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Silva Figueiredo et al.

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(54) **READ CURVED VISUAL MARKS**

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G06K 1/12	(2006.01)

(52) **U.S. Cl.**

CPC ***G06K 19/06037*** (2013.01); ***G06K 1/121*** (2013.01); ***G06K 7/1443*** (2013.01); ***G06K 7/1491*** (2013.01); ***G06K 19/06075*** (2013.01)

(58) **Field of Classification Search**

CPC G06K 19/06131; G06K 19/06075; G06K 7/1417; G06K 7/1456; G06K 7/1419

(Continued)

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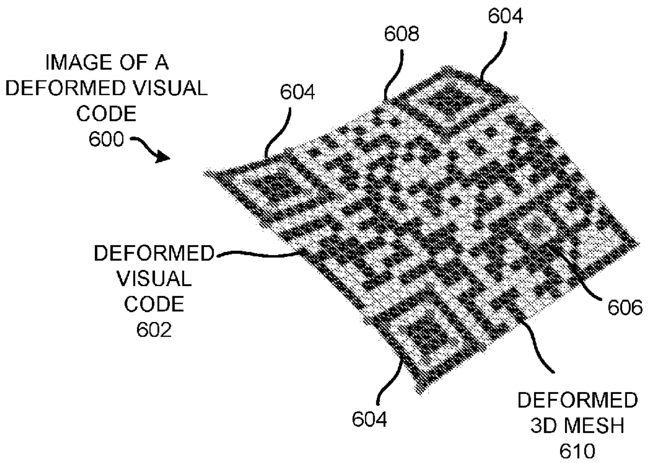
(57) **ABSTRACT**

According to examples, an apparatus may include a processor and a non-transitory computer readable medium on which is stored instructions that may cause the processor to create a 2D reference mesh for an image of a curved visual mark, establish correspondences between finder pattern points in the curved visual mark and points of the 2D reference mesh, and determine a curved 3D mesh having a

(Continued)



radius that results in a minimal reprojection error of a projective transform estimated for correspondences between the 2D reference mesh and the curved 3D mesh while the radius remains below a predefined upper limit. The instructions may also cause the processor to sample components of the curved visual mark in elements of the determined curved 3D mesh to form a 2D planar image of the curved visual mark and analyze the 2D planar image of the curved visual mark to read the curved visual mark.

20 Claims, 12 Drawing Sheets

(58) **Field of Classification Search**
USPC 235/462.1, 462.09
See application file for complete search history.

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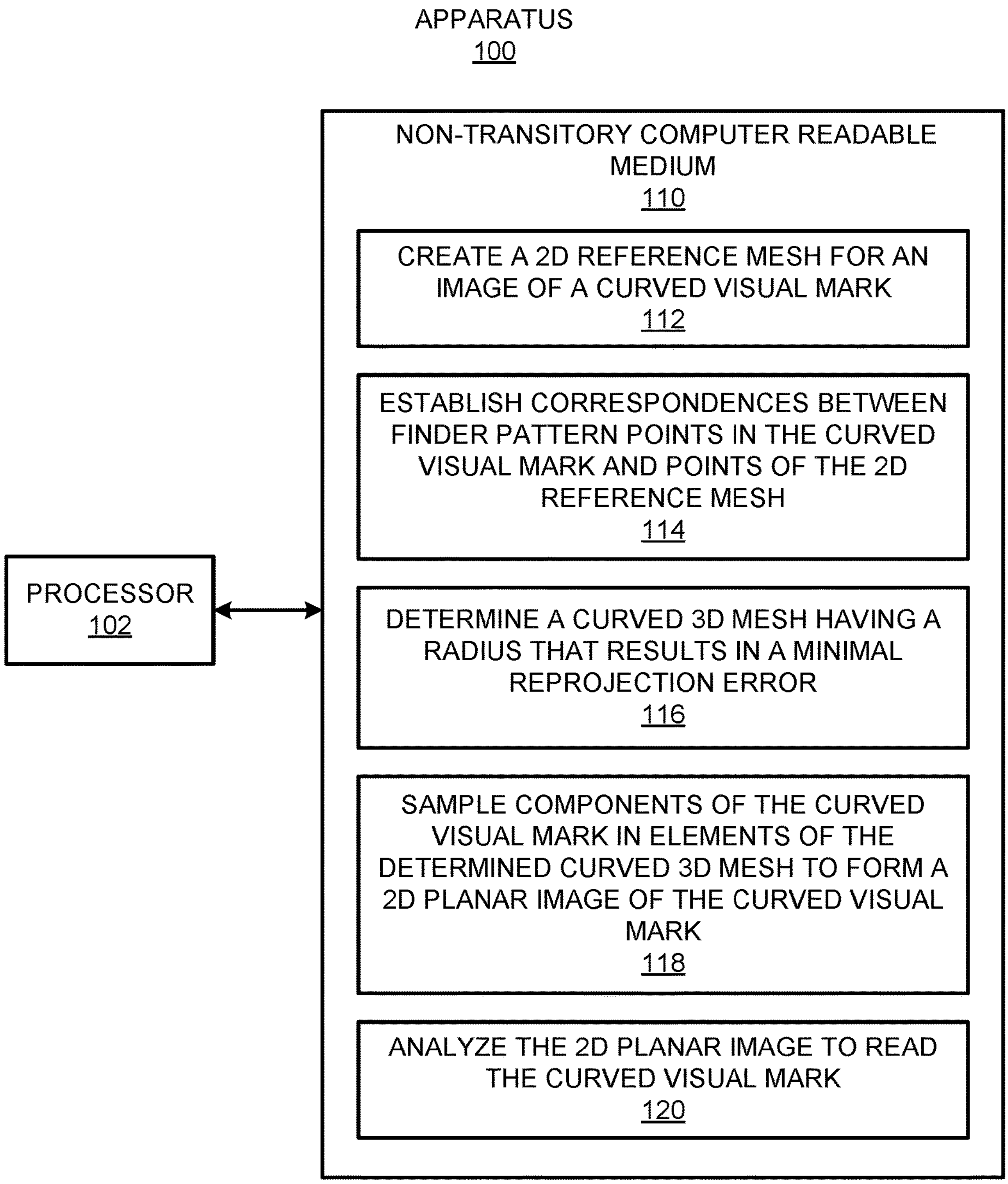
APPARATUS
100
NON-TRANSITORY COMPUTER READABLE MEDIUM
110
CREATE A 2D REFERENCE MESH FOR AN IMAGE OF A CURVED VISUAL MARK
112
ESTABLISH CORRESPONDENCES BETWEEN FINDER PATTERN POINTS IN THE CURVED VISUAL MARK AND POINTS OF THE 2D REFERENCE MESH
114
DETERMINE A CURVED 3D MESH HAVING A RADIUS THAT RESULTS IN A MINIMAL REPROJECTION ERROR
116
SAMPLE COMPONENTS OF THE CURVED VISUAL MARK IN ELEMENTS OF THE DETERMINED CURVED 3D MESH TO FORM A 2D PLANAR IMAGE OF THE CURVED VISUAL MARK
118
ANALYZE THE 2D PLANAR IMAGE TO READ THE CURVED VISUAL MARK
120
PROCESSOR
102

FIG. 1

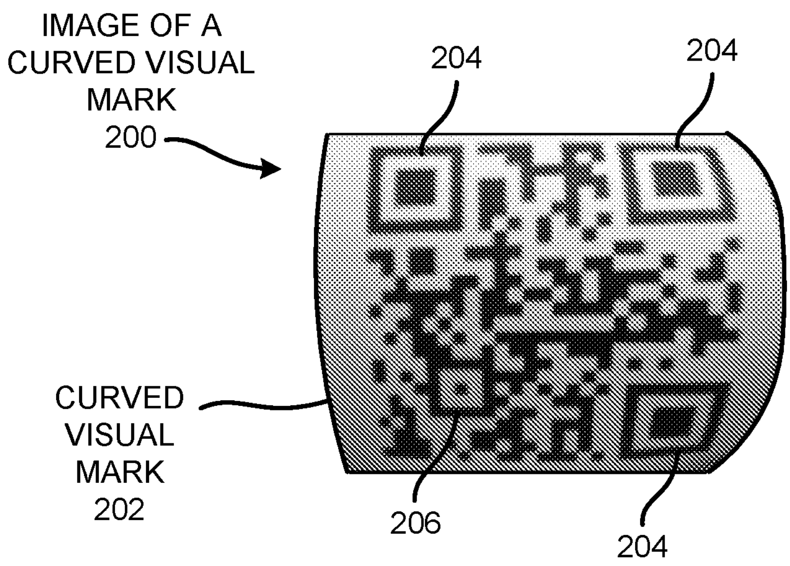
IMAGE OF A
CURVED VISUAL
MARK
200
204
204
CURVED
VISUAL
MARK
202
206
204

FIG. 2A

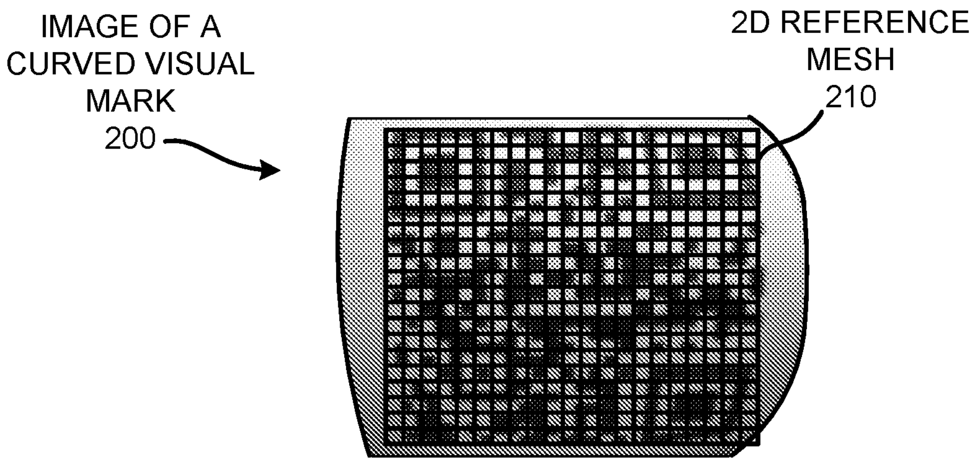
IMAGE OF A
CURVED VISUAL
MARK
200
2D REFERENCE
MESH
210

FIG. 2B

300

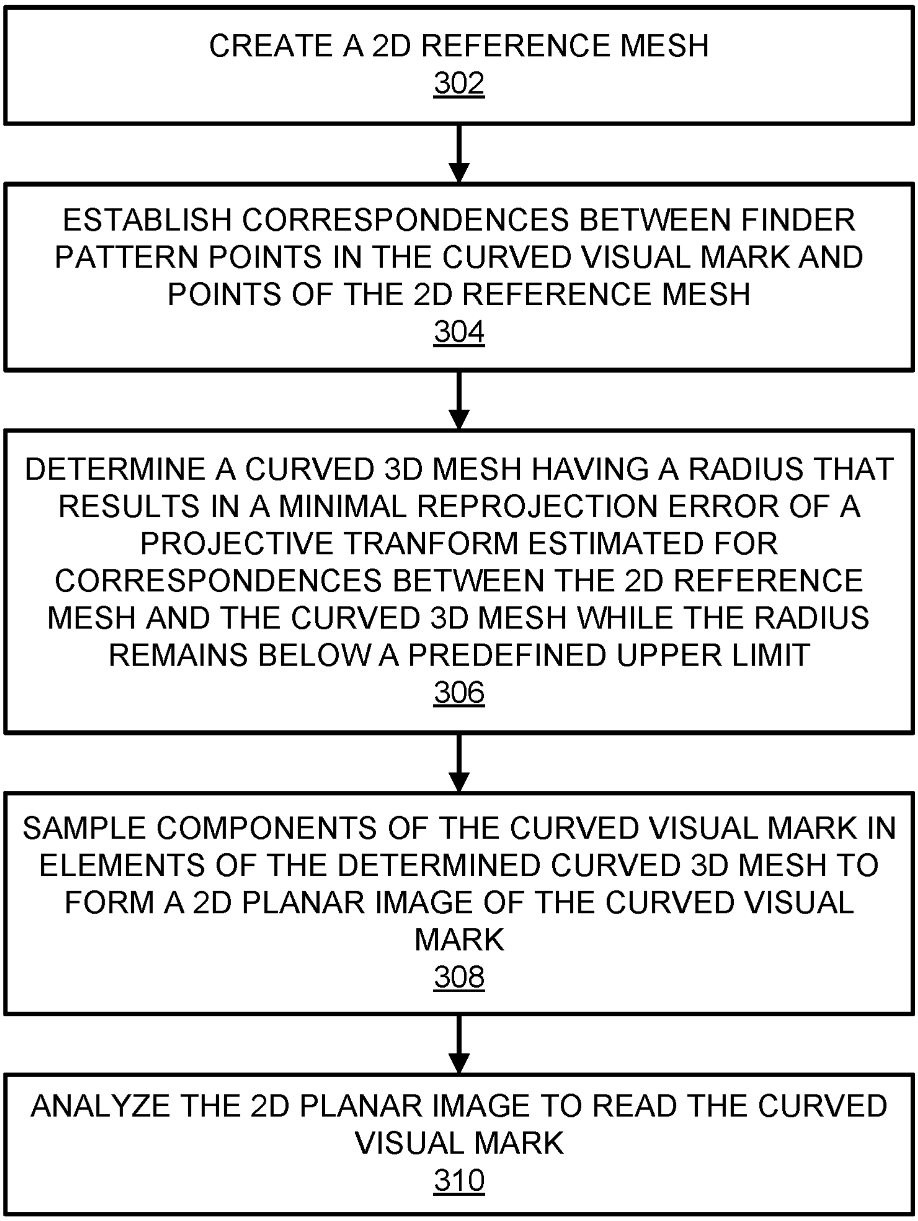
CREATE A 2D REFERENCE MESH
302
ESTABLISH CORRESPONDENCES BETWEEN FINDER PATTERN POINTS IN THE CURVED VISUAL MARK AND POINTS OF THE 2D REFERENCE MESH
304
DETERMINE A CURVED 3D MESH HAVING A RADIUS THAT RESULTS IN A MINIMAL REPROJECTION ERROR OF A PROJECTIVE TRANFORM ESTIMATED FOR CORRESPONDENCES BETWEEN THE 2D REFERENCE MESH AND THE CURVED 3D MESH WHILE THE RADIUS REMAINS BELOW A PREDEFINED UPPER LIMIT
306
SAMPLE COMPONENTS OF THE CURVED VISUAL MARK IN ELEMENTS OF THE DETERMINED CURVED 3D MESH TO FORM A 2D PLANAR IMAGE OF THE CURVED VISUAL MARK
308
ANALYZE THE 2D PLANAR IMAGE TO READ THE CURVED VISUAL MARK
310

FIG. 3

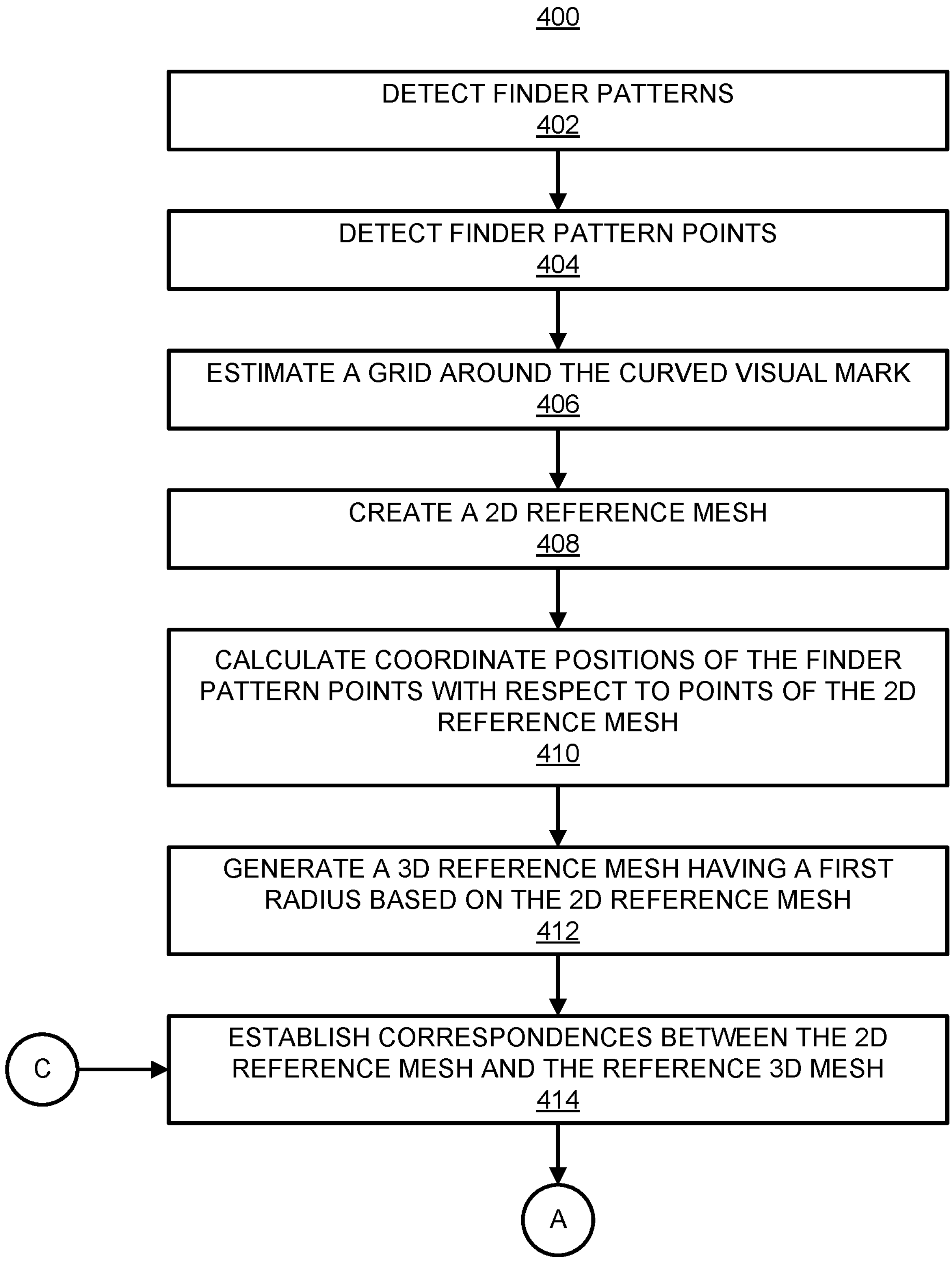
400
DETECT FINDER PATTERNS
402
DETECT FINDER PATTERN POINTS
404
ESTIMATE A GRID AROUND THE CURVED VISUAL MARK
406
CREATE A 2D REFERENCE MESH
408
CALCULATE COORDINATE POSITIONS OF THE FINDER PATTERN POINTS WITH RESPECT TO POINTS OF THE 2D REFERENCE MESH
410
GENERATE A 3D REFERENCE MESH HAVING A FIRST RADIUS BASED ON THE 2D REFERENCE MESH
412
C
ESTABLISH CORRESPONDENCES BETWEEN THE 2D REFERENCE MESH AND THE REFERENCE 3D MESH
414
A

FIG. 4A

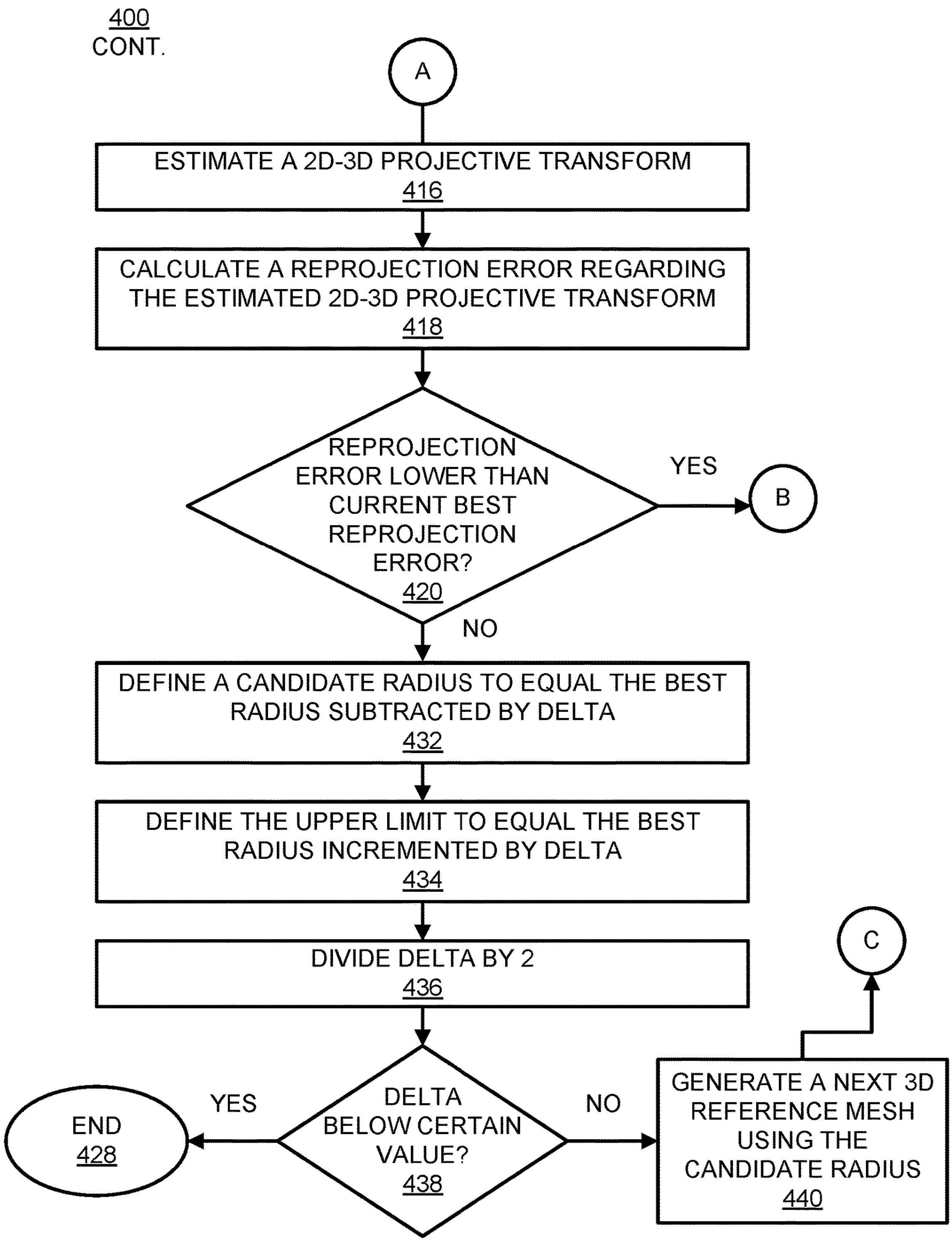
400
CONT.
A
ESTIMATE A 2D-3D PROJECTIVE TRANSFORM
416
CALCULATE A REPROJECTION ERROR REGARDING THE ESTIMATED 2D-3D PROJECTIVE TRANSFORM
418
REPROJECTION ERROR LOWER THAN CURRENT BEST REPROJECTION ERROR?
420
YES
B
NO
DEFINE A CANDIDATE RADIUS TO EQUAL THE BEST RADIUS SUBTRACTED BY DELTA
432
DEFINE THE UPPER LIMIT TO EQUAL THE BEST RADIUS INCREMENTED BY DELTA
434
DIVIDE DELTA BY 2
436
C
DELTA BELOW CERTAIN VALUE?
438
YES
END
428
NO
GENERATE A NEXT 3D REFERENCE MESH USING THE CANDIDATE RADIUS
440

FIG. 4B

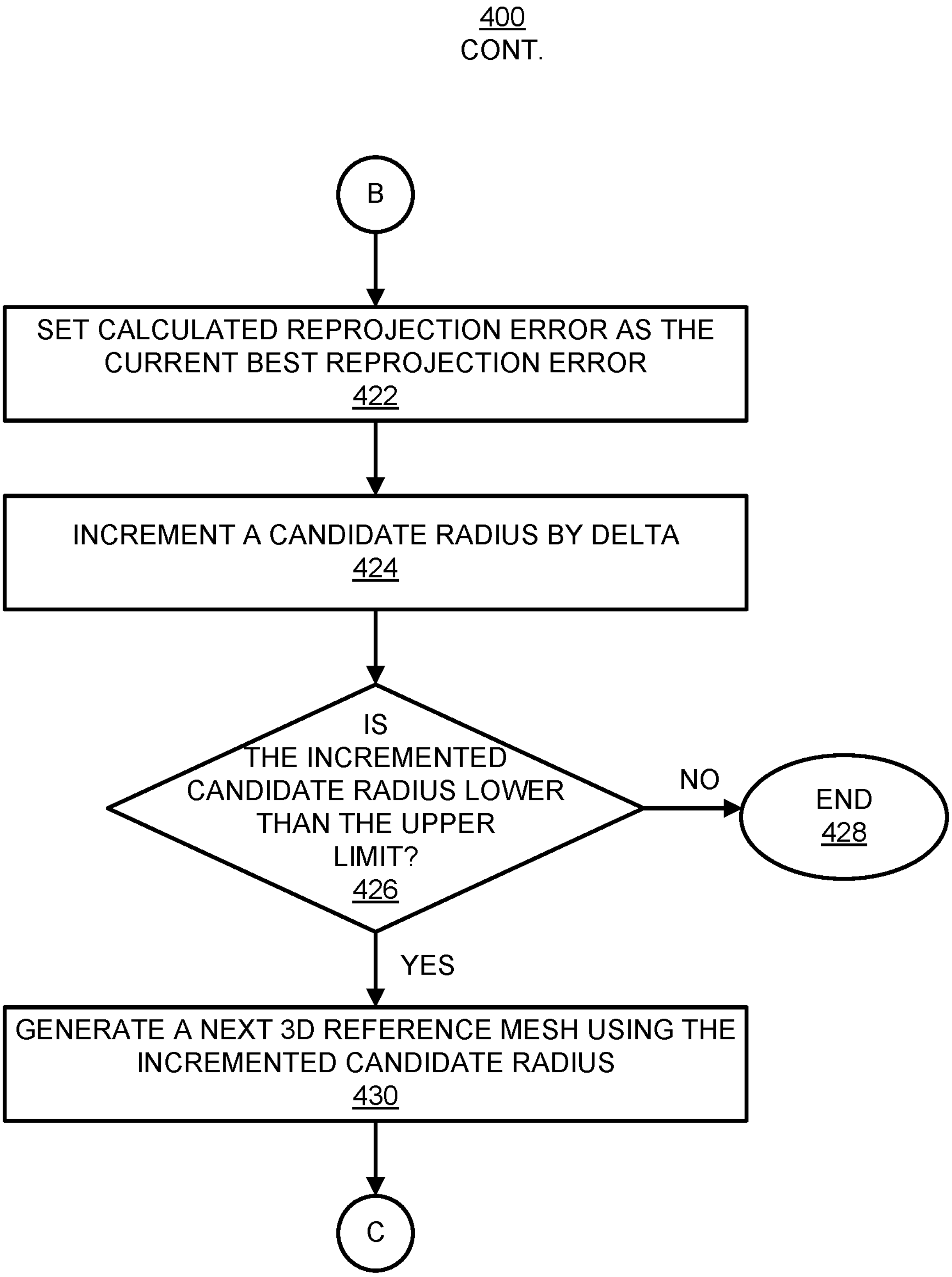
400
CONT.
B
SET CALCULATED REPROJECTION ERROR AS THE CURRENT BEST REPROJECTION ERROR
422
INCREMENT A CANDIDATE RADIUS BY DELTA
424
IS THE INCREMENTED CANDIDATE RADIUS LOWER THAN THE UPPER LIMIT?
426
NO
END
428
YES
GENERATE A NEXT 3D REFERENCE MESH USING THE INCREMENTED CANDIDATE RADIUS
430
C

FIG. 4C

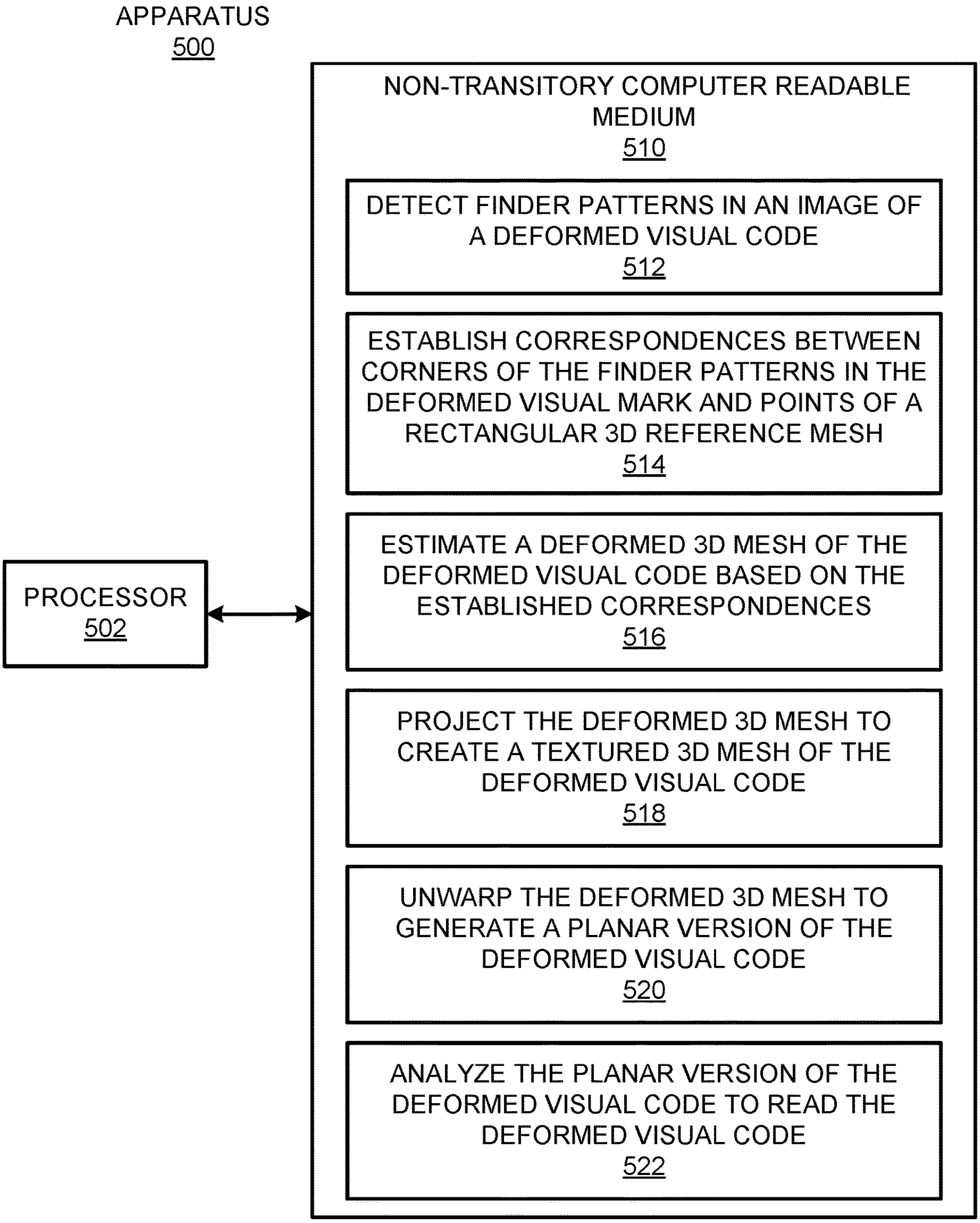
APPARATUS
500
NON-TRANSITORY COMPUTER READABLE MEDIUM
510
DETECT FINDER PATTERNS IN AN IMAGE OF A DEFORMED VISUAL CODE
512
ESTABLISH CORRESPONDENCES BETWEEN CORNERS OF THE FINDER PATTERNS IN THE DEFORMED VISUAL MARK AND POINTS OF A RECTANGULAR 3D REFERENCE MESH
514
ESTIMATE A DEFORMED 3D MESH OF THE DEFORMED VISUAL CODE BASED ON THE ESTABLISHED CORRESPONDENCES
516
PROJECT THE DEFORMED 3D MESH TO CREATE A TEXTURED 3D MESH OF THE DEFORMED VISUAL CODE
518
UNWARP THE DEFORMED 3D MESH TO GENERATE A PLANAR VERSION OF THE DEFORMED VISUAL CODE
520
ANALYZE THE PLANAR VERSION OF THE DEFORMED VISUAL CODE TO READ THE DEFORMED VISUAL CODE
522
PROCESSOR
502

FIG. 5

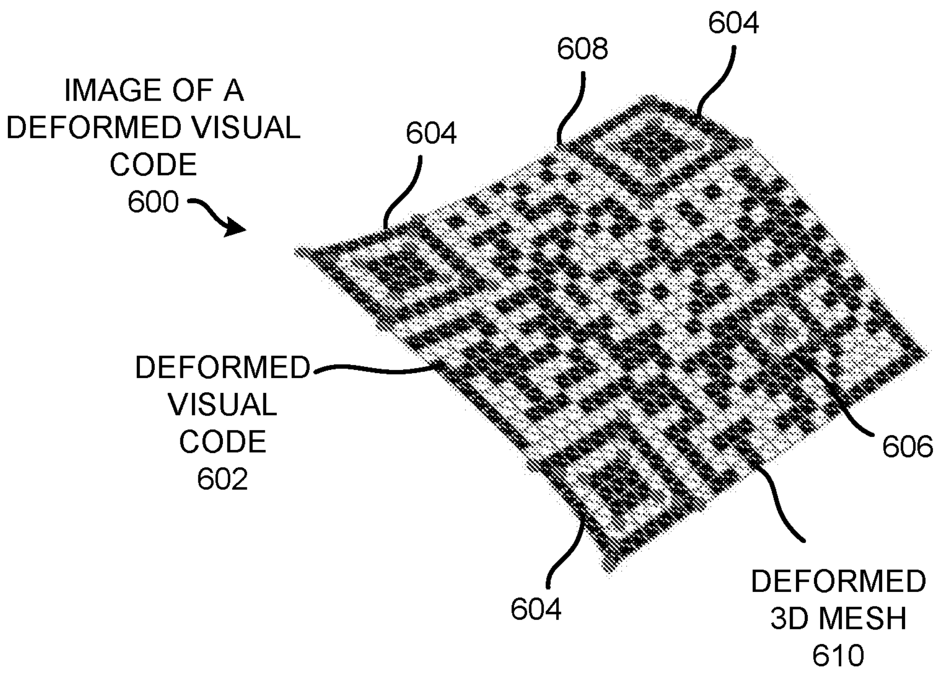

IMAGE OF A DEFORMED VISUAL CODE 600
604
608
604
DEFORMED VISUAL CODE 602
606
604
DEFORMED 3D MESH 610

FIG. 6

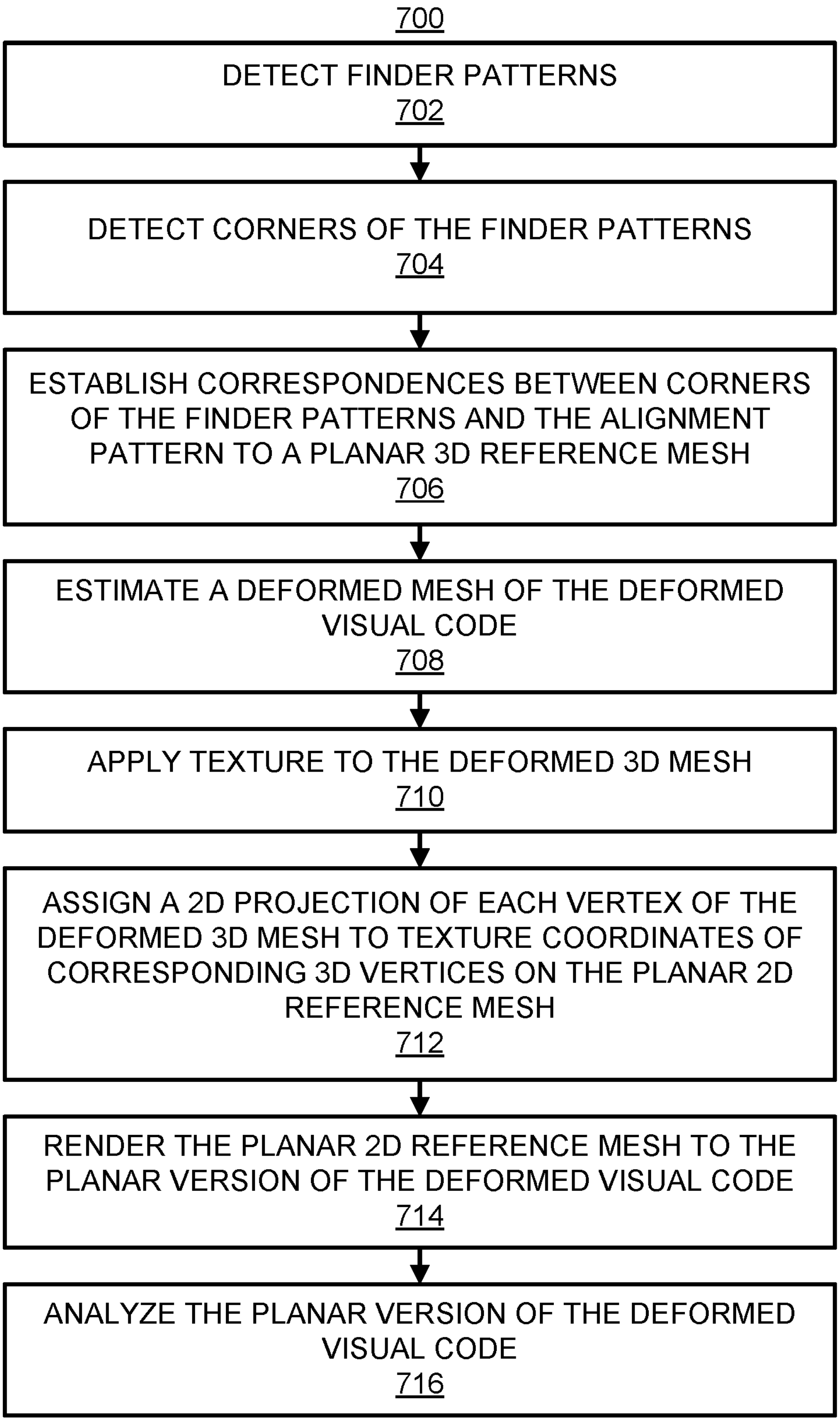
700
DETECT FINDER PATTERNS
702
DETECT CORNERS OF THE FINDER PATTERNS
704
ESTABLISH CORRESPONDENCES BETWEEN CORNERS OF THE FINDER PATTERNS AND THE ALIGNMENT PATTERN TO A PLANAR 3D REFERENCE MESH
706
ESTIMATE A DEFORMED MESH OF THE DEFORMED VISUAL CODE
708
APPLY TEXTURE TO THE DEFORMED 3D MESH
710
ASSIGN A 2D PROJECTION OF EACH VERTEX OF THE DEFORMED 3D MESH TO TEXTURE COORDINATES OF CORRESPONDING 3D VERTICES ON THE PLANAR 2D REFERENCE MESH
712
RENDER THE PLANAR 2D REFERENCE MESH TO THE PLANAR VERSION OF THE DEFORMED VISUAL CODE
714
ANALYZE THE PLANAR VERSION OF THE DEFORMED VISUAL CODE
716

FIG. 7

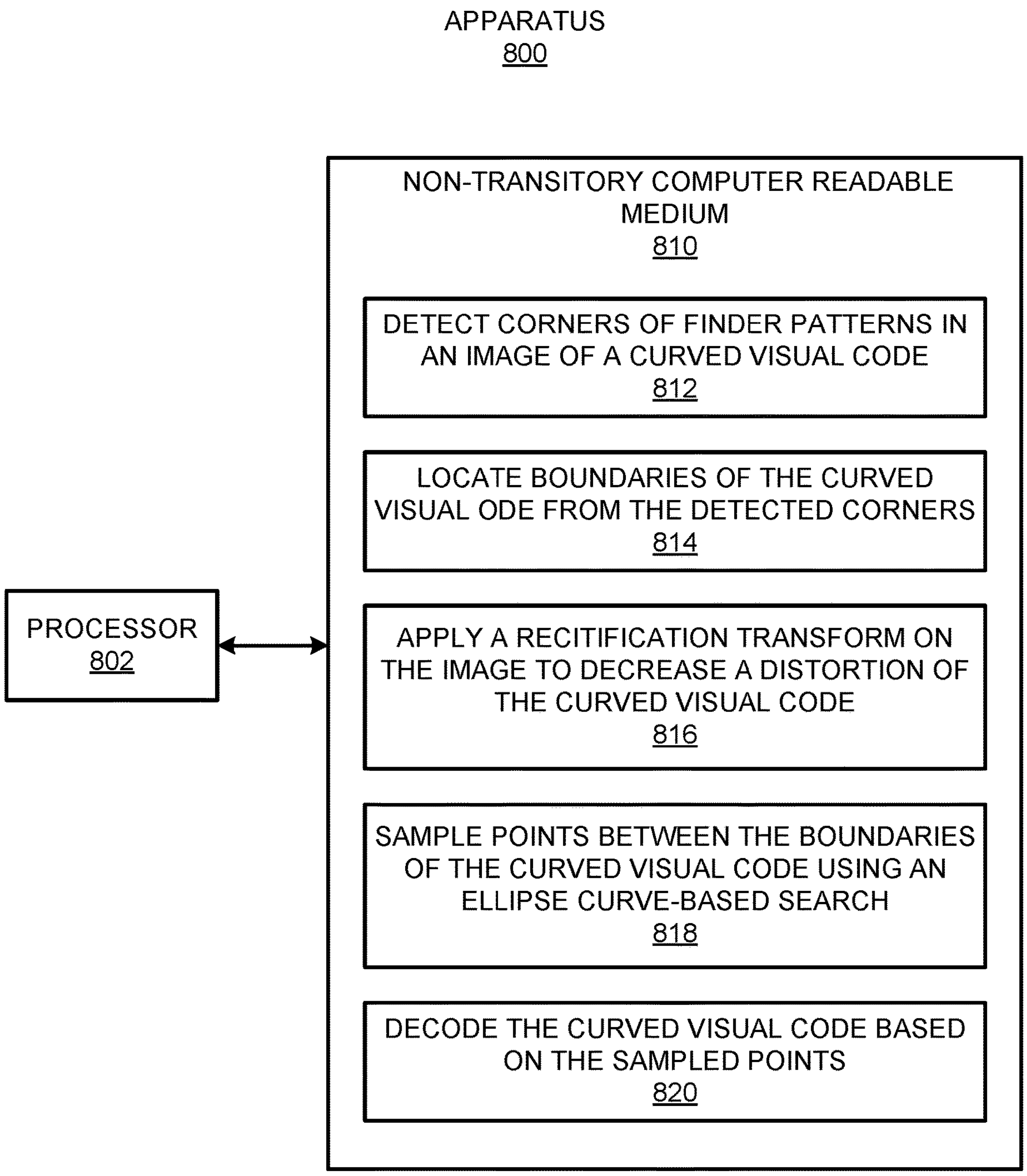
APPARATUS
800
NON-TRANSITORY COMPUTER READABLE MEDIUM
810
DETECT CORNERS OF FINDER PATTERNS IN AN IMAGE OF A CURVED VISUAL CODE
812
LOCATE BOUNDARIES OF THE CURVED VISUAL ODE FROM THE DETECTED CORNERS
814
APPLY A RECITIFICATION TRANSFORM ON THE IMAGE TO DECREASE A DISTORTION OF THE CURVED VISUAL CODE
816
SAMPLE POINTS BETWEEN THE BOUNDARIES OF THE CURVED VISUAL CODE USING AN ELLIPSE CURVE-BASED SEARCH
818
DECODE THE CURVED VISUAL CODE BASED ON THE SAMPLED POINTS
820
PROCESSOR
802

FIG. 8

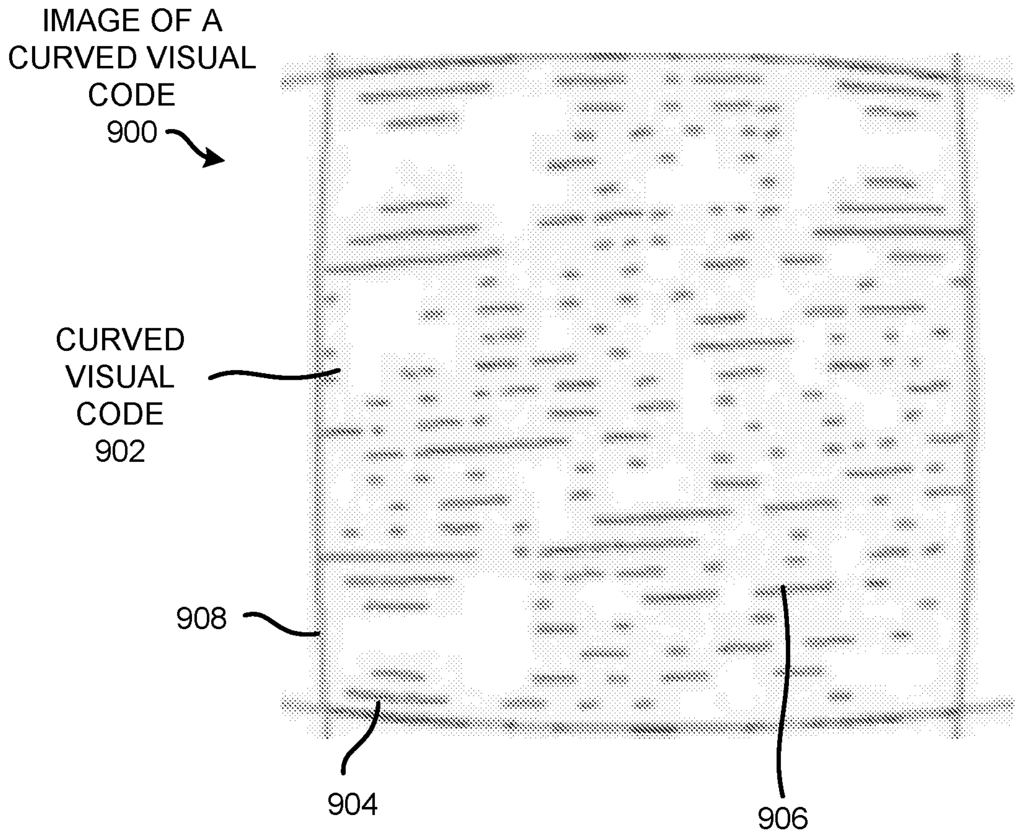
IMAGE OF A CURVED VISUAL CODE 900
CURVED VISUAL CODE 902
908
904
906

FIG. 9A

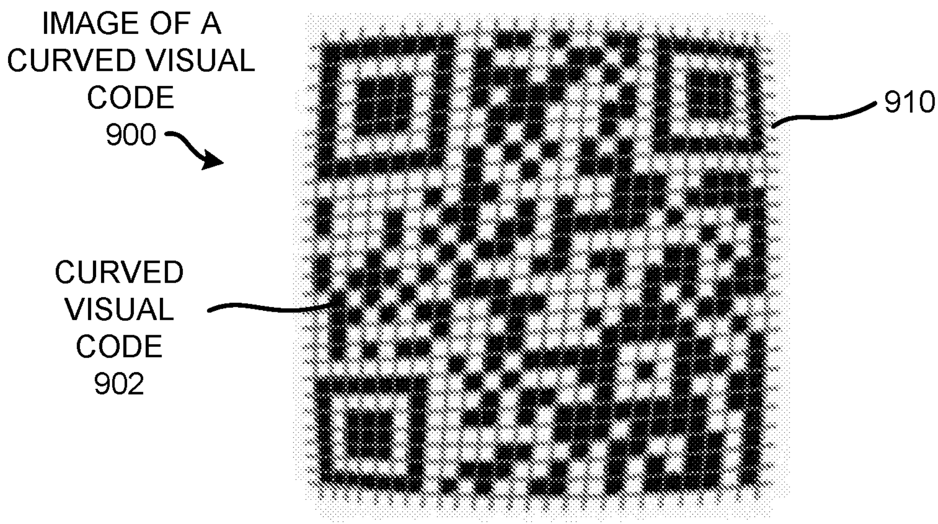
IMAGE OF A CURVED VISUAL CODE 900
CURVED VISUAL CODE 902
910

FIG. 9B

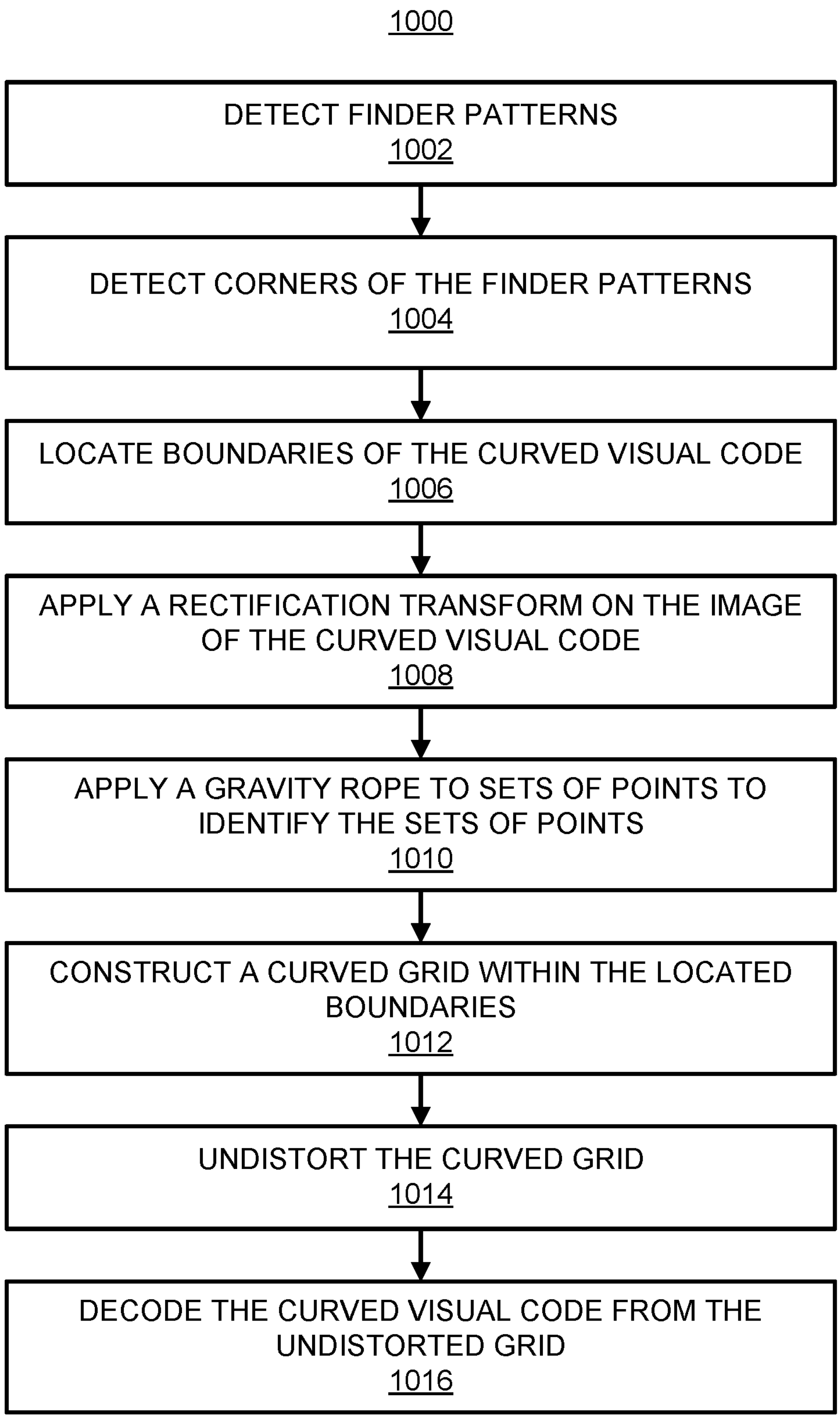
1000
DETECT FINDER PATTERNS
1002
DETECT CORNERS OF THE FINDER PATTERNS
1004
LOCATE BOUNDARIES OF THE CURVED VISUAL CODE
1006
APPLY A RECTIFICATION TRANSFORM ON THE IMAGE OF THE CURVED VISUAL CODE
1008
APPLY A GRAVITY ROPE TO SETS OF POINTS TO IDENTIFY THE SETS OF POINTS
1010
CONSTRUCT A CURVED GRID WITHIN THE LOCATED BOUNDARIES
1012
UNDISTORT THE CURVED GRID
1014
DECODE THE CURVED VISUAL CODE FROM THE UNDISTORTED GRID
1016

FIG. 10

READ CURVED VISUAL MARKS

BACKGROUND

Visual marks, such as quick release (QR) codes, watermarks, and barcodes, may be provided on various physical objects, such as documents and other products. The visual marks may represent code that may provide information regarding the physical objects to which the visual marks are provided or other information. For instance, the visual marks may represent code that may be used for identification of the physical objects and/or for tracking the physical objects. In other examples, the visual marks may represent code that may be used to direct users to particular websites or to provide information regarding a product or service to users.

BRIEF DESCRIPTION OF THE DRAWINGS

Features of the present disclosure are illustrated by way of example and not limited in the following figure(s), in which like numerals indicate like elements, in which:

FIG. **1** shows a diagram of an example apparatus that may process an image of a curved visual mark to read information encoded in the curved visual mark;

FIG. **2**A shows a diagram of an example image of a curved visual mark;

FIG. **2**B shows a diagram of an example 2D reference mesh overlaying the example image depicted in FIG. **2**A;

FIG. **3** shows an example method for processing an image of a curved visual mark to read information encoded in the curved visual mark;

FIGS. **4**A-**4**C, collectively, depict an example method for determining a curved 3D mesh having a radius that results in a minimal reprojection error for use in reading a curved visual mark;

FIG. **5** shows a block diagram of an example apparatus that may process an image of a deformed visual code to read information encoded in the deformed visual code;

FIG. **6** shows a diagram of an example image of a deformed visual code with a deformed mesh overlaying the image;

FIG. **7** depicts an example method for processing an image of a deformed visual code to read information encoded in the deformed visual code;

FIG. **8** shows a block diagram of an example apparatus that may process an image of a curved visual code to read information encoded in the curved visual code;

FIG. **9**A shows a diagram of an example image of a curved visual code following preprocessing of the image;

FIG. **9**B shows a diagram of an example curved grid laid over the image of the curved visual code shown in FIG. **9**A; and

FIG. **10** depicts an example method for processing an image of a curved visual code to read information encoded in the curved visual code.

DETAILED DESCRIPTION

A visual code, such as a barcode, a QR code, or the like, may be printed or attached to various surfaces. The surfaces on which the visual codes may be provided may be flat or may be curved or deformed. In instances in which the surfaces are flat, the visual codes may easily be identified and the information represented by the visual codes may easily be read. However, in instances in which the surfaces are curved or deformed, the visual code may also be curved or deformed, which may make it more difficult for the visual code to be identified and read.

Disclosed herein are apparatuses and methods for reading curved and/or deformed visual codes, which are also referenced herein as marks. Particularly, a processor of an apparatus disclosed herein may execute instructions that may process an image of curved or deformed visual code to render the code that the visual code represents to accurately be read. The processor may execute one of the sets of instructions disclosed herein or may execute a plurality of the sets of instructions disclosed herein to process and read the visual code.

Through implementation of the apparatuses and methods disclosed herein, visual codes printed or otherwise provided on curved or deformed surfaces may accurately be read. In one regard, the accurate reading of the visual codes afforded through implementation of the apparatuses and methods disclosed herein may enable the visual codes to be provided on a large number of variously shaped objects. As a result, the use of visual codes, which are used in a number of different applications, may be expanded to additional applications, which may increase the usefulness of the visual codes. In addition, the accurate reading of the visual codes may reduce processing resource consumption as, for instance, multiple operations to read the visual codes may be reduced or eliminated.

Before continuing, it is noted that as used herein, the terms "includes" and "including" mean, but is not limited to, "includes" or "including" and "includes at least" or "including at least." The term "based on" means "based on" and "based at least in part on."

Reference is first made to FIGS. **1**, **2**A, and **2**B. FIG. **1** shows a block diagram of an example apparatus **100** that may process an image **200** of a curved visual mark **202** to read information encoded in the curved visual mark **202**. FIG. **2**A shows a diagram of an example image **200** of a curved visual mark **202** and FIG. **2**B shows a diagram of an example 2D reference mesh **210** overlaying the example image **200** depicted in FIG. **2**A. It should be understood that the apparatus **100** depicted in FIG. **1**, the image **200** of the curved visual mark **202** depicted in FIG. **2**A, and/or 2D reference mesh **210** depicted in FIG. **2**B may include additional components and that some of the components described herein may be removed and/or modified without departing from the scopes of the apparatus **100**, the image **200** of the curved visual mark **202**, and/or the 2D reference mesh **210** disclosed herein.

The apparatus **100** may be a computing apparatus, e.g., a personal computer, a laptop computer, a tablet computer, a smartphone, a server computer, or the like. As shown in FIG. **1**, the apparatus **100** may include a processor **102** that may control operations of the apparatus **100**. The processor **102** may be a semiconductor-based microprocessor, a central processing unit (CPU), an application specific integrated circuit (ASIC), a field-programmable gate array (FPGA), a graphics processing unit (GPU), and/or other hardware device. The apparatus **100** may also include a non-transitory computer readable medium **110** that may have stored thereon machine readable instructions **112**-**120** (which may also be termed computer readable instructions) that the processor **102** may execute. The non-transitory computer readable medium **110** may be an electronic, magnetic, optical, or other physical storage device that contains or stores executable instructions. The non-transitory computer readable medium **110** may be, for example, Random Access memory (RAM), an Electrically Erasable Programmable Read-Only Memory (EEPROM), a storage device, an optical disc, and the like. The term "non-transitory" does not encompass transitory propagating signals.

According to examples in which the curved visual mark **202** is a curved quick response (QR) code as shown in FIG. 2A, the curved visual mark **202** may include a plurality of finder patterns **204**. The curved visual mark **202** may have a rounded or a cylindrical curvature. In a flat version of the curved visual mark **202**, the finder patterns **204** may have the same patterns and sizes with respect to each other and may be positioned at or near the edges of the QR code. The finder patterns **204** may thus define the boundaries of the QR code. In addition, each of the finder patterns **204** may include a smaller square element surrounded by a larger square element, with a gap between the elements. In some examples, the QR code may also include an alignment pattern **206** positioned near a corner of the QR code at which the finder patterns **204** are not provided. The alignment pattern **206** may also include a smaller square element surrounded by a larger square element, with a gap between the elements. The QR code may further include patterns that may represent or encode various information.

The processor **102** may fetch, decode, and execute the instructions **112** to create a two-dimensional (2D) reference mesh **210** (FIG. **2**) for the image **200** of the curved visual mark **202**. The 2D reference mesh **210** may correspond to a flat version of the visual mark **202**. In addition, the processor **102** may create the 2D reference mesh **210** based on detected finder patterns **204** in the curved visual mark **202**. For instance, the processor **102** may detect the finder patterns **204** in the image **200** through a suitable detection technique, such as contour detection and checking of the quad geometries of the finder patterns **204**, by scanlines over the image **200** to identify the finder patterns **204**, or the like.

According to examples, the processor **102** may detect the finder patterns **204** of the curved visual mark **202** by applying a threshold on the image **200** to make the features in the image **200** that are below a certain threshold white and the features in the image **200** that are above the certain threshold black, or vice versa. In addition, the processor **102** may find contours in the image **200** following application of the threshold and storing points that are part of a contour. The processor **102** may determine patterns in the contour and may identify the patterns that have the finder pattern **204** shapes to find the locations of the finder patterns **204**.

The processor **102** may also estimate a N×N grid size of the curved visual mark **202** based on the detected finder patterns **204**. For instance, the processor **102** may detect the sizes of the finder patterns **204** in the image **200** and based on the detected sizes, the processor **102** may determine the distances between the finder patterns **204**. In addition, the processor **102** may estimate the N×N grid size of the curved visual mark **202** based on the detected finder patterns, e.g., the determined distances between the finder patterns **204**. That is, for instance, the processor **102** may estimate the N×N grid size to be within or equal to the boundaries of the curved visual mark **202** as determined from the locations of the finder patterns **204** with respect to each other. The processor **102** may create the 2D reference mesh **210** to have a size that may correspond to the estimated N×N grid size.

The processor **102** may also initiate a projective transform, e.g., that may be a 3×4 projective transform that contain only zeros, and may identify a best projective transform error that may be as big as the image **200**. The processor **102** may store the identified best projective transform error as a current best projective transform error.

The processor **102** may fetch, decode, and execute the instructions **114** to establish correspondences between points, e.g., corners, of each of the finder patterns **204** in the curved visual mark **202** and points of the 2D reference mesh **210**. For instance, the processor **102** may calculate coordinate positions of the finder pattern **204** corners with respect to points, e.g., vertices or intersections of the horizontal and vertical lines of the 2D reference mesh **210**. By way of example, the processor **102** may relate each of the corners of the finder patterns **204** to coordinates of the 2D reference mesh **210**.

The processor **102** may fetch, decode, and execute the instructions **116** to determine a curved three-dimensional (3D) mesh having a radius that results in a minimal reprojection error of a projective transform estimated for correspondences between the 2D reference mesh and the curved 3D mesh while the radius remains below a predefined upper limit. The predefined upper limit may be defined as a radius that is sufficiently large to be near a flat surface. In some examples, the predefined upper limit may be a radius that results in a flat surface. In other examples, the predefined upper limit of the radius may be determined through testing, simulations, etc. Various operations that the processor **102** may execute to determine the curved 3D mesh are described in greater detail herein below.

The processor **102** may fetch, decode, and execute the instructions **118** to sample components of the curved visual mark **202** in elements of the determined curved 3D mesh to form a 2D planar image of the curved visual mark **202**. That is, for instance, the processor **102** may sample the square elements of the curved visual mark **202** based on the curved 3D mesh having a radius that resulted in the minimal reprojection error and may form the 2D planar image from that curved 3D mesh.

The processor **102** may fetch, decode, and execute the instructions **120** to analyze the 2D planar image of the curved visual mark **202** to read the curved visual mark **202**. For instance, the processor **102** may identify the locations of the visual mark squares in the 2D planar image of the curved visual mark **202** and may determine the information represented by the visual mark squares based on the locations of the squares.

Various manners in which the processor **102** may be implemented are discussed in greater detail with respect to the methods **300** and **400** depicted in FIGS. **3** and **4A-4C**. Particularly, FIG. **3** depicts an example method **300** for processing an image **200** of a curved visual mark **202** to read information encoded in the curved visual mark **202**. FIGS. **4A-4C**, collectively, depict an example method **400** for determining a curved 3D mesh having a radius that results in a minimal reprojection error for use in reading a curved visual mark **202**. It should be apparent to those of ordinary skill in the art that the methods **300** and **400** may represent generalized illustrations and that other operations may be added or existing operations may be removed, modified, or rearranged without departing from scopes of the methods **300** and **400**.

The descriptions of the methods **300** and **400** are made with reference to the apparatus **100** illustrated in FIG. **1** as well as to the features depicted in FIGS. **2A** and **2B** for purposes of illustration. It should be understood that apparatuses having configurations other than that shown in FIG. **1** may be implemented to perform the methods **300** and/or **400** without departing from scopes of the methods **300** and/or **400**. In addition, the method **400** may be related to the method **300** in that the method **400** may include more detailed operations for blocks **302-304** in the method **300**.

At block **302**, the processor **102** may create a two-dimensional (2D) reference mesh **210** for an image **200** of a curved visual mark **202**. At block **304**, the processor **102** may establish correspondences between finder pattern **204** points in the curved visual mark **202** and points, e.g., vertices, of the 2D reference mesh **210**. At block **306**, the processor **102** may determine a curved three-dimensional (3D) mesh having a radius that results in a minimal reprojection error of a projective transform estimated for correspondences between the 2D reference mesh **210** and the curved 3D mesh while the radius remains below a predefined upper limit. At block **308**, the processor **102** may sample components of the curved visual mark **202** in elements of the determined curved 3D mesh to form a 2D planar image of the curved visual mark **202**. At block **310**, the processor **102** may analyze the 2D planar image of the curved visual mark **202** to read the curved visual mark **202**.

Turning now to FIGS. **4A-4C**, and particularly to FIG. **4A**, at block **402**, the processor **102** may detect finder patterns **204** in an image **200** of a curved visual mark **202**. The processor **102** may detect the finder patterns **204** in any of the manners discussed herein. At bock **404**, the processor **102** may detect points of the finder patterns **204**. For instance, the processor **102** may detect the corners of the detected finder patterns **204**.

At block **406**, the processor **102** may estimate a grid around the curved visual mark **202**, for instance, as discussed herein. At block **408**, the processor **102** may create a 2D reference mesh **210**, for instance, as discussed herein. At block **410**, the processor **102** may calculate coordinate positions of the finder pattern **204** points with respect to points of the 2D reference mesh **210**. For instance, the processor **102** may determine the intersections of the 2D reference mesh **210** to which the corners of the finder patterns **204** are the closest.

At block **412**, the processor **102** may generate a 3D reference mesh based on the 2D reference mesh, the 3D reference mesh having a first radius. The 3D reference mesh may have a curved profile and may have a size that corresponds to the size of the 2D reference mesh **210**. The 3D reference mesh may also include lines that are spaced apart at similar distances as the 2D reference mesh **210**.

At block **414**, the processor **102** may establish correspondences between the 2D reference mesh **210** and the 3D reference mesh. That is, for instance, the processor **102** may determine correspondences between respective intersection points, e.g., vertices, of the 2D reference mesh **210** and the 3D reference mesh. The correspondences may relate to the distances between the respective intersection points of the 2D reference mesh **210** and the 3D reference mesh along a particular direction. In one regard, because the 3D reference mesh may be created using coordinates of the 2D reference mesh **210** as input, the correspondences between the respective intersections may be known a priori.

At block **416**, the processor **102** may estimate a 2D-3D projective transform from the established correspondences between the 2D reference mesh **210** and the 3D reference mesh. For instance, the processor **102** may use sixteen 2D-3D correspondences found between the 2D reference mesh **210** and the 3D reference mesh having the first radius, e.g., as may be represented by the cylinder, at block **412**. The sixteen 2D-3D correspondences found between the 2D reference mesh **210** and the 3D reference mesh may include four correspondences from each of the three finder patterns **204** and four correspondences from the alignment pattern **206**. In addition, the processor **102** may estimate the 2D-3D projective transform through application of a perspective-n-point (PnP) method that may estimate the pose of a calibrated camera given a set of n 3D points and their corresponding 2D projections in an image. By way of particular example, the processor **102** may apply an Efficient perspective-n-point (EPnP) camera pose estimation to estimate the 2D-3D projective transform of the 2D reference mesh **210** and the 3D reference mesh.

At block **418** (FIG. **4B**), the processor **102** may calculate a reprojection error regarding the estimated 2D-3D projective transform. The processor **102** may calculate the reproduction error based on a summation of the distances of the projected 3D points of the 3D reference mesh to their corresponding 2D points of the 2D reference mesh given by the previously detected finder pattern **204** corners.

At block **420**, the processor **102** may determine whether the calculated reprojection error is lower than a current best projection error. During an initial iteration of the method **400**, the best projection error may be equivalent to the projective transform error that may be as big as the image **200** as discussed above.

Based on a determination that the calculated reprojection error is lower than the current best projection error, at block **422** (FIG. **4C**), the processor **102** may set the calculated reprojection error as the current best reprojection error. At block **424**, the processor **102** may increment a candidate radius by a predefined value (delta). The predefined value may be based on empirical testing, through simulations, and/or the like. At block **426**, the processor **102** may determine whether the incremented candidate radius is lower than a predefined upper limit. Based on a determination that the incremented candidate radius is not lower than the upper limit, the method **400** may end as indicated at block **428**. In addition, the processor **102** may sample the components of the curved visual mark in elements of the determined curved 3D mesh having the current lowest radius to form the 2D planar image of the curved visual mark **202** as discussed herein with respect to block **308**.

However, based on a determination that the incremented candidate radius is lower than the upper limit, at block **430**, the processor **102** may generate a next reference 3D mesh using the candidate radius incremented at block **424**. In addition, the processor **102** may repeat blocks **414-430** until the incremented candidate radius is not lower than the upper limit at block **426** or delta is below a certain value as discussed below with respect to block **438**.

With reference back to block **420** (FIG. **4B**), based on a determination that the reprojection error calculated at block **418** is not lower than the current best reprojection error, at block **432**, the processor **102** may define a candidate radius to equal the radius corresponding to the current best reprojection error subtracted by the predefined value (delta). In some examples, the delta may be equivalent to the delta discussed above with respect to block **424**, while in other examples, the delta at block **432** may differ from the delta at block **424**.

At block **434**, the processor **102** may define the upper limit to equal the radius corresponding to the current best reprojection error (e.g., the best radius), incremented by delta. At block **436**, the processor **102** may divide delta by two. In addition, at block **438**, the processor **102** may determine whether delta divided by two is below a certain value. The certain value may be determined empirically and may correspond to an acceptably small reprojection error. By way of particular example, the certain value may be an average of 5 pixels or less. Based on a determination that delta is below the certain value, the method **400** may end as indicated at block **428**. However, based on a determination that delta is not below the certain value, at block **440**, the processor **102** may generate a next reference 3D mesh using the candidate radius defined at block **432**. In addition, the processor **102** may repeat blocks **414-440** until the incremented and/or defined candidate radius is not lower than the upper limit at block **426** or delta is below the certain value as discussed with respect to block **438**.

Reference is now made to FIGS. **5** and **6**. FIG. **5** shows a block diagram of an example apparatus **500** that may process an image **600** of a deformed visual code **602** to read information encoded in the deformed visual code **602**. FIG. **6** shows a diagram of an example image **600** of a deformed visual code **602** with a deformed mesh **610** overlaying the image **600**. It should be understood that the apparatus **500** depicted in FIG. **5** and the image **600** of the deformed visual code **602** depicted in FIG. **6** may include additional components and that some of the components described herein may be removed and/or modified without departing from the scopes of the apparatus **500** disclosed herein.

The apparatus **500** may be a computing apparatus, similar to the apparatus **100** depicted in FIG. **1**. As shown in FIG. **5**, the apparatus **500** may include a processor **502** that may control operations of the apparatus **500**. The processor **502** may be similar to the processor **102** depicted in FIG. **1**. The apparatus **500** may also include a non-transitory computer readable medium **510** that may have stored thereon machine readable instructions **512-522** (which may also be termed computer readable instructions) that the processor **502** may execute. The non-transitory computer readable medium **510** may be similar to the non-transitory computer readable medium **110** depicted in FIG. **1**.

According to examples in which the deformed visual code **602** is a quick response (QR) code as shown in FIG. **6**, the deformed visual code **602** may include a plurality of finder patterns **604**. In a flat version of the deformed visual code **602**, the finder patterns **604** may have the same patterns and sizes with respect to each other and may be positioned at or near the edges of the QR code. The finder patterns **604** may each have respective sets of corners **608** as denoted by the dots shown in FIG. **6**, in which some of the corners **608** of the finder patterns **604** may define the boundaries of the QR code. In addition, each of the finder patterns **604** may include a smaller square element surrounded by a larger square element, with a gap between the elements. In some examples, the QR code may also include an alignment pattern **606** positioned near a corner of the QR code at which the finder patterns **604** are not provided. The alignment pattern **606** may also include a smaller square element surrounded by a larger square element, with a gap between the elements. The QR code may further include patterns that may represent various information.

The processor **502** may fetch, decode, and execute the instructions **512** to detect the finder patterns **604** in an image **600** of a deformed visual code **602**, each of the finder patterns **604** including a respective set of corners **608**. As shown in FIG. **6**, each of the finder patterns **604** may include 12 corners. In addition, the deformed visual code **602** may have an arbitrary deformation, e.g., not a cylindrical or round deformation. The processor **502** may detect the finder patterns **602** and the corners **608** in any of the manners discussed above with respect to the processor **102**.

The processor **502** may fetch, decode, and execute the instructions **514** to establish correspondences between the corners **608** of the finder patterns **604** and points of a planar 2-dimensional (2D) reference mesh (not shown). The planar 2D reference mesh may be a triangular mesh that may be sized according to a detected size of the deformed visual code **602**. That is, the size of the planar 2D reference mesh may be based on a size of the deformed visual code **602**, as may be calculated from the sizes and locations of the finder patterns **604** in the image **600**. In addition, the processor **502** may calculate coordinate positions of the corners **608** of the finder patterns **604** with respect to points, e.g., intersections of the vertices of the 2D reference mesh. By way of example, the processor **502** may relate each of the corners **608** of the finder patterns **604** to coordinates of the 2D reference mesh.

The processor **502** may fetch, decode, and execute the instructions **516** to estimate a deformed 3D mesh **610** of the deformed visual code **602** based on the established correspondences. For instance, the processor **502** may estimate the deformed 3D mesh of the deformed visual code **602** through use of the Laplacian mesh method based on the established correspondences.

The processor **502** may fetch, decode, and execute the instructions **518** to project the deformed 3D mesh on top of the deformed visual code **602** in the image **600** to create a textured 3D mesh of the deformed visual code **602**. In addition, the processor **502** may unwarp the deformed 3D mesh **602** to generate a planar version of the deformed visual code **602**. Moreover, the processor **502** may analyze the planar version of the deformed visual code **602** to read the deformed visual code **602**.

Various manners in which the processor **502** may be implemented are discussed in greater detail with respect to the method **700** depicted in FIG. **7**. Particularly, FIG. **7** depicts an example method **700** for processing an image **600** of a deformed visual code **602** to read information encoded in the deformed visual code **602**. It should be apparent to those of ordinary skill in the art that the method **700** may represent a generalized illustration and that other operations may be added or existing operations may be removed, modified, or rearranged without departing from the scope of the method **700**.

The description of the method **700** is made with reference to the apparatus **500** illustrated in FIG. **5** as well as to the features depicted in FIG. **6** for purposes of illustration. It should be understood that apparatuses having configurations other than that shown in FIG. **5** may be implemented to perform the method **700** without departing from the scope of the method **700**.

At block **702**, the processor **502** may detect finder patterns **604** in an image **600** of a deformed visual code **604**, each of the finder patterns **604** including a respective set of corners **608**. At block **704**, the processor **502** may detect the corners **608** of the finder patterns. In some examples, at block **702**, the processor **502** may also detect an alignment pattern **606** and at block **704**, the processor **502** may detect the corners **608** of the alignment pattern.

At block **706**, the processor **502** may establish correspondences between the corners **608** of the finder patterns **604** and points of a planar 2-dimensional (2D) reference mesh. At block **708**, the processor **502** may estimate a deformed 3D mesh **610** of the deformed visual code **602** based on the established correspondences. At block **710**, the processor **502** may apply texture to the deformed 3D mesh **610**. At block **712**, the processor **502** may assign a 2D projection of each vertex of the deformed 3D mesh **610** to texture coordinates of corresponding 3D vertices on the planar 2D reference mesh.

At block **714**, the processor **502** may render the planar 2D reference mesh to the planar version of the deformed visual code. For instance, the processor **502** may render the planar 2D reference mesh to the planar version of the deformed visual code **602** using a graphics pipeline with texturing enabled. At block **716**, the processor **502** may analyze the planar version of the deformed visual code **602** to read the deformed visual code **602**.

Reference is now made to FIGS. **8** and **9**A. FIG. **8** shows a block diagram of an example apparatus **800** that may process an image **900** of a curved visual code **902** to read information encoded in the curved visual code **902**. FIG. **9**A shows a diagram of an example image **900** of a curved visual code **902** following preprocessing of the image **900**. It should be understood that the apparatus **800** depicted in FIG. **8** and the image **900** of the curved visual code **902** depicted in FIG. **9**A may include additional components and that some of the components described herein may be removed and/or modified without departing from the scopes of the apparatus **500** disclosed herein.

The apparatus **800** may be a computing apparatus, similar to either or both of the apparatuses **100**, **500** respectively depicted in FIGS. **1** and **5**. As shown in FIG. **8**, the apparatus **800** may include a processor **802** that may control operations of the apparatus **800**. The processor **802** may be similar to either or both of the processors **102**, **502** respectively depicted in FIGS. **1** and **5**. The apparatus **800** may also include a non-transitory computer readable medium **810** that may have stored thereon machine readable instructions **812**-**820** (which may also be termed computer readable instructions) that the processor **802** may execute. The non-transitory computer readable medium **810** may be similar to either or both of the non-transitory computer readable mediums **110** and **510** respectively depicted in FIGS. **1** and **5**.

The curved visual code **902** may be preprocessed version of the curved visual mark **202** depicted in FIG. **2**A and may include a plurality of finder patterns **904**. In a flat version of the deformed visual code **902**, the finder patterns **904** may have the same patterns and sizes with respect to each other and may be positioned at or near the edges of the QR code. The finder patterns **904** may each have respective sets of corners, in which some of the corners of the finder patterns **904** may define the boundaries of the QR code. The QR code may further include patterns that may represent various information.

The processor **802** may fetch, decode, and execute the instructions **812** to detect corners of finder patterns **904** in an image **900** of the curved visual code **902**. The processor **802** may detect the corners of the finder patterns **904** in any of the manners discussed herein. The processor **802** may fetch, decode, and execute the instructions **814** to locate boundaries **908** of the curved visual code **902** from the detected corners of the finder patterns **904**. That is, the processor **802** may locate the boundaries **908** as laying at the outermost corners of the detected finder patterns **904**.

The processor **802** may fetch, decode, and execute the instructions **816** to apply a rectification transform on the image **900** of the curved visual code **902** to decrease a distortion of the curved visual code **902**. The processor **802** may fetch, decode, and execute the instructions **818** to sample points between the located boundaries **908** of the curved visual code **902** using an ellipse curve-based search, in which each of the sampled points corresponds to a cell of the curved visual code **902**. The processor **802** may fetch, decode, and execute the instructions **820** to decode the curved visual code **902** based on the sampled points.

Various manners in which the processor **802** may be implemented are discussed in greater detail with respect to the method **1000** depicted in FIG. **10**. Particularly, FIG. **10** depicts an example method **1000** for processing an image **900** of a curved visual code **902** to read information encoded in the curved visual code **902**. It should be apparent to those of ordinary skill in the art that the method **1000** may represent a generalized illustration and that other operations may be added or existing operations may be removed, modified, or rearranged without departing from the scope of the method **1000**.

The description of the method **1000** is made with reference to the apparatus **800** illustrated in FIG. **8** as well as to the features depicted in FIG. **9** for purposes of illustration. It should be understood that apparatuses having configurations other than that shown in FIG. **8** may be implemented to perform the method **1000** without departing from the scope of the method **1000**.

At block **1002**, the processor **802** may detect the finder patterns **904** and the alignment pattern **906** in the image **900** of the curved visual code **902**. The processor **802** may detect the finder patterns **904** through any suitable detection technique as discussed herein. For instance, the processor **802** may preprocess an image **900** of the curved visual code **902** with RGB conversion and thresholding.

At block **1004**, the processor **802** may detect corners of the finder patterns **904** in the image **900** of the curved visual code **902**. At block **1006**, the processor **802** may locate boundaries **908** of the curved visual code **902**. At block **1008**, the processor **802** may apply a rectification transform on the image of the curved visual code **902**.

At block **1010**, the processor **802** may apply a gravity rope to sets of points in the image **900** of the curved visual code **902** to identify the sets of points. Particularly, for instance, the processor **802** may find the points of the curved visual code **902** that a virtual rope contacts as the virtual rope is dropped from a top of the curved visual code **902**. Thus, for instance, the gravity rope may contact a first set of points shown in FIG. **9** that are located at an uppermost position of curved visual code **902** and those points may be identified as being located along a common line. The first set of points may be removed and the gravity rope may contact a second set of points that are located at the remaining uppermost position of the curved visual code **902** to identify the second set of points. The processor **802** may repeat this process to identify the sets of points along the remaining lines of the curved visual code **902**.

In addition, or in other examples, the processor **802** may identify the sets points in a direction other than from top to bottom. In these examples, the processor **802** may identify the sets from a side of the curved visual code **902** and/or from a bottom to the top of the curved visual code **902**. In some examples, the processor **802** may combine the results from the searches in the different directions to find common points and testing the different possibilities found.

At block **1012**, the processor **802** may construct a curved grid **910** within the located boundaries **908** of the curved visual code **902**. An example curved grid **910** laid over the image **900** of the curved visual code **902** is depicted in FIG. **9**B. According to examples, the curved grid **910** may be reconstructed to match the curvature of the curved visual code **902** based on curves (defined as segments of ellipses) that fit the identified sets of boundary points in the curved visual code **902**. In addition, at block **1014**, the processor **802** may undistort the curved visual code **902**, e.g., the curved grid. That is, for instance, the processor **802** may flatten the curved grid **910** and the image **900** to undistort the curved visual code **902**. In addition, at block **1016**, the processor **802** may decode the curved visual code **902** from the undistorted curved grid.

According to examples, a processor **102**, **502**, **802** may select one or a combination of the techniques disclosed herein to read a curved or deformed visual code **202**, **602**,

902. For instance, the processor **102**, **502**, **802** may perform each of the techniques on the same curved visual code **202**, **602**, **902** and may compare error levels of the results of each of the techniques. In these examples, the processor **102**, **502**, **802** may select the read code information from the technique that resulted in the lowest error level. In addition or alternatively, the processor **102**, **502**, **802** may employ the technique that is best suited for the type of distortion applied to the visual curved visual code **202**, **602**, **902**. For instance, in instances in which the curved visual code **202**, **602**, **902** is curved in a cylindrical or spherical contour, the processor **102**, **502**, **802** may select the technique disclosed herein with respect to FIGS. **1-4**C. However, in instances in which the curved visual code **202**, **602**, **902** is deformed in other manners, the processor **102**, **502**, **802** may select the technique disclosed with respect to FIGS. **5-7**.

Although described specifically throughout the entirety of the instant disclosure, representative examples of the present disclosure have utility over a wide range of applications, and the above discussion is not intended and should not be construed to be limiting, but is offered as an illustrative discussion of aspects of the disclosure.

What has been described and illustrated herein is an example of the disclosure along with some of its variations. The terms, descriptions and figures used herein are set forth by way of illustration only and are not meant as limitations. Many variations are possible within the spirit and scope of the disclosure, which is intended to be defined by the following claims—and their equivalents—in which all terms are meant in their broadest reasonable sense unless otherwise indicated.

What is claimed is:

1. An apparatus comprising:
a processor; and
a non-transitory computer readable medium on which is stored instructions that when executed by the processor, are to cause the processor to:
create a two-dimensional (2D) reference mesh for an image of a curved visual mark;
establish correspondences between finder pattern points in the curved visual mark and points of the 2D reference mesh;
determine a curved three-dimensional (3D) mesh having a radius that results in a minimal reprojection error of a projective transform estimated for correspondences between the 2D reference mesh and the curved 3D mesh while the radius remains below a predefined upper limit;
sample components of the curved visual mark in elements of the determined curved 3D mesh to form a 2D planar image of the curved visual mark; and
analyze the 2D planar image of the curved visual mark to read the curved visual mark.

2. The apparatus of claim **1**, wherein the curved visual mark includes finder patterns, each of the finder patterns including respective finder pattern points, and wherein the instructions are further to cause the processor to:
detect the finder patterns in the image of the curved visual mark;
detect the finder pattern points of the detected finder patterns;
estimate a grid around the curved visual mark based on the detected finder pattern points of the finder patterns; and
create the 2D reference mesh based on the estimated grid.

3. The apparatus of claim **2**, wherein the instructions are further to cause the processor to:
estimate a N×N grid size of the curved visual mark based on sizes of the detected finder patterns;
create the 2D reference mesh based on the estimated N×N grid size of the curved visual mark; and
calculate coordinate positions of the finder pattern points with respect to points of the 2D reference mesh.

4. The apparatus of claim **3**, wherein the instructions are further to cause the processor to:
generate a 3D reference mesh based on the 2D reference mesh, the 3D reference mesh having a first radius;
establish correspondences between the 2D reference mesh and the 3D reference mesh;
estimate a 2D-3D projective transform from the established correspondences between the 2D reference mesh and the 3D reference mesh;
calculate a reprojection error regarding the estimated 2D-3D projective transform;
determine whether the calculated reprojection error is lower than an initial projective transform error for the 2D reference mesh; and
based on a determination that the calculated reprojection error is lower than a current best reprojection error, set the radius of the 3D reference mesh as the current best reprojection error;
increment a candidate radius by a predefined value;
determine whether the incremented candidate radius is lower than an upper limit; and
based on the incremented candidate radius being lower than the upper limit, generate a next 3D reference mesh using the incremented candidate radius.

5. The apparatus of claim **4**, wherein the instructions are further to cause the processor to:
establish correspondences between the 2D reference mesh and the next 3D reference mesh;
estimate a second 2D-3D projective transform from the established correspondences between the 2D reference mesh and the next 3D reference mesh;
calculate a second reprojection error regarding the second estimated 2D-3D projective transform;
determine whether the second calculated reprojection error is lower than the second calculated reprojection error; and
based on a determination that the second calculated reprojection error is lower than the calculated reprojection error, set the radius of the next 3D reference mesh as the radius of the determined curved 3D mesh.

6. The apparatus of claim **4**, wherein the instructions are further to cause the processor to:
based on a determination that the calculated reprojection error is not lower than the best reprojection error,
define a candidate radius to equal the radius corresponding to the current best reprojection error subtracted by the predefined value;
define the upper limit to equal the radius corresponding to the current best reprojection error;
divide the predefined value by two; and
determine whether the predefined value divided by two is below a certain value.

7. The apparatus of claim **6**, wherein the instructions are further to cause the processor to:
based on a determination that the predefined value divided by two is below the certain value, generate a next 3D reference mesh using the defined candidate radius;
establish correspondences between the 2D reference mesh and the next 3D reference mesh;

estimate a second 2D-3D projective transform from the established correspondences between the 2D reference mesh and the next 3D reference mesh;

calculate a second reprojection error regarding the second estimated 2D-3D projective transform;

determine whether the second calculated reprojection error is lower than the second calculated reprojection error; and

based on a determination that the second calculated reprojection error is lower than the calculated reprojection error, set the radius of the next 3D reference mesh as the radius of the determined curved 3D mesh.

8. The apparatus of claim **1**, wherein the curved visual mark comprises a curved quick response code.

9. A method comprising:

detecting, by a processor, finder patterns in an image of a deformed visual code, each of the finder patterns including a respective set of corners;

establishing, by the processor, correspondences between the corners of the finder patterns and points of a planar 2-dimensional (2D) reference mesh;

estimating, by the processor, a deformed 3D mesh of the deformed visual code based on the established correspondences;

projecting, by the processor, the deformed 3D mesh on top of the deformed visual code to create a textured 3D mesh of the deformed visual code;

unwarping the deformed 3D mesh to generate a planar version of the deformed visual code; and

analyzing the planar version of the deformed visual code to read the deformed visual code.

10. The method of claim **9**, wherein unwarping the deformed 3D mesh further comprises:

assigning a 2D projection of each vertex of the deformed 3D mesh to texture coordinates of corresponding 3D vertices on the planar 2D reference mesh; and

rendering the planar 2D reference mesh to the planar version of the deformed visual code.

11. The method of claim **10**, wherein rendering the planar 2D reference mesh to the planar version of the deformed visual code further comprises rendering the planar 2D reference mesh to the planar version of the deformed visual code using a graphics pipeline with texturing enabled.

12. The method of claim **9**, further comprising using a Laplacian mesh method to estimate the deformed 3D mesh.

13. A non-transitory computer readable medium on which is stored machine readable instructions that when executed by a processor are to cause the processor to:

create a two-dimensional (2D) reference mesh for an image of a curved visual mark;

establish correspondences between finder pattern points in the curved visual mark and points of the 2D reference mesh;

determine a curved three-dimensional (3D) mesh having a radius that results in a minimal reprojection error of a projective transform estimated for correspondences between the 2D reference mesh and the curved 3D mesh while the radius remains below a predefined upper limit;

sample components of the curved visual mark in elements of the determined curved 3D mesh to form a 2D planar image of the curved visual mark; and

analyze the 2D planar image of the curved visual mark to read the curved visual mark.

14. The non-transitory computer readable medium of claim **13**, wherein the curved visual mark includes finder patterns, each of the finder patterns including respective finder pattern points, and wherein the instructions are further to cause the processor to:

detect the finder patterns in the image of the curved visual mark;

detect the finder pattern points of the detected finder patterns;

estimate a grid around the curved visual mark based on the detected finder pattern points of the finder patterns; and

create the 2D reference mesh based on the estimated grid.

15. The non-transitory computer readable medium of claim **14**, wherein the instructions are further to cause the processor to:

estimate a N×N grid size of the curved visual mark based on sizes of the detected finder patterns;

create the 2D reference mesh based on the estimated N×N grid size of the curved visual mark; and

calculate coordinate positions of the finder pattern points with respect to points of the 2D reference mesh.

16. The non-transitory computer readable medium of claim **15**, wherein the instructions are further to cause the processor to:

generate a 3D reference mesh based on the 2D reference mesh, the 3D reference mesh having a first radius;

establish correspondences between the 2D reference mesh and the 3D reference mesh;

estimate a 2D-3D projective transform from the established correspondences between the 2D reference mesh and the 3D reference mesh;

calculate a reprojection error regarding the estimated 2D-3D projective transform;

determine whether the calculated reprojection error is lower than an initial projective transform error for the 2D reference mesh; and

based on a determination that the calculated reprojection error is lower than a current best reprojection error, set the radius of the 3D reference mesh as the current best reprojection error;

increment a candidate radius by a predefined value;

determine whether the incremented candidate radius is lower than an upper limit; and

based on the incremented candidate radius being lower than the upper limit, generate a next 3D reference mesh using the incremented candidate radius.

17. The non-transitory computer readable medium of claim **16**, wherein the instructions are further to cause the processor to:

establish correspondences between the 2D reference mesh and the next 3D reference mesh;

estimate a second 2D-3D projective transform from the established correspondences between the 2D reference mesh and the next 3D reference mesh;

calculate a second reprojection error regarding the second estimated 2D-3D projective transform;

determine whether the second calculated reprojection error is lower than the second calculated reprojection error; and

based on a determination that the second calculated reprojection error is lower than the calculated reprojection error, set the radius of the next 3D reference mesh as the radius of the determined curved 3D mesh.

18. The non-transitory computer readable medium of claim **16**, wherein the instructions are further to cause the processor to:

based on a determination that the calculated reprojection error is not lower than the best reprojection error, define a candidate radius to equal the radius corresponding to the current best reprojection error subtracted by the predefined value;

define the upper limit to equal the radius corresponding to the current best reprojection error;

divide the predefined value by two; and

determine whether the predefined value divided by two is below a certain value.

19. The non-transitory computer readable medium of claim **18**, wherein the instructions are further to cause the processor to:

based on a determination that the predefined value divided by two is below the certain value, generate a next 3D reference mesh using the defined candidate radius;

establish correspondences between the 2D reference mesh and the next 3D reference mesh;

estimate a second 2D-3D projective transform from the established correspondences between the 2D reference mesh and the next 3D reference mesh;

calculate a second reprojection error regarding the second estimated 2D-3D projective transform;

determine whether the second calculated reprojection error is lower than the second calculated reprojection error; and

based on a determination that the second calculated reprojection error is lower than the calculated reprojection error, set the radius of the next 3D reference mesh as the radius of the determined curved 3D mesh.

20. The non-transitory computer readable medium of claim **13**, wherein the curved visual mark comprises a curved quick response code.

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