

UNIVERSIDADE FEDERAL DE PERNAMBUCO

CENTRO DE CIÊNCIAS SOCIAIS APLICADAS

PROGRAMA DE PÓS-GRADUAÇÃO EM ECONOMIA

**The Impacts of Monetary Policy Shocks on the Volatility of
Inflation, Unemployment and Exchange Rate in Brazil**

MATHEUS GIROLA MACEDO BARBOSA

Recife – PE

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RESUMO

Esse estudo avalia os impactos de um choque contracionista de política monetária na volatilidade da inflação, da taxa de desemprego e da taxa de câmbio no Brasil no período de 2000-2020. Esses efeitos foram analisados através da estimação de funções de resposta a impulso e da decomposição da variância do erro de previsão a partir de um modelo Bayesiano Autoregressivo Vetorial. Os resultados mostram que, após um choque de política monetária, a volatilidade da inflação e do desemprego são reduzidas em 15% e do câmbio em 30%. Estes resultados sugerem que a redução da volatilidade após o choque é devido ao aumento da confiança dos agentes no compromisso do Banco Central do Brasil em atingir as metas de inflação.

Palavras-Chaves: Macroeconomia; incerteza; metas de inflação;

ABSTRACT

This study evaluates the impacts of a contractionary monetary policy shock on inflation, unemployment rate, and exchange rate volatility in Brazil during the period of 2000-2020. These effects were analyzed through the estimation of impulse response functions and the decomposition of forecast error variance using a Bayesian Vector Autoregressive Model (BVAR). The results indicate that, following a monetary policy shock, inflation and unemployment volatility decrease by 15%, and exchange rate volatility decreases by 30%. These findings suggest that the reduction in volatility after the shock is attributed to increased confidence among agents in the commitment of the Central Bank of Brazil to achieving inflation targets.

Keywords: Macroeconomics; uncertainty; inflation target regimes.

*Dedicado ao Dr. Daniel Silveira, Rafaella
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1 Introduction

The role of monetary policy in the economy is one of the most important topics of debate and research in economics. Traditionally the focus is on the effects of monetary policy shocks on the level of variables such as unemployment, inflation, and output, among others (Christiano; Eichenbaum; Evans, 2005; Romer; Romer, 2004; Bernanke; Boivin; Elias, 2005; Barackian; Crowe, 2013; Ahmadi; Uhlig, 2015; Gertler; Karadi, 2015). However, monetary policy shocks also affect the uncertainty of the agents about present and future economic conditions, which causes a significant impact on economic fluctuations, especially in labor search-matching, and investment (Mumtaz; Theodoridis, 2020; Salisu; Gupta, 2021; Haan; Freund; Rendahl, 2021). This is particularly important to emerging economies that face higher levels of uncertainty and concerns related to their monetary policy credibility.

In this context, this paper aims to measure the impact of a contractionary monetary policy shock on the volatility of CPI inflation, unemployment rate, and real-dollar exchange rate in Brazil. We treat this effect in volatility as a measure of the monetary policy impacts on the endogenous uncertainty of the economic agents.¹ Brazil is of particular interest because of its not-so-distant past of high inflation and exchange rate depreciation. Besides, monetary policy in the past was plagued with political interventions that affected its monetary policy credibility² (Bogdanski; Tombini; Werlang, 1999; Fraga; Goldfajn, 2002; Minella et al., 2003; Issler; Soares, 2023).

To address this problem, we adopt the methodology of Mumtaz and Theodoridis (2020), employing a structural Bayesian VAR with stochastic volatility, extended to allow for feedback from endogenous variables to volatility. The empirical model is evaluated with robustness tests with different identification schemes, a time-varying coefficients model, and sensitivity analysis. To investigate the significance of monetary policy shocks, we construct a forecast error variance decomposition (FEVD) following the method described by Lanne and Nyberg (2016) for non-linear models.

Our results show that after a contractionary monetary policy level shock (corresponding to an increase in the Selic rate - the monetary policy instrument of the Brazilian Central Bank - BCB) the volatility of all aforementioned endogenous variables decreases for the whole period after the shock. The volatility of the Selic rate, unemployment rate, and inflation decreases by 15% and the volatility of the exchange rate decreases by 30%. The time-varying parameter version of the model shows that over the years the volatility

¹ A high level of uncertainty leads to a decrease in economic growth because of a rise in precautionary savings - decrease in consumption - and a reduction in production by firms taking longer periods to make decisions (Haan; Freund; Rendahl, 2021).

² The more recent crisis occurred in 2015 when Alexandre Tombini was the president of the Brazilian Central Bank (2011-2016). In this period, the BCB was more complaisant with higher levels of output growth than inflation control. The results of this policy culminated in 2015 with an inflation of 10.7% and retraction of -3.5% in the GDP (Nunes, 2019).

decreases more after the shock. However, the contribution for the FEVD is small, only 1% in the first year, and increases to 6% in 5 years. The monetary policy shock is responsible for 30%-27% of the total contribution of all level shocks to the volatility. We argue that the decrease in volatility is that agents may perceive an increase in interest rates as a signal of higher commitment to price stabilization, reducing uncertainty and therefore, reducing the volatility of the variables.

In the monetary policy shock literature, our work is related to [Mumtaz and Theodoridis \(2020\)](#) and [Salisu and Gupta \(2021\)](#), which estimated the same model for the United States and the United Kingdom, respectively. We contribute to the literature by showing that contractionary monetary policy shocks in an emerging economy such as Brazil can lead to a decrease in volatility - the opposite of the results obtained by both papers. Our results indicate that increases in interest rates are mostly perceived as a higher commitment of the monetary policy authorities to control inflation, resulting in a lower level of uncertainty, a higher level of Central Bank credibility, and market expectations closer to the target midpoint.

For the Brazilian case, researchers have not yet addressed this issue. The bulk of the literature focuses on the impact of the monetary policy shock on the level of inflation and output ([Jawadi; Mallick; Sousa, 2016](#); [Minella; Souza-Sobrinho, 2013](#); [Mello; Moccero, 2011](#)). The closest paper related to ours is [Fasolo \(2019\)](#), which examines the impact of monetary policy *volatility shocks* on the level of the industrial production, dollar-real exchange rate, and inflation.

The paper is structured as follows. Section [2](#) presents some stylized facts about the Brazilian economy after the adoption of the inflation targeting regime. In section [3](#) we do a survey on the literature regarding monetary policy shocks. Section [4](#) presents the methodology. Section [5](#) displays the main results and analysis and section [6](#) brings the main conclusions.

2 Stylized Facts

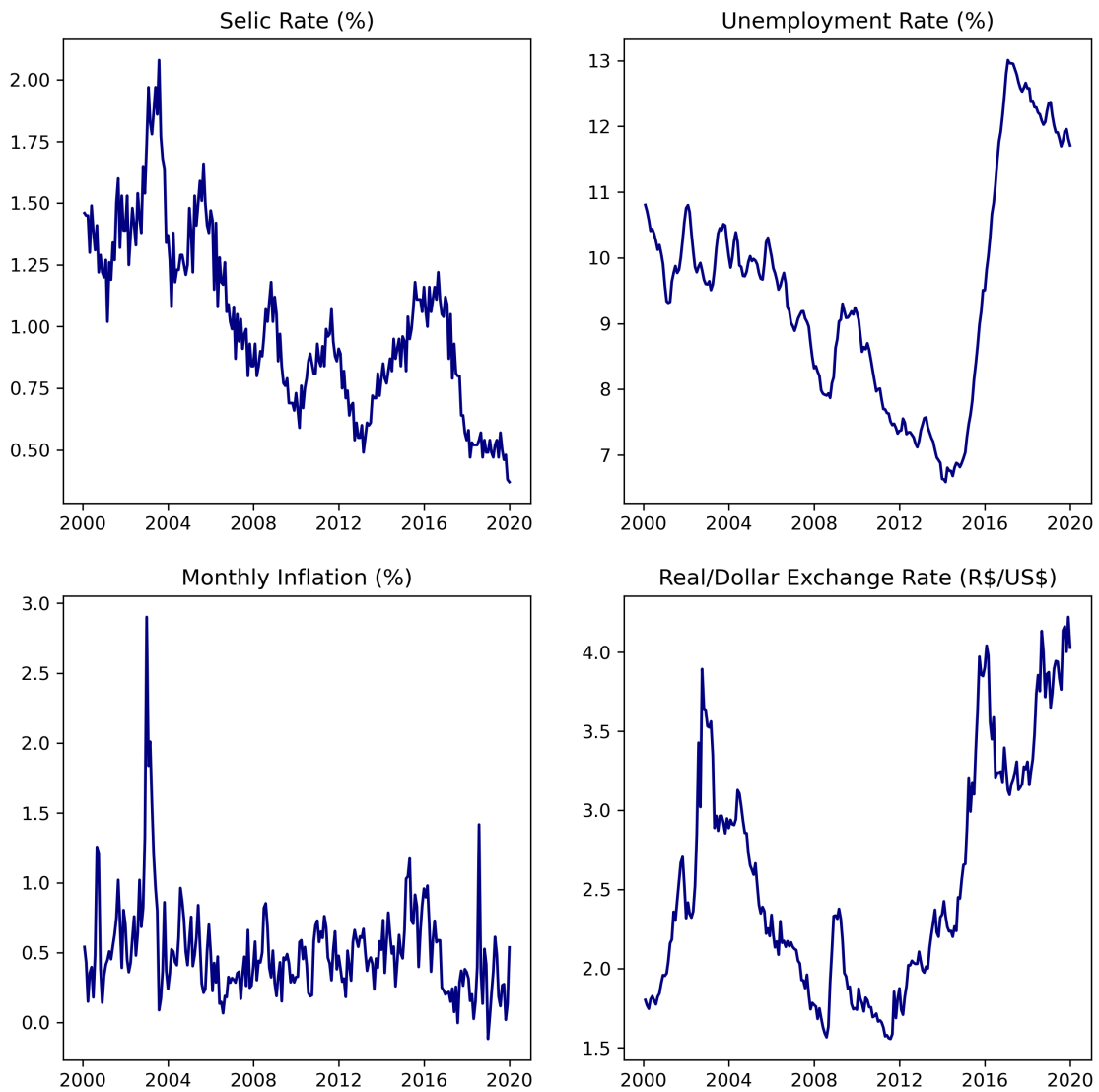
The Central Bank of Brazil adopted the inflation target regime in July 1st, 1999. At the time, the country had already went through other two important macroeconomic policy changes. The first one was the adoption of the Real Plan in 1994, which both ended the hyperinflation crisis and created the now used *real* as the official Brazilian coin. Since this wasn't the first time that a new coin was created to solve the inflation crisis the country, one of the key aspects of the program was the adoption of the crawling peg to the dollar as way to assure the population the value of the new coin being used.

According to [Bogdanski, Tombini and Werlang \(1999\)](#), this stabilization program was accompanied by a comprehensive package of economic reforms, including the privatization of state-owned enterprises, trade liberalization, and restructuring of the financial

system. However, many lingering structural issues were addressed with a gradualist approach, leaving some unresolved. This rendered the Brazilian economy vulnerable to confidence crises, particularly after the Russian moratorium of August 1998, which led to a gradual erosion of market confidence until January 1999. Subsequently, the second major macroeconomic policy change occurred: due to a shortage of dollars in the reserve bank, the Central Bank was compelled to float the currency on January 15, 1999.

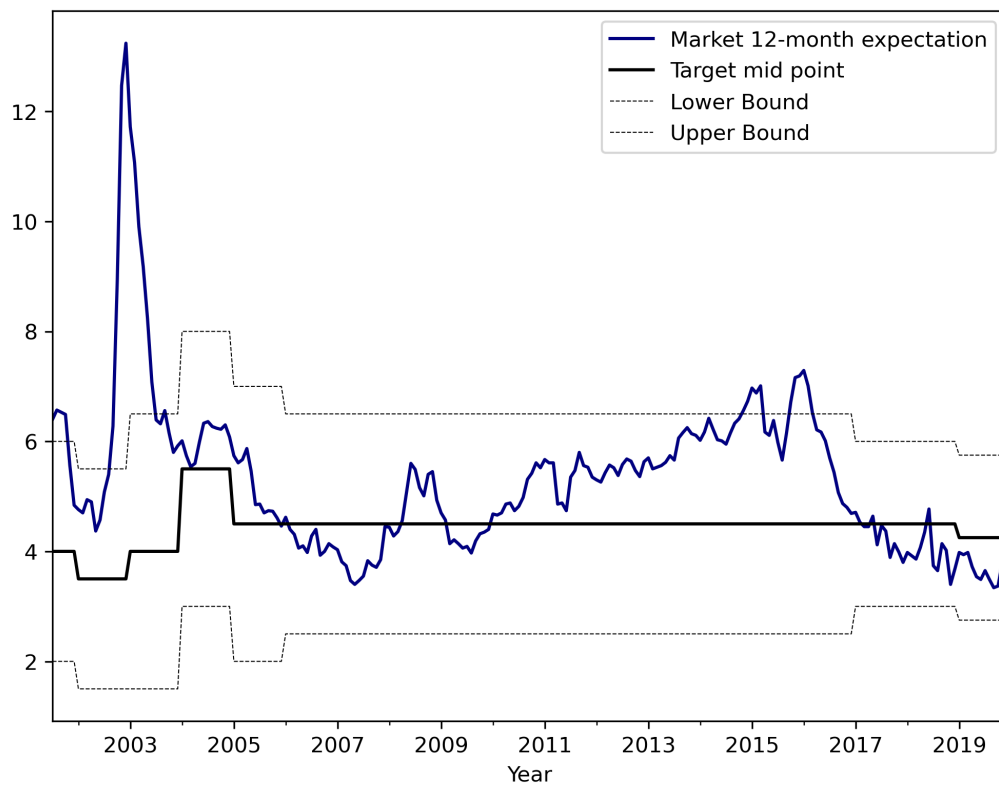
After the abandonment of the crawling peg, most of the Central Bank's Board of Directors was replaced. Inflation started to rise up again and the real quickly depreciated in relation to the dollar. The new board of director's assumed in March, and after some careful study of the situation, the inflation target regime was adopted. Figure (1) shows the time series of the Selic Rate (the monetary policy instrument of the BCB), the unemployment rate, monthly inflation and real/dollar exchange rate.

Figure 1 – Time Series of the Variables (2000 - 2020)



One of the most important aspects of inflation target regimes is building confidence and credibility in the commitment to the policy. This is especially true for emerging countries like Brazil. As we saw, when the inflation targeting regime was implemented, only five years had passed since the hyperinflation crisis ended with the implementation of the Real Plan.³ Furthermore, an even more recent confidence crisis led the country to face a significant depreciation of its currency, which the government had to make quick changes in both exchange rate and monetary policy to prevent further complications. But, the country would face even more economic and confidence crisis that the BCB would have to take in consideration when align the market expectations. Figure (2) provides a comparison between the inflation target midpoint and the market's 12-month expectations, as collected by the Focus survey.

Figure 2 – Focus Fixed Horizon Inflation Forecasts



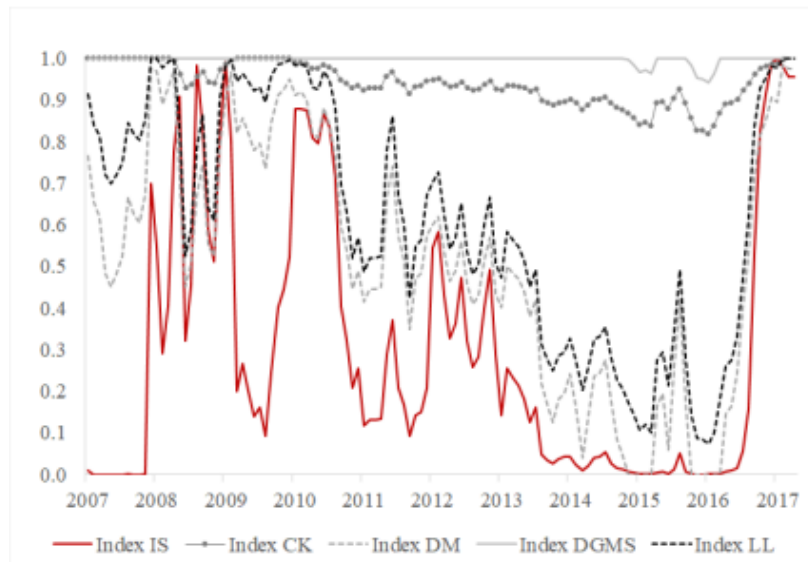
Instances where expectations deviate further from the midpoint coincide with peaks of uncertainty during this period. The onset of the new century was characterized by numerous domestic and external shocks to the economy, resulting in a wave of depreciation and an uptick in inflation. Post-2003, the Brazilian economy entered a phase of stable growth. By this time, both Fraga and Goldfajn (2002) and Minella et al. (2003) observed a considerable decrease in the variance of inflation, output, and the Selic rate. Additionally,

³ Credibility was also one of the main reasons the headline index was used as a target. Unfortunately, Brazilian society has experienced several price index manipulations in the not-so-distant past, and so would be suspicious about any change related to suppressing items from the target index (Bogdanski; Tombini; Werlang, 1999).

the Central Bank demonstrated agility in responding to shifts in expectations during this period. This stability endured until the political-economic crises of 2015, precipitating a severe recession that persisted until the conclusion of 2016. Finally, the highly uncertain political landscape during the 2018 elections manifested in increased uncertainty regarding inflation and exchange rates.

Figure (3) presents a comparison of different credibility indexes for the central bank during the period 2007-2017.⁴ The figure illustrates that a minority of the indexes consistently show high credibility throughout the entire period, some exhibit a loss of credibility during the 2008 crisis, while the majority experienced a gradual decline until 2015, after which credibility began to rise again.

Figure 3 – Comparison Between Credibility Indexes Proposed in the Literature

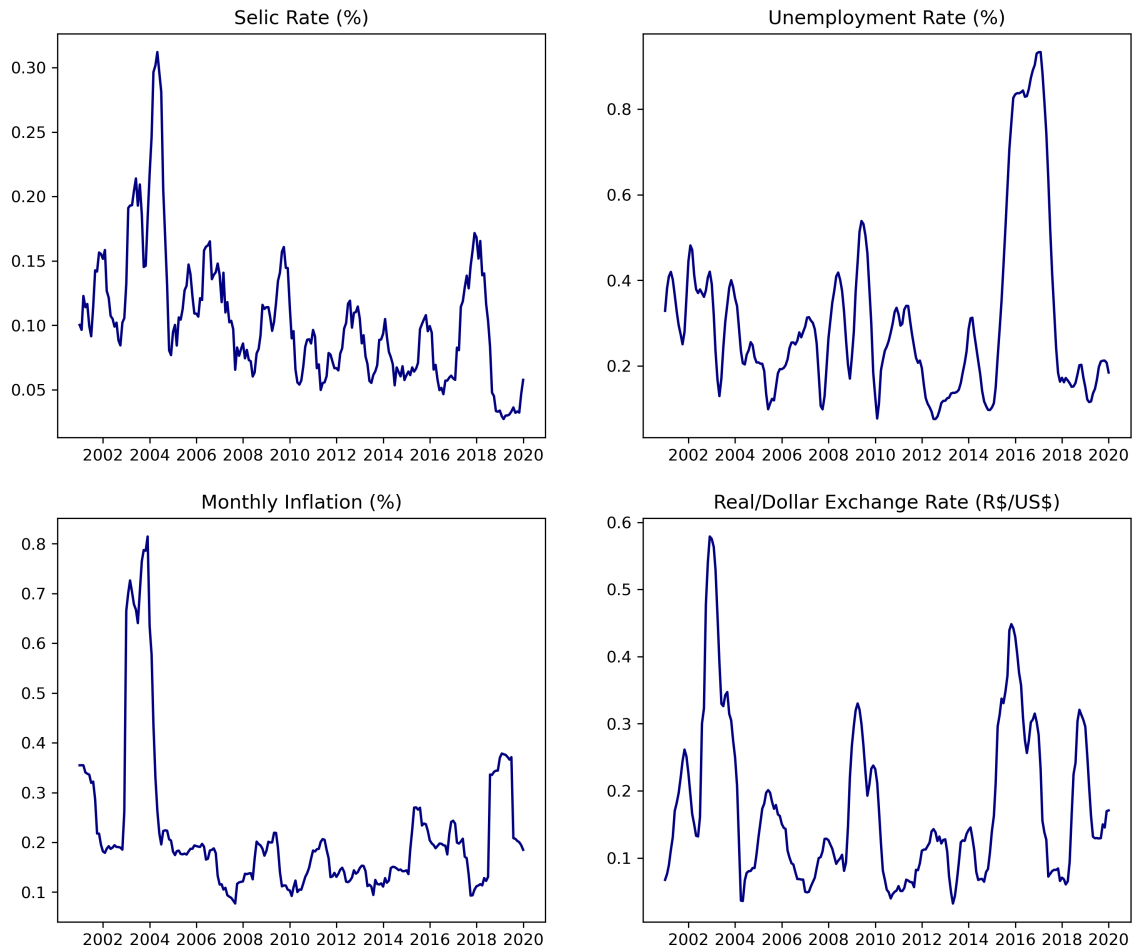


Note: The values closer to 1 indicate more credibility. Source: Issler and Soares (2023)

Lastly, Figure (4) show the 12-month rolling window standard deviation of our variables of interest.

We can utilize the standard deviation of these variables to gauge the level of uncertainty in the economy. Across all the aforementioned plots, it is evident that the standard deviation of these variables exhibits significant fluctuations throughout the entire time frame. However, notable peaks in standard deviation coincide with periods of heightened uncertainty, as previously discussed. In 2003, there were major spikes in the Selic Rate, monthly inflation, and the real/dollar exchange rate. During the 2008-2009

⁴ Other authors, such as Val et al. (2017), also estimated the index for this period because both the target midpoint and the bands remained constant throughout the entire sample, at 4.5% and 2%, respectively. Although these indexes do not cover our entire analysis period, they are still significant for several reasons, they still encompass major crises that Brazil faced, including the 2008 and 2015 crises.

Figure 4 – Rolling Window Standard Deviation of the Variables (2000 - 2020)

Note: each point in it's curve depicts the standard deviation of the last 12 months. For example, the peak in 2004 actually represents the standard deviation of the last 12 months - which corresponds to the year 2003.

crisis, spikes were particularly pronounced in the exchange rate and the unemployment rate. The 2015 crisis witnessed the largest spike in the unemployment rate, a substantial increase in the exchange rate, and a moderate one in monthly inflation. Lastly, spikes in inflation and the exchange rate were observed during the 2018 election, reflecting heightened uncertainty during that period.

This visual analysis suggests a correlation between the efficacy of the inflation targeting regime and the level of economic uncertainty. When market expectations were nearer to the midpoint and central bank credibility was stronger, uncertainty in the Brazilian economy tended to be lower. Naturally, this raises the question of the impact of monetary policy shocks on economic uncertainty.

3 Literature Survey

In this section, we present a brief survey of monetary policy shocks literature. We recommend the interested reader to [Christiano, Eichenbaum and Evans \(1999\)](#) for a

review on the literature from late 1970s to 1990s, Ramey (2016) for a more recent and broader survey on shocks, and Stock and Watson (2016) for a more in-depth analysis of the methodology used in the field.

The meaning of the *shocks* we wish to estimate is closely related to *VAR innovations* and *instruments* (which we will talk about in a moment). Furthermore, often in the literature, these terms are used as if they have the same meaning, but they are not necessarily identical. More precisely, Bernanke (1986) define shocks as *primitive exogenous forces that are economically meaningful and uncorrelated with each other*⁵. The most frequently discussed source of monetary policy shocks are shifts in central bank preferences, caused by changing weights on inflation versus unemployment in the loss function, or by political influences in the central bank (Ramey, 2016).

Concerning the estimation of the impact of these shocks, the literature predominantly features two models: Structural VAR and Dynamic Stochastic General Equilibrium (DSGE) models. The SVAR, introduced by Blanchard and Watson (1986) and Bernanke (1986), rewrites the VAR⁶ model to represent the endogenous variables in terms of underlying structural shocks. These structural shocks represent unexpected exogenous disturbances to structural economic relationships and allow the researcher to establish a link between innovations and theoretical shocks. However, the SVAR faces the *SVAR identification problem* or the problem of identifying the structural shocks. Researchers must impose additional restrictions on the parameters to correctly identify the model. The most common approach is to use *short-run restrictions* on contemporaneous coefficients, also known as *Cholesky decomposition* or *recursive identification scheme*. Other forms of identification methods include narrative methods, sign restrictions, long-run restrictions, high-frequency identification, and external instruments (Ramey, 2016; Stock; Watson, 2016).

Now that the object of interest is defined, we need to look for data to measure it. There is an ongoing debate in the literature on the correct measure of monetary policy shocks. During the 1970s and much of the 1980s, shocks to the stock of money were the main target of investigation (Sims, 1972; Barro, 1977; Barro, 1978). While most of these works show an important contribution from the shocks in output fluctuation, further developments by Sims (1980b) and Litterman and Weiss (1985), with the inclusion of interest rates in the VAR model, found no relevant contribution from the monetary policy

⁵ For Ramey (2016) they should also be the empirical counterparts from the shocks discussed in theory. More rigorously, shocks need to have the following characteristics: 1) should be exogenous concerning the other current and lagged endogenous variables in the model; 2) they should be uncorrelated with other exogenous shocks; and 3) they should represent either unanticipated movements in exogenous variables or news about future movements in exogenous variables.

⁶ The VAR model, introduced by Sims (1980a), revolutionized the study of systems driven by random impulses by linking innovations to a linear system with macroeconomic shocks (Ramey, 2016). Using this method, it became easier to discuss identification assumptions, estimate impulse response functions, and perform innovation accounting through forecast error decomposition.

shocks in explaining economic fluctuations.

The late 1980s saw a reemerge in this debate. [Romer and Romer \(1989\)](#) developed a narrative series on monetary policy shocks, by identifying in FOMC minutes dates in which the Fed imposed a contractionary monetary policy. They found that industrial production decreased significantly after one of these dates⁷. [Bernanke and Blinder \(1992\)](#) showed that interest rate, especially the Fed funds rate, were in fact the key indicator of monetary policy and not the money stock. The Fed fund rates, the 3-month treasury bill, and the 10-month treasury bond outperformed both M1 and M2 in forecast error decomposition.

During the 1990s, several papers focused on the accurate identification of the monetary policy function. For instance, [Christiano and Eichenbaum \(1992\)](#) utilized non-borrowed reserves, while [Strongin \(1995\)](#) proposed isolating a portion of non-borrowed reserves that is orthogonal to total reserves. [Bernanke and Mihov \(1998\)](#) considered regime shifts in the choice of monetary instruments targeted. Another pertinent issue that emerged during this era was the “Price Puzzle”, a term introduced by [Eichenbaum \(1992\)](#) to describe the common observation that a contractionary shock to monetary policy seemed to temporarily increase the price level.

By the 2000s, [Romer and Romer \(2004\)](#) derived a new measure of monetary policy shocks using narrative methods. They first collected changes in the interest rate of the FED funds discussed in the meetings of the Federal Open Market Committee (FOMC). This resulting series removed a significant portion of endogeneity between the interest rate and economic activity. Next, they performed a regression of these changes in the interest rate around the forecasted dates of inflation and real economic activity made by the FED. The residuals from this regression show the intended changes in interest rates that did not take into account the information the FED had about the future of the economy, thereby removing a substantial portion of anticipated movements. Continuing along these lines, [Barackian and Crowe \(2013\)](#) point out that monetary policy in the USA has increasingly relied on the outcomes of its forecasts in decision-making. Therefore, identification schemes that disregard the role of these forecasts end up misspecifying the reaction function of policymakers. To address this issue, they estimate a new measure of monetary policy shocks by using factors derived from changes in prices of Fed Funds futures contracts on the day following the announcements of strategies discussed in FOMC meetings. Both papers found a significant impact of the monetary policy shock on output and inflation

Around this time, authors started also to employ different models from the regular

⁷ This methodology was criticized by both [Shapiro \(1994\)](#) and [Leeper \(1997\)](#), who showed that the Romer and Romer dummy variables were predictable from lagged values of other variables because the narrative method did not properly identify the exogenous shock on the monetary policy. See [Romer and Romer \(1997\)](#) for a response to this claims.

VAR and SVAR models. Some examples are the regime-switching model, which is based on the idea that monetary policy is driven not just by shocks but also by changes in the policy parameters, but most of them found that changes in monetary policy regime are not significant to explain economic fluctuation (Owyang; Ramey, 2004; Primiceri, 2005; Sims; Zha, 2006). Bernanke, Boivin and Elias (2005) utilize a Factor-Augmented VAR (FAVAR), which combines the advantages of VAR models with factor analysis, allowing for better identification of monetary policy shocks and their effects on macroeconomic variables. They conclude that monetary policy has significant and persistent effects on output, consumption, prices, and other macroeconomic variables. This approach is also employed by Boivin, Kiley and Mishkin (2010) along with a DSGE model to study the evolution of the transmission mechanisms of monetary policy shocks over time. The results show that the monetary transmission mechanism evolved significantly during the analyzed period, with credit and expectations channels starting to play an increasingly important role after the 1980s.

A growing part of the literature started to employ DSGE models to better evaluate the transmission channel of the shocks, such as Christiano, Eichenbaum and Evans (2005). In this paper, they address the impact of monetary policy shocks in an economy with nominal rigidity. They conclude that nominal rigidity can deepen the effects of monetary policy shocks on the economy, leading to more persistent fluctuations in output and employment. Smets and Wouters (2007) go further and include in their model frictions in goods, labor, and credit markets, as well as supply shocks, demand shocks, monetary shocks, and fiscal shocks. They conclude that monetary policy shocks are the main source of short-term fluctuations⁸.

Interestingly enough, even with the number of monetary policy instruments presented so far, most of the papers above used the Cholesky Decomposition as the benchmark identification strategy in the VAR models (Christiano; Eichenbaum; Evans, 1999; Ramey, 2016), including the ones that developed external instruments and FAVAR models (Romer; Romer, 2004; Barackian; Crowe, 2013; Bernanke; Boivin; Elias, 2005; Boivin; Kiley; Mishkin, 2010). An important part of the recursive assumption is that shocks to the monetary policy have restricted effects in the short run - which may not be the case in the real world - and quite often the price puzzle appears in the results (Ramey, 2016). Papers that do not apply the recursive strategy generally use sign restrictions - or set identification -, for example, by restricting that a contractionary monetary policy shock cannot raise prices and/or decrease unemployment after the shock. Faust (1998) and Uhlig (2005) argue that this has the benefit of a direct connection with broad economic theories. The standard approach to set identification in SVARs is to use Bayesian meth-

⁸ On the other hand, Krause and Lubik (2007) point out that introducing search and matching frictions to a New Keynesian model with sticky prices is insufficient to replicate persistent effects of a monetary policy shock. Introducing sticky real wages improves labor market behavior but does not help explain the persistent effects of shocks and practically does not affect the inflation dynamics.

ods, however, this introduces new econometric issues for both computation and inference (Stock; Watson, 2016). More recently, Ahmadi and Uhlig (2015) estimated a FAVAR using sign restrictions and found significant impacts of monetary policy shocks on economic fluctuations.

After the 2008 economic crisis, some authors started to give more attention to alternative transmission channels such as the credit channel, the expectation channel, and the uncertainty channel. For instance, Gertler and Karadi (2015) estimate the impacts of monetary policy shocks on credit costs. The authors use a VAR with Fed funds rates and high-quality corporate bond rates to create an external instrument and estimate the effects of monetary policy shocks. They find that modest movements in short-term interest rates lead to larger movements in credit costs, primarily caused by reactions in the term premium and credit spreads. Additionally, the effects of shocks are amplified during periods of financial stress, and the transmission of effects can be slower when credit markets are frozen or illiquid. Wu and Xia (2016) tried to overcome the zero lower bound problem by using a multifactor Shadow Rate Term Structure Model to estimate a shadow federal funds rate. This shadow rate can capture additional features, such as quantitative easing.

Most papers focused on estimating the impacts of the monetary policy shocks on the level of the macroeconomic variables, neglecting the effects on the volatility of macroeconomic variables. Mumtaz and Theodoridis (2020) estimate a structural BVAR with sign restrictions to assess the impact that monetary policy shocks have on the volatility of macroeconomic variables in the United States. The results indicate that a monetary policy shock has a significant effect on macroeconomic volatility in the short term. A 1% increase in the monetary policy instrument (monetary policy shock) increases unemployment and inflation volatility by about 15%. To investigate the transmission channels of the shocks, the authors use a DSGE model with search and matching frictions and Epstein-Zin preferences. Model simulations show that these effects on volatility occur due to the coexistence of fear of unemployment and agents' uncertainties about the monetary authority's ability to reverse deviations from the policy path.

Salisu and Gupta (2021) follow the same approach as Mumtaz and Theodoridis (2020) and evaluate the impact of monetary policy shocks on the volatility of macroeconomic variables in the post-World War II period in the United Kingdom. The main difference is that the authors include the period of unconventional monetary policy with the zero lower bound. The estimates show that a 1% increase in the monetary policy instrument (policy rate), on average, increases the volatility of inflation and unemployment by 10%, with peaks during local and global crises. The authors point out that during economically uncertain periods, a monetary policy with few changes is ideal for reviving the economy.

In the case of Brazil, [Minella \(2003\)](#) was one of the early researchers to estimate the impact of monetary policy shocks. Using a VAR with recursive identification schemes, Minella segmented the sample period into three different intervals of Brazilian history: moderately increasing inflation (1975-1985), high inflation (1985-1994), and low inflation (1994-2000). The results show that monetary policy shocks did not induce a reduction in the inflation rate during the first two periods. However, there are indications that they gained the power to affect prices after the launch of the Real Plan in July 1994. Additionally, these shocks reduced the output level across all three periods.

[Mello and Moccero \(2011\)](#) estimated a New Keynesian Model and a VAR model to study the effects of the inflation target regime with floating exchange rate regimes adopted by Latin American countries, including Brazil. The sample period was 1996 to 2006 for all countries, with the sample split at the respective month each country adopted these policies. The IRFs indicate that, following a contractionary policy shock, a price puzzle and a significant decrease in industrial production occurred in the period from 1996 to July 1999. Moreover, from July 1999 to 2006, both price levels and industrial production decreased. Additionally, their findings suggest that the adopted regime was associated with a greater responsiveness of the monetary authority to changes in expected inflation in Brazil and a lower interest rate volatility. However, the change in the monetary regime had not yet resulted in a reduction in output volatility.

[Minella and Souza-Sobrinho \(2013\)](#) estimates a semi-structural model to decompose the monetary policy effects into four individual channels, using quarterly data from 1999Q3 to 2008Q2. Their results indicate that the household interest rate channel plays the most crucial role in influencing output dynamics. Meanwhile, the combination of the household interest rate channel and the exchange rate channel holds the greatest relevance for understanding inflation dynamics. However, when considering the expectations channel in the decomposition, it emerges as the most significant one for comprehending the responses of inflation to monetary policy decisions.

[Jawadi, Mallick and Sousa \(2016\)](#) estimated the impact of both fiscal and monetary policy shocks on the five BRICS members: Brazil, Russia, India, China, and South Africa. They employed a Panel VAR (PVAR) and a unified framework to track developments and potential spillovers between both policies, using quarterly data spanning from 1990:1 to 2013:2. The results reveal that a monetary policy shock exerts a contractionary impact on real economic activity, leading to a gradual decrease in the price deflator and tighter conditions in the liquidity market.

[Fasolo \(2019\)](#), building on the work of [Mumtaz and Zanetti \(2013\)](#), examines the impact of monetary policy volatility shocks on the industrial production, dollar-real exchange rate and inflation. The results show a significant and persistent effect on macroeconomic variables, particularly on the exchange rate and inflation. The study finds that

an increase in the volatility of shocks leads to higher prices, in contrast to the results of [Mumtaz and Zanetti \(2013\)](#) for the USA. Furthermore, the study highlights that monetary policy is more effective when volatility is lower, and the transmission of volatility shocks to the real economy occurs within a period of up to one year. Therefore, to our knowledge, we are the first to assess the impacts of monetary policy shocks on the volatility of inflation, unemployment and exchange rate in an emerging economy.

4 Methodology ⁹

4.1 Benchmark Model

We use an SVAR model with stochastic volatility, with the following observation equation:

$$Z_t = c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K b_k \tilde{h}_{t-k} + \Omega_t^{1/2} \epsilon_t, \epsilon_t \sim N(0, I_N) \quad (1)$$

Where in equation [\(1\)](#), Z_t is a $N \times 1$ vector of endogenous variables and \tilde{h}_{t-k} is a $N \times 1$ logarithmic vector of stochastic volatilities. The parameter c is the constant, and β_j and b_k are $N \times N$ matrices while I_N denotes the identity matrix of dimension $N \times N$. The covariance matrix of the residuals is time-varying and factored as:

$$\begin{aligned} \Omega_t &= A^{-1} H_t A^{-1'} \\ H_t &= \text{diag}(\exp(\tilde{h}_t)) \end{aligned} \quad (2)$$

Where H_t is a $N \times N$ diagonal matrix that holds the stochastic volatilities of the orthogonalized shocks ($\tilde{h}_t = (h_{1t}, \dots, h_{Nt})$), we use the matrix A to model the contemporary relationship amongst the reduced form shocks.

The transition equation for the stochastic volatilities is given by:

$$\tilde{h}_t = \alpha + \theta \tilde{h}_{t-1} + \sum_{j=1}^K d_j Z_{t-j} + \eta_t, \eta_t \sim N(0, Q), E(\epsilon_t, \eta_t) = 0, \quad (3)$$

where α is the $N \times 1$ constant vector and θ is the $N \times N$ matrix of coefficients on lags. The $N \times N$ matrices d_j allow lagged endogenous variables to impact the log variances. If these coefficients are different than zero then shocks to equation [\(1\)](#) impact \tilde{h}_t , and therefore Ω_t and measures of the unconditional variance of Z_t .

4.2 Data

The data are monthly and run from 2000m1 to 2019m12. We use seasonally adjusted monthly inflation [\[10\]](#), with the IPCA as the consumer price index. For the unemployment rate, we use the retropolated PNADC computed by [Vaz and Barreira \(2021\)](#) also seasonally adjusted. This allowed us to increase the time period of the model for the whole period after the adoption of the inflation targeting regime up to the Covid-19 pandemic. As the monetary policy instrument, we use the Selic accumulated in the month.

⁹ This section is based upon [Mumtaz and Theodoridis \(2020\)](#)

Finally, for the exchange rate we used the commercial real-dollar buy value at the end of the period.

4.3 Estimation and Impulse Response

The model is estimated with Bayesian methods, using Gibbs sampling to approximate the posterior distribution. The sampling procedure together with the prior distribution is stated in detail in Appendix A. We used the first three years to train the model. The estimation was carried over 2003m1-20019m12 with 50,000 draws, burning the first 45,000.

In the benchmark model, we use 2 lags for the endogenous variables in equation (1), 2 lags for the stochastic volatilities in the same equation, and 2 lags for the endogenous variables in the transition equation (3). In the sensitivity checks, we experimented with different lag settings.

To identify shocks in monetary policy we use sign restrictions in the columns of A^{-1} . We identify a contractionary monetary policy shock as a shock that decreases the exchange rate (domestic currency appreciation) and inflation and increases the unemployment rate.

The impulse response function is defined as the difference between the following conditional expectations:

$$IRF_t = E(\log var(Z_{t+k})|\Psi_t, Z_{t-1}, \mu) - E(\log var(Z_{t+k})|\Psi_t, Z_{t-1}) \quad (4)$$

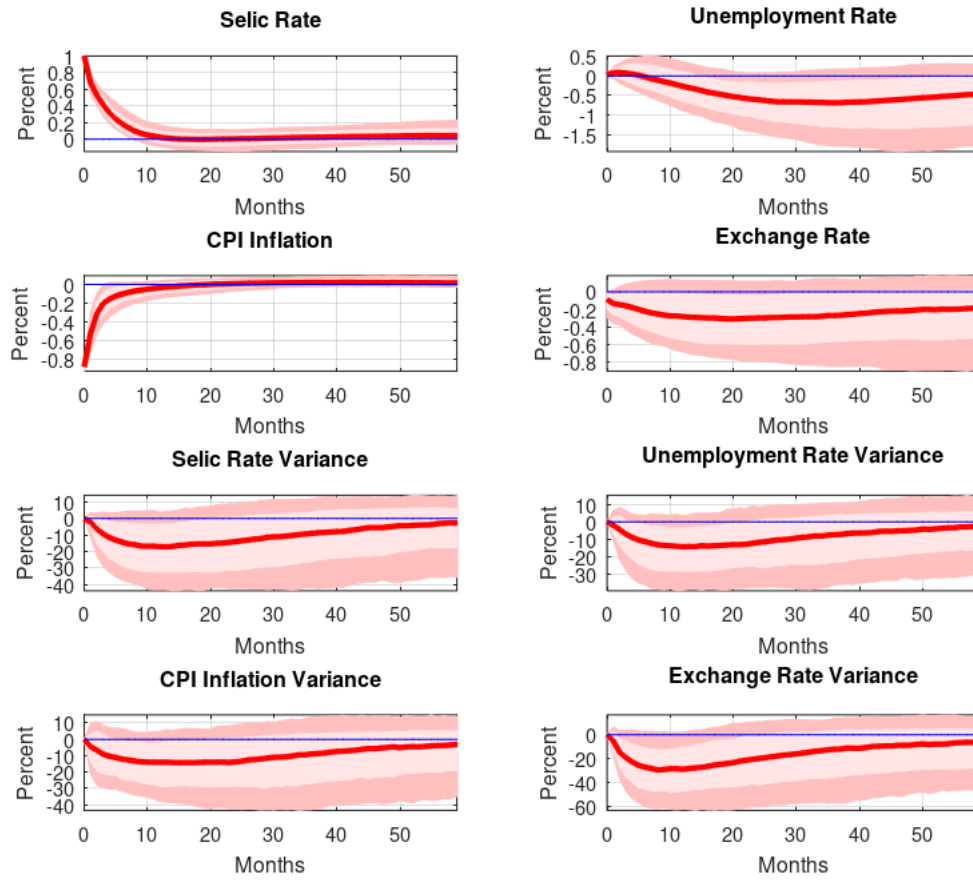
Where Ψ_t are the model parameters and state variables and μ is the monetary policy shock. The first term in equation (4) denotes a forecast of the log volatility conditioned on one of the structural shocks μ . The second term is also the forecast of the log variance but is conditioned on the scenario where the shock equals zero. We approximated this conditional expectation with stochastic simulations. We use 100 simulations to calculate IRF_t repeating this for 500 retained Gibbs draws. In order to account for the history dependence of the non-linear responses, the calculation is done for every 12th month in the sample and we report the median together with 68% and 95% bands.

5 Results

5.1 Impulse Response to a Monetary Policy Shock

Figure (5) displays the Impulse Response Functions (IRF) resulting from a monetary policy shock. The first two rows of the figure show the impact on the level of the variables. Following the shock, inflation decreases by approximately 1%, and this initial impact diminishes over the course of around 5 months, gradually converging back to equilibrium in 15 months. The initial impact on unemployment is negligible, and after 10

¹⁰ We attempted to use the annual CPI inflation as Mumtaz and Theodoridis (2020), but the results showed the existence of a price puzzle. Using the monthly CPI inflation solved this issue.

Figure 5 – Impulse Response to a Monetary Policy Shock

Note: The solid line is the median. The light shaded area is the 68% error band while the dark shaded area is the 95% error band.

months, it continues to decrease for the remainder of the period. As for the exchange rate, there is an appreciation of the Real for up to 20 months, after which it slowly begins to converge back to equilibrium.

The final two rows represent the impact of the shock on unconditional volatility. The volatility of all endogenous variables decreases throughout the entire period after the shock. Specifically, the volatilities of the Selic rate unemployment rate, and inflation decrease by approximately 15% in the first 10 months before gradually increasing. On the other hand, the volatility of the exchange rate experienced a more significant decrease, reaching around 30% in the initial months before it began to rise again.

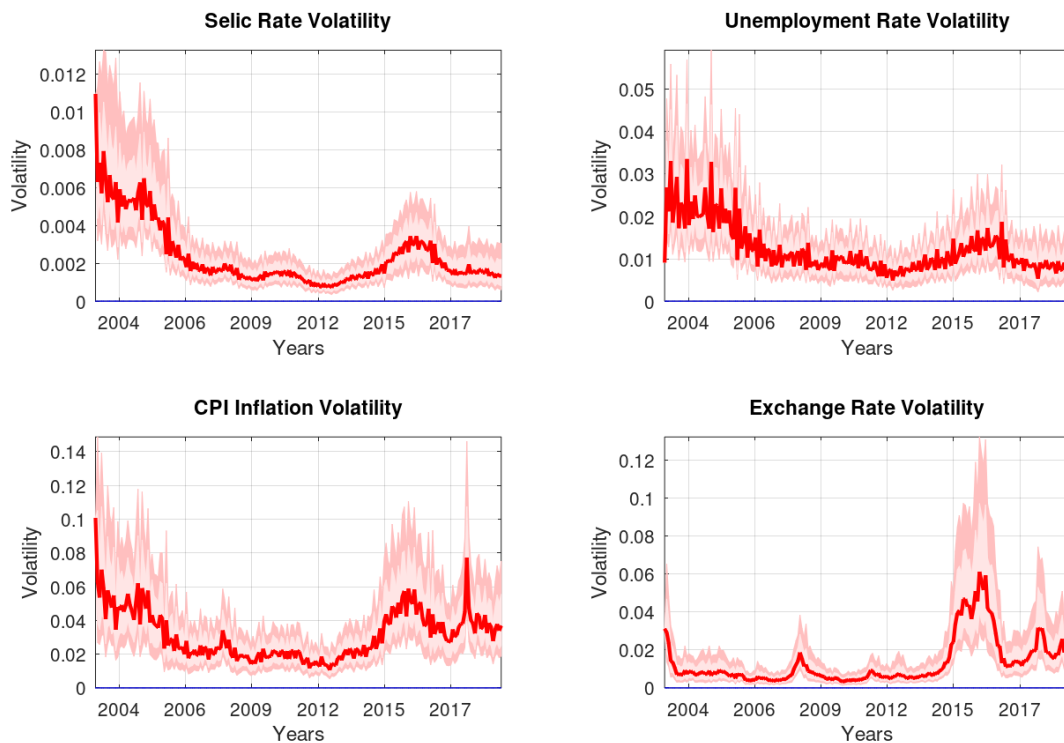
The IRF results differ from those obtained by both [Mumtaz and Theodoridis \(2020\)](#) and [Salisu and Gupta \(2021\)](#). To facilitate a better comparison, we estimated both models for the USA and the UK, using the same settings as ours (the results can be found in Appendix B), and the disparities remained consistent.

As previously discussed in the literature survey, [Mumtaz and Theodoridis \(2020\)](#)

explained the rise in volatility as a consequence of the coexistence of fear of unemployment and agents' uncertainties about the monetary authority's ability to reverse deviations from the policy path. In the Brazilian case, based on the empirical results, this fear of unemployment does not appear to be present¹¹. Unemployment experiences a slight increase in the short term but starts decreasing in less than a year. Regarding the second point, it seems that in our case, agents may perceive a contractionary policy as a step in the right direction for monetary policy.

We argue that the decrease in volatility following a monetary policy shock is a result of increased confidence among agents in the central bank's commitment to inflation-targeting policies. Given the recent history of elevated inflation rates and structural challenges, a rise in the Selic rate can be interpreted as a reaffirmation of the commitment to the adopted policy, which heavily relies on confidence and credibility from the agents. Lastly, Figure (6) illustrates the implicit volatility of the benchmark index during this period.

Figure 6 – Brazilian Implicit Volatility



Note: The solid line is the median. The light shaded area is the 68% error band while the dark shaded area is the 95% error band.

¹¹ One could also argue that the absence of a significant impact on the unemployment rate means a good monetary policy from the Central Bank, which effectively targets the inflation while also reducing the impact on the unemployment. However, here we don't have enough results to support such claims.

In Figure (6), two observations stand out. Firstly, there are noticeable short-term fluctuations in volatility throughout the entire period, which becomes particularly striking when compared to the implicit volatility for the USA and UK, both of which do not exhibit these fluctuations (see Appendix B for reference)¹². Additionally, there is a discernible trend in the volatility of the endogenous variables, which is consistent with both the uncertainty events and the inflation market expectation we mentioned in section 2. This trend began in 2003, declined until 2010, started rising again, reaching its peak in 2015, and eventually declining once more. Additionally, the inflation and exchange rate volatilities display peaks in 2008 - the financial crisis - and in 2018 - a presidential election year in Brazil. This shows that the model was successful in replicating the underlying uncertainty of the economy.

5.2 Robustness Check

5.2.1 Recursive Identification Scheme

We reestimate the benchmark model but now with a recursive identification scheme using Cholesky decomposition. The endogenous variable entered the Cholesky decomposition in the following sequence: Selic rate; unemployment rate; monthly CPI inflation; and real-dollar exchange rate. Figure (7) shows the results:

There are notable differences in the level of response. We have a lower and less persistent impact on the inflation and exchange rate and a rise in unemployment in the first 20 months. However, the results in the unconditional volatility are similar to the sign restriction scheme.

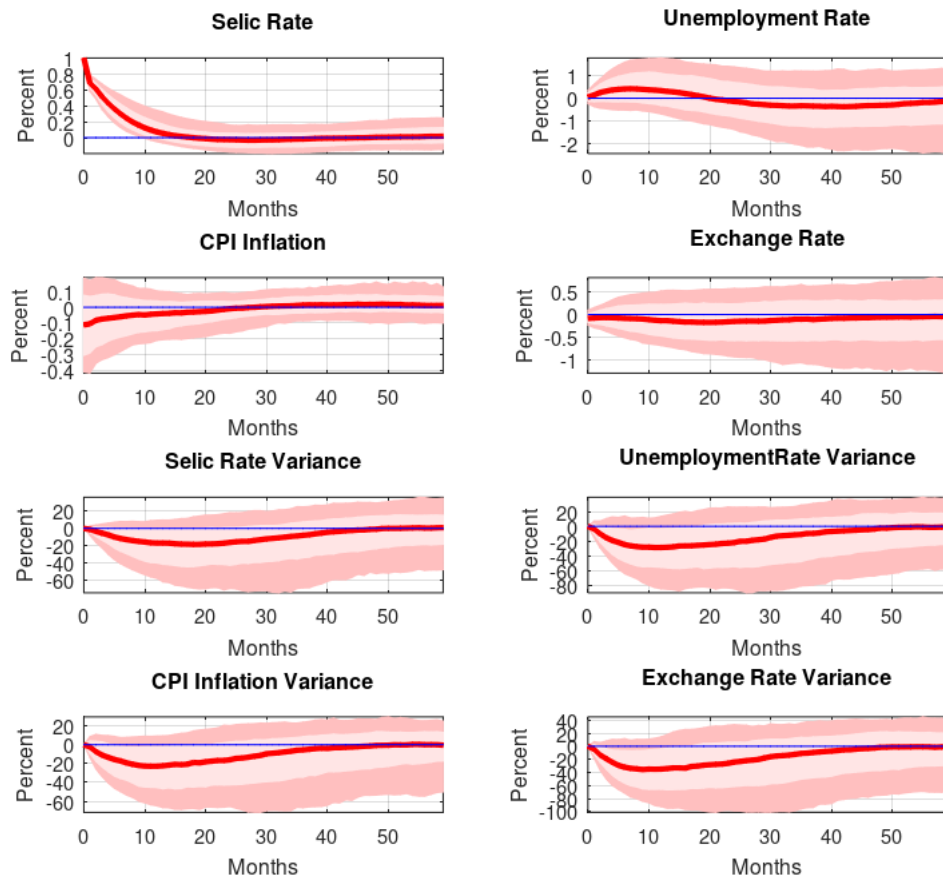
5.2.2 Time Varying Parameters Model

To check the structural stability of the results, we modify the benchmark model to allow the coefficients to vary over time. The modifications made are presented in Appendix A.2. Figure (8) reports the median of the impulse response after the monetary policy shock.

The impact on the level of the endogenous variable remains stable throughout the entire period. However, there are noticeable differences in the impact on volatility. Following the shock, there is a short-term decrease in volatility, which gradually increases until it reaches its peak at the ten-month horizon, stabilizing afterward.

In more recent years, a further and more persistent decrease in volatility is evident. Both the peak at the ten-month horizon and the level of stability are lower than in past years. This trend is particularly pronounced in the volatilities of inflation and the exchange rate.

¹² This is consistent, however, with the rolling window standard deviation we plotted section 2.

Figure 7 – Impulse response with Cholesky Decomposition

Note: The solid line is the median. The light shaded area is the 68% error band while the dark shaded area is the 95% error band.

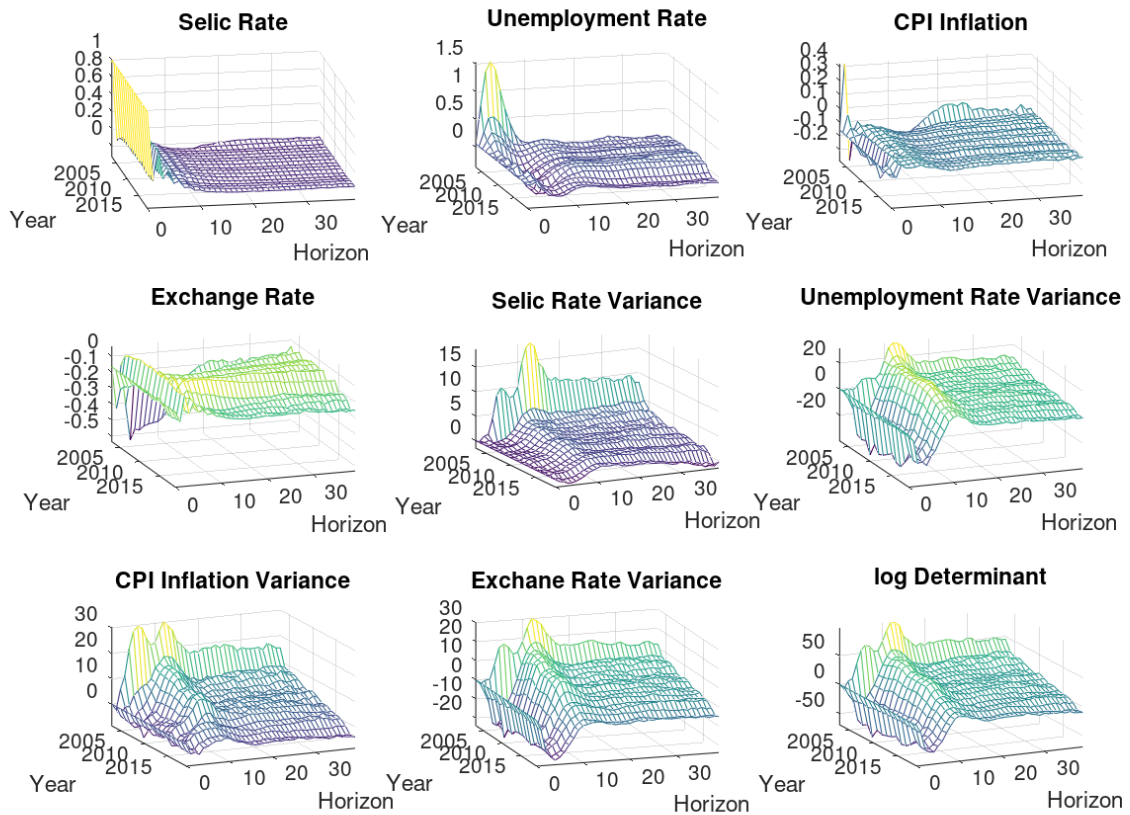
5.2.3 Sensitivity Checks

We explored various settings to assess the robustness of the model. The results remained consistent when using more lags¹³: 6 lags for the endogenous variables in equation (1), 3 lags for the stochastic volatility in the same equation and 3 lags for the endogenous variables in the transition equation (1). Additionally, increasing the number of particles to 300 in the Particle Gibbs Sample did not alter the results. Expanding the training period to cover the years 2000 to 2008 also did not yield any changes in the results.

5.3 Variance Decomposition

Tables (1) and (2) show the importance of the level shock and the volatility shock of all variables to the FEVD, respectively. In the first table, we can see that the

¹³ The benchmark model with fewer lags (2-2-2) had a better fit than the model with more lags (6-3-3) based on the deviance information criterion (DIC). The DIC for the benchmark model is -2200.5 and for the model with more lags is -1754.1.

Figure 8 – Impulse Response with Time-Varying Parameters Model

Note: The impulse response is calculated every 12th month in the sample.

contribution from the monetary policy shock is approximately 2% in the first 12 months, 3.5% in 24 months, and 6.5% in 60 months. This is slightly higher than shocks on unemployment and inflation but below the contribution of the exchange rate to volatility.

These results show a negligible contribution from the monetary policy shock in the short period and with an increase, but still modest, contribution in the long horizon. In relative terms, the Selic shock accounts for around 30% in the total contribution of level shocks and decreases to 25%-27% in the long run, where exchange rates take over most of the contribution. Furthermore, the FEVD results from [Mumtaz and Theodoridis \(2020\)](#) indicate a decrease in the contribution of monetary policy as the time horizon increases, whereas our results show an increase.

The contributions from the volatility shocks paint a similar picture. Shocks to the volatility of monetary policy have the most significant impact on the Selic rate and inflation, followed by the exchange rate, which is also quite important. Conversely, for unemployment and the exchange rate, the most crucial shocks are those affecting the volatility of the exchange rate, followed by the Selic rate.

Table 1 – Contribution from the level shocks to FEV of volatility

Level Shock - FEV Volatility Contribution (%)					
Variable	h	Selic Shock	Unemployment Shock	Inflation Shock	Exchange Rate Shock
Selic	12	1.94 (0.88, 3.86)	1.60 (0.76, 3.46)	2.00 (0.95, 3.78)	2.77 (1.12, 7.62)
	24	3.52 (1.91, 6.56)	3.27 (1.68, 6.10)	3.71 (2.07, 7.07)	5.67 (2.53, 15.41)
	60	6.58 (3.67, 10.88)	5.96 (3.77, 9.35)	6.50 (4.01, 11.03)	9.63 (4.94, 20.58)
Unemployment	12	2.19 (0.93, 5.06)	1.56 (0.60, 3.33)	1.91 (0.75, 4.24)	4.20 (1.18, 10.68)
	24	3.48 (1.75, 7.72)	2.84 (1.44, 6.42)	3.62 (1.79, 7.56)	7.88 (2.85, 19.84)
	60	6.55 (3.25, 11.21)	5.46 (3.21, 9.68)	6.38 (3.62, 11.55)	11.62 (5.27, 24.53)
Inflation	12	1.75 (0.78, 4.09)	1.48 (0.68, 3.12)	1.79 (0.75, 3.92)	4.07 (1.32, 9.53)
	24	3.21 (1.63, 6.80)	2.95 (1.49, 6.48)	3.46 (1.74, 6.93)	7.00 (2.61, 18.20)
	60	6.24 (3.24, 10.86)	5.80 (3.41, 10.09)	5.98 (3.41, 11.21)	10.88 (4.92, 22.86)
Exchange Rate	12	2.24 (0.89, 5.60)	1.47 (0.59, 3.33)	1.93 (0.73, 4.45)	4.37 (1.21, 11.19)
	24	3.66 (1.75, 7.91)	2.67 (1.41, 6.18)	3.57 (1.71, 8.10)	8.18 (2.71, 20.02)
	60	6.51 (3.32, 10.89)	5.38 (3.08, 9.37)	6.33 (3.42, 11.61)	11.90 (5.33, 25.51)

Note: 68 percent error bands in parenthesis.

Table 2 – Contribution from the volatility shocks to FEV of volatility

Volatility Shock - FEV Volatility Contribution (%)					
Variable	h	Selic Shock	Unemployment Shock	Inflation Shock	Exchange Rate Shock
Selic	12	30.76 (14.38, 50.61)	10.48 (4.70, 22.61)	15.87 (7.34, 27.32)	22.00 (7.26, 46.26)
	24	25.26 (13.48, 40.01)	10.70 (5.28, 19.46)	16.29 (7.86, 28.46)	19.11 (7.61, 35.93)
	60	18.50 (10.85, 29.71)	11.33 (6.52, 17.20)	14.73 (8.23, 23.33)	16.23 (8.84, 26.60)
Unemployment	12	15.13 (8.37, 26.63)	7.51 (2.84, 17.34)	8.48 (2.82, 22.82)	47.87 (32.25, 63.39)
	24	14.36 (7.86, 24.65)	7.87 (3.46, 15.73)	9.65 (3.68, 21.23)	36.86 (23.80, 51.27)
	60	12.53 (7.42, 21.99)	9.57 (4.98, 15.32)	11.16 (5.70, 19.64)	24.94 (15.27, 36.80)
Inflation	12	28.34 (13.81, 43.45)	8.74 (3.48, 17.41)	24.13 (8.54, 38.12)	20.77 (7.20, 42.37)
	24	22.64 (12.31, 35.04)	8.74 (3.93, 16.40)	22.01 (9.30, 34.28)	18.06 (7.52, 34.89)
	60	17.18 (9.86, 26.92)	9.94 (5.43, 16.16)	17.01 (9.15, 27.13)	15.61 (8.02, 26.27)
Exchange Rate	12	13.76 (6.66, 24.72)	7.13 (2.67, 15.70)	7.06 (2.26, 19.51)	51.23 (37.66, 65.41)
	24	13.26 (7.21, 23.13)	7.46 (3.34, 14.65)	8.54 (3.28, 18.22)	39.45 (28.00, 54.54)
	60	12.12 (6.46, 21.18)	8.99 (4.70, 14.82)	10.59 (4.94, 18.64)	26.85 (16.83, 39.81)

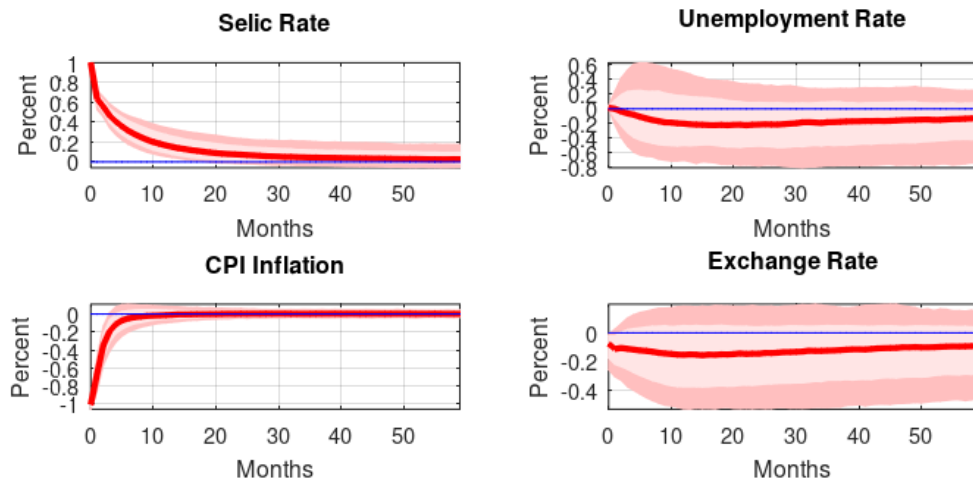
Note: 68 percent error bands in parenthesis.

Finally, Figure (9) displays the impulse response from the restricted version of the model, where the effects of level shocks on second moments are set to be equal to zero (achieved by setting $d_j = 0$ in equation (1)). When compared to Figure (5), it becomes apparent that the shocks on the unemployment rate and inflation are less persistent in the restricted case. This implies that the cumulative change in these variables is estimated to be much smaller when the impact of policy shocks on volatility is assumed away.¹⁴

6 Conclusion

In this paper, we evaluate the impacts of monetary policy level shocks on the volatility of inflation, unemployment rate, and real-dollar exchange rate in Brazil. To do this, we estimate a BVAR model with stochastic volatilities that allows lagged endogenous variables to impact the log variances.

¹⁴ According to the DIC, the benchmark model (-2200.5) also has a better fit than the restricted model (-2122.2).

Figure 9 – Impulse Response with Restricted Model

Note: The solid line is the median. The light shaded area is the 68% error band while the dark shaded area is the 95% error band.

Our results show that after a contractionary monetary policy shock, the unconditional volatility of all variables decreased in the five-year horizon. The variance of inflation, unemployment rate, and Selic rate was reduced by 15% and for the exchange rate, it was 30%. This result was robust across different model settings and shock identification schemes. The IRF from the TVP version of the model shows that the volatility decreases even further in later years when compared to early years in the sample. The FEVD indicates that the contribution from the level shocks is only 1% in the one-year horizon but it increases to 6% in 5 years. The presence of policy-level shocks in the model was shown to be significant to the cumulative change of the variables. These results differ from the ones obtained in the United States and the United Kingdom. In both cases, after a monetary policy shock, volatility increases.

We argue that the decrease in volatility, in the Brazilian case, is due to an increase in the credibility of the BCB in its objective to reach price stability. The implicit volatility of the model is positively correlated with the difference between the market's expectations of inflation and the target midpoint of the BCB. And the volatility is negatively correlated with the credibility indexes of the Central Bank. After a not-so-distant past of hyperinflation in Brazil, agents may perceive contractionary shocks as a signal of higher commitment to price stabilization, lowering uncertainty and, therefore, reducing the volatility of the variables.

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APPENDIX A

Model estimation

1 Benchmark Model

Consider the following VAR model:

$$\begin{aligned} Z_t &= c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K b_k \tilde{h}_{t-k} + \Omega_t^{1/2} \epsilon_t, \epsilon_t \sim N(0, I_N) \\ \Omega_t &= A^{-1} H_t A^{-1'} \\ \tilde{h}_t &= \alpha + \theta \tilde{h}_{t-1} + \sum_{j=1}^K d_j Z_{t-j} + \eta_t, \eta_t \sim N(0, Q), E(\epsilon_t, \eta_t) = 0 \end{aligned} \tag{5}$$

Where $\tilde{h}_t = (h_{1t}, \dots, h_{Nt})$, $H_t = \text{diag}(\exp(\tilde{h}_t))$.

1.1 Priori distributions and initial values

1.1.1 VAR coefficients

Define $\Gamma := (c, \beta_j, b_k)$ as the vector of coefficients from the equation observation (5). Following Banbura, Reichlin and Giannone(2007), we implement a normal prior distribution using dummy variables, y_D and x_D , defined as:

$$y_D = \begin{bmatrix} \frac{\text{diag}(\gamma_1 s_1 \dots \gamma_n s_n)}{\tau} \\ 0_{N(P-1) \times N} \\ 0_{EX \times N} \end{bmatrix}, \quad x_D = \begin{bmatrix} \frac{J_P \otimes \text{diag}(s_1 \dots s_n)}{\tau} 0_{NP \times EX} \\ 0_{N \times (NP+EX)} \\ 0_{EX \times NP} \quad I_{EX} \cdot 1/c \end{bmatrix}$$

Where: γ_i denotes the means of the priors for the parameters in the first lag obtained from individual AR(1) regression estimates; τ is the degree of tightness of the prior on the coefficients of the VAR; c is the degree of restriction of the prior on the exogenous and predetermined regressors; EX is the number of exogenous and predetermined regressors; N is the number of exogenous variables; and P the lag length.

We use $\tau = 0.1$ and $c = 0.1$ for the coefficients of the lagged volatilities, and for

the intercept, we use a loose prior with $c = 1000$.

Thus, the coefficient's prior distribution is $N(\Gamma_0, P_0)$ where $\Gamma_0 = (x_D' x_D)^{-1} (x_D' y_D)$ and $P_0 = S \otimes (x_D' x_D)^{-1}$ with S as a diagonal matrix diagonal with an estimate of the variance of Z_t (obtained using the training sample described below) on the main diagonal.

1.1.2 Elements of H_t

Following [Cogley and Sargent\(2005\)](#) we use a training sample (36 initial values, from January 2000 to January 2003) to establish prior beliefs for the elements of the transition equation. Let \hat{v}^{ols} be the covariance matrix of the VAR model estimated in the training sample using Ordinary Least Squares (OLS). The prior for \tilde{h}_t in $t = 0$ is $\log \tilde{h}_0 \sim N(\log \mu_0, I_4)$, where μ_0 is a vector with the elements of the main diagonal of the Cholesky decomposition of \hat{v}^{ols} .

1.1.3 Elements of A

The prior for the elements outside the diagonal of matrix A is $A_0 \sim N(\hat{a}^{ols}, V(\hat{a}^{ols}))$, where \hat{a}^{ols} are the off-diagonal elements of \hat{v}^{ols} with each row scaled by the corresponding element on the diagonal. $V(\hat{a}^{ols})$ is, by assumption, a diagonal matrix with elements equal to 100. We use a tighter prior for the elements when we impose sign restrictions.

1.1.4 Parameters of the transition equation

For the transition equation, the prior for the coefficients and the error covariance are implemented with dummy variables, reducing each equation to an autoregressive process. This artificial dataset also includes dummy variables that implement the inverse Wishart distribution as a prior for Q and for the coefficients in the predetermined regressors. A narrowness level of 0.05 is used for both the priors on the coefficients of lagged volatilities and the lags of the predetermined variables.

1.2 Simulating the posterior distribution

1.2.1 VAR coefficients

The distribution of the VAR coefficients, Γ , conditional on all other parameters, Ξ , and the stochastic volatility, \tilde{h}_t , is linear and Gaussian: $\Gamma|Z_t, \tilde{h}_t, \Xi \sim N(\Gamma_{T|T}, P_{T|T})$, where $\Gamma_{T|T} = \mathbb{E}(\Gamma_T|Z_t, \tilde{h}_t, \Xi)$, $P_{T|T} = Cov(\Gamma_T|Z_t, \tilde{h}_t, \Xi)$. Following [Carter and Kohn\(1994\)](#) we use the Kalman filter to estimate $\Gamma_{T|T}$ and $P_{T|T}$, taking into account that the covariance matrix of the VAR residuals changes through time. Since the coefficients are conditioned on \tilde{h}_t and A , the form of heteroscedasticity is known. To use the Kalman filter, we rewrite the VAR in state-space form:

$$\begin{aligned}
y_t &= x_t \Gamma_t + (A^{-1} H_t A^{-1'})^{1/2} e_t \\
\Gamma_t &= \Gamma_{t-1}
\end{aligned} \tag{6}$$

The filter is initialized in Γ_0 and P_0 , and the recursions are given by the following equations:

$$\begin{aligned}
\Gamma_{t|t-1} &= \Gamma_{t-1|t-1} \\
P_{t|t-1} &= P_{t-1|t-1} \\
\eta_{t|t-1} &= y_t - x_t \Gamma_{t|t-1} \\
f_{t|t-1} &= x_t P_{t|t-1} x_t' + (A^{-1} H_t A^{-1'}) \\
K_t &= P_{t|t-1} x_t' f_{t|t-1}^{-1} \\
\Gamma_{t|t} &= \Gamma_{t|t-1} + K_t \eta_{t|t-1} \\
P_{t|t} &= P_{t|t-1} - K_t x_t P_{t|t-1}
\end{aligned} \tag{7}$$

The final iteration of the Kalman filter at time T results in $\Gamma_{T|T}$ e $P_{T|T}$. This application of the [Carter and Kohn\(1994\)](#) algorithm to this heteroscedastic VAR model is equivalent to a GLS transformation of the model.

1.2.2 Elements of A_t

Given a drawn of Γ and \tilde{h}_t , the VAR model can be rewritten as:

$$A'(\tilde{Z}_t) = e_t \tag{8}$$

where:

$$\tilde{Z}_t = Z_t - c + \sum_{j=1}^P \beta_j Z_{t-j} = v_t \tag{9}$$

$$\text{VAR}(e_t) = H_t$$

this is a system of linear equations with a known form of heteroscedasticity. The conditional distributions for a simple linear regression apply to each equation of the system after a simple Generalized Least Squares (GLS) transformation to make the errors homoscedastic. The $i - th$ equation of the system is given by:

$$v_{it} = -\alpha v_{-it} + e_{it} \tag{10}$$

Where the subscript i denotes the $i - th$ column while $-i$ denotes the columns from 1 to $i - 1$. The variance of e_{it} changes through time and is given by $\exp(\tilde{h}_{it})$. A GLS transformation is performed by diving both sides of the equation by $(\exp(\tilde{h}_{it}))^{1/2}$, resulting in:

$$v_{it}^* = -\alpha v_{-it}^* + e_{it}^* \tag{11}$$

where $*$ denotes the transformed variables and $\text{var}(e_{it}^*) = 1$. The posterior distribution of α is a normal distribution with mean M^* and variance V^* given by:

$$\begin{aligned}
M^* &= (V(\hat{a}^{ols})^{-1} + v_{-it}^{*'} v_{-it}^*)^{-1} (V(\hat{a}^{ols})^{-1} \hat{a}^{ols} + v_{-it}^{*'} v_{-it}^*) \\
V^* &= (V(\hat{a}^{ols})^{-1} + v_{-it}^{*'} v_{-it}^*)^{-1}
\end{aligned} \tag{12}$$

1.2.3 Elements of H_t

Conditioned on the coefficients of the VAR and the parameters of the transition equation, the model can be presented as a multivariate and nonlinear state-space model. To sample from the posterior distribution of \tilde{h}_t , we use the Gibbs particle filter with ancestor sampling.

The Gibbs particle filter was introduced in the work of [Andrieu, Doucet and Holenstein \(2010\)](#). The authors demonstrate how a version of the particle filter, conditioned on a fixed trajectory of one of the particles, can be used to generate samples resulting in a Markov kernel with an invariant target distribution.

However, a common problem in the particle filter is the degeneracy of the paths, which can lead to poor mixing in the original version of the Gibbs particle filter. Fortunately, there are small modifications that can be made to the algorithm to significantly alleviate this issue. In particular, [Lindsten, Jordan and Schön \(2014\)](#) propose adding a step for sampling “ancestors” - indices associated with the particle on which they are conditioned. This ancestor sampling breaks the reference path into pieces, causing the particle system to collapse towards something different from the reference path. The authors demonstrate that this step results in a substantial improvement in the mixing of the algorithm, even with a small number of particles.

Let $\tilde{h}_t^{(i-1)}$, with $t = 1, \dots, T$, be the fixed trajectory obtained in the previous generation of the Gibbs algorithm, Ξ the parameters of the model and $j = 1, 2, \dots, M$ the particles. The particle filter with ancestor sampling proceeds with the following steps:

1. For $t = 1$:

- a) Draw $(\tilde{h}_1^{(j)} | \tilde{h}_0^{(j)}, \Xi)$ for $j = 1, 2, \dots, M - 1$. Fix $\tilde{h}_1^{(M)} = \tilde{h}_1^{(i-1)}$
- b) Compute the normalised weights $p_1^{(j)} = \frac{w_1^{(j)}}{\sum_{j=1}^M w_1^{(j)}}$ where $w_1^{(j)}$ denotes the conditional likelihood: $|\Omega_1^{(j)}|^{-0.5} - 0.5 \exp\left(\tilde{e}_1 \left(\Omega_1^{(j)}\right)^{-1} \tilde{e}_1'\right)$ where $\tilde{e}_1 = Z_t - (c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K b_k \tilde{h}_{1,[-k]}^{(j)})$ and $\Omega_1^{(j)} = A^{-1} H_1^{(j)} A^{-1'}$ with $H_1^{(j)} = \text{diag}(\exp(\tilde{h}_{1,[0]}^{(j)}))$. The subscript $[0]$ denotes the contemporaneous value in the state vector while $[-k]$ denotes the k lagged states.

2. For $t = 2$ to T :

- a) Resample $\tilde{h}_{t-1}^{(j)}$ for $j = 1, 2, \dots, M - 1$ using indices $a_t^{(j)}$ with $\Pr(a_t^{(j)} = j) \propto p_{t-1}^{(j)}$
- b) Draw $(\tilde{h}_t^{(j)} | \tilde{h}_{t-1}^{(a_t^{(j)})}, \Xi)$ for $j = 1, 2, \dots, M - 1$ using the transition equation of the model. Note that $\tilde{h}_{t-1}^{(a_t^{(j)})}$ denotes the resampled particles in step a) above.
- c) Fix $\tilde{h}_t^{(M)} = \tilde{h}_t^{(i-1)}$

d) Sample $a_t^{(M)}$ with $\Pr(a_t^{(M)} = j) \propto p_{t-1}^{(j)} \Pr(\tilde{h}_t^{(i-1)} | \tilde{h}_{t-1}^{(j)}, \alpha, \theta, d, Q)$ where the density $\Pr(\tilde{h}_t^{(i-1)} | \tilde{h}_{t-1}^{(j)}, \alpha, \theta, d, Q)$ is computed as $|Q|^{-0.5} \exp(\tilde{\eta}_t^{(j)}(Q)^{-1} \tilde{\eta}_t^{(j)})$ with $\tilde{\eta}_t^{(j)} = \tilde{h}_t^{(i-1)} - (\alpha + \theta \tilde{h}_{t-1}^{(j)} + \sum_{j=1}^K d_j Z_{t-j})$. This constitutes the ancestor sampling step. If $a_t^{(M)} = M$ then the algorithm collapses to the simple particle Gibbs.

e) Update the weights $p_t^{(j)} = \frac{w_t^{(j)}}{\sum_{j=1}^M w_t^{(j)}}$ where $w_1^{(j)}$ denotes the conditional likelihood: $|\Omega_t^{(j)}|^{-0.6} - 0.5 \exp(\tilde{e}_t (\Omega_t^{(j)})^{-1} \tilde{e}_t')$ where $\tilde{e}_t = Z_t - (c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K b_k \tilde{h}_{t,[-k]}^{(j)})$ and $\Omega_t^{(j)} = A^{-1} H_t^{(j)} A^{-1'}$ with $H_t^{(j)} = \text{diag}(\exp(\tilde{h}_{t,[0]}^{(j)}))$.

3. End

4. Sample $\tilde{h}_t^{(i)}$ with $\Pr(\tilde{h}_t^{(i)} = \tilde{h}_t^{(j)}) \propto p_T^{(j)}$ to obtain a draw from the conditional posterior distribution

We use $M = 50$ particles, with the aforementioned μ_0 values initializing the filter.

1.2.4 Parameters of the transition equation

Conditional on the draws of volatilities, the vector with the parameters of the transition equation $\bar{B} := (\alpha, \theta, d_1, \dots, d_j)$ has a normal conditional posterior distribution. Let y and x be the left and the right side of the transition equation, respectively. The posterior distribution is:

$$G(\bar{B} | \Xi) \sim N(B^*, Q \otimes (x^{*'} x^*)^{-1})$$

Where x^* and y^* are x and y appended with dummy observations, and $B^* = (x^{*'} x^*)^{-1} x^{*'} y^*$. The conditional posterior Q is a inverse Wishart distribution given by:

$$G(Q | \Xi) \sim IW(S^*, T^*)$$

Where T^* is the total number of observations — including the dummy variables — and $S^* = (y^* - x^* B^*)' (y^* - x^* B^*)$.

2 Model with Time Varying Parameters

We also estimate the following version of the model:

$$\begin{aligned} Z_t &= c + \sum_{j=1}^P \beta_{t,j} Z_{t-j} + \sum_{k=1}^K b_{t,k} \tilde{h}_{t-k} + \Omega_t^{1/2} \epsilon_t, \epsilon_t \sim N(0, I_N) \\ \Omega_t &= A^{-1} H_t A^{-1'} \\ \tilde{h}_t &= \alpha + \theta \tilde{h}_{t-1} + \sum_{j=1}^K d_{t,j} Z_{t-j} + \eta_t, \eta_t \sim N(0, Q), E(\epsilon_t, \eta_t) = 0 \end{aligned} \tag{13}$$

Defining the vectors:

$$\underbrace{\Theta_t}_{N(NP+NK+1) \times 1} := (c_t, \beta_{t,1}, \dots, \beta_{t,P}, b_{t,1}, \dots, b_{t,K})$$

$$\underbrace{\Psi_t}_{N(N+NK+1) \times 1} := (\alpha_t, \theta_t, d_{t,1}, \dots, d_{t,K})$$
(14)

The evolution of the coefficients is determined by two additional transition equations:

$$\begin{aligned}\Theta_t &= \Theta_{t-1} + \tilde{Q}_1^{1/2} v_{1t} \\ \Psi_t &= \Psi_{t-1} + \tilde{Q}_2^{1/2} v_{2t}\end{aligned}$$
(15)

The prior distribution of $\tilde{Q}_1^{1/2}$ and $\tilde{Q}_2^{1/2}$ is a inverse Wishart, with time varying parameters following [Benati and Mumtaz \(2007\)](#):

$$\begin{aligned}P(\tilde{Q}_1) &\sim IW(\tilde{Q}_{1,0}, \tilde{T}_{1,0}) \\ P(\tilde{Q}_2) &\sim IW(\tilde{Q}_{2,0}, \tilde{T}_{2,0})\end{aligned}$$

where $\tilde{T}_{1,0} = \dim(\tilde{Q}_{1,0} + 1)$, $\tilde{T}_{2,0} = \dim(\tilde{Q}_{2,0} + 1)$ are scalar matrices computed with:

$$\begin{aligned}\tilde{Q}_{1,0} &= V_{1,0} \cdot T_0 \cdot \kappa \\ \tilde{Q}_{2,0} &= V_{2,0} \cdot T_0 \cdot \kappa\end{aligned}$$

Where $V_{1,0}$ and $V_{2,0}$ are covariance matrices of Θ and Ψ , respectively, obtained via an OLS estimation of a time inverting VAR model, with a training sample $T_0 = 36$ and an initial value of \tilde{h}_t . Lastly, κ is a scaling parameter, with $\kappa = 0.0001$.

The Gibbs algorithm requires a modification in the generation of the coefficients of the VAR and the coefficients of the transition equation. Conditioned on the remaining parameters, the model has a linear state-space representation, and the Carter and Kohn algorithm can be used to generate Θ_t and Ψ_t . Thus, given Θ_t and Ψ_t , the covariances \tilde{Q}_1 and \tilde{Q}_2 are generated from the inverse Wishart distribution:

$$\begin{aligned}\tilde{Q}_1 &\sim IW((\Theta_t - \Theta_{t-1})'(\Theta_t - \Theta_{t-1}), \tilde{T}_{1,0} + T) \\ \tilde{Q}_2 &\sim IW((\Psi_t - \Psi_{t-1})'(\Psi_t - \Psi_{t-1}), \tilde{T}_{2,0} + T)\end{aligned}$$

APPENDIX B

Further results

3 Estimating the model for the USA and UK

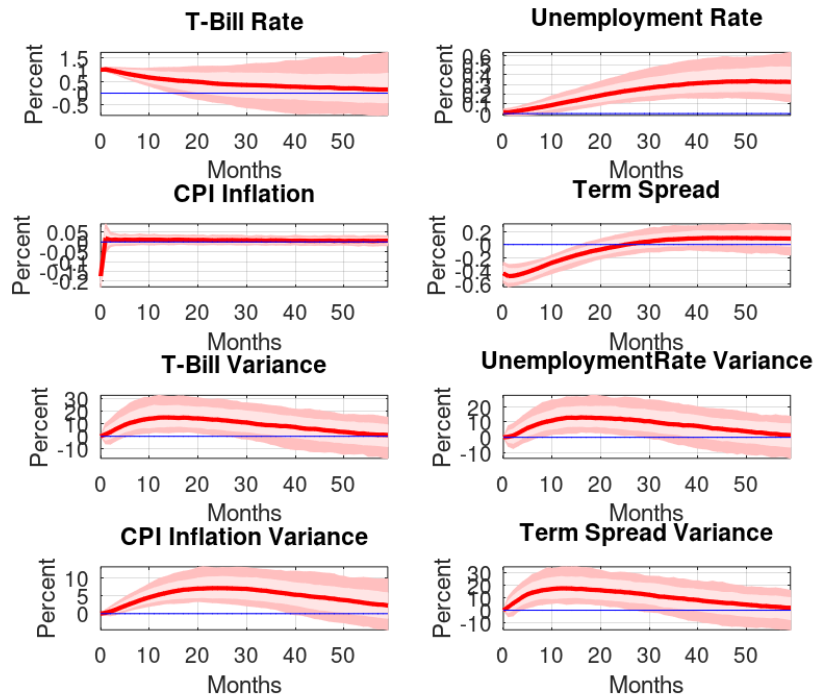
We estimate a model for the USA and UK economies restricted to our time period for better result comparison. There are notable differences between our version and the original estimated by Mumtaz and Theodoridis (2020) and Salisu and Gupta (2021). First is the sample restriction to 2000m1-2019m12. Second, we utilized the same lag setting of our model with 36 month training sample. Third, we used the monthly inflation instead of the annual inflation. Lastly, we follow Salisu and Gupta (2021) and use the Shadow Rate estimated by Wu and Xia (2016)¹⁵ instead of the 3 month T-bill rate for both USA and UK for the months after 2008m12 to account for the zero lower bound.

Figures (10) and (11) show the IRF from the benchmark models. Figures (12) and (13) show the implicit volatility of the models.

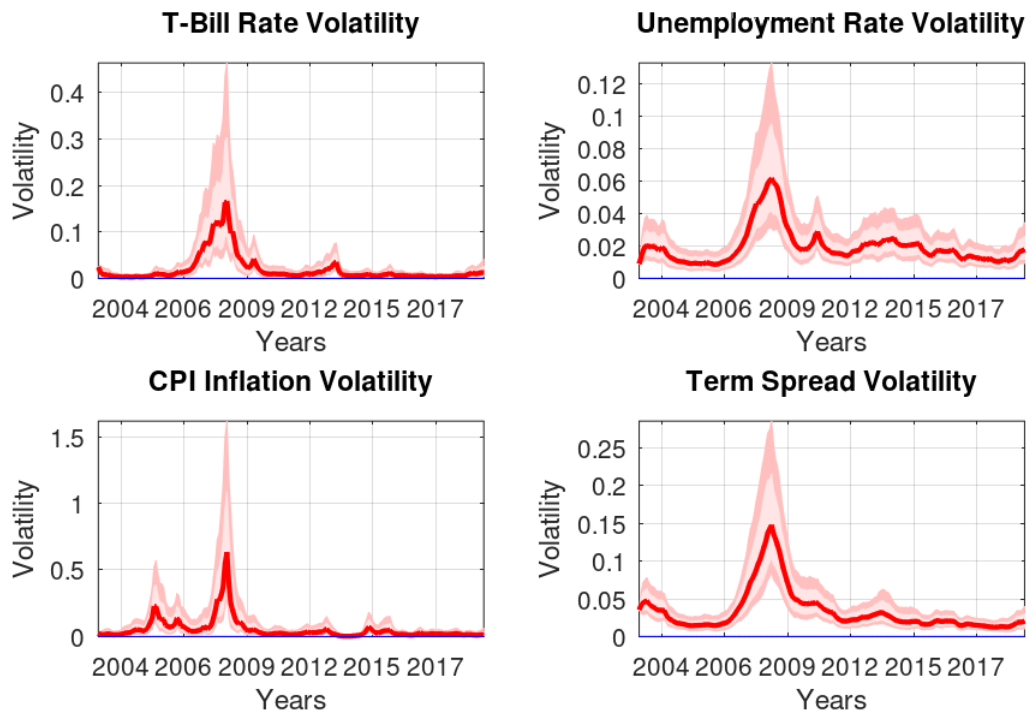
¹⁵ The data can be visualized and downloaded at <<https://sites.google.com/view/jingcynthiawu/shadow-rates?authuser=0>>

Figure 10 – Impulse Response Function for the USA

Note: The solid line is the median. The light shaded area is the 68% error band while the dark shaded area is the 95% error band.

Figure 11 – Impulse Response Function for UK

Note: The solid line is the median. The light shaded area is the 68% error band while the dark shaded area is the 95% error band.

Figure 12 – USA Implicit Volatility**Figure 13 – UK Implicit Volatility**