GALASSI, M.; DAVIES, J.; THEILER, J.; GOUGH, B.; JUNGMAN, G.; ALKEN, P.; BOOTH, M.; ROSSI, F. *GNU Scientific Library Reference Manual*. 3rd. ed. [S.l.], 2009. Available at: <<https://www.gnu.org/software/gsl/>>. Accessed on: 23 Mar. 2026.

GARCÍA-PALACIOS, J. L.; LÁZARO, F. J. Langevin-dynamics study of the dynamical properties of small magnetic particles. *Physical Review B*, College Park, v. 58, n. 22, p. 14937, 1998. Available at: <<https://doi.org/10.1103/PhysRevB.58.14937>>. Accessed on: 23 Mar. 2026.

GARRIGA, J.; VERDAGUER, E. Cosmic strings and einstein-rosen soliton waves. *Physical Review D*, College Park, v. 36, n. 8, p. 2250–2258, 1987. Available at: <<https://doi.org/10.1103/PhysRevD.36.2250>>. Accessed on: 17 Jan. 2026.

GELL-MANN, M. A schematic model of baryons and mesons. *Physics Letters*, Amsterdam, v. 8, n. 3, p. 214–215, 1964. Available at: <[https://doi.org/10.1016/S0031-9163\(64\)92001-3](https://doi.org/10.1016/S0031-9163(64)92001-3)>. Accessed on: 21 mar. 2026.

GERLACH, W.; STERN, O. Der experimentelle nachweis der richtungsquantelung im magnetfeld. In: *Walther Gerlach (1889–1979) Eine Auswahl aus seinen Schriften und Briefen*. Berlin: Springer, 1989. p. 26–29. Available at: <<https://doi.org/10.1007/BF01326983>>. Accessed on: 21 Mar. 2026.

GIL, G. *Átimo de Pó*. Rio de Janeiro: [s.n.], 1997. Available at: <<https://gilbertogil.com.br/conteudo/musicas/?busca=atimo>>. Accessed on: 19 mar. 2025.

GILBERT, T. L. A phenomenological theory of damping in ferromagnetic materials. *IEEE Transactions on Magnetism*, New York, v. 40, n. 6, p. 3443–3449, 2004. Available at: <<https://doi.org/10.1109/TMAG.2004.836740>>. Accessed on: 21 mar. 2026.

GILBERT, W. *On the Magnet, Magnetic Bodies, and on the Great Magnet the Earth*. New York: Dover Publications, 1900. Originally published in 1600 as *De Magnete*. Available at: <[https://en.wikisource.org/wiki/Index:On_the_Magnet_-_Gilbert_\(1900_translation_of_1600_work\).djvu](https://en.wikisource.org/wiki/Index:On_the_Magnet_-_Gilbert_(1900_translation_of_1600_work).djvu)>. Accessed on: 21 Mar. 2026.

GÖBEL, B. et al. Magnetic bimerons as skyrmion analogues in in-plane magnets. *Physical Review B*, College Park, v. 99, n. 6, p. 060407, 2019. Available at: <<https://doi.org/10.1103/PhysRevB.99.060407>>. Accessed on: 21 Mar. 2026.

GOMPPER, G. et al. The 2025 motile active matter roadmap. *Journal of Physics: Condensed Matter*, Bristol, v. 37, 2025. Available at: <<https://doi.org/10.1088/1361-648X/adac98>>. Accessed on: 07 jan. 2026.

GORDON, B. Degaussing: The demagnetisation of ships. *Electronics and Power*, London, v. 30, n. 6, p. 473–476, 1984. Available at: <<https://doi.org/10.1049/ep.1984.0246>>. Accessed on: 21 Mar. 2026.

- GREEF, J. Van der. *Murmuration*. Washington, DC: [s.n.], 2015. Available at: <<https://www.smithsonianmag.com/smart-news/how-waves-rippling-through-bird-flocks-help-them-escape-predators-180954792/>>. Accessed on: 21 mar. 2026.
- GRIFFITHS, D. J. *Introduction to Electrodynamics*. 4th. ed. Boston: Pearson, 2013.
- GRIFFITHS, D. J.; SCHROETER, D. F. *Introduction to Quantum Mechanics*. 3rd. ed. Cambridge: Cambridge University Press, 2018.
- GRIGORIEV, S. V. et al. Magnetic structure of mnsi under an applied field probed by polarized small-angle neutron scattering. *Physical Review B*, College Park, v. 74, n. 21, p. 214414, 2006. Available at: <<https://doi.org/10.1103/PhysRevB.74.214414>>. Accessed on: 21 mar. 2026.
- GRÜNBERG, P. Layered magnetic structures: History, highlights, applications. *Physics Today*, Melville, v. 54, n. 5, p. 31–37, 2001. Available at: <<https://doi.org/10.1063/1.1381100>>. Accessed on: 21 Mar. 2026.
- GRÜNBERG, P. Kopplung macht den widerstand. *Physik Journal*, Weinheim, v. 6, n. 8/9, p. 33–39, 2007. Available at: <<https://pro-physik.de/zeitschriften/download/17051>>. Accessed on: 21 Mar. 2026.
- GRÜNBERG, P. et al. Layered magnetic structures: Evidence for antiferromagnetic coupling of fe layers across cr interlayers. *Physical Review Letters*, College Park, v. 57, n. 19, p. 2442–2445, 1986. Available at: <<https://doi.org/10.1103/PhysRevLett.57.2442>>. Accessed on: 21 mar. 2026.
- HASSANI, S. *Mathematical physics: a modern introduction to its foundations*. New York: Springer, 2013.
- HAYNES, W. M. (Ed.). *Handbook of Chemistry and Physics*. 91st. ed. Boca Raton: CRC Press, 2010.
- HEISENBERG, W. Zur theorie des ferromagnetismus [on the theory of ferromagnetism]. *Zeitschrift für Physik*, Berlin, v. 49, p. 619–636, 1928. Translated by D. H. Delphenich. Available at: <<https://doi.org/10.1007/BF01328601>>. Accessed on: 21 Mar. 2026.
- HIROSAWA, T.; KLINOVAJA, J.; LOSS, D.; DÍAZ, S. A. Laser-controlled real- and reciprocal-space topology in multiferroic insulators. *Physical Review Letters*, College Park, v. 128, n. 3, p. 037201, 2022. Available at: <<https://doi.org/10.1103/PhysRevLett.128.037201>>. Accessed on: 21 Mar. 2026.
- HU, X. F. et al. Discontinuous properties of current-induced magnetic domain wall depinning. *Scientific Reports*, London, v. 3, n. 1, p. 3080, 2013. Available at: <<https://doi.org/10.1038/srep03080>>. Accessed on: 21 Mar. 2026.
- ISHIKAWA, Y.; SHIRANE, G.; TARVIN, J. A.; KOHGI, M. Magnetic excitations in the weak itinerant ferromagnet mnsi. *Physical Review B*, College Park, v. 16, n. 11, p. 4956–4970, 1977. Available at: <<https://doi.org/10.1103/PhysRevB.16.4956>>. Accessed on: 21 Mar. 2026.

ISHIKAWA, Y.; TAJIMA, K.; BLOCH, D.; ROTH, M. Helical spin structure in manganese silicide mnsi. *Solid State Communications*, Amsterdam, v. 19, n. 6, p. 525–528, 1976. Available at: <[https://doi.org/10.1016/0038-1098\(76\)90057-0](https://doi.org/10.1016/0038-1098(76)90057-0)>. Accessed on: 21 Mar. 2026.

ISTOCK. *Globo envolto* [*Wrapped globe*]. Calgary: [s.n.], 2025. Available at: <<https://www.istockphoto.com/br/foto/globo-envolto-gm157328581-5959690>>. Accessed on: 26 dez. 2025.

IWASAKI, J.; MOCHIZUKI, M.; NAGAOSA, N. Universal current-velocity relation of skyrmion motion in chiral magnets. *Nature Communications*, London, v. 4, n. 1, p. 1463, 2013. Available at: <<https://doi.org/10.1038/ncomms2442>>. Accessed on: 21 mar. 2026.

JACKSON, J. D. *Classical Electrodynamics*. 3rd. ed. New York: John Wiley & Sons, 1999.

JIANG, W. et al. Direct observation of the skyrmion hall effect. *Nature Physics*, London, v. 13, n. 2, p. 162–169, 2017. Available at: <<https://doi.org/10.1038/nphys3883>>. Accessed on: 21 mar. 2026.

JILES, D. *Introduction to magnetism and magnetic materials*. 3rd. ed. Boca Raton: CRC Press, 2015. Available at: <<https://doi.org/10.1201/b18948>>. Accessed on: 21 mar. 2026.

JONIETZ, F. et al. Spin transfer torques in mnsi at ultralow current densities. *Science*, Washington, DC, v. 330, n. 6011, p. 1648–1651, 2010. Available at: <<https://doi.org/10.1126/science.1195709>>. Accessed on: 21 Mar. 2026.

JUGE, R.; SISODIA, N.; LARRAÑAGA, J. U.; ZHANG, Q.; PHAM, V. T.; RANA, K. G.; SARPI, B.; MILLE, N.; STANESCU, S.; BELKHOU, R.; MAWASS, M.-A.; NOVAKOVIC-MARINKOVIC, N.; KRONAST, F.; WEIGAND, M.; GRÄFE, J.; WINTZ, S.; FINIZIO, S.; RAABE, J.; ABALLE, L.; FOERSTER, M.; BELMEGUENAI, M.; BUDA-PREJBEANU, L. D.; PELLOUX-PRAYER, J.; SHAW, J. M.; NEMBACH, H. T. Skyrmions in synthetic antiferromagnets and their nucleation via electrical current and ultra-fast laser illumination. *Nature Communications*, v. 13, p. 4807, 2022. Available at: <<https://doi.org/10.1038/s41467-022-32525-4>>. Accessed on: 24 Mar. 2026.

KAPLAN, I. G. Pauli exclusion principle and its theoretical foundation. *arXiv:1902.00499*, Ithaca, 2019. Available at: <<https://arxiv.org/abs/1902.00499>>. Accessed on: 21 Mar. 2026.

KASMAN, A. A brief history of solitons and the KdV equation. *Current Science*, Indian Academy of Sciences, v. 115, n. 8, p. 1487–1496, 2018. Available at: <<https://doi.org/10.18520/cs/v115/i8/1486-1496>>. Accessed on: 21 mar. 2026.

KÉZSMÁRKI, I. et al. Néel-type skyrmion lattice with confined orientation in the polar magnetic semiconductor GaV₄S₈. *Nature Materials*, London, v. 14, n. 11, p. 1116–1122, 2015. Available at: <<https://doi.org/10.1038/nmat4402>>. Accessed on: 21 Mar. 2026.

KIM, J.-V.; GARCIA-SANCHEZ, F.; SAMPAIO, J.; MOREAU-LUCHAIRE, C.; CROS, V.; FERT, A. Breathing modes of confined skyrmions in ultrathin magnetic dots. *Physical Review B*, College Park, v. 90, n. 6, p. 064410, 2014. Available at: <<https://doi.org/10.1103/PhysRevB.90.064410>>. Accessed on: 21 mar. 2026.

KINGDON, A. *The South Pointing Spoon*. 2019. Hakai Magazine. Available at: <<https://hakaimagazine.com/article-short/the-south-pointing-spoon/>>. Accessed on: 19 Jan. 2026.

KITTEL, C. *Introduction to Solid State Physics*. 9th. ed. Hoboken, NJ: Wiley, 2018.

KOSHIBAE, W.; NAGAOSA, N. Theory of skyrmions in bilayer systems. *Scientific Reports*, London, v. 7, n. 1, p. 42645, 2017. Available at: <<https://doi.org/10.1038/srep42645>>. Accessed on: 21 mar. 2026.

LANCASTER, T. Skyrmions in magnetic materials. *Contemporary Physics*, London, v. 60, n. 3, p. 246–261, 2019. Available at: <<https://doi.org/10.1080/00107514.2019.1699352>>. Accessed on: 21 mar. 2026.

LANDAU, L. D.; LIFSHITZ, E. M. *Electrodynamics of Continuous Media*. 1st. ed. Oxford: Pergamon Press, 1960. (Course of Theoretical Physics, v. 8). Translated by J. B. Sykes and J. S. Bell.

LANDAU, L. D.; LIFSHITZ, E. M. On the theory of the dispersion of magnetic permeability in ferromagnetic bodies. In: HAAR, D. T. (Ed.). *Collected Papers of L. D. Landau*. Oxford: Pergamon Press, 1965. p. 101–114. Available at: <<https://doi.org/10.1016/B978-0-08-036364-6.50008-9>>. Accessed on: 21 mar. 2026.

LANDAU, L. D.; LIFSHITZ, E. M. *Fluid Mechanics*. 2nd. ed. Oxford: Pergamon Press, 1987. (Course of Theoretical Physics, v. 6).

LANGEVIN, P. Sur la théorie du magnétisme. *Journal de Physique Théorique et Appliquée*, Paris, v. 4, n. 1, p. 678–693, 1905. Available at: <<https://doi.org/10.1051/jphysap:019050040067800>>. Accessed on: 21 Mar. 2026.

LEGRAND, W. et al. Room-temperature stabilization of antiferromagnetic skyrmions in synthetic antiferromagnets. *Nature Materials*, London, v. 19, n. 1, p. 34–42, 2020. Available at: <<https://doi.org/10.1038/s41563-019-0468-3>>. Accessed on: 21 mar. 2026.

LITZIUS, K.; LEMESH, I.; KRÜGER, B.; BASSIRIAN, P.; CARETTA, L.; RICHTER, K.; BÜTTNER, F.; SATO, K.; TRETIKOV, O. A.; FÖRSTER, J.; REEVE, R. M.; WEIGAND, M.; BYKOVA, I.; STOLL, H.; SCHÜTZ, G.; BEACH, G. S. D.; KLÄUI, M. Skyrmion hall effect revealed by direct time-resolved x-ray microscopy. *Nature Physics*, v. 13, n. 2, p. 170–175, 2017. Available at: <<https://doi.org/10.1038/nphys4000>>. Accessed on: 24 Mar. 2026.

LIU, L.; CHEN, W.; ZHENG, Y. Emergent mechanics of magnetic skyrmions deformed by defects. *Physical Review Letters*, College Park, v. 131, n. 24, p. 246701, 2023. Available at: <<https://doi.org/10.1103/PhysRevLett.131.246701>>. Accessed on: 21 mar. 2026.

LIU, L.; SUN, F.; REN, J.; CHEN, W.; ZHENG, Y. Flexoresponses of synthetic antiferromagnetic systems hosting skyrmions. *Physical Review Letters*, College Park, v. 128, n. 25, p. 257201, 2022. Available at: <<https://doi.org/10.1103/PhysRevLett.128.257201>>. Accessed on: 21 Mar. 2026.

LIU, Z. T.; SHI, Y.; ZHAO, Y.; CHATÉ, H.; SHI, X.-q.; ZHANG, T. H. Activity waves and freestanding vortices in populations of subcritical quincke rollers. *Proceedings of the National Academy of Sciences*, Washington, DC, v. 118, n. 40, p. e2104724118, 2021. Available at: <<https://doi.org/10.1073/pnas.2104724118>>. Accessed on: 21 mar. 2026.

LONSKY, M.; HOFFMANN, A. Coupled skyrmion breathing modes in synthetic ferri- and antiferromagnets. *Physical Review B*, v. 102, n. 10, p. 104403, 2020. Available at: <<https://doi.org/10.1103/PhysRevB.102.104403>>. Accessed on: 24 Mar. 2026.

LYONS, D. W. 1.3: *Stereographic Projection*. Davis: [s.n.], 2021. Available at: <https://math.libretexts.org/Bookshelves/Abstract_and_Geometric_Algebra/Introduction_to_Groups_and_Geometries_%28Lyons%29/01%3A_Preliminaries/1.03%3A_Stereographic_projection>. Accessed on: 26 dez. 2025.

MA, M. et al. Optical control of RKKY coupling and perpendicular magnetic anisotropy in a synthetic antiferromagnet. *Nature Communications*, London, v. 16, n. 1, p. 4401, 2025. Available at: <<https://doi.org/10.1038/s41467-025-59689-z>>. Accessed on: 21 Mar. 2026.

MAKAROV, A. *Modeling of Emerging Resistive Switching Based Memory Cells*. Phd Thesis (PhD Thesis) — Technische Universität Wien, Vienna, 2014. Available at: <<https://repositum.tuwien.at/handle/20.500.12708/20326>>. Accessed on: 20 Jan. 2026.

MANH, T. L.; BENITEZ, J. A. P.; HERNÁNDEZ, J. H. E.; LÓPEZ, J. M. H. Correlation between barkhausen noise and magnetocrystalline anisotropy energy. In: *Barkhausen Noise for Nondestructive Testing and Materials Characterization in Low-Carbon Steels*. Amsterdam: Elsevier, 2020. p. 147–182. Available at: <<https://doi.org/10.1016/B978-0-08-102800-1.00006-X>>. Accessed on: 21 Mar. 2026.

MANTON, N. S. *Skyrmions: A Theory of Nuclei*. Singapore: World Scientific, 2022. Available at: <<https://doi.org/10.1142/q0368>>. Accessed on: 21 Mar. 2026.

MARUYAMA, T.; SHIOTA, Y.; NOZAKI, T.; OHTA, K.; TODA, N.; MIZUGUCHI, M.; TULAPURKAR, A. A.; SHINJO, T.; SHIRAISHI, M.; MIZUKAMI, S.; ANDO, Y.; SUZUKI, Y. Large voltage-induced magnetic anisotropy change in a few atomic layers of iron. *Nature Nanotechnology*, London, v. 4, n. 3, p. 158–161, 2009. Available at: <<https://doi.org/10.1038/nnano.2008.406>>. Accessed on: 21 mar. 2026.

MCCORD, J. Progress in magnetic domain observation by advanced magneto-optical microscopy. *Journal of Physics D: Applied Physics*, Bristol, v. 48, n. 33, p. 333001, 2015. Available at: <<https://doi.org/10.1088/0022-3727/48/33/333001>>. Accessed on: 21 Mar. 2026.

MCMULLIN, E. The origins of the field concept in physics. *Physics in Perspective*, Berlin, v. 4, p. 13–39, 2002. Available at: <<https://doi.org/10.1007/s00016-002-8357-5>>. Accessed on: 21 mar. 2026.

MELFOS, V.; HELLY, B.; VOUDOURIS, P. The ancient greek names “magnesia” and “magnetes” and their origin from the magnetite occurrences at the mavrovouni mountain of thessaly, central greece. *Archaeological and Anthropological Sciences*, Berlin, v. 3, p. 165–172, 2011. Available at: <<https://doi.org/10.1007/s12520-010-0048-6>>. Accessed on: 21 mar. 2026.

MISHRA, K. K. et al. Magnetic skyrmion: from fundamental physics to pioneering applications. *Applied Physics Reviews*, Melville, v. 12, n. 1, p. 011315, 2025. Available at: <<https://doi.org/10.1063/5.0223004>>. Accessed on: 21 Mar. 2026.

MITCHELL, J. G.; PEARSON, L.; DILLON, S. Variability in motility characteristics among marine bacteria. *Archives of Microbiology*, Berlin, v. 163, n. 5, p. 379–387, 1995. Available at: <<https://doi.org/10.3354/ame028229>>. Accessed on: 21 mar. 2026.

MIYAWAKI, J. et al. Dzyaloshinskii-moriya interaction in α - Fe_2O_3 measured by magnetic circular dichroism in resonant inelastic soft x-ray scattering. *Physical Review B*, College Park, v. 96, n. 21, p. 214420, 2017. Available at: <<https://doi.org/10.1103/PhysRevB.96.214420>>. Accessed on: 21 Mar. 2026.

MOON, K.-W. et al. Skyrmion motion driven by oscillating magnetic field. *Scientific Reports*, London, v. 6, n. 1, p. 20360, 2016. Available at: <<https://doi.org/10.1038/srep20360>>. Accessed on: 21 Mar. 2026.

MOORE, G. E. Cramming more components onto integrated circuits. *Electronics*, New York, v. 38, n. 8, p. 114–117, 1965. Available at: <<https://doi.org/10.1109/N-SSC.2006.4785860>>. Accessed on: 21 Mar. 2026.

MORAN, J. L.; POSNER, J. D. Active colloids as models, materials, and machines. *Annual Review of Chemical and Biomolecular Engineering*, Palo Alto, v. 11, n. 1, p. 269–290, 2020. Available at: <<https://doi.org/10.1146/annurev-chembioeng-101121-084939>>. Accessed on: 07 jan. 2026.

MORIYA, T. Anisotropic superexchange interaction and weak ferromagnetism. *Physical Review*, College Park, v. 120, n. 1, p. 91–99, 1960. Available at: <<https://doi.org/10.1103/PhysRev.120.91>>. Accessed on: 21 Mar. 2026.

MÜHLBAUER, S. et al. Skyrmion lattice in a chiral magnet. *Science*, Washington, DC, v. 323, n. 5916, p. 915–919, 2009. Available at: <<https://doi.org/10.1126/science.1166767>>. Accessed on: 21 Mar. 2026.

NAGAOSA, N.; TOKURA, Y. Topological properties and dynamics of magnetic skyrmions. *Nature Nanotechnology*, London, v. 8, n. 12, p. 899–911, 2013. Available at: <<https://doi.org/10.1038/nnano.2013.243>>. Accessed on: 21 Mar. 2026.

NAMBA, K.; VONDERVISZT, F. Molecular architecture of bacterial flagellum. *Quarterly Reviews of Biophysics*, Cambridge, v. 30, n. 1, p. 1–65, 1997. Available at: <<https://doi.org/10.1017/S0033583596003319>>. Accessed on: 21 mar. 2026.

National Oceanic and Atmospheric Administration. *Morning glory cloud in the Gulf of Carpentaria, Northern Australia*. Washington, DC: [s.n.], 1998. Available at: <https://commons.wikimedia.org/wiki/File:Morningglory_satellite_picture.gif>. Accessed on: 25 dez. 2025.

NEEDHAM, J. *Science and Civilisation in China*. Cambridge: Cambridge University Press, 1962. Cited in: MALMSTROM, V. H. Lodestone compass: Chinese or Olmec primacy. *Science*, Washington, DC, v. 189, n. 4205, p. 753–760, 1975.

NÉEL, L. Magnetism and local molecular field. *Science*, Washington, DC, v. 174, n. 4013, p. 985–992, 1971. Available at: <<https://doi.org/10.1126/science.174.4013.985>>. Accessed on: 21 Mar. 2026.

NEPAL, R.; GÜNGÖRDÜ, U.; KOVALEV, A. A. Magnetic skyrmion bubble motion driven by surface acoustic waves. *Applied Physics Letters*, Melville, v. 112, n. 11, p. 112404, 2018. Available at: <<https://doi.org/10.1063/1.5013620>>. Accessed on: 21 Mar. 2026.

OHKI, Y.; MOCHIZUKI, M. Fundamental theory of current-induced motion of magnetic skyrmions. *Journal of Physics: Condensed Matter*, Bristol, v. 37, n. 2, p. 023003, 2025. Available at: <<https://doi.org/10.1088/1361-648X/ad861b>>. Accessed on: 21 Mar. 2026.

PARKIN, S. S. P. The magic of magnetic multilayers. *IBM Journal of Research and Development*, Armonk, v. 42, n. 1, p. 3–6, 1998. Available at: <<https://doi.org/10.1147/rd.421.0003>>. Accessed on: 21 Mar. 2026.

PARKIN, S. S. P.; MORE, N.; ROCHE, K. P. Oscillations in exchange coupling and magnetoresistance in metallic superlattice structures: Co/ru, co/cr, and fe/cr. *Physical Review Letters*, College Park, v. 64, n. 19, p. 2304–2307, 1990. Available at: <<https://doi.org/10.1103/PhysRevLett.64.2304>>. Accessed on: 21 mar. 2026.

PATHRIA, R. K.; BEALE, P. D. *Statistical Mechanics*. 3rd. ed. Boston: Academic Press, 2011.

PAULI, W. Über den zusammenhang des abschlusses der elektronengruppen im atom mit der komplexstruktur der spektren. *Zeitschrift für Physik*, Berlin, v. 31, n. 1, p. 765–783, 1925. Available at: <<https://doi.org/10.1007/BF02980577>>. Accessed on: 21 Mar. 2026.

PELITI, L.; PIGOLOTTI, S. *Stochastic Thermodynamics: An Introduction*. Princeton: Princeton University Press, 2021. Available at: <<https://press.princeton.edu/books/hardcover/9780691201771/stochastic-thermodynamics>>. Accessed on: 07 jan. 2026.

PEREGRINUS, P. *Epistola de magnete*. London: The Electrician Printing and Publishing Company, 1904. Available at: <<https://archive.org/details/letterofpetrusp00pieriala>>. Accessed on: 21 mar. 2026.

PHAM, V. T. et al. Fast current-induced skyrmion motion in synthetic antiferromagnets. *Science*, Washington, DC, v. 384, n. 6693, p. 307–312, 2024. Available at: <<https://doi.org/10.1126/science.add5751>>. Accessed on: 21 mar. 2026.

PHAM, V. T.; SISODIA, N.; MANICI, I. D.; URRESTARAZU-LARRAÑAGA, J.; BAIRAGI, K. et al. Fast current-induced skyrmion motion in synthetic antiferromagnets. *Science*, v. 384, n. 6693, p. 307–312, 2024. Available at: <<https://doi.org/10.1126/science.add5751>>. Accessed on: 24 Mar. 2026.

PURCELL, E. M. Life at low reynolds number. *American Journal of Physics*, College Park, v. 45, n. 1, p. 3–11, 1977. Available at: <<https://doi.org/10.1119/1.10903>>. Accessed on: 21 mar. 2026.

QIN, G. et al. Dynamics of magnetic skyrmions driven by a temperature gradient in a chiral magnet fege. *Physical Review B*, College Park, v. 106, n. 2, p. 024415, 2022. Available at: <<https://doi.org/10.1103/PhysRevB.106.024415>>. Accessed on: 21 Mar. 2026.

- QIU, L.; SHEN, L.; ZHANG, X.; ZHOU, Y.; ZHAO, G.; XIA, W.; LUO, H.-B.; LIU, J. P. Interlayer coupling effect on skyrmion dynamics in synthetic antiferromagnets. *Applied Physics Letters*, Melville, v. 118, n. 8, p. 082403, 2021. Available at: <<https://doi.org/10.1063/5.0039470>>. Accessed on: 23 Mar. 2026.
- QIU, T. et al. Swimming by reciprocal motion at low reynolds number. *Nature Communications*, London, v. 5, n. 1, p. 5119, 2014. Available at: <<https://doi.org/10.1038/ncomms6119>>. Accessed on: 21 mar. 2026.
- RANDALL, J. T.; BOOT, H. A. H. The cavity magnetron. *Journal of the Institution of Electrical Engineers-Part IIIA: Radiolocation*, London, v. 93, n. 5, p. 928–938, 1946. Available at: <<https://doi.org/10.1049/ji-3a-1.1946.0169>>. Accessed on: 21 Mar. 2026.
- RICHARDSON, O. W. A mechanical effect accompanying magnetization. *Physical Review (Series I)*, College Park, v. 26, n. 3, p. 248–253, 1908. Available at: <<https://doi.org/10.1103/PhysRevSeriesI.26.248>>. Accessed on: 21 Mar. 2026.
- ROBERTSON, M. A.; EFREMOV, F.; PAIK, J. Roboscallop: A bivalve inspired swimming robot. *IEEE Robotics and Automation Letters*, New York, v. 4, n. 2, p. 2078–2085, 2024. Available at: <<https://doi.org/10.1109/LRA.2019.2893165>>. Accessed on: 21 mar. 2026.
- RODRIGUES, D. R.; TOMASELLO, R.; SIRACUSANO, G.; CARPENTIERI, M.; FINOCCHIO, G. Ultra-sensitive voltage-controlled skyrmion-based spintronic diode. *Nanotechnology*, Bristol, v. 34, n. 37, p. 375202, 2023. Available at: <<https://doi.org/10.1088/1361-6528/acdad6>>. Accessed on: 23 Mar. 2026.
- ROESSLER, U. K.; BOGDANOV, A. N.; PFLEIDERER, C. Spontaneous skyrmion ground states in magnetic metals. *Nature*, London, v. 442, n. 7104, p. 797–801, 2006. Available at: <<https://doi.org/10.1038/nature05056>>. Accessed on: 21 mar. 2026.
- RUSSELL, J. S. Report on waves. *Report of the Fourteenth Meeting of the British Association for the Advancement of Science*, London, p. 311–390, 1844. Available at: <<https://www.macs.hw.ac.uk/~chris/Scott-Russell/SR44.pdf>>. Accessed on: 21 Mar. 2026.
- SAMPAIO, J.; CROS, V.; ROHART, S.; THIAVILLE, A.; FERT, A. Nucleation, stability and current-induced motion of isolated magnetic skyrmions in nanostructures. *Nature Nanotechnology*, London, v. 8, n. 11, p. 839–844, 2013. Available at: <<https://doi.org/10.1038/nnano.2013.210>>. Accessed on: 24 Mar. 2026.
- SATTLER, K. D. (Ed.). *21st Century Nanoscience – A Handbook: Exotic Nanostructures and Quantum Systems (Volume Five)*. Boca Raton: CRC Press, 2020. Available at: <<https://doi.org/10.1201/9780429347313>>. Accessed on: 21 Mar. 2026.
- SCHÖNKE, J.; FRIED, E. Stability of vertical magnetic chains. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, London, v. 473, n. 2198, p. 20160703, 2017. Available at: <<https://doi.org/10.1098/rspa.2016.0703>>. Accessed on: 21 Mar. 2026.
- SHANKAR, S. et al. Topological active matter. *Reviews of Modern Physics*, College Park, v. 94, n. 3, p. 035004, 2022. Available at: <<https://doi.org/10.1038/s42254-022-00445-3>>. Accessed on: 21 mar. 2026.

SHEN, C.; JIANG, Z.; LI, L.; GILCHRIST, J. F.; OU-YANG, H. D. Frequency response of induced-charge electrophoretic metallic janus particles. *Micromachines*, Basel, v. 11, n. 3, 2020. Available at: <<https://doi.org/10.3390/mi11030334>>. Accessed on: 21 mar. 2026.

SHEN, Y. Topological bimeronic beams. *Optics Letters*, Washington, DC, v. 46, n. 15, p. 3737–3740, 2021. Available at: <<https://doi.org/10.1364/OL.431122>>. Accessed on: 27 dez. 2025.

SILVA, C. C. de S.; CORREIA, M. V.; VELÁSQUEZ, J. C. P. Emergent self-propulsion of skyrmionic matter in synthetic antiferromagnets. *Physical Review Letters*, College Park, v. 135, n. 8, p. 086701, 2025. Available at: <<https://doi.org/10.1103/c2y9-3cc9>>. Accessed on: 21 mar. 2026.

SKYRME, T. H. R. A non-linear field theory. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, London, v. 260, n. 1300, p. 127–138, 1961. Available at: <<https://doi.org/10.1098/rspa.1961.0018>>. Accessed on: 21 mar. 2026.

SMYTH, H. D. W. *Atomic Energy for Military Purposes: The Official Report on the Development of the Atomic Bomb under the Auspices of the United States Government, 1940-1945*. Princeton: Princeton University Press, 1945. Available at: <https://www.osti.gov/opennet/manhattan-project-history/publications/smyth_report.pdf>. Accessed on: 21 Mar. 2026.

SOGARI, P. *Marble herm depicting Janus Bifrons*. Reggio Emilia: [s.n.], 1576. Palazzo Magnani. Available at: <<https://www.palazzomagnani.it/sedi/palazzo-magnani/>>. Accessed on: 14 jan. 2026.

SON, K.; GUASTO, J. S.; STOCKER, R. Bacteria can exploit a flagellar buckling instability to change direction. *Nature Physics*, London, v. 9, n. 8, p. 494–498, 2013. Available at: <<https://doi.org/10.1038/nphys2676>>. Accessed on: 21 mar. 2026.

SONG, C. et al. Skyrmion-based multi-channel racetrack. *Applied Physics Letters*, Melville, v. 111, n. 19, p. 192413, 2017. Available at: <<https://doi.org/10.1063/1.4994093>>. Accessed on: 21 Mar. 2026.

TAKEUCHI, A.; MIZUSHIMA, S.; MOCHIZUKI, M. Electrically driven spin torque and dynamical dzyaloshinskii-moriya interaction in magnetic bilayer systems. *Scientific Reports*, London, v. 9, n. 1, p. 9528, 2019. Available at: <<https://doi.org/10.1038/s41598-019-46009-x>>. Accessed on: 23 Mar. 2026.

TANG, J.; WU, Y.; WANG, W.; KONG, L.; LV, B.; WEI, W.; ZANG, J.; TIAN, M.; DU, H. Magnetic skyrmion bundles and their current-driven dynamics. *Nature Nanotechnology*, London, v. 16, n. 10, p. 1086–1091, 2021. Available at: <<https://doi.org/10.1038/s41565-021-00954-9>>. Accessed on: 27 dez. 2025.

Technische Universität München. *Skyrmion Lattice in a chiral Magnet*. 2009. Available at: <<https://www.tum.de/en/news-and-events/all-news/press-releases/details/31094>>. Accessed on: 23 Dec. 2025.

Technische Universität Wien. *Soliton on the Scott Russell Aqueduct on the Union Canal near Heriot-Watt University*, 12 July 1995. Viena: [s.n.], 1995. Available at:

<<https://www.tuwien.at/en/phy/ati/nuclear-and-particle-physics/research/fundamental-interactions/solitons>>. Accessed on: 25 dez. 2025.

TED-Ed. *The physics of football in TED-Ed GIFs*. 2016. Available at: <<https://ed.ted.com/blog/2016/09/09/the-physics-of-football-in-ted-ed-gifs>>. Accessed on: 19 Jan. 2026.

TF1. *France vs. Brazil: Tournoi de France*. Lyon: [s.n.], 1997. Television broadcast of the match held on June 3, 1997. Featuring Roberto Carlos's free kick goal. Available at: <<https://www.youtube.com/watch?v=WLgMRyRjVrI>>. Accessed on: 19 Jan. 2026.

THIELE, A. A. Steady-state motion of magnetic domains. *Physical Review Letters*, College Park, v. 30, n. 6, p. 230–233, 1973. Available at: <<https://doi.org/10.1103/PhysRevLett.30.230>>. Accessed on: 21 Mar. 2026.

THOMSON, G. P. *Set of twenty copies of original electron diffraction photographs*. Aberdeen: [s.n.], 1927. Science Museum Group Collection Online. Available at: <<https://collection.sciencemuseumgroup.org.uk/objects/co5292/set-of-twenty-copies-of-original-electron-diffraction-photographs>>. Accessed on: 14 Nov. 2025.

THORNTON, S. T.; MARION, J. B. *Classical Dynamics of Particles and Systems*. 5th. ed. Pacific Grove, CA: Brooks/Cole, 2003.

TOKURA, Y.; KANAZAWA, N. Magnetic skyrmion materials. *Chemical Reviews*, Washington, DC, v. 121, n. 5, p. 2857–2897, 2020. Available at: <<https://doi.org/10.1021/acs.chemrev.0c00297>>. Accessed on: 21 Mar. 2026.

TOSCANO, D. et al. Suppression of the skyrmion hall effect in planar nanomagnets by the magnetic properties engineering: Skyrmion transport on nanotracks with magnetic strips. *Journal of Magnetism and Magnetic Materials*, Amsterdam, v. 504, p. 166655, 2020. Available at: <<https://doi.org/10.1016/j.jmmm.2020.166655>>. Accessed on: 21 Mar. 2026.

TRETIKOV, O. A.; CLARKE, D.; CHERN, G.-W.; BAZALIY, Y. B.; TCHERNYSHYOV, O. Dynamics of domain walls in magnetic nanostrips. *Physical Review Letters*, College Park, v. 100, n. 12, p. 127204, 2008. Available at: <<https://doi.org/10.1103/PhysRevLett.100.127204>>. Accessed on: 21 mar. 2026.

UPADHYAYA, P.; YU, G.; AMIRI, P. K.; WANG, K. L. Electric-field guiding of magnetic skyrmions. *Physical Review B*, College Park, v. 92, n. 13, p. 134411, 2015. Available at: <<https://doi.org/10.1103/PhysRevB.92.134411>>. Accessed on: 21 mar. 2026.

VANSTEENKISTE, A.; LELIAERT, J.; DVORNIK, M.; HELSEN, M.; GARCIA-SANCHEZ, F.; WAEYENBERGE, B. V. The design and verification of mumax3. *AIP Advances*, Melville, v. 4, n. 10, p. 107133, 2014. Available at: <<https://doi.org/10.1063/1.4899186>>. Accessed on: 23 Mar. 2026.

VICSEK, T.; ZAFEIRIS, A. Collective motion. *Physics Reports*, Amsterdam, v. 517, n. 3-4, p. 71–140, 2012. Available at: <<https://doi.org/10.1016/j.physrep.2012.03.004>>. Accessed on: 21 mar. 2026.

VOLPE, G.; BECHINGER, C.; CICHOS, F.; GOLESTANIAN, R.; LÖWEN, H.; SPERL, M.; VOLPE, G. Active matter in space. *npj Microgravity*, London, v. 8, n. 1, p. 54, 2022. Available at: <<https://doi.org/10.1038/s41526-022-00230-7>>. Accessed on: 21 mar. 2026.

VOLPE, G.; GIGAN, S.; VOLPE, G. Simulation of the active brownian motion of a microswimmer. *American Journal of Physics*, College Park, v. 82, n. 7, p. 659–664, 2014. Available at: <<https://doi.org/10.1119/1.4870398>>. Accessed on: 21 mar. 2026.

WALDROP, M. M. The chips are down for Moore's law. *Nature*, London, v. 530, n. 7589, p. 144–147, 2016. Available at: <<https://doi.org/10.1038/530144a>>. Accessed on: 21 Mar. 2026.

Walmart Canada. *Kraft Letter Paper Light Brown Rectangular Vintage Writing 50 Sheets 8.3x11.4in*. Mississauga: [s.n.], 2026. Available at: <<https://www.walmart.ca/en/ip/Kraft-Letter-Paper-Light-Brown-Rectangular-Vintage-Writing-50-Sheets-8-3x11-4in/5PTTXHGPOZGO>>. Accessed on: 26 dez. 2025.

WANG, J.; WANG, G.; CHEN, H.; LIU, Y.; WANG, P.; YUAN, D.; MA, X.; XU, X.; CHENG, Z.; JI, B.; YANG, M.; SHUAI, J.; YE, F.; WANG, J.; JIAO, Y.; LIU, L. Robo-matter towards reconfigurable multifunctional smart materials. *Nature Communications*, London, v. 15, n. 1, p. 8853, 2024. Available at: <<https://doi.org/10.1038/s41467-024-53123-6>>. Accessed on: 21 mar. 2026.

WANG, W.; BEG, M.; ZHANG, B.; KUCH, W.; FANGOHR, H. Driving magnetic skyrmions with microwave fields. *Physical Review B*, College Park, v. 92, n. 2, p. 020403, 2015. Available at: <<https://doi.org/10.1103/PhysRevB.92.020403>>. Accessed on: 21 mar. 2026.

WANG, X. S.; YUAN, H. Y.; WANG, X. R. A theory on skyrmion size. *Communications Physics*, London, v. 1, n. 1, p. 31, 2018. Available at: <<https://doi.org/10.1038/s42005-018-0029-0>>. Accessed on: 21 mar. 2026.

Weinrich Minerals, Inc. *Hematite (specimen 4151358)*. Grover: [s.n.], 2025. Available at: <<https://www.weinrichmineralsinc.com/products/hematite-4151358.php>>. Accessed on: 21 mar. 2026.

WHITTAKER, E. T. *A History of the Theories of Aether and Electricity, Volume 1: The Classical Theories*. Revised. London: Thomas Nelson & Sons, 1951. Available at: <<https://archive.org/details/historyoftheorie00whitrich>>. Accessed on: 21 Mar. 2026.

Wikimedia Commons. *Seagate ST33232A hard disk head and platters detail*. 2005. Available at: <https://en.wikipedia.org/wiki/Disk_read-and-write_head#/media/File:Seagate_ST33232A_hard_disk_head_and_platters_detail.jpg>. Accessed on: 21 Mar. 2026.

Wikimedia Commons. *Superexchange in MnO*. San Francisco: [s.n.], 2015. Available at: <https://commons.wikimedia.org/wiki/File:Superexchange_in_MnO.jpg>. Accessed on: 18 dez. 2025.

Wikimedia Commons. *Time reversible flow demonstration in a Taylor-Couette system*. 2018. Available at: <https://commons.wikimedia.org/wiki/File:Time_reversible_flow_demonstration_in_a_Taylor-Couette_system.png>. Accessed on: 14 jan. 2026.

WOO, S.; LITZIUS, K.; KRÜGER, B.; IM, M.-Y.; CARETTA, L.; RICHTER, K.; MANN, M.; KRONE, A.; REEVE, R. M.; WEIGAND, M.; AGRAWAL, P.; LEMESH, I.; MAWASS, M.-A.; FISCHER, P.; KLÄUI, M.; BEACH, G. S. D. Observation of room-temperature magnetic skyrmions and their current-driven dynamics in ultrathin metallic ferromagnets. *Nature Materials*, v. 15, p. 501–506, 2016. Available at: <<https://doi.org/10.1038/nmat4593>>. Accessed on: 24 Mar. 2026.

XIE, L.; ALTINDAL, T.; CHATTOPADHYAY, S.; WU, X.-L. Reverse and flick: Hybrid locomotion in bacteria. *Proceedings of the National Academy of Sciences*, Washington, DC, v. 108, n. 6, p. 2246–2251, 2011. Available at: <<https://doi.org/10.1073/pnas.1019199108>>. Accessed on: 21 mar. 2026.

YANG, L.; ZHANG, Y.; WANG, Q.; WONG, P.-W.; LI, T. Micro/nanorobots for precise drug delivery via targeted transport and triggered release: A review. *International Journal of Pharmaceutics*, Amsterdam, v. 616, p. 121551, 2022. Available at: <<https://doi.org/10.1016/j.ijpharm.2022.121551>>. Accessed on: 07 jan. 2026.

YOSIDA, K. *Theory of Magnetism*. Berlin: Springer, 1996. Available at: <<https://d-nb.info/94686523X/04>>. Accessed on: 21 mar. 2026.

YU, X.; MORIKAWA, D.; TOKUNAGA, Y.; KUBOTA, M.; KURUMAJI, T.; OIKE, H.; NAKAMURA, M.; KAGAWA, F.; TAGUCHI, Y.; ARIMA, T.-h.; KAWASAKI, M.; TOKURA, Y. Current-induced nucleation and annihilation of magnetic skyrmions at room temperature in a chiral magnet. *Advanced Materials*, v. 29, n. 21, p. 1606178, 2017. Available at: <<https://doi.org/10.1002/adma.201606178>>. Accessed on: 24 Mar. 2026.

YUAN, H. Y.; WANG, X. S.; YUNG, M.-H.; WANG, X. R. Wiggling skyrmion propagation under parametric pumping. *Physical Review B*, College Park, v. 99, n. 1, p. 014428, 2019. Available at: <<https://doi.org/10.1103/PhysRevB.99.014428>>. Accessed on: 21 mar. 2026.

ZABUSKY, N. J.; KRUSKAL, M. D. Interaction of "solitons" in a collisionless plasma and the recurrence of initial states. *Phys. Rev. Lett.*, College Park, v. 15, n. 6, p. 240–243, 1965. Available at: <<https://doi.org/10.1103/PhysRevLett.15.240>>. Accessed on: 21 mar. 2026.

ZAHED, I.; BROWN, G. E. The skyrme model. *Physics Reports*, Amsterdam, v. 142, n. 1, p. 1–102, 1986. Available at: <[https://doi.org/10.1016/0370-1573\(86\)90142-0](https://doi.org/10.1016/0370-1573(86)90142-0)>. Accessed on: 21 mar. 2026.

ZHANG, J.; ZHANG, Y.; GAO, Y.; ZHAO, G.; QIU, L.; WANG, K.; DOU, P.; PENG, W.; ZHUANG, Y.; WU, Y. et al. Magnetic skyrmions in a hall balance with interfacial canted magnetizations. *Advanced Materials*, Montreal, v. 32, n. 38, p. 1907452, 2020. Available at: <<https://doi.org/10.1002/adma.201907452>>. Accessed on: 23 Mar. 2026.

ZHANG, L. et al. Collective behaviors of magnetic active matter: Recent progress toward reconfigurable, adaptive, and multifunctional swarming micro/nanorobots. *Accounts of Chemical Research*, Washington, DC, v. 54, n. 15, p. 3003–3014, 2021. Available at: <<https://doi.org/10.1021/acs.accounts.1c00619>>. Accessed on: 07 jan. 2026.

ZHANG, S.; LI, Z. Roles of nonequilibrium conduction electrons on the magnetization dynamics of ferromagnets. *Physical Review Letters*, College Park, v. 93, n. 12, p. 127204, 2004. Available at: <<https://doi.org/10.1103/PhysRevLett.93.127204>>. Accessed on: 21 mar. 2026.

ZHANG, X.; MOCHIZUKI, M. Skyrmions as active matter. *Physics*, College Park, v. 18, p. 146, 2025. Available at: <<https://physics.aps.org/articles/v18/146>>. Accessed on: 21 Mar. 2026.

ZHANG, X.; ZHOU, Y.; EZAWA, M. Magnetic bilayer-skyrmions without skyrmion hall effect. *Nature Communications*, London, v. 7, n. 1, p. 10293, 2016. Available at: <<https://doi.org/10.1038/ncomms10293>>. Accessed on: 21 Mar. 2026.

ZHU, D. et al. Skyrmion racetrack memory with random information update/deletion/insertion. *IEEE Transactions on Electron Devices*, New York, v. 65, n. 1, p. 87–95, 2018. Available at: <<https://doi.org/10.1109/TED.2017.2769672>>. Accessed on: 21 Mar. 2026.

ZWEIG, G. *An $SU(3)$ model for strong interaction symmetry and its breaking*. Geneva, 1964. 24 p. Version I. Available at: <<https://http://dx.doi.org/10.17181/CERN-TH-401>>. Accessed on: 21 mar. 2026.

APPENDIX A – LIST OF PUBLICATIONS

Publications related to the thesis are indicated by asterisks (*).

1. *SILVA, Clécio C.S.; CORREIA, Matheus V.; VELÁSQUEZ, J.C. Piña. Emergent self-propulsion of skyrmionic matter in synthetic antiferromagnets. **Physical Review Letters**, v. 135, n. 8, p. 086701, 2025.
2. *CORREIA, Matheus V.; VELÁSQUEZ, J.C. Piña; SILVA, Clécio C.S. Stability and dynamics of synthetic antiferromagnetic skyrmions in asymmetric multilayers. **Physical Review B**, v. 110, n. 9, p. 094430, 2024.
3. CORREIA, Matheus V. et al. Structural phases of classical 2D clusters with competing two-body and three-body interactions. **Journal of Physics: Condensed Matter**, v. 35, n. 41, p. 415404, 2023.

APPENDIX B – PHYSICAL ORIGIN OF ATOMIC MAGNETISM

The current loop analogy helps us visualize an important phenomenon called the **gyromagnetic effect**, stating that magnetic bodies possess a magnetic moment μ convertible to a mechanical angular momentum $\mathbf{L}$ (and vice versa). Its manifestation is found in **Einstein–de Haas** and **Barnett effects**. The former was predicted by Richardson (1908) and observed by Einstein and de Haas (EINSTEIN; HAAS, 1915): a change in the magnetic moment of a ferromagnet causes the body to rotate by conservation of $\mathbf{L}$. The latter is the corresponding inverse summarized in a paper by Samuel Barnett as a change in magnetization observed for a rotating ferromagnet (BARNETT, 1915).

The gyromagnetic effect can be further described by considering electrons with mass m_e orbiting the loop with a velocity $\mathbf{v}$ and angular momentum $\mathbf{L}$. Then, one obtains

$$\mu = \gamma \mathbf{L} \quad (\text{B.1})$$

where γ is defined as the **gyromagnetic ratio**. For the electron, $\gamma_e \approx 1.76 \times 10^{11} \text{ s}^{-1} \text{ T}^{-1}$ (JILES, 2015, p. 59). We can determine γ in a classical approach by first calculating $\mathbf{L}$, as follows.

For a given electron in the current loop, $\mathbf{L} = \mathbf{r} \times \mathbf{p} = m_e \mathbf{r} \times \mathbf{v}$, where $\mathbf{r}$ is the particle's position and $\mathbf{p} = m_e \mathbf{v}$ its linear momentum. Considering a loop made of a material with density ρ , we integrate contributions within volume V as

$$\mathbf{L} = m_e \iiint_V (\mathbf{r} \times \mathbf{v}) \rho d^3r. \quad (\text{B.2})$$

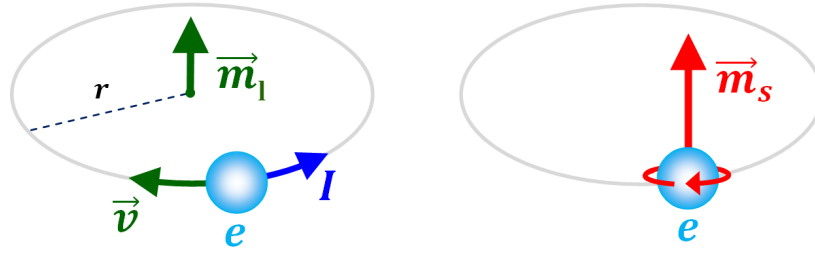
Since $\mathbf{j} = e\rho\mathbf{v}$ is the current density, we use Eq. 2.2 to obtain

$$\mathbf{L} = \frac{m_e}{e} \iiint_V (\mathbf{r} \times \mathbf{j}) d^3r = \frac{2m_e}{e} \boldsymbol{\mu} \Rightarrow \boldsymbol{\mu} = \frac{e}{2m_e} \mathbf{L}, \quad (\text{B.3})$$

So, the gyromagnetic ratio of the electron is given by $\gamma = e/2m_e \approx 8.805 \times 10^{10} \text{ s}^{-1} \text{ T}^{-1}$, smaller than experimental data told us. Thus, Eq. B.3 is not accurate for atoms in nature as we cannot consider orbital motion of electrons as the only source of magnetic moments in matter, as we shall discuss later.

An isolated magnetic moment μ is disturbed by the presence of a nearby magnetic

Figure 57 – The sources of magnetism in electrons: orbital magnetic moment m_L (left) and intrinsic spin moment m_S (right).



Source: Adapted from Encyclopedia Magnetica (2023).

field $\mathbf{B}$. In fact, in the absence of energy dissipation, $\boldsymbol{\mu}$ experiences a torque given by

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} = \gamma \mathbf{L} \times \mathbf{B}. \quad (\text{B.4})$$

Besides a torque, the external field sets a potential energy $\mathcal{E}_Z$ to the system, which is known as the **Zeeman energy** and given by

$$\mathcal{E}_Z = -\boldsymbol{\mu} \cdot \mathbf{B}, \quad (\text{B.5})$$

with the lowest energy found for parallel alignment between $\boldsymbol{\mu}$ and $\mathbf{B}$, when $\boldsymbol{\tau} = 0$.

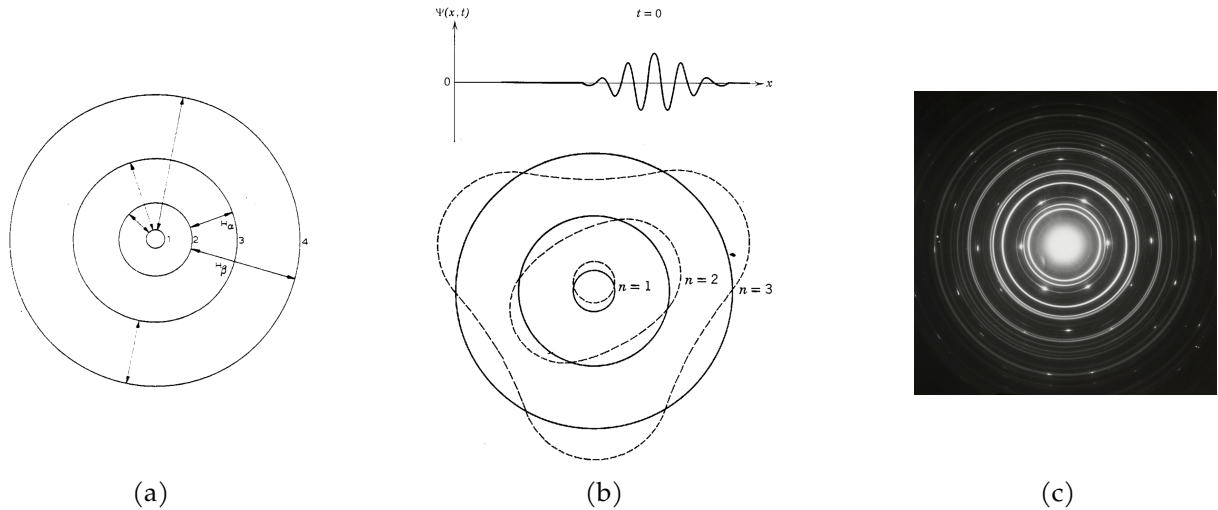
By using Newton's second law for rotations, we obtain the time evolution of $\mathbf{L}$ as

$$\frac{d\mathbf{L}}{dt} = \boldsymbol{\tau} = \gamma \mathbf{L} \times \mathbf{B}. \quad (\text{B.6})$$

Equation B.6 describes the **Larmor precession** of $\boldsymbol{\mu}$ around the axis defined by the direction of $\mathbf{B}$, similar to a tilted gyroscope precessing around the rotation axis in classical mechanics. $\boldsymbol{\mu}$ precesses around $\mathbf{B}$ with a frequency given by $\omega_L = |\gamma B|$, known as the **Larmor frequency** and named after Irish physicist Joseph Larmor.

We see from Eqs. B.3 and B.6 that angular and magnetic momenta are connected in nature. In the detailed structure of matter, electrons actually possess two main sources of angular momentum: **orbital** and **spin**. With exception situations such as the nuclear magnetic resonance, nuclear magnetism can be often neglected. So we proceed in associating magnetism in materials to electron's orbital and spin moments (Fig. 57).

Figure 58 – (a) Bohr’s diagram of stationary circular orbits for the hydrogen atom with arrows indicating the Balmer and Lyman series transitions. This is found in his Nobel Lecture of 1922. (b) Top: example of a matter wave for a particle in a given instant of time. Bottom: standing matter waves are fitted to the electron orbits in Bohr’s atom. (c) Photograph by G.P. Thomson (1925) exhibiting a Debye-Scherrer pattern. This was the first confirmation for the electron diffraction and, consequently, for the wavelike behavior predicted by de Broglie.



Source: Bohr (1922, p. 17) (a), Eisberg and Resnick (1985) (b), and Thomson (1927) (c).

B.1 Orbital magnetic moment

The **orbital** contribution can be visualized by thinking about semiclassical **Niels Bohr's atomic model** [Fig. 58(a)] in a similar way as in the current loop analogy. The electron is orbiting an atomic nucleus of charge Ze (Z is the atomic number) under the Coulomb potential $U(r) = -Ze/4\pi\epsilon_0 r$, where ϵ_0 is the electric permittivity of the vacuum and r is a fixed radius of the orbit. The circular motion works as a current loop since the negative charge e revolves with a velocity v around the nucleus (Fig. 57, left). The period is given by $T = 2\pi r/v$, so the electric current is $I = -e/T = -ev/2\pi r$. Using Eq. 2.2, this must generate a magnetic moment

$$\boldsymbol{\mu}_L = I(\pi r^2) = -\left(\frac{ev}{2\pi}\right)(\pi r^2) = -\left(\frac{e}{2m_e}\right)\mathbf{L}, \quad (\text{B.7})$$

which is the exact result of Eq. B.3 and $\boldsymbol{\mu}_L$ is known as the **orbital magnetic moment**. However, Bohr postulated in his model that the values of $\mathbf{L}$ (and $\boldsymbol{\mu}_L$) are limited and **only some magnitudes are permitted**. So now we enter in the realm of *quantum* rules.

As Louis de Broglie showed in 1924, we cannot think of electrons only as classical, charged point particles. Instead, they behave as a particle or a wave according to the

circumstances they are exposed to in experiments, which is summarized as the **wave-particle duality**. De Broglie extended the concept already used for light phenomena, considering that the linear momentum of the electron is given by

$$p = h/\lambda_e \quad (\text{B.8})$$

where λ_e is the wavelength of the particle's associated wave (matter wave) (BROGLIE, 1924, p. 99). Equation B.8 implies that electrons are also subjected to undulatory phenomena [Fig. 58(b)], such as interference and diffraction. This wave-like nature was first demonstrated by J.J. Thomson's son¹, George P. Thomson, in the late 1920s. He scattered highly energetic electrons through thin films of gold and aluminum, producing a distinct, wavelike diffraction pattern [Fig. 58(c)].

The restrictions imposed by the undulatory behavior predicted by de Broglie justify Bohr's postulate on the quantization of the angular momentum L of the electron. In the collection of articles in 1913 (BOHR, 1913a; BOHR, 1913b; BOHR, 1913c), Niels Bohr postulates² that L is restricted to integer multiples of $\hbar = h/2\pi$, i.e., $L = n\hbar$, where $n = 1, 2, 3, \dots$ is known as the **principal quantum number** and $h = 6.62 \times 10^{-34}$ J s is the Planck's constant and each electronic shell is represented by n . Finally, de Broglie's relation (Eq. B.8) associated the quantization of L to standing waves with such a wavelength that an integer number of λ_e could fit the electron's orbit [Fig. 58(b)].

What about the permitted values for orbital magnetic moment μ_L ? For that we need the modern approach of quantum mechanics. The Bohr model was a good approximation for the hydrogen atom, however it was wrong in explaining the fundamentals of atomic particles. Consider, for example, the following crucial issue: **classical descriptions cannot account for magnetism**, as Bohr himself realized (BOHR, 1972) and as we shall see in detail later. So Bohr model's limitations arise from its hybrid nature, superimposing *ad hoc* quantization upon a classical framework of deterministic orbits.

The advent of modern quantum mechanics was initiated by Werner Heisenberg who developed **matrix mechanics** in 1925. Concurrently, Erwin Schrödinger, in the following year, building on de Broglie's wave-particle duality, formulated **wave mechanics**,

¹ Ironically, J.J. Thomson won the Nobel Prize for discovering the electron as a *particle*, while his son was later awarded the prize for demonstrating the exact opposite: that the electron also behaves as a *wave*, confirming de Broglie's theory (EISBERG; RESNICK, 1985).

² This quantization condition was required for the semiclassical model to agree with the experimental results on the spectrum of the hydrogen atom (EISBERG; RESNICK, 1985).

which describes the electron via the probabilistic wave function $\Psi(\mathbf{r}, t)$ whose time evolution is governed by the differential equation

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \hat{H} \Psi(\mathbf{r}, t), \quad (\text{B.9})$$

where $\hat{H}$ is the Hamiltonian operator. Together, these two approaches replaced Bohr's paradigm of discrete, deterministic orbits with that of **orbitals**, regions where the electron's probability distribution $|\Psi(\mathbf{r}, t)|^2$ gives limited access to the position of the particle. Moreover, the quantum numbers are no longer postulates, but necessary constants when we solve Schrödinger's equation (Eq. B.9). As already exemplified in Eq. B.9 with $\hat{H}$, wave mechanics associates a mathematical hermitian operator to each physical variable, which acts on the wave function $\Psi(\mathbf{r}, t)$ to reproduce the act of measurement. The result is one of the possible values obtained when solving the *eigenvalue* problem³. Consider, for example, the angular momentum operator $\hat{\mathbf{L}}^2$ acting on Ψ , then we obtain

$$\hat{\mathbf{L}}^2 \Psi = \hbar^2 l(l+1) \Psi, \quad (\text{B.10})$$

which implies the quantization of the angular momentum, since l is an integer. The magnitude of $\mathbf{L}$ is determined by this new quantum number: the **orbital** (or **azimuthal**) **quantum number** (l), which explains the shapes of different orbitals and whose value ranges from 0 to $n-1$. From Eq. B.10, the magnitude of $\mathbf{L}$ reads $L = \|\mathbf{L}\| = \sqrt{l(l+1)} \hbar$. Considering the magnitude of μ_L in Eq. B.7, we obtain the contribution to the electronic magnetic moment as

$$\|\mu_L\| = \frac{\mu_B}{\hbar} \|\mathbf{L}\| = \sqrt{l(l+1)} \mu_B, \quad (\text{B.11})$$

where $\mu_B = e\hbar/2m_e = 9.274 \times 10^{-24} \text{ A m}^2$ is a natural unit for the electronic magnetic moment known as the **Bohr magneton**. Equation B.11 gives the correct value of ($\|\mu_L\| = 0$) for the $1s$ ($l = 0$) electronic state, which is not possible using Bohr's model.

The vector μ_L itself precesses around an applied field and cannot be fully determined as its components do not commute, i.e., according to **Heisenberg's uncertainty principle**, the components cannot be measured simultaneously with infinite precision. So we rely on its projection onto an external magnetic field, typically in the z axis, de-

³ Hermitian operators possess a complete set of real eigenvalues. This mathematical property makes them suitable to work as physical quantities that can be measured, that is, as observables. (HASSANI, 2013)

terminated by the **magnetic orbital quantum number** m_L , which describes orbital's orientation in space and takes the values $-l, -l + 1, \dots, 0, \dots, l - 1, l$. Thus,

$$(\mu_L)_z = -\mu_B m_L. \quad (\text{B.12})$$

B.2 Spin

The second contribution to the magnetic moment of the electron is the intrinsic **spin** moment ⁴, represented in Fig. 57 (right). The existence of the spin was first demonstrated in the famous **Stern-Gerlach experiment** (Fig. 59), where silver atoms — although possessing no orbital angular momentum — were somehow deflected in two orientations due to a magnetic field gradient (GERLACH; STERN, 1989; BAUER, 2023). The cause was an intrinsic moment, the spin, which is a feature of many fundamental particles.

Similarly as $\mathbf{L}$, we introduce a quantized spin angular moment vector $\mathbf{S}$ as

$$\hat{\mathbf{S}}^2 \Psi = \hbar^2 s(s+1) \Psi, \quad (\text{B.13})$$

with magnitude $\|\mathbf{S}\| = \sqrt{s(s+1)} \hbar$, where s is the **spin quantum number** which takes the half-integer multiples of $1/2$ ($s = 0, 1/2, 3/2, \dots$). For the electron, $s = 1/2$. Also, projecting onto the z-axis we obtain

$$S_z = m_S \hbar \quad (\text{B.14})$$

where m_S is the **spin magnetic quantum number** and corresponds to the orientations observed in the Stern-Gerlach experiment. The two possibilities for m_S are typically referred as "spin-up" and "spin-down" states. We find the contribution of $\mathbf{S}$ to the spin magnetic moment μ_S of the electron as

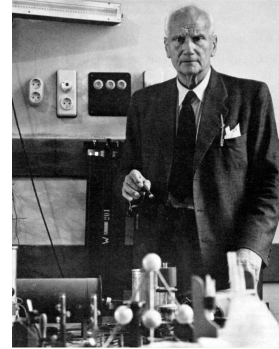
$$\mu_S = -g_S \left(\frac{\mu_B}{\hbar} \right) \mathbf{S}, \quad (\text{B.15})$$

⁴ Although its name and the illustration may evoke the analogy of a spinning body, the electron radius is smaller than 10^{-20} m so we consider it to be a point particle (COEY, 2010).

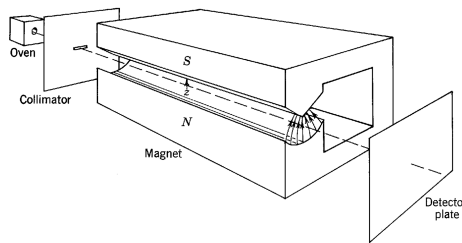
Figure 59 – The Stern-Gerlach experiment on spin: Otto Stern (a) in Hamburg (1930) and Walther Gerlach (b) in Munich (1950), both with a cigar in hand. Their famous experiment apparatus, based on a spatially inhomogeneous magnetic field where silver atoms passed through, is shown in (c). In (d), a postcard sent by Gerlach to Niels Bohr in 1922 describing the results: the beam splitting of silver atoms proving the existence of the quantized spin component of the angular momentum.



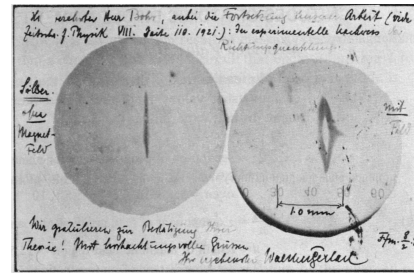
(a)



(b)



(c)



(d)

Source: Eisberg and Resnick (1985) (c) and Friedrich and Herschbach (2003) (a–b, d).

and its z component is, accordingly,

$$(\mu_S)_z = -g_S \mu_B m_S. \quad (\text{B.16})$$

where $g_S \approx 2$ is the g -factor of the electron. The g -factor is the dimensionless ratio of the magnetic moment to that expected of a classical particle with the same charge and angular momentum. We omitted the correspondent g -factor for μ_L since $g_L = 1$. g_S cannot be understood classically, it is a property arising from the relativistic quantum mechanical treatment of the electron by **Dirac's equation** (DIRAC, 1928). Indeed, we cannot think of the electron spin as a classical entity.

In summary, the state of an electron in an atom is expressed by a set of four quantum numbers: n, l, m_L and m_S . Also, since $s = 1/2$ is a half-integer, we know that electrons are **fermions**. As a result of the antisymmetric fermionic wave function, Wolfgang Pauli stated his important **exclusion principle** for fermions in 1925, which forbids two parti-

cles from “occupying” the same quantum state, i.e., from having all the same quantum numbers (PAULI, 1925 apud KAPLAN, 2019).

The orbital and spin components of the angular momentum may couple due to a relativistic effect called the **spin-orbit interaction**. We explain it by using an analogy, changing the point of view between the electron and the atomic nucleus. In the electron’s reference frame, the electron is at rest while the nucleus is orbiting it instead. Since it is a moving charge of magnitude Ze , the nucleus establishes an electric current $I_n = Zev/2\pi r$ and a magnetic field $\mathbf{B}_{so} = \mu_0 I_n / 2r$ (COEY, 2010). Thus, the spin-orbit coupling $\mathcal{E}_{so}$ reflects the interaction of $\boldsymbol{\mu}_S$ with $\mathbf{B}_n$ as⁵

$$\mathcal{E}_{so} = -\frac{1}{2} \boldsymbol{\mu}_S \cdot \mathbf{B}_{so} = -\frac{1}{2} (g\mu_B m_s) \left(\frac{\mu_0 Zev}{4\pi r^2} \right). \quad (\text{B.17})$$

We use an approximation to determine how $\mathcal{E}_{so}$ scales with the atomic number Z . Considering Bohr’s model for the hydrogen atom, for tightly bound core electrons $r \approx a_0/Z$, where $a_0 = 4\pi\epsilon_0 \hbar^2 / m_e e^2$ is the Bohr’s radius. Also, we combine de Broglie’s relation (Eq. B.8) with the standing wave condition, $m_e v r \approx \hbar$, finally approximating Eq. B.17 to

$$\mathcal{E}_{so} \approx -\frac{1}{2} (g\mu_B m_s) \left(\frac{\mu_0 e \hbar Z^4}{2\pi a_0^3} \right) \propto Z^4. \quad (\text{B.18})$$

Therefore, the spin-orbit interaction is much stronger for heavy atoms and for electrons in the inner shells. This interaction is of great relevance, as we shall see later, in the stabilization of magnetic skyrmions.

Since spin-orbit interaction has its way in the composition of angular momentum, we now define the **total angular momentum** $\mathbf{J} \equiv \mathbf{L} + \mathbf{S}$ for the electron. Analogous quantization rules are applied to $\mathbf{J}$, so

$$\|\mathbf{J}\| = \hbar \sqrt{j(j+1)}, \quad (\text{B.19})$$

where j is an integer, the **total angular momentum quantum number**, that ranges from $|l-s| < j < l+s$. So, the total magnetic moment $\boldsymbol{\mu}$ can be written as

$$\boldsymbol{\mu} = -g_J \mu_B \mathbf{J}, \quad (\text{B.20})$$

where $g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$ is the g-factor if $g_S \approx 2$.

⁵ The factor of 1/2 comes from the **Thomas precession**, a relativistic correction taking the time dilation between electron and nucleus into account (COEY, 2010).

So we can define analogous **total quantum numbers** with capital letters

$$S' = \sum_i s_i, \quad L' = \sum_i l_i, \quad M'_S = \sum_i (m_S)_i, \quad \text{and} \quad M'_L = \sum_i (m_L)_i. \quad (\text{B.21})$$

According to the electronic shell distribution, we may obtain different values of energy for a given atom. In order to obtain the ground state of a multielectron atom, we use the **Hund's rules**, very instructive to "fill" the electronic orbitals of atoms with small and intermediate Z (EISBERG; RESNICK, 1985).

The first rule is the maximum multiplicity: electrons prefer to occupy "free" orbitals first, filling them in pairs afterwards. Due to Pauli's exclusion principle, each orbital can only host up to two electrons ($m_S = \pm 1/2$). The lowest energy is then reached for maximum S' , as the **multiplicity** of a state is $2S' + 1$, diminishing Coulomb repulsion with maximum number of unpaired electrons. The second rule states that, for a fixed multiplicity, the lowest energy corresponds to the largest L' , so more electrons orbit the nucleus in the same sense, also reducing Coulomb repulsion. The last rule relies on the assumption that the spin-orbit interaction, so far neglected, is weak. In this regime⁶, known as the **Russell-Saunders** or **LS coupling**, L' and S' combine to form the total angular momentum J' such that $\|J'\| = \sqrt{J'(J' + 1)}\hbar$. So we determine the value of $J = \|J'\|$ to minimize the energy as $J = L \pm S$ [lowest (highest) J], if the valence shell is less (more) than half full, and $J = S$ if it is exactly half full.

B.3 Orbital quenching

While Hund's rules successfully predict the ground state of an isolated ion by maximizing the total orbital angular momentum L to minimize Coulomb repulsion, this description requires a change when the ion is embedded within a crystal lattice. In $3d$ transition metals, electrons occupy outer orbitals directly exposed to non-spherical electrostatic potential generated by neighboring ligands, the **crystal field**. This field breaks the atomic spherical symmetry, energetically splitting orbital states and locking the electron density into standing waves with fixed spatial orientations.

⁶ In the other case, that of heavier atoms (typically $Z \geq 30$), we adopt a different approach with the **jj coupling**, that "involves first treating the relatively strong coupling of the spin and orbital angular momenta of each optically active electron" (EISBERG; RESNICK, 1985, p. 356) and later combine the electrons' J .

As a result, $\mathbf{L}$ effectively averages to zero ($\langle \mathbf{L} \rangle \approx 0$), a phenomenon known as the **quenching of the orbital moment**. The quenching of $\mathbf{L}$ simplifies the quantum mechanical description of magnetic ordering, since the macroscopic magnetic moment arises almost exclusively from the intrinsic $\mathbf{S}$, generally decoupled from the crystal field. So $\mathbf{S}$ acts as the driving vehicle for ferromagnetism, while $\mathbf{L}$ is reintroduced only as a perturbation via spin-orbit coupling.

In this section, we focused on the phenomena relating the angular momentum components of the electron and the magnetic moment μ of an isolated atom, with the exception of the orbital quenching. In Appendix C, we discuss the magnetic order in materials, including ferromagnetism and the exchange interaction.

APPENDIX C – COLLECTIVE BEHAVIOR AND MAGNETISM IN MATTER

Here, we summarize how to characterize magnetic moments in matter and what types of magnetic order we can find in materials. Typically, we consider three main magnetic response mechanisms: diamagnetism, paramagnetism and ferromagnetism (and its two other siblings, ferri and antiferromagnetism). Before detailing each type, we need to consider how to characterize magnetic fields in matter.

We describe the magnetic moments in a material using the **magnetization** field $\mathbf{M}$ defined as

$$\mathbf{M} = \frac{d\mathbf{m}}{dV} \quad (\text{C.1})$$

where $d\mathbf{m}$ is a given infinitesimal magnetic moment, dV is a volume element of the material, and $\mathbf{M}$ is measured in SI units of A/m. The magnetization is a pseudovector¹ and can be visualized as a volume distribution of the magnetic moments in a given material. As we discuss in Appendix. B, the physical source for the magnetic moments, and then for $\mathbf{M}$, is the electron's orbital motion and its intrinsic spin. These microscopic details are crucial for the response of a given material to an applied magnetic field, which is macroscopically revealed by the $\mathbf{M}$ field. The field approach implies a microscopic averaging on the atomic magnetic moments, however tuned in such a way that $\mathbf{M}$ vary across the material and reproduce its magnetic features. Therefore, it is important for us to consider the relation between the magnetization $\mathbf{M}$, the auxiliary magnetic field $\mathbf{H}$, and the magnetic induction $\mathbf{B}$.

A material with magnetization $\mathbf{M}$ produces a magnetic field by its own. This magnetic field, as shown in Griffiths (2013), is the same that would be produced by both a volume current density $\mathbf{J}_b = \nabla \times \mathbf{M}$ and a surface current density $\mathbf{K}_b$ on the boundary of the material. $\mathbf{J}_b$ and $\mathbf{K}_b$ are known as **bound currents**. A given region of space may have current density contributions arising from the outside of the material, which we call $\mathbf{J}_{\text{free}}$. This leads to the total current density $\mathbf{J} = \mathbf{J}_{\text{free}} + \mathbf{J}_b$. Recalling Ampère's law

¹ A pseudovector, also known as an axial vector, is a quantity that behaves like a vector under regular rotations. However, under discontinuous transformations such as a reflection, a pseudovector does not flip its sign, unlike usual (polar) vectors do. This behavior is because pseudovectors typically arise from the cross products of actual vector physical quantities. Angular velocity is another common example. (ARFKEN; WEBER, 2005)

for steady currents (Eq. 2.4), we calculate

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} = \mu_0 (\mathbf{J}_{\text{free}} + \mathbf{J}_b) = \mu_0 \mathbf{J}_{\text{free}} + \mu_0 \nabla \times \mathbf{M},$$

that is rearranged to $\nabla \times \mathbf{H} = \mathbf{J}_{\text{free}}$, where $\mathbf{H} \equiv (1/\mu_0)\mathbf{B} - \mathbf{M}$ is known as the **auxiliary magnetic field** (or simply “H-field”). In free space, we note that $\mathbf{J} = \mathbf{J}_{\text{free}}$. $\mathbf{J}_{\text{free}}$ is the quantity which we can control in the laboratory, while $\mathbf{J}_b$ is an intrinsic property of the material in response to the applied magnetic field provided *a priori* for a given region of space. In that regard, $\mathbf{H}$ is typically named “magnetic field” for practical purposes, since it is the quantity controlled in the laboratory by experimentalists via $\mathbf{J}_{\text{free}}$.

Another important quantity is the **magnetic susceptibility** χ , a dimensionless constant relating $\mathbf{M}$ and $\mathbf{H}$. Some materials do not hold a magnetization such that $\mathbf{M} \neq 0$ unless an external field $\mathbf{H}$ is applied. For these nonmagnetic materials, when low values of $\mathbf{H}$ and high temperatures are considered, $\mathbf{M} \approx \chi \mathbf{H}$. The sign of χ allows us to define diamagnetism ($\chi < 0$) and paramagnetism ($\chi > 0$).

Table 1 – Magnetic susceptibilities χ of selected diamagnetic and paramagnetic materials, for pressure of 1 atm and temperature of 20 °C (unless stated otherwise).

Diamagnetic		Paramagnetic	
Material	Susceptibility	Material	Susceptibility
Bismuth	-1.7×10^{-4}	Oxygen (O ₂)	1.7×10^{-6}
Gold	-3.4×10^{-5}	Sodium	8.5×10^{-6}
Silver	-2.4×10^{-5}	Aluminum	2.2×10^{-5}
Copper	-9.7×10^{-6}	Tungsten	7.0×10^{-5}
Water	-9.0×10^{-6}	Platinum	2.7×10^{-4}
Carbon Dioxide	-1.1×10^{-8}	Liquid Oxygen (−200° C)	3.9×10^{-3}
Hydrogen (H ₂)	-2.1×10^{-9}	Gadolinium	4.8×10^{-1}

Source: Adapted from Haynes (2010 apud GRIFFITHS, 2013, p. 285).

C.1 Diamagnetism

Diamagnetism appears in all matter., it is the tendency of a material to repel an external magnetic field, which can be explained by Lenz’s law. As one induces a magnetic moment opposed to the applied field, $\chi < 0$. Typical values of χ for diamagnetic materials are shown in Table 1 (left) and indicate that, although very common, diamagnetism is a very weak effect. It can be better observed in atoms with no unpaired

electrons, where paramagnetism — which is typically slightly stronger — and ferromagnetism — which is much stronger! — are absent.

Although correct in the prediction of diamagnetic susceptibility, as described by Langevin (1905), **classical physics does not predict any magnetism at all**: we obtain no diamagnetism, paramagnetism or ferromagnetism from there. As a matter of fact, Niels Bohr (1911) and later Hendrika van Leeuwen (1919) obtained a vanishing thermal average of M using classical and statistical mechanics. Essentially, the Lorentz force for an electron under a magnetic field $\mathbf{B}$, $\mathbf{F} = e(\mathbf{v} \times \mathbf{B})$, is perpendicular to the electron's velocity $\mathbf{v}$ and, therefore, cannot change the energy and magnetization. The results are summarized as the **Bohr-van Leeuwen theorem** (COEY, 2010). In the end, the Langevin model only worked because it is based on the false assumption that a magnetic moment is produced by electron's classical orbit.

C.2 Curie paramagnetism

Diamagnetism is overshadowed in materials with atoms with partially filled shells, when **paramagnetism** becomes relevant. In Table 1 (right) we show magnetic susceptibility of some paramagnetic materials. Paramagnetism describes the tendency of a material to become weakly magnetized in the direction of an applied field $\mathbf{B}$. Unfilled electron shells form magnetic dipoles because the electrons possess uncanceled magnetic moments. Since only part of the electrons are influenced by $\mathbf{B}$, paramagnetism is typically weak. When $\mathbf{B}$ is applied, it exerts a torque $\boldsymbol{\tau}$ on $\boldsymbol{\mu}$ given by $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$. If $\boldsymbol{\mu}$ is not aligned with $\mathbf{B}$, $\boldsymbol{\tau}$ tends to align it with the field, creating a net magnetization $\mathbf{M} \parallel \mathbf{B}$. For $\|\mathbf{B}\| = 0$, these permanent dipoles are randomly oriented due to thermal fluctuations, with no net macroscopic magnetization ($\mathbf{M} = 0$).

The alignment is always incomplete, as it is constantly opposed by the randomizing effects of thermal energy proportional to $k_B T$, k_B the Boltzmann's constant. Consequently, paramagnetism is strongly dependent on temperature. Pierre Curie discovered that, for many paramagnetic materials at sufficiently high T , the susceptibility is inversely proportional to the absolute temperature as

$$\chi_p = \frac{C}{T} \tag{C.2}$$

This is known as the **Curie's law**, where C is the material-specific Curie constant. This $1/T$ dependence is a hallmark of paramagnetism and stands in stark contrast to diamagnetism, which is essentially independent of temperature — it depends solely on the moment induced by the external field.

A classical (and quantum) statistical mechanics treatment provides the theoretical basis for the experimental Curie's law. Classically, Paul Langevin modeled the system as N non-interacting magnetic dipoles μ per unit volume, free to rotate. Using Boltzmann distribution, which gives us the probability $p_i \propto \exp\left(\frac{-E_i}{k_B T}\right)$ of finding a system at temperature T to be found in a state i with energy E_i , he derived the magnetization

$$M = N\mu \left[\coth(x) - \frac{1}{x} \right] = N\mu \mathcal{L}(x),$$

where $x = \mu B / k_B T$ and the function $\mathcal{L}(x)$ is known as the **Langevin function** (COEY, 2010). In the typical Curie's limit of high temperatures and low fields ($x \ll 1$, i.e., $k_B T \gg \mu B$), the function can be approximated as $\mathcal{L}(x) \approx x/3$. So we approximate M as

$$M \approx \frac{N\mu^2 B}{3k_B T}$$

Comparing this with $M = \chi_p H \approx (\chi_p / \mu_0) B$, we recover Eq. C.2 with a microscopic expression for the Curie constant, namely $C \approx \mu_0 N \mu^2 / (3k_B)$.

As we establish in Appendix B, however, $\mathbf{L}$ is quantized. A full quantum mechanical treatment, first performed by Brillouin, replaces the classical μ with the quantized $\mathbf{J} = \mathbf{L} + \mathbf{S}$ and the continuous Boltzmann integral with a discrete sum over the $2J + 1$ allowed m_J states. This leads to a more complex, but more accurate, expression for the magnetization involving the **Brillouin function** $B_J(x')$ (GRIFFITHS, 2013). Nonetheless, in the high- T limit, the quantum result simplifies and also yields Curie's law, $M \propto B/T$, with the effective magnetic moment μ replaced by its quantum equivalent, $\mu_{\text{eff}} = g_J \mu_B \sqrt{J(J+1)}$. This leads to the following expression for the Curie constant:

$$C = \frac{\mu_0 N g_J^2 \mu_B^2 J(J+1)}{3k_B}. \quad (\text{C.3})$$

The quantum model correctly predicts, for example, the magnetic moments of paramagnetic salts, such as those of the rare-earth ion Gadolinium listed in Table 1.

C.3 Pauli paramagnetism

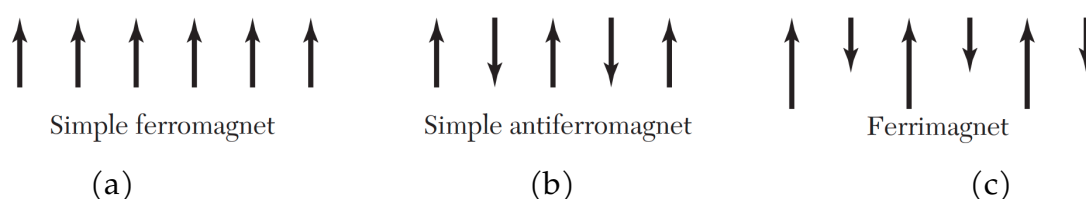
In metals, we consider a different form of paramagnetism, the **Pauli paramagnetism**. The Langevin paramagnetism is applied to dilute paramagnets, such as the hydrated salts of transition metal ions (JILES, 2015, p. 262). However, in **non-magnetic metals** like aluminum, we observe a smaller paramagnetic susceptibility that is temperature independent. A major difference is that one of the assumptions of the Langevin model is not met here: not *all* the electrons are free to rotate when subjected to an external field. Electrons actually occupy energy bands — determined by the Pauli exclusion principle — that are split when an external magnetic field is applied, creating an imbalance in magnetization which ultimately produces a temporary, weak magnetic moment in the material.

C.4 Ferro-, antiferro- and ferrimagnetism

The most crucial and powerful phenomenon for **magnetic** materials is certainly **ferromagnetism**, although only relatively few materials are ferromagnetic — iron, cobalt, nickel, gadolinium, and the majority of their alloys, and some rare-earth metals, such as neodymium. In order to explain ferromagnetic phenomena we need to account for electron-electron interaction, which is not the case of the Pauli theory, that explains the weak response to an external field, but not how a material like iron creates its own permanent field. The missing ingredient is the **exchange interaction**, a powerful quantum-mechanical force that makes electrons align their spins with each other [Fig. 60(a)] — the quantum theory of ferromagnetism is discussed in detail later.

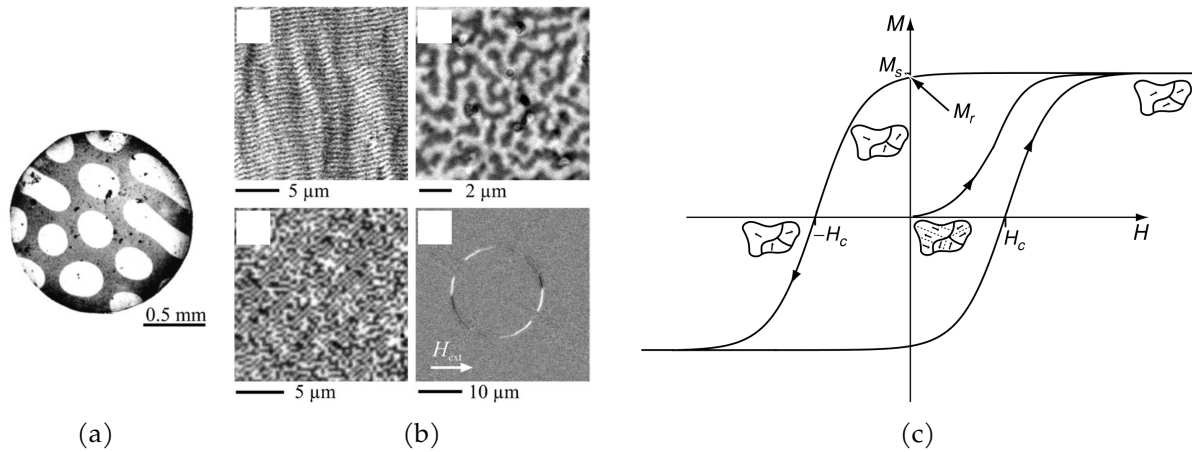
In the Pauli model, an external field causes only a few spins close to the Fermi level to

Figure 60 – Examples of ordering for a group of magnetic moments: (a) aligned and pointing to the same direction in ferromagnetism. Equal (b) and different (c) sublattices producing the antiparallel alignment in antiferromagnetism and ferrimagnetism, respectively. The combination of different sublattices leads to alternating moments with different magnitude in (c).



Source: Adapted from Kittel (2018, p. 322).

Figure 61 – (a) Early magnetic domain imaging obtained from yttrium orthoferrite crystal by magneto-optic Faraday effect (MOFE) in 1959. (b) Sub-micrometer modern imaging of several magnetic domain patterns via magneto-optic Kerr effect (MOKE). (c) Hysteresis loop for a ferromagnetic material showing the magnetization M as a function of the applied magnetic field H . It exhibits the virgin branch, the saturation magnetization M_s , the remanence M_r , and the coercivity H_c . Small drawings show orientation of magnetic domains.



Source: Adapted from McCord (2015) (a–b) and original in Coey (2010) (c).

flip. In a ferromagnetic material, these few flips start a cooperative, domino effect strong enough to ultimately align multiple spins and produce a large, spontaneous magnetization M . This happens as inside the ferromagnet we find structures known as **magnetic domains** [Fig. 61(a)–(b)]: regions of locally aligned magnetic dipoles (around 10^{12} to 10^{15} of them) possessing their own magnetic field and with a more or less random spatial distribution. When the majority of the domains are aligned to the external field, they produce magnetic order reaching high values of the material-dependent **magnetic permeability** μ , defined as $B = \mu H$, measured in units of H/m. To compare μ to the free space permeability $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$ we define the relative permeability $\mu_r = \mu / \mu_0$. Values of μ_r range from 10 to 10^5 , the highest values occurring for materials such as the nickel-iron alloys of permalloy ($\mu_r = 10^5$) and supermalloy ($\mu_r = 8 \times 10^5$) (JILES, 2015).

How does the magnetization **hold** in a ferromagnet? These materials feature memory! In the sense that they recall their magnetic history. This property is known as **magnetic hysteresis**. If a given ferromagnetic material is exposed for the first time to an external field H , its magnetization M behaves as shown in Fig. 61(c). From $H = 0$, their magnetic domains are randomly oriented, but as soon as $H > 0$ the magnetization increases via the virgin branch (middle curve). Magnetization reaches its maximum value at $M = M_s$, known as the **saturation magnetization**. In this situation all

the magnetic domains are aligned to the field. Now we decrease H [see the top line of Fig. 61(c)], but note that the path is different and magnetization still holds for $H = 0$. This is the residual magnetization M_r which we call **remanence**. Following the curve even further means to apply negative values of H , i.e., in the opposite direction. This disturbs the magnetic domains' alignment until $M = 0$ at $H = -H_c$. H_c is known as **coercivity**, the magnitude of the field necessary to fully remove magnetization. Following the curve to the right by the increase of H we define a closed *hysteresis loop*, which tells us that the magnetic history of the material is as important as the magnitude of H .

This collective behavior can be destroyed by thermal noise, just as in paramagnets. However, thanks to the magnetic domains, this happens at the large material-dependent **Curie temperature** (T_c). The susceptibility in the ferromagnetic phase is often too complex to summarize as a single-valued function. However, once becoming a paramagnet, χ follows a slightly different version of the Curie law, the **Curie-Weiss law**,

$$\chi = \frac{C}{T - T_c} \quad \text{for } T > T_c. \quad (\text{C.4})$$

For example, $T_c = 770^\circ\text{C}$ for iron and 1130°C for cobalt (JILES, 2015). Above this temperature, thermal energy causes the breaking of the alignment, and the material becomes a paramagnet.

The **Weiss mean-field theory** first described magnetic domains in 1906 [see Néel (1971)], providing a phenomenological explanation for the ferromagnetic-paramagnetic transition and the remaining magnetization in the ferromagnet. It uses the approximation that a given spin feels an average internal magnetic field, called the **Weiss molecular field** (B_e), created by all the other aligned spins and responsible for sustaining the alignment. In the original Weiss theory, B_e is proportional to M ($B_e = \alpha M$). So, the magnetic field experienced by each magnetic moment is $B = B_{\text{ext}} + \alpha M$, where B_{ext} is an external field. This is known as the **mean-field approximation** and consider each magnetic moment interacting equally with the others. It is a good qualitative description of paramagnetic phase in ferromagnetic materials ($T > T_c$), where thermal fluctuations destroy long-range order but short-range interactions persist, although does not work in the ferromagnetic phase since M is inhomogeneous at large scale.

Nevertheless, the mean-field approach is well-suited *within* each magnetic domain, so $B_e = \alpha M_d$ where $\alpha = T_c/C$ is the temperature-independent mean-field constant

and M_d is the magnetization within the domains such that $M_d = M_s$ at $T = 0$. Considering the definition of α and Eq. C.3, we calculate

$$\alpha = \frac{T_c}{C} = \frac{3k_B T_c}{\mu_0 N g_S^2 S(S+1) \mu_B^2}. \quad (\text{C.5})$$

If $\alpha > 0$, we have ferromagnetic alignment inside the domain. In order to check Weiss theory we use $g_S \approx 2$ and, for iron, $T_C \approx 1000$ K and $S = 1$, to reach $\alpha \approx 5 \times 10^3$. With $M_s \approx 1.7 \times 10^6$ A/m, $\|\mathbf{B}_e\|$ is approximately 10^3 T (KITTEL, 2018), which is a field much larger than what is found among actual magnetic ions.

In the case of $\alpha < 0$, we have **antiferromagnetism**, when magnetic moments organize themselves as a “chess board”, alternating antiparallel orientations [see Fig. 60(b)]. This is due to the lattice topology: the material is arranged in sublattices whose magnetic moments are compensated, making the net magnetization to be zero. When two sublattices are present to cancel each other perfectly, we refer to the material as a Néel antiferromagnet. In the **Néel temperature** T_N the magnetic order is broken due to the thermal energy, when the material becomes a paramagnet also following the Curie-Weiss law (Eq. C.3), however with a positive sign in the denominator. Examples of such materials are the hematite (Fe_2O_3), chromium, and some alloys and oxides such as iron manganese (FeMn) and nickel oxide (NiO).

Antiferromagnetism can be actually considered a special case of **ferrimagnetism** [see Fig. 60(c)]. Just as antiferromagnets, the antiparallel alignment between neighbor spins is favored, however the sublattices are not completely equivalent. For example, the sublattices can be formed by different elements or different amounts of elements at distinct crystallographic sites. The magnetite (Fe_3O_4) is a very common iron ore that exemplifies it by having a mixture of Fe^{2+} and Fe^{3+} ions in different sites A and B — aligned antiparallel —, but only canceling out the Fe^{3+} ions magnetization at A sites (COEY, 2010). In this way, the Fe^{2+} ions at B sites remain uncompensated and a spontaneous magnetization emerges.

The Weiss theory — although introduces the concept of magnetic domains, explains the existence of spontaneous magnetization, and predicts the behavior of susceptibility around ferromagnetic-to-paramagnetic phase transition — still resorts to **localized magnetic moments** such as the Langevin theory does for paramagnets. For metals with their itinerant electrons in the outer shells, this hypothesis does not hold. Observations

of the magnetic moment μ of the iron, for example, show an average value of $2.2 \mu_B$ per atom. Localized electrons should account for integer values of μ . This fact suggested that electrons are not localized, but instead that they flow together in an electron sea.

The **Stoner Model**, developed by British physicist Edmund Clifton Stoner, is a quantum explanation for such ferromagnetic materials — examples include nickel and cobalt. It relies on the Pauli exclusion principle and on Fermi-Dirac statistics to reveal how the energy bands work in this itinerant ferromagnetism. We consider the material to have an electron gas in the lattice, where kinetic and exchange interactions compete in order to determine the ground state. In metals, at $T = 0$, the ground state is such that electrons fill the energy levels up to the Fermi level with energy E_F . In order to prepare a material to be magnetic we need to align a great number of spins. So to split a spin-down to a spin-up electron, energy is required since the electrons are filling all the available energy states up to E_F . Simultaneously, electrons repel each other via Coulomb interaction and also follow the rule imposed by the Pauli exclusion principle — which gives rise to the **exchange interaction**. The exclusion principle forbids electrons to be spatially close to each other, so this ends up diminishing Coulomb repulsion.

The exchange interaction gives the fundamental explanation the Weiss theory was not able to offer on the origin of the strong “molecular” field within ferromagnetic materials. So, spontaneously and permanently, the system shifts the energy bands (“up” to lower energy and “down” to higher energy), causing electrons to fill all the new available states and establishing a difference in the number of spin-up n_+ and spin-down n_- electrons. Thus the excess population produces a net magnetic moment proportional to $n_+ - n_-$. The **Stoner criterion** tells us in which situation this happens: for $\mathcal{E}_{ex}N(E_F) > 1$, where $\mathcal{E}_{ex}$ is the exchange interaction and $N(E_F)$ is the density of states (DOS) at the Fermi level. Taking the iron again as an example, it has both a large exchange and a great amount of levels available at E_F , which makes it a ferromagnet. Copper, on the contrary, has a low DOS and, therefore, it is not ferromagnetic.

While the Stoner model successfully provides a quantum explanation for the fractional magnetic moments observed in itinerant metals like iron, it assumes that electrons are delocalized waves. However, the band theory approach faces difficulties when describing materials where electronic correlations are **strong**, such as transition metal oxides (e.g., the antiferromagnetic MnO) or rare-earth metals. In these systems, the Coulomb repulsion is strong enough to localize magnetic moments at specific atomic

sites. In this regime, we must transition from the itinerant picture back to a model of localized spins: the **Heisenberg model**. Proposed by Werner Heisenberg (HEISENBERG, 1928), this model formalizes the exchange interaction without relying on the motion of charges. Instead, it projects the intricate many-body interactions between electrons onto an effective, isotropic Hamiltonian involving only spin operators $\mathbf{S}$ localized at the lattice sites i and j :

$$\mathcal{E}_{\text{ex}} = - \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j, \quad (\text{C.6})$$

where the summation is restricted to the nearest neighbors $\langle i, j \rangle$. The crucial parameter here is the exchange integral J_{ij} which accounts for energy originated by the overlapping of electron's wave functions and reads

$$J_{ij} = \iint \psi_i^*(\mathbf{r}_1) \psi_j^*(\mathbf{r}_2) \frac{e^2}{4\pi\epsilon_0 |\mathbf{r}_1 - \mathbf{r}_2|} \psi_i(\mathbf{r}_2) \psi_j(\mathbf{r}_1) d^3\mathbf{r}_1 d^3\mathbf{r}_2, \quad (\text{C.7})$$

where ψ_i (ψ_j) is the spatial orbital that the electron i (j), at position $\mathbf{r}_1$ ($\mathbf{r}_2$), is occupying. Depending on the sign of J , the model describes the ground state for ferromagnetism ($J > 0$) or antiferromagnetism ($J < 0$).

It is important to note that while the materials used in skyrmion research, such as cobalt, are metallic — and thus itinerant —, the Heisenberg model remains fundamental for the study of magnetic textures. Indeed, the itinerant nature of electrons in metals like Co is governed by energy band theory, but dynamics occur on timescales where $\|\mathbf{M}\|$ is well-defined and robust. This means that the magnitude of local $\mathbf{M}$ is typically fixed and we put our attention to the angles between neighboring magnetic moments as the remaining degree of freedom. Consequently, the energy cost of spatial variations in the direction of $\mathbf{M}$ is universally described by the Heisenberg exchange formalism, regardless of localized or itinerant origin of the moment.

Alone, the exchange favors a uniform, collinear alignment of spins. However, this cannot tell us much about the formation of complex non-collinear configurations such as skyrmions, which would never stabilize without other interactions. In fact, Landau and Lifshitz (KITTEL, 2018) showed that magnetic domains are a natural consequence of energy minimization of competing contributions: exchange, anisotropy, dipolar interaction, and the antisymmetric Dzyaloshinskii-Moriya interaction (DMI), for example. These fundamental interactions are addressed in Sec. 2.2 of the main text.