UNIVERSIDADE FEDERAL DE PERNAMBUCO
PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA DE PRODUÇÃO

A NEW MODEL FOR IMPERFECT MAINTENANCE UNDER DEPENDENT CORRECTIVE AND PREVENTIVE ACTIONS

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Advisor: Prof. Enrique López Droguett

RECIFE, NOVEMBER / 2011

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"A NEW MODEL FOR IMPERFECT MAINTENANCE UNDER DEPENDENT CORRECTIVE AND PREVENTIVE ACTIONS"

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A comissão examinadora, composta pelos professores abaixo, sob a presidência do(a) primeiro(a), considera o candidato **RICARDO JOSÉ FERREIRA APROVADO.**

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RESUMO

O presente trabalho propõe um modelo para a manutenção imperfeita sob ações corretivas e preventivas dependentes. O modelo é baseado no acoplamento entre um modelo de Riscos Competitivos (o modelo de alerta de reparação) e Processo de Renovação Generalizado. Normalmente, os Processos de Renovação Generalizados são capazes de capturar a qualidade da manutenção executada como sendo perfeita, mínima ou imperfeita, porém eles não conseguem distinguir entre os tipos de falhas. Por outro lado, Riscos Competitivos é uma metodologia que é capaz de captar os diferentes tipos de falhas - e, consequentemente, os diferentes tipos de manutenção (corretiva e preventiva) - não podendo, no entanto, estudar sistemas reparáveis. Portanto, o modelo híbrido proposto neste trabalho é capaz de preencher a lacuna das duas metodologias através de um modelo robusto. O modelo proposto é caracterizado através da construção de funções probabilísticas, o desenvolvimento de um teste de hipótese para verificar a sua aplicabilidade, e estimadores de máxima verossimilhança obtidos através da técnica conhecida como otimização por enxame de partículas. Um exemplo de aplicação de dados de falha de um sistema de compressor *offshore* é fornecido e os resultados mostram que o modelo proposto prevê um melhor ajuste aos dados de falha do que outro modelo disponível na literatura com base na abordagem *Intensity Proportional Repair Alert+Brown Proschan*.

Palavras-chave: *Riscos Competitivos, Processos de Renovaçao Generalizados, Análise de Confiabilidade.*

ABSTRACT

The present work proposes a model for imperfect maintenance under dependent corrective and preventive actions. The model is based on the coupling between a Competing Risks model (the repair alert model) and Generalized Renewal Process. Typically, Generalized Renewal Processes are able to capture the quality of maintenance performed as being perfect, minimal or imperfect, however they cannot distinguish between types of failures. On the other hand, Competing Risk is a methodology that is able to capture the different types of failures - and consequently the different types of maintenance (corrective and preventive) - not being able, however, to study repairable systems. Therefore, the hybrid model proposed in this work is capable of filling the gap of the two methodologies through a robust model. The proposed model is characterized via the construction of probabilistic functions, the development of a hypothesis test to verify its applicability, and maximum likelihood estimators obtained through the optimization technique known as Particle Swarm Optimization. An example of application to failure data of an offshore compressor system is provided and the results show that the proposed model provides a better fit to the failure data than other models available in the literature based on the Intensity Proportional Repair Alert + Brown-Proschan framework.
Keywords: *Competing Risks, Generalized Renewal Process, Reliability Analysis.*

SUMMARY

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LIST OF ACRONYMS

CR – Competing Risks

RSC – Random Signs Censoring

RA – Repair Alert model

IPRA – Intensity Proportional Repair Alert model

BP – Brown and Proschan model

GRP – Generalized Renewal Processes

PSO – Particle Swarm Optimization

CDF – Cumulative Distribution Function

PDF – Probability Density Function

LIST OF SYMBOLS

X = time between critical failures;

Z = time between degraded/incipient failures;

Y = min(X, Z);

J = indicator variable that assumes the value 0 for critical failures and 1 for degraded/incipient failures;

r = P(Z<X);

w_i = the virtual age at the *i*th failure;

q = rejuvenation parameter;

ω = hazard function;

Ω = accumulated hazard function.

1 INTRODUCTION

Literature shows recent advances in theory and applications of stochastic point process models in reliability engineering since first approaches through Renewal Processes (RP) until virtual age models. These methodologies are applied in the study of the behavior of a system after a failure – the state of operation of a system after a repair action.

Maintenance actions and their effectiveness can lead the equipment to a different state after its return to operation. Three well-defined states after a repair are often treated – the perfect repair brings the equipment to a state as good as new; the minimal repair brings the equipment to a state it had just before the failure; and the imperfect repair brings an equipment to a state better than before but not so good as a new one. The perfect repair is treated in literature through the use of Renewal Processes (RP), whereas the minimal repair is treated by Non-Homogeneous Poisson Processes (NHPP). The imperfect repair can be modeled via virtual age models. Kijima & Sumita (1986b), Krivtsov (2000) and Yañez et al. (2002) developed some works that treat this kind of situation. Among the models proposed to analyze the maintenance effectiveness, one can cite the Generalized Renewal Process (GRP), which is capable to analyze the failure time distribution through a concept called virtual age (originally presented by Kijima & Sumita (1986b)). The virtual age is measured through the rejuvenation parameter q included for the development of the GRP. With this parameter, the GRP is capable to treat all three situations described above, and also two more unusual situations – the equipment can be repaired to a state better than new or the equipment can be brought back to operation worse than before the failure.

However, any of these processes, including GRP, cannot distinguish among different types of failure causes and then among different types of maintenance (corrective or preventive).

Amongst these formal techniques used to perform a reliability analysis, one known by Competing Risks (CR) is discussed in this work. Basically, CR can provide a complete analysis of the probabilistic behavior of failures as many other methodologies presented in the literature. However, CR has an additional feature addressing not only failure times but also their causes through a pair of observations. Basically, CR treats the scenario where two (or more) risks (failure modes, preventive maintenance, etc.) are competing to be responsible for the equipment's stoppage. Crowder (2001) presents a complete description of CR theory, presenting its main characteristics, functions and properties.

CR is widely applied in fields of medicine, economics, engineering and so forth. The use of CR in medical area has many similarities with its application in engineering context, since the theoretical basis is related to survival/reliability analysis for each area. In the reliability context, the main challenge is to properly relate critical and degraded failures – the former is often treated as corrective maintenance and the latter, as preventive maintenance. This relationship is expected to be of dependence – preventive maintenance tends to cause a delay in the occurrence of critical failure, avoiding more costly maintenance.

Besides the situation of dependence described above, CR can also model relationships of independence between risks. The independence case can be illustrated where there is interest to study failure time and its possible failure modes. Cooke (1996b) presents a review about CR concepts and its main models treating independence and dependence situations.

In a real scenario, the dependence relationship between failures and maintenance deserve some attention. Cooke (1993a) shows concern about this problem and presented a model capable of capturing the relationship between critical and degraded failures through the so-called Random Signs Censoring (RSC) model. Basically, this model states that preventive maintenance can “censure” corrective action by being executed when the failure is on the degraded state. Cooke (1996b) presents several models on CR context where RSC is also presented. Cooke & Paulsen (1997) show an application related to preventive and corrective maintenance through CR methodology. Several applications of CR can be seen in Bunea *et al.* (2006), Jiang (2010), Sarhan *et al.* (2010), Ferreira *et al.* (2010), Ferreira *et al.* (2011), Christen *et al.* (2011).

After the occurrence of an event, the system status is not investigated by CR. In Reliability Engineering, this corresponds to study a system only until the moment of a failure – critical or degraded – and the operational status of the system after a repair action, for example, is not investigated. Therefore, the system is always treated as non-repairable – after the occurrence of a failure, a new one is replaced.

Langseth & Lindqvist (2003; 2005) present a model capable of modeling the quality of a maintenance using the Brown & Proschan (1983) model – called as the BP model – where perfect repairs are considered to occur with probability p and minimal repair, with probability $(1-p)$ coupled with a CR model known by Repair Alert (RA) model. However, this joint model is not able to capture correctly the imperfect repair. Doyen & Gaudoin (2006b) discuss in details the use of virtual age models in a general framework of CR. They present the importance of properly treating the imperfect maintenance (through virtual age models) and show the advantages when such a model is considered in a CR scenario. Ferreira *et al.* (2010) present a hybrid model capable of modeling a CR situation using the RA model developed in Lindqvist *et al.* (2005) and the GRP, characterizing its main functions and developing a hypothesis test to verify the applicability of the proposed model and comparing results with Langseth & Lindqvist (2005) in terms of best fitting.

Ferreira *et al.* (2011) present the likelihood function of Ferreira *et al.* (2010) model and develop the Maximum Likelihood Estimators (MLE) for a Weibull distribution, showing results related with reliability metrics.

The failure behavior is of great importance, given its impact on equipment's longevity of equipment. Furthermore, equipments in industrial applications are usually more expensive to repair or replace than preventively maintaining them. In this way, understanding failure behavior can avoid high costs and improve the longevity of equipment. In order to do this, specific information about failures is needed to make a consistent analysis possible. One way to obtain this type of information is Reliability Databases (RDB). RDB have been developed since the mid of twentieth century and Fragola (1996) presents a historical discussion about their development.

With the amount of information present in RDB, a reliability analysis earns in robustness and its metrics are more realistic. Cooke (1996a) makes a review of the main concepts concerning the use of RDB. Bedford & Cooke (2002) discuss the importance of using a structured database with well-defined parameters in qualitative and quantitative field. Bunea *et al.* (2006) discuss the growing interest of developing or applying formal techniques to analyze modern reliability database in a military scenario. Thus, it can be seen that structured information can provide more confident metrics used in a study and its use is on a rising demand.

The likelihood function presented in Ferreira *et al.* (2011) is extended considering that each type of maintenance action taken – corrective or preventive – has different effectiveness. Thus, the likelihood function now presents one more parameter considering the GRP parameter as two – one q is corresponding to corrective maintenance (q_x) and another to preventive maintenance (q_z). That is, the corrective and preventive maintenance actions effectiveness are explicitly taken into account.

The validation process is made on the joint probability density function (PDF) of the proposed model and some example applications are also provided to estimate the parameters.

Due to the complexity of the likelihood function, numerical methods are necessary to achieve the required estimates

In order to obtain the MLE for the proposed model likelihood function, evolutionary algorithms are employed to estimate the likelihood parameters. The main advantage of these methods against Monte Carlo simulation, as used in Yañez *et al.* (2002), is the efficiency. Among the methods of evolutionary algorithm methods, one can cite the use of Genetic Algorithms (GA) and Particle Swarm (PS). Their main goal is to optimize a function of interest, called *fitness*. GA treats the optimum solution as the best individual – the strongest one of a population studied – using the evolutionary theory as basis. PS tries to achieve the optimum solution considering the behavior of a set of particles, where the optimum solution is defined to be the information shared by all (or a set of) particles. Bratton & Kennedy (2007) bring a discussion about PS optimization (PSO), whereas Firmino *et al.* (2007) present an application of PSO to estimate GRP parameters trying to compare this method with the one presented one in Yañez *et al.* (2002).

In this work, Particle Swarm Optimization (PSO), introduced by Kennedy & Eberhart (1995), is used. For more details, see Lins *et al.* (2010) and Bratton & Kennedy (2007).

1.1 Justification

This work presents the development of a new model for imperfect maintenance under dependent corrective and preventive actions. The proposed model is based on dependent CR and GRP. CR is a methodology that allows relating possible causes to their failure times creating a dependence relationship between these causes. Specifically, these causes can be defined as failure severity, which has direct influence on the type of maintenance made: critical failures are treated with corrective maintenance and degraded/incipient failures, with preventive maintenance.

CR methodology usually analyzes non-repairable systems disregarding the status of a system after a maintenance action. This kind of situation is treated in the literature by Stochastic Process where it is assumed that the equipment can be brought back to operation as a new one (Renewal Process modeling), as bad as before the failure (NHPP modeling) and better than old but worse than new (virtual age models). However, these methodologies are only capable of modeling one type of maintenance at a time.

Thus, the proposed model fills the gap between these two methodologies aiming to provide a more comprehensive modeling approach for cases where a system is subjected to imperfect maintenance actions and where both (possibly coupled) corrective and preventive maintenance are performed more specifically.

1.2 Objectives

1.2.1 Main Objective

The main objective of this dissertation is to develop and characterize an imperfect maintenance model under dependent corrective (critical failures) and preventive (degraded failures) maintenance actions. The dependence between critical and degraded failures regarding the quality of repair after each maintenance action taken is achieved by the use of the CR methodology and the GRP is used to model the quality of repairs.

1.2.2 Specific Objectives

To achieve what is proposed, the following specific objectives are established:

- Development the imperfect maintenance model based on the CR methodology and the GRP;
- Development of specific functions of the proposed model and its main characteristics;
- Validation of the PDF governing the proposed model;
- Development of a hypothesis test to verify the applicability of the proposed model;
- Derivation of the likelihood function of the proposed model considering the GRP modeling;
- Implement these function to calculate the ML estimators through Particle Swarm Optimization;
- Perform comparisons between existing models in literature and the proposed model.

1.3 Dissertation Layout

This dissertation is organized as follows:

- **Chapter 2**: brings the theoretical background discussing concepts of Competing Risks and its main models, including the used model. GRP is also presented and discussed with its main modeling characteristics. Furthermore, a brief discussion about PSO is presented since it is the numerical method used in this work.
- **Chapter 3**: presents the whole development of the proposed model, from hypothesis test to the proposed likelihood.
- **Chapter 4**: presents some applications aiming to validate the model and compare it with another one presented in literature.
- **Chapter 5**: brings conclusions about the developed work.

2 THEORETICAL BACKGROUND

The theory of CR, GRP and its main definitions are presented in next Sections.

2.1 Competing Risks

In order to present a definition of easy understanding about the meaning of a competing risk scenario, consider the following example. Let a vehicle be an operating system of interest that a costumer acquired. This vehicle has constant use and is operational at most part of the day. An undesired event for the user is, for example, the car does not work when he needs to go to work. In a car, there are several kinds of failures – a tire can blow during the trip or it can run out of gas – which are independent events. Here it is characterized a situation of independent risks – a blown tire or the lack of gas are competing to be responsible for the car's stoppage.

In other situation, consider again that the user drives his car every day to his work. In a real situation, cars must pass for reviewing processes to do some maintenance in items that can result in critical failures. During this review, the mechanic can discover items that are in a degraded state of use and he/she decides, or not, to perform a maintenance on these items. With this, it is expected that, after this review, the car is able to operate for a longer time without breaking. Thus, this situation represents a dependence relationship between preventive and corrective actions.

In the CR context, risks that are trying to stop an operation causing an undesirable event are called bad risks, whereas risks that bring some kind of stoppage delay are called good risks. In the reliability context, failure modes are treated as bad risks, whereas preventive actions are treated as good risks.

Given the intuitive vision of a scenario of competing risks, consider now the following definition.

Definition 1: *Let X and Z be the time to failure and time until preventive maintenance of the system, respectively. Let Y be the time until the occurrence of the first event in the set of relevant events during the observation period, that is to say, Y = min (X, Z) - the minimum time between the two competing risks in relation to the unit under study. It is also important to define as J (= 1, 2 ,...., k) the cause of the event that occurred.*

For convenience, Y is considered in this work as the time between the occurrence of events (with X being the operating time until a failure and Z the operating time until a preventive maintenance to fix a degraded/incipient failure). Thus, Z is often treated as preventive maintenance or degraded failure (where degraded include the concept of incipient due to their similarities). Figure 2.1 presents the classification aforementioned. Here, *p(t)* represents the performance of the system which can designate a degraded/incipient level through the occurrence of a degraded failure or a critical level with the occurrence of a critical failure.

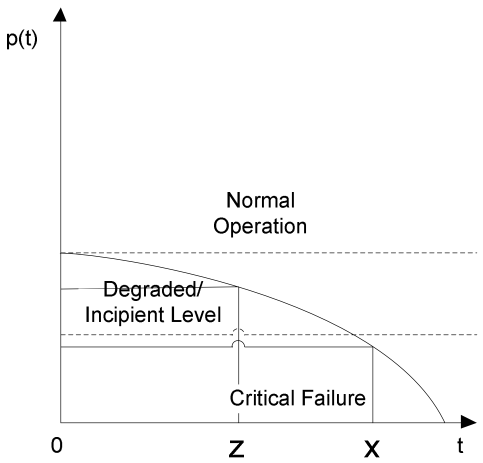


Figure 2.1 – Classification of failures according with the state of operation

It is also taken into account that only two types of risks are competing against each other - critical failures and degraded failures. As previously mentioned, this scenario is then treated with dependent competing risks.

Some additional definitions are required.

Definition 2*: Given that Y = min (X, Z), the joint probability* $P(X < Z, X \le t)$, *represented by* $F_X^*(t)$, *is known as sub-distribution of X for studying the event "a critical failure occurs before the preventive maintenance and it occurs until the time t". Similarly,* $P(Z < X, Z \le t)$ *is known as sub-distribution of Z, being represented by* $F_Z^*(t)$.

Definition 3: *Consider the probability given by* $P(X < Z, X > t)$ *the sub-reliability (or sub-survival) function of X and it is represented by* $R_X^*(t)$. *Similarly,* $P(Z < X, Z > t)$ *represents the sub-reliability of Z, being represented by* $R_Z^*(t)$.

These functions have different characteristics and properties of probability distributions. $F_X^*(t) + R_X^*(t) = p$ where $p = pr(X < Z) = F_X^*(\infty) = R_X^*(0)$ is one of its characteristics.

Bedford & Cooke (2002), Lindqvist *et al.* (2005), Crowder (2001) and other authors describe in detail such features.

One of the most simple and widely used model to deal with dependence is the RSC model. The RSC consists on relating preventive maintenance actions to the occurrence of failures. So, when a failure is latent, the component emits some kind of “signal” (inspection, in general) that alerts the maintenance crew to make a preventive maintenance. Consider the definition as follows.

Definition 4*(Bedford & Cooke (2002))***:** *Let X and Z be life variables with* $Z = X - \xi$ *where* ξ *is a random variable,* $\xi \le X$, $P(\xi = 0) = 0$ *, whose sign is independent of X. The variable* $Y = (\min(X, Z), J)$ *is called a random signs censoring of X by Z.*

From the definition above, one can see that Z (maintenance action) aims at preceding X (the critical failure), if the random quantity ξ is greater than zero. Otherwise, Z occurs after X and one can see a situation where Z is censored by X, which is not desirable.

Models can be chosen depending on some characteristics. Cooke (1993) presents a theorem that captures some important information about the random signs censoring.

Theorem 1:*Let* (R_1^*, R_2^*) *be a pair of continuous strictly monotonic sub-survival functions; then the following are equivalent:*

(1) *There exist random variables* ξ *and* X , $X \perp \text{sgn}(\xi)$, *such that* $R_1^*(t) = P(X > t, \xi < 0)$ *and* $R_2^*(t) = P(X - \xi > t, \xi > 0)$.

(2) *For all* $t > 0$, $\dfrac{R_1^*(t)}{R_1^*(0)} > \dfrac{R_2^*(t)}{R_2^*(0)}$.

From (1), one can observe the orthogonal relationship between the variable X and the signal from ξ (sgn). Therefore, the interpretation of the sub-survival functions can be seen from values that ξ assume. Furthermore, one can observe that the ratio of sub-survival functions for an earlier moment is greater than for a latter moment.

From this theorem one can observe that the conditional probability of censoring, $\phi(t) = P(Z < X \mid Z \wedge X > t) = \dfrac{R_Z^*(t)}{R_Z^*(t) + R_X^*(t)}$, is maximal at the origin, where $Z \wedge X$ represents the minimum between Z and X.

This is important because it becomes possible to estimate the sub-survival function from data and then to adjust $\phi(t)$. Thus, if $\phi(t)$ has the characteristic above, the data follows the RSC model.

The practical implication of this model is the possibility of preventing a critical failure through some kind of signal that alerts the failure's proximity, and performing a preventive maintenance to avoid the critical failure.

Lindqvist et al. (2005) present a model based on RSC, namely Repair Alert (RA). RA approach performs the same analysis as that of RSC, but it also analyzes the "alertness" of the maintenance crew through a function called repair alert function.

$$P\left(Z \le z \mid X = x, Z < X\right) = \frac{G(z)}{G(x)}, \qquad 0 \le z \le x \tag{2-1}$$

where G(.) is an increasing function with G(0) = 0. Lindqvist *et al.* (2005) states that the repair alert function is meant to reflect the reaction of the maintenance crew. More precisely *g(t)* ought to be high at times *t* for which failures are expected and the alert therefore should be high. When the hazard function is used, then Equation (2-1) characterizes the Intensity Proportional Repair Alert (IPRA) model. In the proposed model (see Chapter 3), this model is used to capture the Competing Risks situation.

The practical interpretation of this function can be briefly defined as the proportionality between intensity functions of risks Z and X. This probability can be interpreted as the conditional probability that a preventive maintenance action (PM) be performed until z, given that there would be a failure at time $X = x$ and given that the maintenance team will perform a PM before a failure occurs (Z<X). Such a probability is proportional to the repair alert function (LINDQVIST et.al, 2005). So, it can be seen that the repair alert function reflects the reaction of the maintenance team against the failure risk. The choice of this function depends on the problem under consideration. Thus, the proposed model is capable to capture the dependence between preventive maintenances and critical failures. Furthermore, it can analyze the maintenance crew "alertness", as shown in Figure 2.2. Here, the closer the preventive maintenance is performed of the critical failure, the better the alertness of the team. Thus, Z' represents the best situation in Figure 2.2. With the value of Z' (or Z), corresponding values of U (U') are captured in H(X) (which represents the hazard function). Therefore, values of H^{-1}(U) (as to U') are identified through time, and then, the probability shown in Equation (2-1) can be drawn.

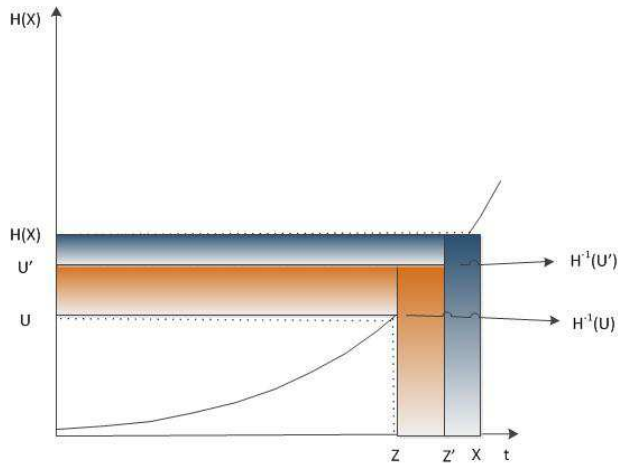


Figure 2.2 – Behavior of conditional probability of preventive maintenance

Table 2.1 summarizes the main concepts and notation of the CR methodology that is used here.

Table 2.1– Notations and definitions used in CR methodology.

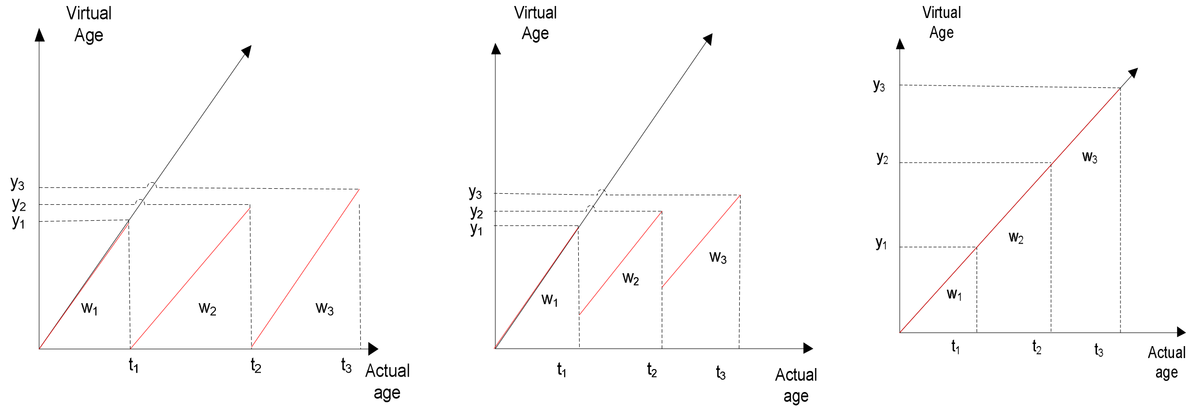
Expression	Description	Main properties
X	Time of a critical failure	x > 0
Z	Time of a degraded failure	z > 0
Y = min(X, Z)	Time of occurrence of an event	y > 0
J = (0,1)	An indicator variable	Discrete variable
(Y,J)	Observed pair from data	–
$F_{Y,J}(y, j)$	Joint distribution of Y and J	Joint CDF properties
$f_{Y,J}(y, j)$	Joint density of Y and J	Joint PDF properties
$F_Y(y)$	CDF of Y	CDF properties
$f_Y(y)$	PDF of Y	PDF properties
$R^*_X(x)$	Sub-survival of X	$R^*_X(\infty) = 0$; $R^*_X(0) = 1 - R^*_Z(0)$
$R^*_Z(z)$	Sub-survival of Z	$R^*_Z(\infty) = 0$; $R^*_Z(0) = 1 - R^*_X(0)$

As can be seen, CR methodology can model the relationship between an occurred event and its related cause. However, in terms of the maintenance actions taken, it is interesting to analyze the quality of maintenance. This issue is not directly treated by CR methodology. However, there are some methodologies capable of doing this in the Stochastic Processes area. One of them is known as Generalized Renewal Processes and is presented in the next Section.

2.2 The Generalized Renewal Process

The GRP model was developed to study the stochastic nature of repairs. Besides of dealing with multiple repair types (minimal, perfect, imperfect, worse or better than before), its advantage is on the fact that the concept of virtual age is introduced. Moura et al. (2007) present a complete discussion about the main definitions and implications of GRP.

It is necessary to define the concept of virtual age for the complete understanding of the GRP. The virtual ages y_i and w_i correspond to the calculated age of the equipment before and after the *i*th repair action, respectively, whereas the actual age t_i corresponds to the chronological time (also known as process time). Figure 2.3 shows the relationship between real and virtual ages y_i and w_i, according to the quantity *q*, which is called the rejuvenation parameter and will be used here to assess the degree of effectiveness of a repair action.



(a) $q = 0$ (b) $0 < q < 1$ (c) $q = 1$

Figure 2.3 – Relationship between real age and virtual age.

The virtual age w_i represents the reduction in the actual age of the system promoted by the *i*th repair action and it is directly related to the parameter *q*. The values assumed by this parameter allow for the representation of the types of repairs cited in the start of this Section. It can be done as follows:

• q = 0: corresponds to a perfect repair, since the virtual age w_i is always restarted after the *i*th repair action;

• 0 <q<1: corresponds to an imperfect repair, since w_i is a fraction of the actual age you;

• q = 1: corresponds to a minimal repair, since w_i is exactly equal to the actual age t_i.

Other values for the parameter q are also possible, as $q< 0$ and $q> 1$ corresponding to "fix better" and to "fix worst," respectively.

Kijima & Sumita (1986b) proposed two types of virtual age models (for other models see Christen *et al.* (2011)). The first one (Equation (2-2)) is commonly called a Kijima type I model and consists fundamentally of the idea that repair works only with failures that occur in the range of exposure immediately prior to repair. Thus, being $t_1, t_2,...$ successive accumulated failure times, the virtual age of the system suffers proportional increments over time:

$$w_i = w_{i-1} + q \cdot h_i = qt_i, \tag{2-2}$$

where h_i is the time between the occurrence of $(i-1)$th and the *i*th failure.

The Kijima Type II model [Equation (2-3)] assumes that the repair works in order to recover the system failures resulting from all the previous intervals of exposure since the beginning of system operation. In this model, the virtual age suffers proportional increments throughout the range of cumulative exposure:

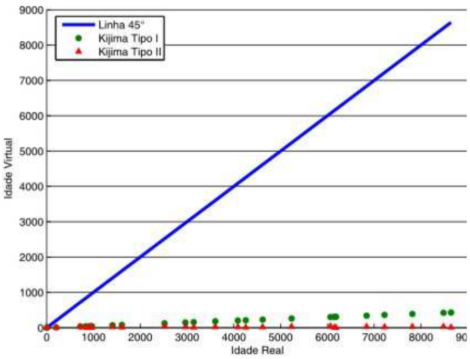
$$w_i = q(h_i + w_{i-1}) = q(q^{i-1}h_1 + q^{i-2}h_2 + \ldots + h_i). \tag{2-3}$$

Thus, the Kijima Type I model assumes that the *i*th repair cannot remove all the damage incurred to the *i*th failure only by reducing the additional age h_i, whilst the Kijima Type II model assumes that for the *i*th repair, the virtual age was accumulated in $w_{i-1} + h_i$. Thus, the *i*th repair will remove the accumulated damage due to failures during the last interval of exposure as well as in the previous intervals.

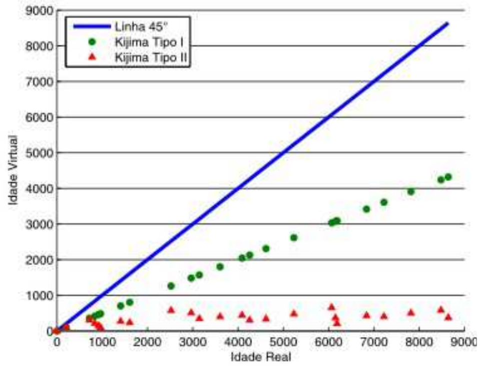
Ferreira *et al.* (2010) and Doyen & Gaudoin (2006b) argue that the choice between model Kijima type I or type II is directly related to the scope of the repair. Thus, the following recommendations are presented:

1. For individual components, the most appropriate model is the Kijima Type I.
2. For complex systems, the Kijima Type II is more appropriate.

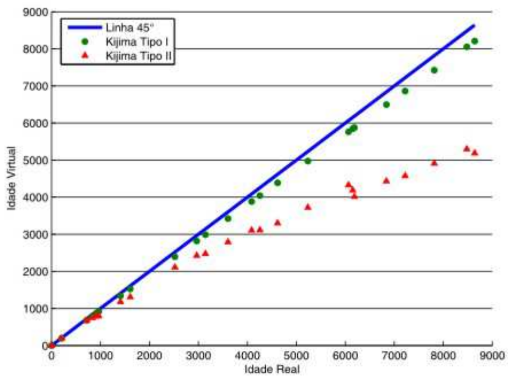
To illustrate the difference between these models, Figure 2 shows the relationship between actual age and virtual age w for different values of parameter q.



(a) q = 0,05



(b) q = 0,50



(c) q = 0,95

Figure 2.4 – Relationship between the Kijima type I and type II models, according to the parameter q.

Note in Figure 2.4 (a) that the difference between the models Kijima Type I and Type II is barely noticeable for a low value of *q*, which represents a high efficiency of repair actions. In other words, the repair is so effective that little would represent the difference between a repair located in an equipment or system as a whole. Times of virtual ages for the two models become more discrepant when the efficiency is decreased, as seen in Figure 2.4 (b). When the repair becomes very inefficient in their actions, the virtual ages approach from each other (Figure 2.4 (c)). In this case, the repair action does not reflect improvements in equipment.

Regardless of the Kijima model, one can evaluate the *i*th failure time t_i through the cumulative distribution function conditioned on the virtual age w_{i-1}, as in Equation (2-4):

$$F(t_i \mid w_{i-1}) = P\left(T \le t_i \mid T > w_{i-1}\right) = \frac{F(t_i) - F(w_{i-1})}{1 - F(w_{i-1})}, \tag{2-4}$$

where $F(\cdot)$ is the cumulative distribution function of time until the first failure.

Note that GRP can analyze the quality of a repair. However, this methodology treats all maintenance actions as corrective repairs. The implications of this characteristic involve costs with maintenance, the quality of the maintenance crew related with critical situations of failure and non-critical situations. This kind of relationship will be modeled by CR models in the proposed model.

2.3 Particle Swarm Optimization

Introduced by Kennedy & Eberhart (1995), this computational algorithm has as basic element a particle. This particle can traverse the entire search space towards an optimum by using its own information as well as the information provided by other particles comprising its neighborhood. Bratton & Kennedy (2007) state that this neighborhood is a subset of particles with which the particle is able to communicate. The optimum information of each particle is given by its *fitness*.

Basically, there exist two kinds of model for particles to achieve the optimum. The *gbest* model allows each particle to communicate with every other particle in the swarm, obtaining information from them. The *lbest* model allows a particle to communicate with a limited number of particles in the swarm. Bratton & Kennedy (2007) state that the *gbest* model usually converges more rapidly than the *lbest* model. However, this can make the *gbest* model achieves a premature convergence which is not desirable for multi-modal functions.

Firmino *et al.* (2007) present an application of a PS algorithm to estimate the MLE for a Weibull distribution considering the GRP parameter. Basically, they treat the log-likelihood function as the fitness to be maximized and define the search space to the involved parameters’ range.

This dissertation makes use of this methodology to estimate the parameters of the likelihood of the proposed model. In fact, CR and GRP are combined: in the proposed model presented in the next Chapter, CR models analyze the relationship between failure times and maintenance actions taken, whereas GRP analyze the quality of repair made. Thus a hybridism approach will be shown to be a convenient way to jointly treat dependent corrective and preventive actions as well as explicitly quantifying the repair effectiveness.

3 PROPOSED MODEL FOR IMPERFECT MAINTENANCE UNDER DEPENDENDENT CORRECTIVE AND PREVENTIVE ACTIONS

As stated in Chapter 1, the proposed model is based on the hybridism between CR and GRP. In particular, the model put forward in this Section takes as a starting point the Intensity Proportional Repair Alert (IPRA) model proposed by Langseth & Lindqvist (2003). Thus, this Chapter starts by providing some details about the IPRA model in the next Section, and then builds up on it in order to develop the proposed imperfect maintenance model.

3.1 The IPRA model

The IPRA model is a particular case of the RA model to model perfect maintenance through the dependence of corrective and preventive maintenance. As it shall be clear at the end of this Section, the IPRA model is not capable of capturing the corrective and preventive maintenance effectiveness.

The RSC model is appropriated for cases where one wants to analyze a system where the occurrence of (undesirable) events will be advised from a kind of signal and then be censored by another event (a desirable one). In this context, Langseth & Lindqvist (2003) construct a model that studies the “alertness” of the maintenance team against failure events.

They developed the RA model from the RSC by introducing a function, called $G(\cdot)$ – the cumulative repair alert function, which represents the conditional probability shown in Equation (2-1). This model is known as IPRA model when g(.) is the conditional intensity function.

From this, the joint model for (X, Z) can be given as follows:

(i) X has hazard rate $\omega(x)$;

(ii) $\{Z < X\}$ and X are stochastically independent;

(iii) Z given $Z < X$ and $X = x$ has distribution function as in Equation(2-1).

With these requirements, Langseth & Lindqvist (2003) determine the distribution of the observed pair (Y, J) as follows. First, (*ii*) provides:

$$P(y \le Y \le y+dy, J=0) = P(y \le X \le y+dy, X < Z) = (1-r)\,\omega(y)\exp(-\Omega(y))\,dy \qquad (3\text{-}1)$$

Next:

$$P(y \le Y \le y+dy, J=1) = P(y \le Z \le y+dy, Z < X) = \int_y^{\infty} \begin{matrix} P(y \le Z \le y = dy \mid X = x, Z < x) \\ \times P(Z < X \mid X = x)\,\omega(x)\exp(-\Omega(x))\,dx \end{matrix} = \tag{3-2}$$

$$= r\omega(y)\,dy \int_y^{\infty} \frac{\omega(x)\exp(-\Omega(x))}{\Omega(x)}\,dx = r\omega(y)\,Ie(\Omega(y))\,dy$$

Thus, it is straight forward to establish the joint density function of (Y,J) as follows:

$$f(y,j) = \omega(y)(1-r)^{1-j}\exp(-\Omega(y))^{1-j}\,r^j\,Ie(\Omega(y))^j \tag{3-3}$$

where the function $Ie(\Omega(y))$ is known as exponential integral and is defined by $Ie(t) = \int_t^{\infty} \exp(-u)/u\,du$, the parameter r represents the probability of occurring a preventive maintenance before a critical failure ($r = P(Z < X)$), and $\Omega(y)$ represents the cumulative hazard function.

The function in Equation (3-3) yields the density function for Y as can be seen below:

$$f(y) = \sum_{j=0}^{1} f(y,j) = \omega(y)(1-r)\exp(-\Omega(y)) + \omega(y)\,r\,Ie(\Omega(y)) \tag{3-4}$$

Thus, Equation (3-4)allows the analysis of event times regardless of event causes. The CDF can be obtained from Equation(3-4):

$$F(y) = 1 - \exp(-\Omega(y)) + r\Omega(y)\,Ie(\Omega(y)) \tag{3-5}$$

From Equation (3-5) one can obtain the reliability function of the inter-event times as follows.

$$R(y) = \exp(-\Omega(y)) - r\Omega(y)\,Ie(\Omega(y)) \tag{3-6}$$

Equation (3-6)represents the case where neither a critical nor a degraded failures are observed up to time y. The first part on the right side of Equation (3-6) corresponds to the contribution of the critical failures, and the second part is the contribution of degraded failures. Langseth & Lindqvist (2005) show that the second part is stochastically smaller than the first part. Furthermore, note that when r = 0, Equation (3-6) corresponds to the reliability function – without distinguishing between critical and degraded failure.

The IPRA model ensures the study of the reliability function of critical failure direct from data since this model has the same property of the RSC. Since the RA model is derived from the RSC model, the subsurvival function of failure times is identifiable from data. Lindqvist et al. (2005) show that the subdistribution of data can be interpreted as $F_X^*(x) = (1-r)F_Y(y)$. However, this is not true for Z.

Thus, it is reasonable to consider two cases when J assumes specific values in Equation (3-3):

$$f\left(y_i,0\right)=\left(1-r\right)\omega\left(y\right)\exp\left(-\Omega\left(y\right)\right) \tag{3-7}$$

$$f\left(y_i,1\right)=r\omega\left(y\right)\mathrm{Ie}\left(\Omega\left(y\right)\right) \tag{3-8}$$

From Equation (3-7), it is possible to study critical failures. This expression yields the equivalent study to the reliability function of critical failures. Equation (3-8) corresponds to the influence of degraded failures.

The next Section discusses the proposed imperfect model when GRP is coupled with IPRA in order to explicitly capture corrective and maintenance actions effectiveness (quality).

3.2 The proposed model

One of the major limitations of the IPRA model as put forward by Langseth & Lindqvist (2005) is that it considers only dual repair types: minimal or perfect repairs, that is, the parameter p, which is the probability of perfect repair and (1-p) which is the probability of minimal repair. This model cannot represent the intermediate stages of repair.

Ferreira *et al.* (2010) presented the implementation of the GRP parameter on the IPRA model and this process is described as follows. First of all, it is necessary to find the PDF for the hybrid IPRA + GRP model. Considering that the PDF of the time until first failure is equal in both models (since none suffer any kind of intervention) this function is given by Equation (3-3).

From the second event, the PDF is modified since it becomes a conditional PDF. This conditional PDF differs from the PDF found by Langseth & Lindqvist (2003) due to the GRP parameters involved. Thus the way to calculate a conditional distribution function to the nth event (a failure or corrective maintenance action) is given below.

$$\begin{aligned}F\left(y_n\,|\,w_{n-1}\right)&=P\left(Y<y_n\,|\,Y>w_{n-1}\right)=1-P\left(Y>y_n\,|\,Y>w_{n-1}\right)=\\&=1-\frac{P\left(Y>y_n,Y>w_{n-1}\right)}{P\left(Y>w_{n-1}\right)}=1-\frac{P\left(Y>y_n+w_{n-1}\right)}{P\left(Y>w_{n-1}\right)}=1-\frac{R\left(y_n+w_{n-1}\right)}{R\left(w_{n-1}\right)}\end{aligned} \tag{3-9}$$

To obtain the conditional density function, one just needs to differentiate Equation (3-9) which results in the following expression:

$$f\left(y_n\,|\,w_{n-1}\right)=\frac{\partial}{\partial y_n}\left(1-\frac{R\left(y_n+w_{n-1}\right)}{R\left(w_{n-1}\right)}\right)=\frac{1}{R\left(w_{n-1}\right)}\left(-\frac{\partial}{\partial y_n}R\left(y_n+w_{n-1}\right)\right)=\frac{f\left(y_n+w_{n-1}\right)}{R\left(w_{n-1}\right)} \tag{3-10}$$

Since the reliability function can be obtained from the CDF, Equation (3-10) yields the following function:

$$f(y_n|w_{n-1})=\frac{(1-r)\,\omega(y_n+w_{n-1})\exp(-\Omega(y_n+w_{n-1}))+r\omega(y_n+w_{n-1})\,Ie(\Omega(y_n+w_{n-1}))}{\exp(-\Omega(w_{n-1}))-r\Omega(w_{n-1})\,Ie(\Omega(w_{n-1}))} \tag{3-11}$$

Considering the joint PDF, from the second event, the following expressions can be obtained for J =0,1:

$$f((y_n,0)\,|\,w_{n-1})=\frac{(1-r)\,\omega(y_n+w_{n-1})\exp(-\Omega(y_n+w_{n-1}))}{\exp(-\Omega(w_{n-1}))-r\Omega(w_{n-1})\,\mathrm{Ie}(\Omega(w_{n-1}))} \tag{3-12}$$

$$f((y_n,1)\,|\,w_{n-1})=\frac{r\omega(y_n+w_{n-1})\,\mathrm{Ie}(\Omega(y_n+w_{n-1}))}{\exp(-\Omega(w_{n-1}))-r\Omega(w_{n-1})\,\mathrm{Ie}(\Omega(w_{n-1}))} \tag{3-13}$$

Equation (3-12) and Equation (3-13) have a similar interpretation as Equations (3-7) and (3-8) but considering now conditional dependence from the second failure event.

It is important to note that it is necessary to check if these functions found hold the properties of probability densities and cumulative distributions. This was done by simulation, checking whether the functions satisfy the properties of distribution/density functions,[see Gichangi & Vach (2005)]. With these functions and the definition of the GRP, Ferreira *et al.* (2010) show that the GRP parameter has influence on the conditional PDF in Equation (3-11) as follows:

$$f(y_n|w_{n-1})=\frac{(1-r)\,\omega(y_n+w_{n-1})\exp(-\Omega(y_n+w_{n-1}))+r\omega(y_n+w_{n-1})\,Ie(\Omega(y_n+w_{n-1}))}{\exp\left(-\Omega\left(q^*\sum_{j=1}^{n-1}y_j\right)\right)-r\Omega\left(q^*\sum_{j=1}^{n-1}y_j\right)Ie\left(\Omega\left(q^*\sum_{j=1}^{n-1}y_j\right)\right)} \tag{3-14}$$

Equation (3-14) shows that the rejuvenation parameter q acts in denominator (as well as in numerator) as a multiplier of the time passed up to the (n-1)th event. Ferreira *et al.* (2011) use this idea to make inferences about thelikelihood for the proposed model.

However, in Ferreira *et al.* (2011), this is performed without distinguishing weather the repair is the same for corrective or preventive maintenance. It is reasonable to think that corrective maintenance has a different effect on the equipment when compared with the effect of preventive maintenance. Thus, the quality of corrective and preventive maintenances can be captured by different rejuvenation parameters:

$$f(y_n | w_{n-1}) = \frac{(1-r)\,\omega(y_n + w_{n-1})\exp\left(-\Omega(y_n + w_{n-1})\right) + r\omega(y_n + w_{n-1})\,Ie\left(\Omega(y_n + w_{n-1})\right)}{\exp\left(-\Omega\left(q_X\sum_{j=1}^{l} x_j + q_Z\sum_{k=1}^{m} z_k\right)\right) - r\Omega\left(q_X\sum_{j=1}^{l} x_j + q_Z\sum_{k=1}^{m} z_k\right) Ie\left(\Omega\left(q_X\sum_{j=1}^{l} x_j + q_Z\sum_{k=1}^{m} z_k\right)\right)} \quad (3\text{-}15)$$

Equation (3-15) shows the behavior of these two rejuvenation parameters in the PDF where q_X corresponds to the rejuvenation parameter of critical failures and q_Z, of degraded failures with l + m = n-1. The sum of number of critical and degraded failures must be equal to the number of events.

3.3 Verifying the Identifiability of the proposed model parameters

One of the most important steps of a CR model is to prove its identifiability. As can be seen on Lindqvist (2006a), the problem of identifiability turns around the fact that one cannot obtain the distribution of the time between relevant events via the distribution of the pair (Y, J) of observations. In other words, there are several distributions that give rise to the same distribution of the pair (Y, J).

The proposed model has its identifiability problem treated from a parametric stand point due to the functional form of the subdistributions be provided by parametric distributions as the Exponential, Weibull and so forth. According to Langseth & Lindqvist (2003), the parametric identifiability problem can be supplied if $\Psi(\boldsymbol{\theta}) = \Psi(\boldsymbol{\theta}^*)$ where $\boldsymbol{\theta}$ is a vector of parameters and $\Psi(\cdot)$ is the function that represents the distribution of the pair (Y, J).

Based on the proof of the identifiability of the IPRA parameters of Langseth & Lindqvist (2003), the proof of the identifiability of the proposed IPRA+GRP parameter can be given as follows:

Proof of the GRP parameter: *The main difference between the proof of identifiability of IPRA+BP and IPRA+GRP parameters is due to the modeling of imperfect repairs (BP and GRP). It can be noted, from the definitions of the BP and GRP models, that each of their parameters acts on the distribution of time between events. The BP parameters act as a multiplier of the functions whereas the GRP parameter acts on the age of the component. More precisely, one can represent the action of each parameter as follows*

$$f(y_n | y_1, \ldots, y_{n-1}) = \sum_{j=1}^{n} \begin{array}{l} f(y_n | y_1, \ldots, y_{n-1}, d_{j-1} = 0, d_j = \cdots = d_n \\ \times P(D_{j-1} = 0, D_j = \cdots = D_{n-1} = 1) \end{array} \quad (3\text{-}16)$$

In Equation (3-16), the conditional density shows that the BP parameter acts as a probability, where $P(D=0)=p$ *is the probability of a perfect repair and* $P(D=1)=1-p$ *is the probability of a minimal repair. From this, it can be noted that there are only two possibilities of repair, minimal or perfect, but weighted by probabilities.*

For the GRP case, the conditional density can be analyzed as described in Equation (3-10) where the virtual age is captured as described in Equation (3-14).In this case, it can be noted that the parameter acts on the times between events and they do not weight the function of the time. This is important due to two reasons: first is that the parameter q has the capacity to model the quality of the performed repair (involving more possibilities than the BP model as explained before). The latter is that the influence of the GRP parameter is on time and not as a multiplier. Since it was already proved that the functions of the IPRA model owns identifiability property and the GRP parameter is included inside these functions, one can conclude that the identifiability of q was proved together with these functions. This can be seen in Equation (3-14).

Consider the following two types of probabilities

$$P(x \le X \le x+dx, Z > x); x < \tau$$
$$P(z \le Z \le z+dz, X > z); z < \tau$$

As showed by Langseth and Lindqvist (2003), these probabilities can be written respectively as

$$(1-r)*\omega(x)e^{-\Omega(x)}dx; x < \tau$$
$$r\omega(z)Ie(\Omega(z))dz; z < \tau$$

Integrating these expressions from 0 to x and for $x \le \tau$ *it can be concluded that*

$$(1-r)\left(1-e^{-\Omega(x)}\right)=\left(1-r^*\right)\left(1-e^{-\Omega^*(x)}\right)$$
$$r\left(1-e^{-\Omega(x)}+\Omega(x)Ie(\Omega(x))\right)=r^*\left(1-e^{-\Omega^*(x)}+\Omega^*(x)Ie(\Omega^*(x))\right)$$

The proof of the identifiability of the IPRA parameters is made by an argument of contradiction. Supposing that there is an $x_0 \le \tau$ *such that* $\Omega(x_0) \le \Omega^*(x_0)$, *then since both* $1-\exp(-t)$ and $1-\exp(t)+tIe(t)$ *are strictly increasing in t it follows that* $1-r > 1-r^*$ and $r > r^*$. *But this is a contradiction from the same manner if one considers* $\Omega(x_0) > \Omega^*(x_0)$. *So,* $\Omega(x_0)=\Omega^*(x_0)$ *for all* $x \le \tau$ *(thus,* $\omega(x_0)=\omega^*(x_0)$ *for all* $x \le \tau$ *) and hence also* $r = r^*$.

Through these functions, it is important to note that the role of GRP parameters will be present as a multiplier of time (in $\Omega\left(q_X x_0 + q_Z z_0\right)$, *for example). Thus, the GRP parameters can be implemented without harming the identifiability property of the proposed model.* ■

Considering now the case where there are two rejuvenation parameters, the same reasoning is valid, since the action of the new parameters continues to be on the inter-event times.

The next Section describes the hypothesis test developed by Ferreira *et al.* (2010) where a modification is made for the test to account for the new rejuvenation parameter.

3.4 Hypothesis test to verify the proposed model (IPRA+GRP) applicability

This Section presents the development of an hypothesis test for the proposed model (for further details, see Ferreira *et al.* (2010)).

There are two parts on the elaboration of the test. The first one is to test if the IPRA model fits the desirable data, since this model by itself considers the repair made as perfect, whereas the second one is to test if the IPRA+GRP model is suitable to the data.

As presented in Cooke (1996b), each CR model can be adjusted to data through a unique characteristic of them. In the RSC model, Cooke (1996b) states that such characteristic is observed when the probability of maintenance beyond time t is maximum at origin. This probability is defined as follows.

$$\phi(t) = P(Z < X \mid Y > t) \qquad (3\text{-}17)$$

Thus, the described characteristic represents the applicability of RSC to data as $\phi(t) < \phi(0), \forall t > 0$. Equation (3-17) can be rewritten as follows

$$\phi(t) = P(Z < X \mid Y > t) = \frac{P(Z < X, Y >}{P(Y > t)}$$
$$= \frac{P(Z < X, Z > t)}{P(Z < X, Z > t) + P(X < Z, X > t)} \qquad (3\text{-}18)$$

One can define $P(Z < X, Z > t)$ and $P(X < Z, X > t)$ as the subsurvival functions of Z and X respectively, represented as

$$R_Z^*(t) = P(Z < X, Z > t) \qquad (3\text{-}19)$$
$$R_X^*(t) = P(X < Z, X > t)$$

(3-20)

Thus, $\phi(t)$ can be written as

$$\phi(t)=\frac{R_Z^*(t)}{R_Z^*(t)+R_X^*(t)} \tag{3-21}$$

Furthermore, $\phi(t)<\phi(0),\forall t>0$ means that the ratio between these values is a value that belongs to interval (0,1):

$$\frac{\phi(t)}{\phi(0)}=\frac{\phi(t)}{r}\leq 1 \tag{3-22}$$

Since that $\phi(0)=P(Z<X\mid Y>0)=P(Z<X)=r$.

Equation (3-21) is the starting point to the construction of the statistics of the proposed hypothesis test. Since the RA model is derived directly from RSC model, then this characteristic is also applied to RA.

Thus, consider the following assumptions:

i) The RA model must be applicable ($\phi(t)<\phi(0),\forall t>0$);

ii) Amongst the RA possibilities, the chosen one is the IPRA ($G(t)=\Omega(t)$).

With these assumptions in mind, the null and alternative hypotheses must be formulated. Thus, one can test the following hypotheses:

H_0: The IPRA model fits the data

H_1: The IPRA model does not fit the data

The ratio described above can be expressed to the IPRA model, as presented in Langseth & Lindqvist (2005).

$$\frac{\phi(t)}{r}=\frac{\Omega(t)\,\mathrm{Ie}(\Omega(t))-\exp(-\Omega(t))}{r\Omega(t)\,\mathrm{Ie}(\Omega(t))-\exp(-\Omega(t))} \tag{3-23}$$

Consider now $u=\Omega(t)$. Then the expression above can be expressed as follows:

$$v(u;\hat{r})=\frac{uIe(u)-\exp(-u)}{\hat{r}\,u\,Ie(u)-\exp(-u)} \tag{3-24}$$

From Equation (3-24), and considering the non-parametric estimators to $\hat{\phi}(t)=\hat{v}(u;\hat{r})=\{\#z>t\}/(\{\#z>t\}+\{\#x>t\})$, where $\{\#z>t\}$ and $\{\#x>t\}$ are the number of preventive and corrective events beyond t, and $\hat{\Omega}(t)=-\log(\#x>t/\#x>0)$, the calculation of a statistic is possible. It is reasonable to think of a test based on the area between the theoretical statistics and the non-parametric estimates. So this area can be defined as

$$\Delta = \int_0^{\infty} \left| v\left(u;\hat{r}\right) - \hat{v}\left(u;\hat{r}\right) \right| du \tag{3-25}$$

This value indicates that high values of Δ implies that H_0 should be rejected.

But this value also indicates that for "large" values of u, the sampling random fluctuations will dominate and it seems unusual. To outline this problem, one can consider that the portion of failures that happen after time t is given by $\exp\left(-\Omega\left(t\right)\right)$. In this way, the value Δ can be rewritten as

$$\Delta^* = \int_0^{\infty} \left| v\left(u;\hat{r}\right) - \hat{v}\left(u;\hat{r}\right) \right| \exp\left(-u\right) du \tag{3-26}$$

The development presented until here shows the applicability of the IPRA model. Through the estimates presented so far, one can note that the time in question does not receive the influence of repairs when considering only perfect repairs. The challenge is to represent the effects of imperfect repairs, incorporating them in some way in the process described above.

When the effective age of the component is not known at the moment of its failure, it is not possible to calculate $\left\{\# x > t\right\}$ and $\left\{\# z > t\right\}$ directly.

The problem now is how to estimate the quantity $\phi\left(t\right)$ when the component suffers some kind of repair. To represent the influence of repairs, the times involved on the estimation of $\phi\left(t\right)$ is now influenced by the virtual age conditioned in time, as follows.

$$\hat{\phi}^*\left(t\right) = \frac{P\left(Z < X, Z \geq t \mid Z \geq w_i\right)}{P\left(Z < X, Z \geq t \mid Z \geq w_i\right) + P\left(Z > X, X \geq t \mid X \geq w_i\right)} \tag{3-27}$$

In this case, one can observe that there is no problem for identifying the non-parametric estimators to these probabilities when the imperfect repair is treated with GRP as follows.

$$\hat{\phi}^*\left(t\right) = \frac{\left\{\# z > t \mid \# z > w_{n-1}\right\}}{\left\{\# z > t \mid \# z > w_{n-1}\right\} + \left\{\# x > t \mid \# x > w_{n-1}\right\}} \tag{3-28}$$

This is because the virtual age parameters are time related. Thus, the events are now restricted to $\left\{Z < X, Z > t \mid Z > w_{n-1}\right\}$ and $\left\{X < Z, X > t \mid X > w_{n-1}\right\}$ but it still can be observed from data depending only on the value of virtual age at the $\left(n-1\right)$th event.

Furthermore, when the rejuvenation parameter is split into two, one for corrective and another for preventive maintenances, the same reasoning is applicable.

Given the above development, the related statistics in the case of imperfect repair can now be established as follows.

$$\Delta_{imp}^{*} = \int_{0}^{\infty} \left| v(u;\hat{r}) - \hat{v}(u;\hat{r}) \right| \exp(-u)\, du \tag{3-29}$$

As it can be seen, the functional structure of the statistics does not change and the imperfect repair is present in calculation of $\phi(t)^{*}$.

Due to the structure of the functions involved in the calculation of Δ_{imp}^{*}, computational methods were used to estimate the statistics. Also, in order to draw some conclusions concerning Δ_{imp}^{*}, a bootstrap algorithm is used to represent the non-parametric distribution of the statistics. Thus, the p-value used to help in decision about what hypothesis is true is based on the study of the percentile that represents P($\Delta > \Delta^{*}_{imp}$). This algorithm is used basically in three steps: to estimate the parameters of a fitted distribution to data concerning the imperfect repair impact; to estimate the probability of occurring a preventive maintenance before a corrective one; and to calculate the statistics itself. This algorithm is presented in Appendix I and is also explored in Ferreira *et al.* (2010).

3.5 The likelihood function of the proposed model

To develop the likelihood function of the IPRA+GRP model, the pdfs presented in Equation (3-4) and Equation (3-10) are used as basis. According to Yañez *et al.* (2002), considering the failure-terminated process, the likelihood function can be developed as follows.

$$L = f(t_1)\, f(t_2 \mid t_1) \ldots f(t_n \mid t_{n-1}) \tag{3-30}$$

Since the GRP parameter acts in the denominator of the PDF, as shown in Equation (3-15), then it is suitable to admit that it jointly captures the effect of preventive and corrective maintenance. In order to provide an appropriate estimator of *q*, the corresponding likelihood of the hybrid model must be presented as follows

$$L = \omega(y_1)(1-r)^{1-j_1} \exp\left(-\Omega(y_1)\right)^{1-j_1} r^{j_1} Ie\left(\Omega(y_1)\right)^{j_1} \times \\ \times \prod \frac{\omega(y_i + w_{i-1})(1-r)^{1-j_i} \exp\left(-\Omega(y_i + w_{i-1})\right)^{1-j_i} r^{j_i} Ie\left(\Omega(y_i + w_{i-1})\right)^{j_i}}{\exp\left(-\Omega(w_{i-1})\right) - r\Omega(w_{i-1})\, Ie\left(\Omega(w_{i-1})\right)} \tag{3-31}$$

The log-likelihood is given as

$$l = \log\left(\omega\left(y_1\right)\right) + \left(1 - j_1\right)\log\left(1 - r\right) - \left(1 - j\right)\Omega\left(y_1\right) + j_1 \log\left(r\right) + j_1 \log\left(Ie\left(\Omega\left(y_1\right)\right)\right) + \\ + \sum_{i=2}^{n} \left\{ \begin{array}{l} \log\left(\omega\left(y_i + w_{i-1}\right)\right) + \left(1 - j_i\right)\log\left(1 - r\right) - \left(1 - j_i\right)\Omega\left(y_i + w_{i-1}\right) + \\ + j_i \log\left(r\right) + j_i \log\left(Ie\left(\Omega\left(y_i\right)\right)\right) - \log(\exp(-\Omega\left(w_{i-1}\right)) - r\Omega\left(w_{i-1}\right) Ie\left(\Omega\left(w_{i-1}\right)\right)) \end{array} \right\} \tag{3-32}$$

The effectiveness of preventive and corrective maintenances is expected to be different. Thus, depending on the type of observed failure (degraded/incipient or critical), q has a different value. That is,

$$w_n = \delta t_{X_{n-1}} + \left(1 - \delta\right) t_{Z_{n-1}} \tag{3-33}$$

Here, δ is an indicator variable that equals 0 or 1 when degraded/incipient or critical failures occur respectively. Also, $t_{n-1} = w_{n-2} + qh_{n-2}$, which represents the Kijima type I model.

Thus, from Equation (3-32), one can develop the Maximum Likelihood Estimators for the desired parameters. For example, considering the situation where data is adjusted by a Weibull distribution, there exist four parameters to estimate and the log-likelihood can be written as shown in Appendix II. As it can be seen, there is no analytical solution for these equations and a numerical method must be used to estimate the parameters. In this work, it is used the PSO (see Section 2.3) and an application is shown in Chapter 4.

3.6 Exploring the main results of the proposed model

As stated before, the proposed model is the result from the coupling of CR and GRP models, and it is able to explicitly account for the repair quality of corrective and preventive actions. Thus the IPRA+GRP model can provide results concerning critical and degraded failures. Some important results of the IPRA model are presented in Table 3.1.

Table 3.1 – Main functions providing results of IPRA model

Function	Property
$E(Z \mid Z < X) = \int_0^{\infty} \left(1 - \tilde{F}_Z(z)\right) dz = E(X) - E\left[\frac{M(X)}{G(X)}\right]$	Corresponds to expected time of a preventive maintenance – the effectiveness of the maintenance crew.
$E(Y) = E(X) - qE\left[\frac{M(X)}{G(X)}\right]$	Corresponds to the expected time of the occurrence of an event.

with $M(X) = \int_0^x G(t)\,dt$.

Consider now the expected number of failures. Yañez *et al.* (2002) estimate these metrics for RP, NHPP and GRP cases, but only considering critical failures. In the CR context, Lindqvist *et al.* (2005) present only the ML estimators, although discussing the set of possible results of the IPRA model. Finally, for the hybrid model presented by Langseth & Lindqvist (2005) using IPRA+BP model, the estimators provided are only for situations of perfect, minimal and imperfect repair.

Considering Equations (3-12) and (3-13), it can be noted that simple methods to isolate the time variable (using the inverse function) are not capable to achieve an analytical expression for *y*. In this situation, Firmino (2009) presents a review about methods capable of analyzing the sampling values of the underlying distribution which enables the study of the behavior of such distribution.

In fact, the estimation of the reliability function for the critical failures is developed in the next chapter.

4 EXAMPLE OF APPLICATION

In this chapter, the proposed imperfect maintenance model is exemplified by means of an example of an application of a real reliability database and is also compared to the IPRA+BP model proposed by Langseth & Lindqvist (2005) in terms of adequacy in characterizing the system failure and maintenance data obtained from the above mentioned database.

4.1 Data study

The reliability analysis is carried out by making use of the statistical freeware R v2.11.2 and computational algorithms developed in R and C++ languages in order to implement the equations governing the proposed imperfect maintenance model.

The reliability database that is analyzed in this chapter comes from Langseth & Lindqvist (2005) and describes the failure times of an offshore compressor system. The database has information about failure times and their severity, which allows classifying failures in critical and degraded (treated by corrective and preventive maintenance, respectively). The database has 85 failure times, where 55 observations correspond to degraded failure times and 30 to critical failure times, and it is presented in Appendix II

The first step is to check de adequacy of the proposed model, comparing it to the Langseth & Lindqvist (2005) model, to the compressor system dataset. This is achieved by means of the hypothesis test and its corresponding statistic Δ that were developed in Section 3.4. Indeed, the test statistic (calculated according to an algorithm created by the author in the R package v2.9.2 and presented in Appendix I) has a value of $\Delta = 0.1259$. Next, the non-parametric distribution of Δ, namely $\hat{f}^{(B)}(\delta)$, is determined and used to make inferences about the proposed model. Figure 4.1 presents $\hat{f}^{(B)}(\delta)$ with B = 200 (B represents the size of the generated sample through bootstrap). The bootstrap distribution provides $P^{(B)}(\Delta > 0.1259 \mid H_0) = 0.455$. Therefore, the null hypothesis is not rejected: the IPRA+GRP model defines an appropriated class of models to represent this dataset (one has a p-value of 45,5% as a rejection basis). Comparing it to the IPRA+BP model, where the obtained p-value was 25%, one realizes that the proposed IPRA+GRP can model the dataset with more confidence. This result can be explained from the fact that the proposed model makes use of a more general model for describing the repair process, i.e., a GRP instead of the simpler BP. In fact, the GRP approach would converge to a BP model if q = 0 or q=1.

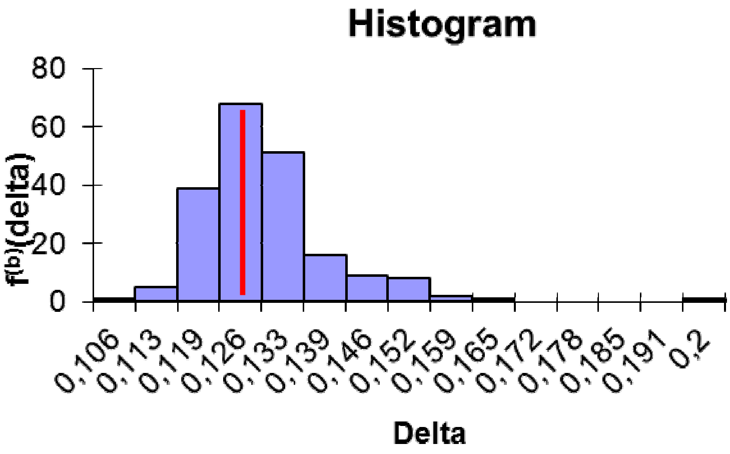


Figure 4.1 – Approximated density to the test statistics via bootstrap.

4.2 Model Parameters Estimation and Reliability Metrics

Based on the results of the previous section, one can now proceed with the estimation the proposed model parameters as well as with the calculation of some reliability metrics considering critical and degraded failure times.

4.2.1 Parameters estimation: optimizing the log-likelihood

The model parameters estimation is achieved by maximizing the log-likelihood. This can be characterized as an optimization problem. Basically, the equation to be optimized corresponds to the following one:

$$\max_{\Theta\in} \quad l\left(Y, J; \Theta\right) \tag{4-1}$$

where Θ is the vector of parameters, and Y and J are the vector of observations. The function *l* is the log-likelihood presented in Appendix II considering data adjustment by a Weibull distribution (the variable Y follows a Weibull).

As stated in Chapter 3, the model parameters are obtained via a Particle Swarm optimization algorithm. The PS parameters are presented in Table 4.1

Table 4.1 – Settings used to perform the PS algorithm.

Criteria	**PS Algorithm**
Type of model	*Gbest*
Population Size	50
Number of runs	30
Stop criteria 1 (epsilon)	1E-12
Stop criteria 2 (number of iterations)	1500

Bratton & Kennedy (2007) state that a reasonable interval to the particle size population is some number between 20 and 100. With the number of runs equal to 30, statistical inference can be done around the estimates such as confidence intervals.

One important feature of the log-likelihood is the presence of the integral exponential (see Section 3.1) as it does not have analytical solution. Therefore, log-likelihood equation is solved by means of Monte Carlo with 5000 iterations. The resulting parameter values for the log-likelihood, and their corresponding upper (95%) and lower (5%) bounds, are presented in Table 4.2.

Table 4.2 – Estimates through repair situations optimizing the IPRA+GRP log-likelihood

	Estimate	**Lower limit**	**Upper limit**	**Likelihood**	**Time spent**
Perfect Repair					
A	10.70	9.46	11.95	-2307.42	3285.49 sec
β	0.0014	0.0012	0.0016		
q_X	0	0	0		
q_Z	0	0	0		
Imperfect Repair					
α	0.3	0.19	0.4	2.4E+30	3362.63 sec
β	5.11	4.76	5.47		
q_X	0.34	0.26	0.44		
q_Z	0.87	0.84	0.89		
Minimal Repair					
α	0.01	0.0056	0.0144	3.21E+25	5954.78 sec
β	5.86	4.9	5.7		
q_X	1	1	1		
q_Z	1	1	1		

Note that the rejuvenation parameters have an important role in the estimation process, since its values have practical interpretations related to the behavior of critical and degraded failures. In fact, since the maximum likelihood estimators are obtained in the imperfect repair case, where alpha and beta parameters have interpretations of short half-life characteristics and accentuated growing of the hazard function, one can say that the perfect repair situation underestimates the alpha and beta parameters. Thus, such a situation could hide the occurrence of failures. Considering the minimal repair case, the alpha parameter indicates more failure occurrence whereas the beta parameter behavior is almost equal compared with the imperfect situation.

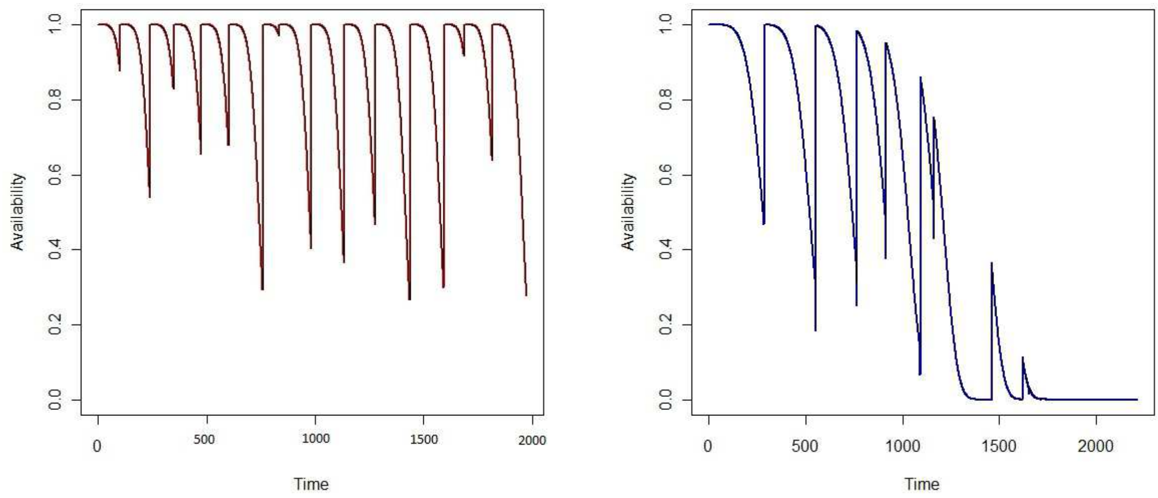
With these estimates, it is now possible to estimate some reliability metrics.

4.2.2 Calculating some reliability metrics

Having estimated the proposed model parameters, one can now proceed to calculate reliability metrics of interest such as the availability function. Note that the estimation procedure is based on the joint PDF, thus taking into account both critical and degraded failures.

Due to the property of the CR model, the reliability function is possible through the analysis of critical failures. Moreover, the repair time is neglected and the availability function turns the same as reliability up to each failure time registered.

Figure 4.2 shows the instantaneous availability predicted for the compressor system based on the proposed IPRA+GRP model discussed in the previous section and with the parameters sets shown in Table 4.2.



(a) Perfect repair (b) Imperfect repair

Figure 4.2 – Availability estimated considering: (a) perfect repair situation and (b) IPRA+GRP modeling.

Provided that the imperfect repair case is more appropriate for the failure data of the compressor (see the likelihood value in Table 4.2), one can argue that it is able to more realistically represent the system availability as compared to the perfect repair, helping to manage the execution of preventive maintenances. Moreover, a reasonable maintenance plan would be to perform corrective maintenances up to 1000 hours of operation, and then to replace the system by a new one. Note that both imperfect and perfect repair cases in Figure 4.3 are based on the same proposed IPRA+GRP model.

For the analysis of the system availability based on the imperfect maintenance model with only one GRP parameter, please refer to Ferreira *et al.* (2011)

5 CONCLUDING REMARKS

A new model for imperfect maintenance was presented where corrective and preventive maintenance actions are dependent was presented. The proposed model results from the coupling of competing risks and general renewal processes. In particular, CR is used in order to treat the dependence between critical (corrective) and degraded (preventive) failures, and GRP is used for properly dealing with the corrective and preventive maintenance effectiveness after a given event (critical or degraded failures).

The proposed model was then used to develop a hypothesis test to verify its applicability to a dataset. Then, the maximum likelihood estimators were presented for the model parameters under the assumption of Weibull hazard rate and separate corrective and maintenance effectiveness. Since the resulting maximum likelihood estimators do not have an analytical closed form expression, they were obtained through probabilistic optimization by means of the implementation of a Particle Swarm algorithm.

The proposed model usage was illustrated by means of an example of application of a reliability database containing a compressor system's failure data. For this particular scenario, the results indicated that the proposed model provided a better description of the system then the alternative model put forward by Langseth & Lindqvist (2005). The main result to state this is about the hypothesis test developed and applied with the proposed model presenting a p-value of 0.455 against a p-value of 0.25 of the IPRA+BP model of the literature. The instantaneous availability function studied provides to the user a configuration where he/she can decide a well-programmed policy of preventive maintenances and the life time of his/her system can be extended.

Thus, the proposed model brings a complete analysis concerning the quality of maintenance made. The virtual age approach coupled in a CR study only brings advantages improving the analysis of maintenance effectiveness.

5.1 FUTURE WORK

The proposed model works with a scenario where two specific risks can be studied – preventive and corrective maintenance actions. The literature brings several studies with cases where one of risks represents a group of failure modes. Thus, the proposed model can also treat this kind of situation being necessary only replacing one of the risks by a group of failure mode. The main effect will be noticed on the failure intensity, where it will be composed by the failure behavior of failure modes jointly. Analysis of expected number of failures in terms of critical and degraded failures requires a more rigorous of the functions of the proposed model, since they have high complexity. This is justified by the structure of the CDF that does not allow calculus for an analytical solution. Therefore it is proposed the development and implementation of a numerical procedure to quantify both the expected number of critical and degraded failures.

Moreover, the maximum likelihood parameters were developed for the model parameters. Thus it is suggested the development of Bayesian estimators for the parameters which might be useful in situations of scarce failure data.

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APPENDIX I

#1 Code used to generate Weibull times

```
generateWeibulltimes<- function(beta, alpha, q)
{
 W <- NULL
 Q <- NULL
 v <- 0
 count <- 0
 U2 <- runif(1)
 teste <- (alpha*((-log(1 - U2))^(1/beta)))
 if( U2 < 0.65)
 {
 W[1] <- teste
 Q[1] <- 0
 v <- q*Q[1] + q*W[1]
 count <- count + 1
 }
 else
 {
 Q[1] <- teste
 W[1] <- 0
 v <- q*Q[1] + q*W[1]
 }
 for(i in 2:85)
 {
        U2 <- 0
        while ((U2 == 0)||(U2 == 1))
        U2 <- runif(1)
        teste <- alpha*((((v/alpha))^(beta))-log(1-U2))^(1/beta) - v
 if( U2 < 0.65)
 {
        W[i] <- teste
```

```
            Q[i] <- 0
            v <- q*Q[i] + q*W[i]
            count <- count + 1
  }
  else
  {
            Q[i] <- teste
            W[i] <- 0
            v <- q*Q[i] + q*W[i]
  }
  }
return (write(count, "C:\\Documents and Settings\\administrator\\Desktop\\Resultados\\contador.txt", sep = "\n", append = TRUE),write(W, "C:\\Documents and Settings\\administrator\\Desktop\\Resultados\\amostra.txt", sep = "\n", append = TRUE), write(Q, "C:\\Documents and Settings\\administrator\\Desktop\\Resultados\\amostraX.txt", sep = "\n", append = TRUE))
}
```

#2 Code used to generate the CR functions

```
weibullHazardCalculusB <- function(beta, alpha, q, iterations1, tempo)
{
  Result <- 0
  for (i in 1:iterations1)
  {
            Z <- runif(1)
            interfunc <- tempo*(beta/alpha)*(((q*tempo*Z)/alpha)^(beta - 1))
            Result <- Result + interfunc
  }
  FinalResult <- Result/iterations1
  return(FinalResult)
}
```

```
alternateCalculusB <- function(beta, alpha , q, iterations1, tempo)
{
  alter <- 0
  for(i in 1:iterations1)
  {
        U3 <- runif(1)
        funca <- -exp(-weibullHazardCalculusB(beta, alpha, q,iterations1, tempo))/(log(exp(-(weibullHazardCalculusB(beta, alpha, q,iterations1, tempo)))*U3))
        alter <- alter + funca
  }
  integAlter <- alter/iterations1
  return(integAlter)
}


RelCalculusZB <- function(betax, alphax, qx, betaz, alphaz, qz, r, iterations, iterations1, tempo)
{

  Result <- 0
  RRR <- 0
  for(i in 1:iterations)
  {
        Z <- runif(1)
        interfunc <- tempo*(r*((betaz/alphaz)*(((qz*tempo*Z)/alphaz)^(betaz -1)))*alternateCalculusB(betaz, alphaz, qz, iterations1, tempo))
        teste <- tempo*((1-r)*((betax/alphax)*(((qx*tempo*Z)/alphax)^(betax -1)))*exp(-weibullHazardCalculusB(betax, alphax, qx, iterations1, tempo)))
        Result <- Result + interfunc
        RRR <- RRR + teste
  }
  Final1 <- Result/iterations
```

```
    Final <- r - Final1
    FFF1 <- RRR/iterations
    FFF <- (1-r) - FFF1
    ReliabilityCalculusZ <- min(Final + FFF, Final + (1 - r))
    return  (ReliabilityCalculusZ)
  }

  RelCalculusXB <- function(betax, alphax, qx, betaz, alphaz, qz, r, iterations, iterations1, tempo)
  {

    Result <- 0
    RRR <- 0
    for(i in 1:iterations)
    {
        Z <- runif(1)
        interfunc <- tempo*(r*((betaz/alphaz)*(((qz*tempo*Z)/alphaz)^(betaz - 1)))*alternateCalculusB(betaz, alphaz, qz, iterations1, tempo))
        teste <- tempo* ((1-r)*((betax/alphax)*(((qx*tempo*Z)/alphax)^(betax - 1)))*exp(-weibullHazardCalculusB(betax, alphax, qx, iterations1, tempo)))
        Result <- Result + interfunc
        RRR <- RRR + teste
    }
    Final1 <- Result/iterations
    Final <- r - Final1
    FFF1 <- RRR/iterations
    FFF <- (1-r) - FFF1
    ReliabilityCalculusX <- min(Final + FFF,FFF + r)
    return (ReliabilityCalculusX)
  }
  subReliabilityCalculusZB <- function(betax, alphax, qx, betaz, alphaz, qz, r, iterations, iterations1, tempo)
  {
```

```
    Result <- 0
    RRR <- 0
    for(i in 1:iterations)
    {
        Z <- runif(1)
        interfunc <- tempo*(r*((betaz/alphaz)*(((qz*tempo*Z)/alphaz)^(betaz - 1)))*alternateCalculusB(betaz, alphaz, qz, iterations1, tempo))
        teste <- tempo*((1-r)*((betax/alphax)*(((qx*tempo*Z)/alphax)^(betax - 1)))*exp(-weibullHazardCalculusB(betax, alphax, qx, iterations1, tempo)))
        Result <- Result + interfunc
        RRR <- RRR + teste
    }
    Final1 <- Result/iterations
    Final <- r - Final1
    return  (Final)
   }

   subReliabilityCalculusXB <- function(betax, alphax, qx, betaz, alphaz, qz, r, iterations, iterations1, tempo)
   {

    Result <- 0
    RRR <- 0
    for(i in 1:iterations)
    {
        Z <- runif(1)
        interfunc <- tempo*(r*((betaz/alphaz)*(((qz*tempo*Z)/alphaz)^(betaz - 1)))*alternateCalculusB(betaz, alphaz, qz, iterations1, tempo))
        teste <- tempo*  ((1-r)*((betax/alphax)*(((qx*tempo*Z)/alphax)^(betax - 1)))*exp(-weibullHazardCalculusB(betax, alphax, qx, iterations1, tempo)))
        Result <- Result + interfunc
        RRR <- RRR + teste
```

```
  }
  Final1 <- Result/iterations
  Final <- r - Final1
  FFF1 <- RRR/iterations
  FFF <- (1-r) - FFF1
  ReliabilityCalculusX <- min(Final + FFF,FFF + r)
  return (FFF)
}
```

#3 Finally, the calculus of test statistics.

```
DeltaCalculusB<- function(betax, alphax, qx, betaz, alphaz, qz, r, iterations, iterations1, i)
{

  Result <- 0
  incr <- 0.01
  for(l in 1:1000)
  {
        cont_x <- 0
        cont_z <- 0
        cont_x_q <- 0
        cont_z_q <- 0
        U <- 0 + l*incr
        teor1 <- (subReliabilityCalculusZB(betax, alphax, 1, betaz, alphaz, 1, r, iterations, iterations1, U)/RelCalculusZB(betax, alphax, qx, betaz, alphaz, qz, r, iterations, iterations1, U))
        teor2 <- (((subReliabilityCalculusXB(betax, alphax, 1, betaz, alphaz, 1, r, iterations, iterations1, U)/RelCalculusXB(betax, alphax, qx, betaz, alphaz, qz, r, iterations, iterations1, U)) + (subReliabilityCalculusZB(betax, alphax, 1, betaz, alphaz, 1, r, iterations, iterations1, U)/RelCalculusZB(betax, alphax, qx, betaz, alphaz, qz, r, iterations, iterations1, U))))
        teor <- teor1/(r*teor2)
```

```
            for (j in 1:trunc((length(X)/1000)))
            {
                  if( X[(i-1)*29+j] > U)
                        cont_x <- cont_x + 1
                  if(X[(i-1)*29+j] > qx*U)
                        cont_x_q <- cont_x_q + 1
            }
            for (k in 1:(trunc(length(Z)/1000)))
            {
                  if( Z[(i-1)*55+k] > U)
                        cont_z <- cont_z + 1
                  if(Z[(i-1)*55+k] > qz*U)
                        cont_z_q <- cont_z_q + 1
            }
            empiric1 <- cont_z/cont_z_q
            empiric2 <- cont_x/cont_x_q + cont_z/cont_z_q
            empiric <- empiric1/(r*empiric2)
            delta <- (abs(teor - empiric) * exp( -
weibullHazardCalculusB(betax,alphax,qx,iterations1,U)))
            Result <- Result + delta


      }
      Final <- Result/1000
      return(write(Final,"C:\\Documents and
Settings\\administrator\\Desktop\\Resultados\\amostra_estatistica.txt", sep = "\n", append =
TRUE))
     }
```

APPENDIX II

The log-likelihood for a Weibull distribution considering the proposed model

$$l=\log(\beta)-\log(\alpha)+(\beta-1)\left(\log(y_1)-\log(\alpha)\right)+(1-j_1)\log(1-r)-\left(\frac{y_1}{\alpha}\right)^{\beta(1-j_1)}+$$

$$+j_1\log(r)+j_1\log\left(\int_{\left(\frac{y_1}{\alpha}\right)^{\beta}}^{\infty}\frac{\exp(-u)}{u}du\right)+$$

$$+\sum_{i=2}^{n}\left\{\begin{array}{l}\log(\beta)-\log(\alpha)+(\beta-1)\log\left(y_i+(1-j_i)q_x\sum_{k=2}^{i}x_{k-1}+j_iq_z\sum_{l=2}^{i}z_{l-1}\right)-\\ -(\beta-1)\log(\alpha)+(1-j_i)\log(1-r)-\left[\frac{y_i+(1-j_i)q_x\sum_{k=2}^{i}x_{k-1}+j_iq_z\sum_{l=2}^{i}z_{l-1}}{\alpha}\right]^{\beta(1-j_i)}+\\ +j_i\log(r)+j_i\log\left(\int_{\left(\frac{y_i+(1-j_i)q_x\sum_{k=2}^{i}x_{k-1}+j_iq_z\sum_{l=2}^{i}z_{l-1}}{\alpha}\right)^{\beta}}^{\infty}\frac{\exp(-u)}{u}du\right)-\log\left(\begin{array}{l}\exp\left(\left[\frac{(1-j_i)q_x\sum_{k=2}^{i}x_{k-1}+j_iq_z\sum_{l=2}^{i}z_{l-1}}{\alpha}\right]^{\beta}\right)-\\ -r\left(\frac{(1-j_i)q_x\sum_{k=2}^{i}x_{k-1}+j_iq_z\sum_{l=2}^{i}z_{l-1}}{\alpha}\right)^{\beta}\times\\ \times\int_{\left(\frac{(1-j_i)q_x\sum_{k=2}^{i}x_{k-1}+j_iq_z\sum_{l=2}^{i}z_{l-1}}{\alpha}\right)^{\beta}}^{\infty}\frac{\exp(-u)}{u}du\end{array}\right)\end{array}\right\}$$

APPENDIX III

Databases used for data study.

Time	Severity	Time	Severity	Time	Severity
220	1	1	0	13	0
13	1	1	0	24	1
1	1	1	1	3	1
6	1	19	1	10	1
25	1	2	0	4	0
5	1	1	1	85	0
3	1	1	1	28	1
6	0	13	1	5	1
6	1	6	1	76	0
2	0	3	1	49	1
7	1	6	1	4	1
1	1	2	1	32	1
5	1	12	1	17	1
25	0	1	1		
3	0	3	1		
5	0	7	1		
32	1	2	1		
3	1	12	0		
1	1	12	0		
12	1	117	0		
36	1	3	0		
1	1	4	0		
11	0	2	0		
10	0	2	0		
4	0	30	0		
1	1	97	0		
1	1	65	0		
32	1	47	1		
14	1	7	1		
1	1	18	1		
12	1	8	1		
7	1	80	0		
28	1	61	0		
10	0	11	0		
24	1	28	1		
8	0	12	1		

APPENDIX IV

Desenvolvimento de um Teste de Hipóteses para um Modelo Híbrido de Riscos Competitivos Dependentes e Processos de Renovação Generalizados

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RESUMO

Banco de Dados de Confiabilidade (BDC) são capazes de fornecer uma quantidade enorme de informações em sistemas diferentes. A Análise da Confiabilidade surge como uma importante ferramenta para o tratamento de informações sobre distribuições de tempo de vida, que decorrem de BDC. Assim, a análise de confiabilidade pode ser realizada através de alguns mecanismos probabilísticos como concorrentes Riscos (CR). modelos CR as relações entre falhas críticas e manutenção preventiva, que competem entre si para provocar a interrupção do sistema. Uma das desvantagens do CR não está tratando a natureza da reparação efectuada, que é tratada por meio de modelos como um conhecido como Processo de Renovação Generalizado (GRP), que não é capaz de modelar a relação entre os riscos que CR faz. Este trabalho o objetivo de estudar as falhas críticas, manutenção preventiva e tipo de reparo realizado por meio de um modelo baseado em um CR e GRP. Um teste de hipótese é desenvolvido para analisar a adaptação do modelo aos dados.

Palavras chave: Riscos Competitivos, Processos de Renovação Generalizados, Testes de Hipóteses.

ABSTRACT

Reliability Databases (RDB) are able to provide a huge amount of information on different systems. Reliability analysis emerges as an important tool for treating information about life time distributions which stem from RDB. Thus, reliability analysis can be accomplished through some probabilistic mechanisms such as Competing Risks (CR). CR models the relationships between critical failures and preventive maintenance which compete with each other to cause system stoppage. One of the disadvantages of CR is not treating the nature of repair performed which is handled by models as one known as Generalized Renewal Process (GRP) which is not capable to model the relationship between risks as CR does. This paper the aim at studying critical failures, preventive maintenance and type of repair performed by using a model based upon a CR and GRP. A hypothesis test is developed to analyze the adjustment of the model to data.

Keywords: Competing Risks, Generalized Renewal Processes, Hypothesis Test.

1 Introdução

A utilização de Bancos de Dados de Confiabilidade (BDC) possui um importante papel no cenário de indústrias de grande porte. Isso se deve ao fato dessas estarem sempre em busca de uma melhor confiabilidade em seu processo como um todo para um melhor desempenho na produção de seu produto final, reduzindo custos e aumentando o atendimento no mercado. Fragola (1996) apresenta uma perspectiva histórica sobre o desenvolvimento de tais bancos desde os anos 50, onde se nota uma evolução natural dos mesmos de acordo com as necessidades que vem surgindo.

A quantidade de análises possíveis a partir de um BDC permite o uso de diversas metodologias para obtenção de métricas de interesse resultando em uma gama de informações valiosas para sistemas sob análise. Uma das metodologias mais usadas se refere à análise de confiabilidade através de métodos probabilísticos.

Uma maneira de realizar a análise de confiabilidade de um equipamento/sistema por ferramentas probabilísticas é conhecida como Riscos Competitivos (RCs). Como pode ser visto em Crowder (2001), tal metodologia utiliza-se do princípio de que qualquer evento que aconteça com o sistema em estudo pode ser relacionado com diversas causas. Em outras palavras, há diversos riscos competindo para ser responsável pelo acontecimento de um evento naquele sistema. A definição do sistema deve ser feita de maneira que a ocorrência seja singular, ou seja, apenas no sistema.

Em particular, é de interesse conseguir relacionar as possíveis falhas de um sistema com seu causador. Tais causadores são conhecidos como causa de uma falha que é acionada por um mecanismo de falha. Problemas dessa natureza estão constantemente presentes na literatura como apresentado em LINDQVIST (2006b), Lindqvist *et al.* (2005) entre outros. Além disso, a relação entre as diversas causas de uma falha pode dar-se de maneira independente ou dependente.

Dentre os modelos existentes, o mais conhecido, desenvolvido por COOKE (1993b), é chamado de Random Signs Censoring (RSC) que tem como princípio modelar falhas críticas e manutenções preventivas através de um tipo de sinal que possa ser emitido pelo sistema em questão.

Um outro modelo que expande o RSC é conhecido como modelo de alerta do reparo (do inglês – Repair Alert Model – RA) o qual é apresentado em Lindqvist *et al.* (2005). Tal modelo é capaz de analisar a capacidade de reflexo da equipe de manutenção através de uma função conhecida como função de alerta de reparo.

Analisando o cenário em discussão, é perceptível ainda uma possível expansão dos modelos de RCs dedicados mais a qualidade de manutenções. Isso porque RCs não levam em consideração a natureza da manutenção realizada. Recomenda-se DOYEN & GAUDOIN (2006a) para uma melhor discussão.

A análise da natureza de manutenções é tratada na literatura através de processos estocásticos pontuais. Ao se realizar uma manutenção em determinado sistema/equipamento, espera-se que o mesmo possa voltar tão bom quanto novo. Para esse caso, a manutenção é denominada de manutenção perfeita e é modelado por processos estocásticos pontuais de Renovação. Ainda, é comum ocorrer situações onde a manutenção realizada leva o equipamento/sistema a um estado de tão ruim quanto antes da falha, conhecido como manutenção mínima. Tal tipo de manutenção é modelado por Processos Não-Homogêneo de Poisson. Em geral, o que se nota após uma manutenção é que o estado do equipamento não é perfeito, porém não é mínimo. A esse estado intermediário dá-se o nome de manutenção imperfeita. Diversos modelos capazes de representar tais estados foram sendo desenvolvidos ao longo do tempo.

Como uma tentativa de trabalhar um modelo de RCs com um modelo de análise da qualidade de manutenções, Langseth & Lindqvist (2003) apresentam o uso do modelo de alerta de reparo junto com o modelo BP. Com relação à aplicabilidade de tais modelos, Langseth & Lindqvist (2005) desenvolvem um teste de hipóteses específico, utilizando técnicas de Estatística Não-Paramétrica, para verificar se o modelo desenvolvido em Langseth & Lindqvist (2003) é aplicável.

Considerando um modelo conhecido como Processos de Renovação Generalizados (PRG), esse trabalho busca apresentar o uso conjunto do modelo de alerta de reparo com o PRG para uma melhor abordagem de problemas de dependência de riscos com a análise da qualidade de um reparo efetuado.

Para verificar a adequacidade dos dados ao modelo, desenvolve-se aqui um teste de hipóteses baseado no teste desenvolvido por Langseth & Lindqvist (2005) utilizando conceitos de Estatística não paramétrica. Segundo WASSERMAN (2006) análises paramétricas são usadas quando se assume que a população em questão pode ser modelada através de distribuições de probabilidade paramétricas. Caso isso não se aplique à população em questão, o uso da estatística não-paramétrica é feito para auxiliar na construção de uma regra de decisão através da estatística do teste.

Uma base de dados do OREDA (do inglês Offshore REliability DAta) – o qual descreve um sistema de compressor de uma instalação marítima – é utilizada para a verificação da adequacidade do modelo proposto. Tal etapa é importante para a correta aplicação e interpretação dos resultados possíveis fornecidos pelo modelo proposto. Esse banco está relacionado diretamente ao interesse em se estudar BDCs de refinarias no cenário nacional.

O trabalho está organizado como segue. A Seção 2 fala brevemente da teoria de Riscos Competitivos e modelos de reparos, além da teoria de testes de hipóteses. A Seção 3 contextualiza o desenvolvimento de um teste de hipóteses e as funções de interesse do modelo proposto. A Seção 4 apresenta os resultados encontrados. Por fim, a Seção 5 apresenta as conclusões.

2 Fundamentação Teórica

A Seção 2.1 apresenta a contextualização de riscos competitivos enquanto que a Seção 2.2 trata de modelos de reparos. A Seção 2.3 apresenta uma discussão sobre testes de hipóteses.

2.1 Riscos Competitivos

O conceito de Riscos Competitivos pode ser ilustrado de diversas formas, assim como visto na introdução deste trabalho. Maiores detalhes de como ilustrar uma situação de Riscos Competitivos podem ser encontrados em Bedford & Cooke (2002), Cooke (1996a), entre outros.

São passadas nesse trabalho apenas as definições mais comumentes usadas no contexto de RC's e frequentemente utilizadas no desenvolvimento dos modelos, incluindo o proposto aqui.

Definição 1: *Seja um sistema em estudo definida por um tempo de operação T. Seja Y definido como o tempo de ocorrência de um evento durante o tempo de observação desse sistema. Defina ainda como X o tempo de falha e Z como o tempo de manutenção preventiva. Logo, $Y = \min(X, Z)$ é considerado como o mínimo entre o tempo de dois riscos competitivos em relação à unidade em estudo. É importante definir ainda como J (= 1, 2,....,k) como a causa do evento que ocorreu.*

Por conveniência Y é considerado nesse trabalho como o tempo entre eventos (sendo X o tempo entre falhas e Z o tempo entre manutenções preventivas). Ainda, nesse trabalho se leva em consideração que se têm apenas dois tipos de riscos competindo entre si – falhas críticas e manutenções preventivas.

É conveniente ainda apresentar algumas funções que são introduzidas a partir de RCs na seguinte definição.

Definição 2: *Dado que Y = min(X, Z), a probabilidade dada por* $P\left(X < Z, X \leq t\right)$ *e representada por* $F_X^*\left(t\right)$ *é conhecida como sub-distribuição de X pois estuda o evento "tempo da falha crítica acontece antes do tempo de uma manutenção" e "tempo da falha crítica é menor ou igual a um tempo t". Analogamente define-se* $P\left(Z < X, Z \leq t\right)$ *como sub-distribuição de Z, sendo representada por* $F_Z^*\left(t\right)$.

Definição 3: *Considera-se a probabilidade dada por* $P\left(X < Z, X > t\right)$ *como sendo a sub-confiabilidade (ou sub-sobrevivência) de X e a mesma é representada por* $R_X^*\left(t\right)$. *Analogamente,* $P\left(Z < X, Z > t\right)$ *representa a sub-confiabilidade de Z, sendo representada por* $R_Z^*\left(t\right)$.

Essas funções possuem características próprias e diferentes das propriedades de distribuições de probabilidades clássicas. Bedford & Cooke (2002), Lindqvist *et al.* (2005), Crowder (2001) e outros autores descrevem detalhadamente tais características.

Tratando falhas críticas do sistema em estudo como um risco (diga-se X), a sua classificação se dá quando o sistema deixa de realizar sua função satisfatoriamente. Com isso uma manutenção corretiva se torna necessária para reativar o sistema. No caso de manutenções preventivas (diga-se Z), sua realização se dá quando o sistema ainda desempenha sua função principal, mas de maneira parcial resultando em um estado de falha degradada. Tal classificação pode ser percebida pela Figura 1.

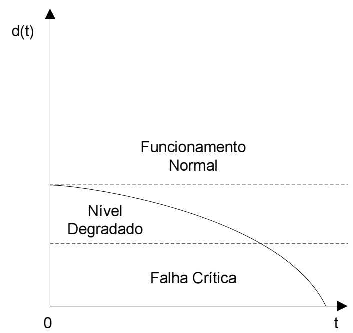


Figura 1 – Representação de falhas críticas e manutenções preventivas para um cenário de riscos competitivos em relação a função desempenho d(t).

Com as principais definições apresentadas, pode-se apresentar os principais modelos que tratam das situações de Riscos Competitivos. Tais situações de RCs vem sendo tratado pela literatura com o passar do tempo, se dando destaque as principais diferenças entre riscos independentes e dependentes. Aos poucos, modelos foram surgindo para cada situação e os mesmos apresentam aplicações em cenários específicos. Bedford & Cooke (2002) apresentam alguns modelos para as duas situações, enquanto que já na década de 90, COOKE (1993b) apresenta um importante modelo dependente.

Ainda na década de 90, Cooke (1996b) apresentou um modelo de riscos dependentes conhecido como Random Signs Censoring e já brevemente discutido em COOKE (1993b).

Por se tratar de um modelo aplicável a uma situação frequente de RC dependentes, o RSC é usado como referência para o desenvolvimento de outro importante modelo de RC que tratam do mesmo tipo de situação. Um modelo conhecido como modelo de alerta de reparo (do inglês *Repair alert model* – modelo RA) foi criado a partir do modelo RSC com o principal diferencial em torno de uma função conhecida como função de alerta de reparo responsável por modelar a capacidade de alerta da equipe de manutenção. Com esse modelo, tem-se a capacidade de analisar a eficiência da equipe de manutenção através dessa característica. Maiores detalhes em Langseth & Lindqvist (2003).

Independente do tipo de modelo de RC utilizado, algumas funções são de constante uso no estudo das relações entre os riscos. Algumas delas estão apresentadas na definição 3 e além daquelas, é importante observar a função densidade conjunta do par (Y,J)

As funções de interesse fornecidas pelo modelo IPRA que se referem ao par de observações (Y,J), que correspondem à função densidade de probabilidade, função de distribuição acumulada e funções sub-confiabilidade e sub-distribuição.

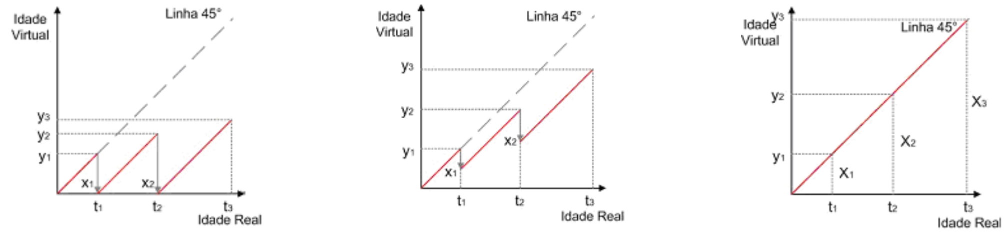
De acordo com cada modelo, as respectivas funções podem ser definidas acerca do interesse das análises em questão.

A partir dessas funções, as métricas de interesse podem ser obtidas para a análise do sistema em questão. Entretanto, além de avaliar o impacto da manutenção preventiva sobre os tempos de falhas do sistema, é interessante estudar a qualidade das manutenções realizadas. Tal estudo se baseia na metodologia de processos estocásticos pontuais. Em particular, esse trabalho trata de uma classe desses modelos conhecida como modelos de idade virtual. Dentre estes, o Processo de Renovação Generalizado é discutido a seguir.

2.2 Modelos de Idade Virtual: Processo de Renovação Generalizado

A análise de dados de falha e de manutenção pode ser feita por diversas metodologias dentre as quais riscos competitivos que foram apresentados na Seção anterior. Como já dito, tais análises respondem apenas como se dá o comportamento dos dados para funções que consideram uma manutenção realizada como perfeita. Tal cenário não retrata a realidade da maneira mais adequada, pois dificilmente uma manutenção consegue retornar um sistema ao estado de tão bom quanto novo. A situação mais encontrada reflete o sistema retornando da manutenção pior que novo, porém melhor que antes da falha.

A discussão nesse trabalho sobre PRG é baseada em Moura *et al.* (2007) onde, além da apresentação do modelo em si, os autores apresentam um método de estimação bayesiana para os parâmetros de interesse.



(a) $q = 0$ (b) $0 < q < 1$ (c) $q = 1$

Figura 2 - Relação entre idade virtual e idade real

A idade virtual xi representa a redução na idade real do sistema promovida pela i-ésima ação de manutenção e está diretamente relacionada com o parâmetro q. Os valores assumidos por este parâmetro possibilitam a representação dos tipos de manutenções citadas o que pode ser feito da seguinte maneira:

q = 0: corresponde a uma manutenção perfeita, já que a idade virtual xi é sempre anulada após a i-ésima ação de manutenção;

0 < q < 1: corresponde a uma manutenção imperfeita, já que xi é uma fração da idade real t_i;

q = 1: corresponde a uma manutenção mínima, já que xi é exatamente igual à idade real t_i.

Outros valores para o parâmetro q são também possíveis, como q < 0 e q > 1 que correspondem a “manutenção melhor” e a “manutenção pior”, respectivamente. Porém, valores realísticos para o parâmetro q estão no intervalo entre 0 e 1, inclusive, uma vez que para q < 0 seria necessário assumir que ocorrem mudanças no projeto ou substituição de componentes por outros melhores do que eram quando novos, por exemplo. Além disso, q > 1 significa supor que a manutenção age de forma contrária ao objetivo de retornar o equipamento a uma condição melhor do que estava antes de ser reparado.

KIJIMA & SUMITA (1986a) propuseram dois tipos de modelos de idade virtual (para outros modelos veja Guo *et al.* (2001)). O primeiro deles (Equação 1) é comumente chamado de modelo Kijima tipo I e consiste fundamentalmente na ideia que a manutenção atua apenas nas falhas que ocorrem no intervalo de exposição imediatamente anterior a manutenção. Deste modo, sendo t1, t2, ... tempos sucessivos entre falhas, a idade virtual do sistema sofre incrementos proporcionais com o tempo:

$$x_i = x_{i-1} + q \cdot h_i = qt_i, \qquad \text{Equação 1}$$

onde hi é o tempo entre a ocorrência da (i-1)-ésima e a i-ésima falha.

Já o modelo Kijima tipo II (Equação 2) assume que a manutenção atua com o objetivo de recuperar o sistema das falhas decorrentes de todos os intervalos anteriores de exposição desde o início da operação do sistema. Neste modelo, a idade virtual sofre incrementos proporcionais durante todo o intervalo de exposição acumulado:

$$x_i = q(h_i + x_{i-1}) = q(q^{i-1}h_1 + q^{i-2}h_2 + \ldots + h_i). \qquad \text{Equação 2}$$

Desta forma, o modelo Kijima Tipo I supõe que a i-ésima manutenção não pode remover todos os danos ocorridos até a i-ésima falha, somente reduzindo a idade adicional hi. Já o modelo Kijima Tipo II supõe que na i-ésima manutenção, a idade virtual esteve acumulada em $x_{i-1} + h_i$. Assim, a i-ésima manutenção removerá os danos acumulados devido às falhas ocorridas durante o último intervalo de exposição assim como nos intervalos precedentes.

Independente do modelo Kijima utilizado, é possível avaliar o i-ésimo tempo ti entre a (i-1) e i-ésima falha através da função de distribuição acumulada condicionada na idade virtual x_{i-1}, como segue:

$$F(t_i \mid x_{i-1}) = P(T \le t_i \mid T > x_{i-1}) = \frac{P(x_{i-1} < T \le t_i)}{P(T > x_{i-1})} = \frac{F(t_i + x_{i-1}) - F(x_{i-1})}{1 - F(x_{i-1})} \qquad \text{Equação 3}$$

onde F($\cdot$) é a função de distribuição acumulada do tempo até a primeira falha e $t_i + x_{i-1}$ representa o tempo acumulado até o i-ésimo tempo entre falhas.

O modelo de PRG utilizado nesse trabalho faz uso da metodologia desenvolvida no modelo Kijima tipo I.

O modelo proposto faz uso do PRG juntamente com o modelo IPRA de RCs, resta analisar se esse modelo é aplicável a uma determinada amostra. Tal análise pode ser realizada pela aplicação de um teste de hipóteses. Na próxima Seção discutem-se conceitos básicos sobre a teoria de testes de hipóteses.

2.3 Testes de Hipóteses

Antes de apresentar as funções de interesse e o desenvolvimento de um teste de hipóteses para o modelo proposto, uma breve discussão sobre a teoria de testes de hipóteses é apresentada nessa Seção a qual é baseada em MOOD *et al.* (1974).

A teoria dos testes de hipóteses é parte integrante da Inferência Estatística e está inteiramente relacionada à teoria de estimação. A idéia é testar determinada afirmação a respeito dos parâmetros da distribuição conjunta das observações, utilizando a informação trazida por $X_1, \ldots, X_n$.

De forma geral, considere o seguinte problema: seja $X_1, \ldots, X_n$ uma amostra aleatória de uma distribuição com parâmetro θ desconhecido. Deseja-se testar se determinada afirmação $H_0 : \theta \in \Theta_0$ é válida ou se uma afirmação alternativa $H_1 : \theta \in \Theta_1$ é válida. Tais expressões são denominadas de hipóteses nulas e alternativas respectivamente com $\Theta = \Theta_0 \cup \Theta_1$. Se Θ_0 é unitário, a hipótese nula é dita ser simples, caso contrário, afirma-se que a hipótese nula é composta.

Deseja-se criar uma regra de decisão, ou seja, quer-se definir uma região $R \subset R^n$ tal que

Se $(X_1, \ldots, X_n) \in R$, rejeita-se H_0;

Se $(X_1, \ldots, X_n) \notin R$, não se rejeita H_0.

R é denominado de região crítica do teste. Definir a regra de decisão equivale a definir R. Na regra de decisão, podem-se cometer dois tipos de erro.

Rejeitar a hipótese nula quando a mesma é verdadeira (ou seja, $\theta \in \Theta_0$ enquanto que o teste indica $\theta \in \Theta_1$).

Não rejeitar a hipótese nula quando a mesma é falsa (ou seja, $\theta \in \Theta_1$ enquanto que o teste indica que $\theta \in \Theta_0$).

Esses erros são conhecidos como erros Tipo I e II, respectivamente.

Quando a distribuição de probabilidade da população em estudo é conhecida, os testes de hipóteses envolvidos são denominados como paramétricos. Em tais testes, as hipóteses envolvidas apenas tratam parâmetros populacionais.

Entretanto, existem diversos casos em que a distribuição da população de interesse é desconhecida. Testes conhecidos como não-paramétricos podem ser efetuados com o objetivo de verificar a qualidade do ajuste de uma distribuição a uma população de interesse (testes de ajuste – Kolmogorov-Smirnov, Qui-quadrado, entre outros).

WASSERMAN (2006) descreve uma técnica conhecida como Bootstrap a qual se utiliza de geração de B amostras com reposição em relação à amostra de interesse. Em se tratando de Bootstrap paramétrico é usado quando se tem que a amostra em questão pode ser modelada por um modelo paramétrico, sendo suas replicações baseadas nos parâmetros desse modelo.

Ainda, tanto em estudos paramétricos como em estudos não-paramétricos, pode-se construir uma regra de decisão através do p-value. O p-value pode ser interpretado como a probabilidade de se observar o efeito observado, dado que a hipótese nula é verdadeira.

Nesse trabalho desenvolve-se uma estatística de teste no qual se verifica a aplicabilidade do modelo proposto (IPRA+PRG) ao banco de dados em estudo, onde a distribuição da estatística do teste é estudada através de Bootstrap paramétrico.

Após o uso de Bootstrap paramétrico, analisa-se o *p-value* para verificar se o modelo é adequado, assumindo que o mesmo o é.

Visto os conceitos da metodologia de Riscos Competitivos e sua capacidade analítica de tempos de falha e manutenções preventivas – cenário representativo de riscos competitivos dependentes – e da metodologia de PRG como modelo de idade virtual, além de uma breve discussão sobre testes de hipóteses, é preciso apresentar como se dá o funcionamento conjunto de tais metodologias. A apresentação das funções de interesse e de um teste de hipóteses capaz de verificar a aplicabilidade do modelo proposto está presente na próxima Seção.

3 Modelo Proposto: Hibridismo entre Riscos Competitivos Dependentes e Processo de Renovação Generalizado

3.1 Funções de Interesse

Basicamente, as funções de interesse são apresentadas na Seção 2.1, mas devido a manutenções agora serem imperfeitas, essas funções sofrem influências dos parâmetros representativos dos modelos de idade virtual.

Quando se trata da pdf para o tempo até a primeira falha, esta se comporta da mesma maneira apresentada na Equação 4.

$$f(y,j)=h(y)(1-r)^{1-j}\exp(-H(y))^{1-j}r^{j}Ie(H(y))^{j} \quad \text{Equação 4}$$

onde a função $Ie(H(y))$ é conhecida como integral exponencial e é definida por $Ie(t)=\int_t^{\infty}\frac{\exp(-u)}{u}du$ e o parâmetro r representa probabilidade de ocorrer uma manutenção preventiva antes de uma falha crítica ($r=P(Z<X)$).

Entretanto, a pdf a partir da segunda falha (ou manutenção) se torna condicionada à qualidade da manutenção realizada. Dessa maneira, de acordo com a definição de probabilidade condicionada apresentada em Yañez *et al.* (2002), a pdf condicional é apresentada da seguinte forma:

$$f((y_n,j_n)\,|\,w_{n-1})=\frac{\partial}{\partial y_n}\left(1-\frac{R(y_n,j_n)}{R(w_{n-1})}\right)=\frac{1}{R(w_{n-1})}\left(-\frac{\partial}{\partial y_n}R(y_n,j_n)\right)=\frac{f(y_n,j_n)}{R(w_{n-1})} \quad \text{Equação 5}$$

onde $w_{n-1}=q\sum_{i=1}^{n-1}y_i$ representa a idade virtual do sistema em questão. Note-se que a pdf condicional depende da pdf no tempo atual e da função de confiabilidade no (n-1) –ésimo tempo de falha (vale ressaltar que as pdf's estudadas para esse modelo satisfazem as propriedades de uma pdf; esse processo de verificação foi realizado via aproximação de integrais Monte Carlo devido à complexidade das funções envolvidas). Tendo em vista que essas funções foram apresentadas anteriormente, a equação acima fornece:

$$f((y_n,j_n)\,|\,w_{n-1})=\frac{(1-r)h(y_n)\exp(-H(y_n))+rh(y_n)Ie(H(y_n))}{\exp(-H(w_{n-1}))-rH(w_{n-1})Ie(H(w_{n-1}))} \quad \text{Equação 6}$$

A apresentação das funções de interesse aqui tem como objetivo demonstrar como se dá a influência do PRG sobre o modelo IPRA. Como foco principal do presente trabalho, a Seção seguinte busca desenvolver um teste de hipóteses e sua estatística para verificar se o modelo proposto é aplicável ao banco de dados desejado.

3.2 Teste para Verificação de Aplicabilidade do Modelo Proposto (IPRA + PRG)

Para o desenvolvimento de um teste de hipóteses capaz de verificar se o modelo é aplicável ou não ao banco de dados em estudo, considere como ponto inicial a discussão presente em Langseth & Lindqvist (2005). Tal discussão é em parte recorrente do caso onde manutenções são perfeitas.

Considerando o caso onde se tem manutenções imperfeitas, a estatística do teste é diferenciada pelo tratamento de manutenções ser feito por PRG nesse trabalho. Nesse trabalho, a estatística é desenvolvida considerando que as manutenções são modeladas via PRG.

Inicialmente, as hipóteses a serem testadas são:

H_0 : O modelo IPRA+PRG é aplicável;

H_1 : O modelo IPRA+PRG não é aplicavel

Os cálculos envolvidos para se estimar $\phi(t)$ fazem uso dos conceitos do PRG considerando a estrutura teórica de $\phi(t)$ apresentada em Cooke (1996b) adicionando um condicionamento do tempo à idade virtual do sistema da seguinte forma:

$$\phi^*(t)=\frac{P(Z<X,Z>t\,|\,Z>t_{n-1})}{P(Z<X,Z>t\,|\,Z>t_{n-1})+P(Z>X,X>t\,|\,Z>t_{n-1})} \qquad \text{Equação 7}$$

onde $t_{n-1}=q\sum_{i=0}^{n-1}y_i$ como já definido na Seção anterior.

Usufruindo da definição de probabilidade condicionada, podem-se obter as seguintes expressões para as probabilidades acima:

$$R_Z^*(t\,|\,t_{n-1})=P(Z<X,Z>t\,|\,Z>t_{n-1})=\frac{P(Z<X,Z>t,Z>t_{n-1})}{P(Z>t_{n-1})}=\frac{P(Z<X,Z>t)}{P(Z>t_{n-1})}=\frac{R_Z^*(t)}{R_Z(t_{n-1})}$$

$$R_X^*(t\,|\,t_{n-1})=P(X<Z,X>t\,|\,X>t_{n-1})=\frac{P(X<Z,X>t,X>t_{n-1})}{P(X>t_{n-1})}=\frac{P(X<Z,X>t)}{P(X>t_{n-1})}=\frac{R_X^*(t)}{R_X(t_{n-1})}$$

O desenvolvimento acima ajuda a definir $\phi^*(t)$ como

$$\phi^*(t)=\frac{\dfrac{R_Z^*(t)}{R_Z(t_{n-1})}}{\dfrac{R_X^*(t)}{R_X(t_{n-1})}+\dfrac{R_Z^*(t)}{R_Z(t_{n-1})}} \qquad \text{Equação 8}$$

Conhecendo a forma funcional das funções presentes na Equação 8 é possível realizar a estimativa de $\phi^*(t)$. As funções $R_Z^*(t)$ e $R_X^*(t)$ podem ser estimadas dos dados como os estimadores $\{\#x>t\}$ e $\{\#z>t\}$ que representam o número de ocorrências de falhas e manutenções preventivas após *t* respectivamente. No caso das funções $R_Z(t_{n-1})$ e $R_X(t_{n-1})$, faz-se o uso dos limites de Peterson apresentados por PETERSON (1975). Ou seja,

$$R_X^*(t)+R_Z^*(t)\le R_X(t)\le R_X^*(t)+(1-r) \qquad \text{Equação 9}$$

$$R_X^*(t)+R_Z^*(t)\le R_Z(t)\le R_Z^*(t)+r \qquad \text{Equação 10}$$

Para a estimativa não-paramétrica, $\phi^*(t)$ pode ser estimado pelos estimadores paramétricos de forma $\{\#x>t\}$, $\{\#z>t\}$, $\{\#U>t\}$ e $\{\#W>t\}$, resultando em:

$$\hat{\phi}^{*}(t)=\frac{\frac{\{\#z>t\}}{\{\#W>t_{n-1}\}}}{\frac{\{\#x>t\}}{\{\#U>t_{n-1}\}}+\frac{\{\#z>t\}}{\{\#W>t_{n-1}\}}}$$ Equação 11

onde U e W são valores estimados da Equação 9 e da Equação 10, respectivamente. Com isso, é possível realizar a estimativa do valor da estatística do teste a qual é apresentada na próxima Seção junto com o resultado do teste de hipóteses.

4 Apresentação dos Resultados

4.1 Caso de Aplicação

Os dados estudados nesse trabalho se referem um sistema de compressão de uma instalação offshore. Tais dados estão presentes em Langseth & Lindqvist (2005) tal como uma breve descrição de suas características.

Os dados são apresentados no Anexo I onde se tem 3 colunas onde a primeira representa o tempo entre ocorrências, a segunda o mecanismo da falha e a terceira a severidade da falha – crítica ou degradada. A depender da severidade, um tipo de risco é identificado. Quando a severidade da falha é de natureza crítica, o risco correspondente é uma falha crítica enquanto que a severidade de natureza degradada tem como risco correspondente uma manutenção preventiva.

4.2 Apresentação dos Resultados

Para realizar os cálculos referentes à estatística de teste, métodos de integração numérica foram usados devido à complexidade de algumas funções envolvidas.

De acordo com os dados contidos no Anexo I e a partir de um teste de aderência verificando que os dados de falha seguem uma distribuição Weibull com parâmetros $\hat{\alpha}$ = 279,3201 e $\hat{\beta}$ = 0,5731 além do parâmetro de rejuvenescimento $\hat{q}$ = 0,0096 – todos estimados a partir dos dados - a estatística do teste é calculada de acordo com o algoritmo apresentado no Anexo II. O software utilizado é o R 2.9.2.

A estatística do teste é calculada através de resoluções computacionais para a integral apresentada a seguir:

$$\Delta=\int_{0}^{\infty}\left|v(u;\hat{r})-v(u;\hat{r})\right|\exp(-u)\,du$$ **Equação 12**

O resultado da estatística calculado da amostra é de $\Delta = 0{,}1259$. A distribuição não-paramétrica da estatística, diga-se $\hat{f}^{(B)}(\delta)$, é utilizada para se realizar inferências formais considerando o modelo. A Figura 3 apresenta $\hat{f}^{(B)}(\delta)$ com um B = 200. A distribuição de bootstrap fornece que $P^{(B)}(\Delta > 0{,}1259 \mid H_0) = 0{,}455$; sendo assim, a hipótese nula não é rejeitada: o modelo IPRA + PRG define uma classe apropriada de modelos para representar esse banco de dados (podendo-se considerar um nível de significância de até 45,5%). Em uso de um modelo híbrido, destaca-se o modelo proposto por Langseth & Lindqvist (2005) o qual é testado ao mesmo banco de dados apresentado aqui e apresenta um p-value de 25%. As diferenças entre os modelos estão no uso de diferentes modelos para a análise da qualidade das manutenções (IPRA + PRG – proposto – versus IPRA + BP – existente) que por conseqüência afeta na maneira de se analisar a estatística do teste por envolver funções diferentes.

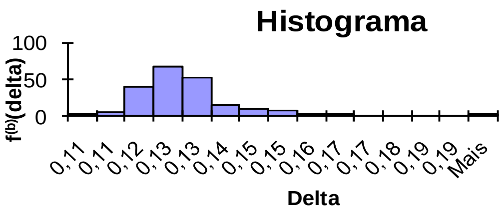


Figura 3 – Densidade aproximada para a estatística do teste via bootstrap.

A partir dos resultados obtidos, o alto *p-value* apresentado demonstra a adequação do modelo proposto ao banco de dados estudado. Ou seja, os dados referentes a tempos de eventos do equipamento offshore podem ser analisados pelo modelo proposto tornando possível análises envolvendo métricas de confiabilidade e a qualidade das manutenções realizadas. Tais métricas de confiabilidade e qualidade são estudadas através das funções estudadas e ainda estão em desenvolvimento tanto algébrico como computacional, sendo assunto para trabalhos futuros.

5 Conclusões

Esse trabalho mostrou a importância do uso de técnicas para análise de confiabilidade em equipamentos/sistemas com uso industrial. Riscos Competitivos são utilizados para realizar tais análises já que são capazes de analisar tempos de interesse e seus causadores, relacionando-os. É realizada então a modelagem da relação existente entre falhas críticas e manutenções preventivas a partir de um modelo conhecido como IPRA considerando a existência de dependência entre essas duas variáveis.

Além disso, o presente trabalho apresentou a importância da modelagem da natureza de manutenções realizadas, característica ausente na análise de RCs. Processos estocásticos pontuais são utilizados para tal modelagem e um modelo conhecido como Processos de Renovação Generalizados é utilizado na relação estudada aqui.

O PRG mostra-se capaz de analisar a natureza de manutenções atribuindo diferentes classificações de acordo com a condição que o equipamento/sistema retorne após a manutenção. Isso é feito através de um parâmetro conhecido como fator de rejuvenescimento onde o mesmo torna possível o cálculo da idade virtual do equipamento considerada como a idade do equipamento influenciada pelas manutenções.

Entretanto, PRG não trata a relação entre os eventos e seus causadores. O uso de PRG juntamente com IPRA mostra-se capaz de realizar os dois tipos de análise descritos anteriormente com mais riqueza nas informações obtidas. As informações, tanto para análise de confiabilidade via RCs como para a natureza das manutenções via PRG, são extraídos de BDCs.

Em particular, esse trabalho tratou de dados referentes a um equipamento *offshore* contendo tempos de paradas e seus causadores – falhas críticas ou manutenções preventivas. Antes de extrair qualquer tipo de informação dos dados a partir do modelo proposto, é preciso saber se o modelo é adequado ao banco de dados utilizado. Para isso, um teste de hipótese foi desenvolvido para verificar a aplicabilidade do modelo IPRA + PRG aos dados.

Durante o desenvolvimento do teste de hipóteses, a construção de uma estatística de teste é necessária, tendo esta que ser capaz de representar alguma informação capaz de auxiliar na regra de decisão. Usando estimadores paramétricos e não paramétricos para o cálculo dessa estatística, a regra de decisão é baseada no uso de bootstrap paramétrico (com geração de uma amostra de tamanho 200) para a identificação de uma pdf aproximada para a estatística.

Após a aplicação do teste desenvolvido, o valor da estatística amostrado dos dados indicou fortes evidências de não rejeição da hipótese nula, ou seja, o modelo IPRA + GRP mostra adequado para modelagem dos dados em questão. Ainda, é importante destacar a margem de rejeição capaz de ser determinada devido a um p-value obtido de 0,545. Em outras palavras, mesmo a margem para se rejeitar a aplicabilidade do modelo seja grande, rejeita-se a hipótese nula apenas quando essa margem é maior que 54,5%.

Apesar do desenvolvimento de um teste de hipóteses capaz de verificar a aplicabilidade do modelo proposto, tal modelo apresenta estimadores como na Equação 8 que não tiveram suas propriedades analisadas (como consistência e suficiência). Ainda, o tamanho da amostra gerada por bootstrap paramétrico (B = 200) pode ser mais bem explorado aumentando o mesmo para resultados mais precisos.

Espera-se em trabalhos futuros tratar os estimadores e suas propriedades além de se trabalhar com uma amostra de bootstrap maior. O estudo da função de verossimilhança tal como sua estimativa de acordo com os resultados obtidos, além das funções de distribuição e confiabilidadlocc e também são de interesse, já que o interesse principal no desenvolvimento do modelo híbrido proposto recai sobre a análise de confiabilidade.

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Application of Competing Risks and Generalized Renewal Processes in Reliability Analysis

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ABSTRACT: The use of Reliability Database is important to ensure longer use of equipment; this can be made through a consistent Reliability and Risk analysis. In this way, besides the study of failure times, their causes are of great importance, in order to understand and control equipment stochastic behavior over time. This paper presents a model that treats the competing risks scenario concerning the effectiveness of repair action taken. By using the so called Repair Alert model and the Generalized Renewal Processes, functions of interest are presented to estimate important reliability and risk metrics. Data about a compression system of a offshore facility is used to illustrate situation in which Competing Risks and Generalized Renewal Process are used separately and together as the proposed model.

APPENDIX V

1 *INTRODUCTION*

In Risk and Reliability Analysis, the failure behavior is of great importance, given its impact on longevity of equipment. Furthermore, equipment in industrial area is more expensive to repair or replace than preventively maintaining it. In this way, understanding failure behavior can avoid high costs and improve the longevity of equipment. In order to do this, specific information about failure is needed to make a consistent analysis possible. One way to obtain this type of information is Reliability Databases (RDB). Fragola (1996) presents a historical discussion about the development of RDB.

RDB possess useful information on stochastic failure behavior and may be used to construct optimal maintenance policies. In this context, we can point out two main characteristics of approaches developed to handle RDB.

Firstly, the actions taken can lead the equipment to a different state after maintenance, according to its effectiveness. Brown & Proschan (1983), Kijima & Sumita (1986b), Krivtsov (2000) and Yañez et al. (2002) developed some works that treat this kind of situation. Among models proposed to analyze the maintenance effectiveness, one can cite the virtual age approach, which is capable to analyze the failure time distribution through a concept called virtual age – which was originally presented by Kijima & Sumita (1986b). This is measured through the rejuvenation parameter q included for the development of Generalized Renewal Processes (GRP), one of the virtual age models.

However, GRP cannot distinguish amidst different types of failure causes, and then among different types of maintenance (corrective or preventive).

On the other hand, Competing Risks (CR) models, firstly presented by Cox (1959) are capable to model not only failure times, but also the group of failure modes that caused failure events. This analysis can be done through the study of a pair of observation (Y, J) where Y is the event time – it is considered in this paper two kinds of events: corrective (a failure) and preventive (a maintenance) actions – and J is an indicator variable (0 for failure and 1 for maintenance). CR models, however, consider that after a repair action (corrective or preventive), SC returns as good as new, what in practice rarely occurs. Bedford & Cooke (2002), Cooke (1996b), Crowder (2001), Langseth & Lindqvist (2003) presented theory and models to treat CR situations.

The gap between these two methodologies may be solved with the construction of a hybrid model. This paper presents the development of a hybrid model of CR and virtual age models by using GRP. Therefore, one can model the relationship between corrective and maintenance actions, besides of analyzing the effectiveness of repair.

The rest of this article is organized as follows: in Section 2 some notation on Generalized Renewal Processes and Competing Risks is presented. In Section 3 the main properties of used models are presented. In Section 4, the proposed model is presented. Section 5 brings an application case. Section 6 concludes remarks.

2 NOTATION AND DEFINITIONS

Some notation and definitions concerning the Competing Risks and Virtual Age models are presented.

2.1 Competing Risks

A CR for Reliability analysis is characterized where there are two or more failure modes competing to fail the equipment. In this paper, there are two competing risks classified in accordance with their severity. Critical failures correspond to corrective maintenance actions whereas degraded or incipient failures are associated to preventive maintenance. The following notation is used throughout this paper.

X is the time of a critical failure; Z is the time of a degraded or incipient failure. Y=min(X, Z) is the minimum between these variables. J is the indicator variable that can assume the values 0 (for a critical failure), 1 (for a degraded or incipient failure) or 2 (for an external event).

One can observe the pair (*Y*, *J*) as well as its probabilistic behavior. The main functions that make use of this pair follow.

- $f_{Y,J}(y,j) \rightarrow$ Joint density of Y and J ;
- $F_{Y,J}(y,j) \rightarrow$ Joint distribution of Y and J ;
- $R_Z(z) \rightarrow$ Subsurvival function of risk Z ;
- $R_X(x) \rightarrow$ Subsurvival function of risk X ;
- $h_Z(z) \rightarrow$ Subhazard function of risk Z ;
- $h_X(x) \rightarrow$ Subhazard function of risk X .

The subsurvival function can be interpreted as $R_Z(z) = P(Z > t, Z < X)$ (the same for risk X) and the subhazard function can be directly obtained from these subsurvival functions.

Crowder (2001) presents a comprehensive discussion concerning the main properties of the above functions mentioned. In spite of the possibility to treat the subsurvival functions in Reliability Analysis, the main interest is about the probabilistic behavior of *X*, as well explained in Lindqvist et al. (2005) due to the future constructions of maintenance policies. Here, the subhazard functions are connected with the CR model used and with the PDF (Probability Density Function) and subsurvival functions.

2.2 Virtual Age Models

A virtual age model, as presented in Yañez et al. (2002), analyzes maintenance effectiveness through a concept called virtual age which is modeled through a rejuvenation parameter, namely *q*, as in Generalized Renewal Processes. The relationship between the virtual age and *q* is described as follows.

$$w_i = w_{i-1} + q \cdot h_i = qt_i. \qquad (1\text{-}1)$$

Here, w_i represents the virtual age up to *i*-th failure, h_i is the time between the (*i*-1)-th and *i*-th failure and t_i is the global time up to *i*-th failure.

The rejuvenation parameter usually belongs to the interval [0, 1], where values 0 and 1 represent perfect and minimal maintenances, respectively, whereas values between both extremes are related to an imperfect repair. The interested reader may consult Kijima & Sumita (1986b), Krivtsov (2000) and Moura et al. (2007) for further details.

3 COMPETING RISKS AND GRP MODELS

3.1 The Intensity Proportional Repair Alert Model

Cooke (1996b) presents an approach that may be considered to model dependence between risks, namely Random Signs Censoring (RSC). Basically, a critical failure can be avoided by a (conditioned) preventive maintenance which is realized by the maintenance crew who noticed some kind of signal. Cooke (1993a) presents the conditions for which the model identifiability is assured as well as ensures that the subsurvival function of risk X can be interpreted as the survival function. Therefore, if a database can be modeled through a RSC model, it is guaranteed to obtain reliability metrics correctly.

Lindqvist et al. (2005) present a model based on RSC, namely Repair Alert (RA). RA approach performs the same analysis that of RSC, but it also analyzes the “alertness” of the maintenance crew through a function called repair alert function.

$$P(Z \le z \mid X = x, Z < X) = G_Z(z)/G_X(x), \;\; 0 \le z \le x. \qquad (1\text{-}2)$$

where G(.) is a increasing function with G(0) = 0. The function here used can be anyone that represents the probabilistic behavior of the maintenance crew. When the hazard function is used, then (1-2) characterizes the *Intensity Proportional Repair Alert* (IPRA) model. In this work, this model is used to capture the Competing Risks situation.

The practical interpretation of this function can be briefly defined as the proportionality between intensity functions of risks Z and X. Thus, this model is capable to capture the dependence between preventive maintenances and critical failures. Furthermore, it can analyze the maintenance crew "alertness", as shown in Figure 3.

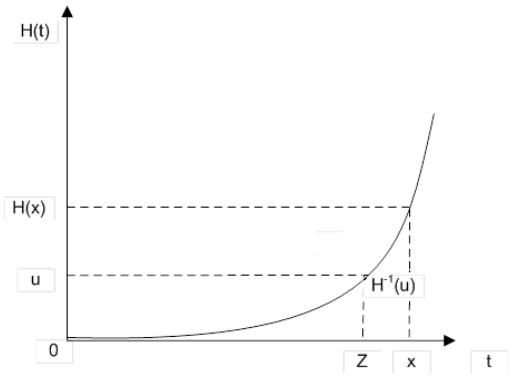

Figure 3. Behavior of conditional probability of preventive maintenance

Since RA model is derived from RSC model, then the subsurvival function of failure times is identifiable from data. Lindqvist et al. (2005) shows that the subdistribution of data can be interpreted as follows.

$F_X^*(x) = (1-r)F_Y(y)$

where $r = P(Z < X)$ is the probability of a preventive maintenance occurs before a critical failure.

3.2 Generalized Renewal Processes

Roughly speaking, GRP can model three different maintenance situations: (*i*) the equipment is repaired and brought to a perfect state; (*ii*) the equipment is recovered only to perform its function under minimal conditions; and (*iii*) the equipment is returned to a suitable state of operation, but nor new neither minimal – the imperfect state.

There exist two ways to analyze maintenance effectiveness: *(i)* one can treat the last failure time occurred and measure the quality of the respective repair made. This analysis is treated by the well-known Kijima Type I model, which is presented in (1-1); *(ii)* on the other hand, the maintenance effect can be considered since beginning of the service, also creating a cumulative effect of the repairs made over time. This situation is modeled by the Kijima Type II model given in (1-3).

$$w_i = q(h_i + w_{i-1}) = q(q^{i-1}h_1 + q^{i-2}h_2 + \ldots + h_i) \quad (1\text{-}3)$$

One can note the main differences between the models presented. In the first case, the rejuvenation parameter only acts on the most recent failure time, the second one presents the influence of *q* through all the failure history.

Once presented the models to be used in this paper, the next Section brings their functionalities working together in a CR scenario considering the maintenance effectiveness.

4 THE PROPOSED MODEL: JOINT USE OF IPRA AND GRP MODELS

The analysis through a CR model allows the correct interpretation of the relationship between two (or more) risks. Thus, the probabilistic functions must represent the robustness of this relationship. Langseth & Lindqvist (2003) present the likelihood for each value of J to the IPRA model. From that, the joint PDF can be developed as follows.

$$f(y,j) = h(y)(1-r)^{1-j}\exp(-H(y))^{1-j} r^j Ie(H(y))^j \quad (1\text{-}4)$$

The term $Ie(\cdot)$ corresponds to the integral exponential and does not have analytical solution.

Lindqvist et al. (2005) present a complete analysis of the properties of the RA model. However, the IPRA model only treats the case where the repair is perfect. This drawback is here overcome by the GRP approach.

Consider the definition for conditional density function presented by Yañez et al. (2002). Eq. (1-5) is a modification considering a CR situation:

$$f((y_n, j_n) \mid w_{n-1}) = \frac{\partial}{\partial y_n}\left(1 - \frac{R(y_n + w_{n-1}, j_n)}{R(w_{n-1})}\right) = \frac{1}{R(w_{n-1})}\left(-\frac{\partial}{\partial y_n}R(y_n + w_{n-1}, j_n)\right) = \frac{f(y_n + w_{n-1}, j_n)}{R(w_{n-1})} \quad (1\text{-}5)$$

From (1-4) – or (1-5) with n = 1, it can be noted that it represents the joint PDF to the first event occurred. After the second failure time, and from (1-5), the failure behavior is measured through the virtual age and the joint PDF is conditioned to the maintenance effectiveness. From (1-4) and (1-5), the expression to analyze times from the second failure from IPRA can be obtained.

$$f\left(\left(y_n, j_n\right) \mid w_{n-1}\right) = \frac{h\left(y_n + w_{n-1}\right)\left(1-r\right)^{1-j} \exp\left(-H\left(y_n + w_{n-1}\right)\right)^{1-j} r^j Ie\left(H\left(y_n + w_{n-1}\right)\right)^j}{\exp\left(-H\left(w_{n-1}\right)\right) - rH\left(w_{n-1}\right) Ie\left(H\left(w_{n-1}\right)\right)} \quad (1\text{-}6)$$

Thus, from (1-6) (n = 2,…), the maintenance effectiveness is captured from the second failure time and inserted in to the IPRA joint PDF, eq. (1-4). This allows analysis of the failure times conditioned on past ones.

The following equations are obtained from eq. (1-6). This can be done when the value of J is known. Thus, when J = 0, eq. (1-7) is obtained; the same for equations (1-8) – when J = 1 – and (1-9) with J = 2.

$$f\left(\left(y_i, 0\right) \mid w_{i-1}\right) = \frac{\left(1-r\right) h\left(y_i + w_{i-1}\right) \exp\left(-H\left(y_i + w_{i-1}\right)\right)}{A} \quad (1\text{-}7)$$

$$f\left(\left(y_i, 1\right) \mid w_{i-1}\right) = \frac{rh\left(y_i + w_{i-1}\right) \mathrm{Ie}\left(H\left(y_i + w_{i-1}\right)\right)}{A} \quad (1\text{-}8)$$

$$f\left(\left(y_i, 2\right) \mid w_{i-1}\right) = \frac{\exp\left(-H\left(y_i + w_{i-1}\right)\right) - rH\left(y_i + w_{i-1}\right) \mathrm{Ie}\left(H\left(y_i + w_{i-1}\right)\right)}{A} \quad (1\text{-}9)$$

where $A = \exp\left(-H\left(w_{i-1}\right)\right) - rH\left(w_{i-1}\right) \mathrm{Ie}\left(H\left(w_{i-1}\right)\right)$. Eqs (1-7) and (1-8) represent the contribution of an observation of a degraded/incipient and critical failure, respectively. Eq. (1-9) represents the situation where an external event is observed, acting as a censoring event.

A computational implementation for these functions is made to prove their PDF condition, using a Weibull as a failure time distribution.

The estimation process used in this article is the same used in Yañez et al. (2002) involving the joint PDF presented here according to steps discussed in Lindqvist et al. (2005) to obtain the estimates of the distribution parameters and *q*, respectively. The estimation of the Competing Risks parameter *r* follows directly from data and is obtained as given in (1-10).

$$\hat{r} = \frac{n\left(z\right)}{N}, \quad (1\text{-}10)$$

where *n(z)* represents the number of preventive maintenances occurred and *N* is the total number of events.

Thus the joint approach is noticed with the use of the CR functions to estimate reliability metrics including the *q* parameter.

With the presentation of the likelihood of the proposed model, one can obtain the estimators of the involved parameters correctly. From this, since data is adjusted with the model, then the reliability metrics can also be obtained. Thanks to the IPRA property, the reliability behavior of risk X can be identified directly from data. Furthermore, properties of the maintenance crew can be analyzed through the repair alert function.

5 EXAMPLE OF APPLICATION

It is here presented the description of dataset as well as the results obtained.

5.1 Dataset description

The database here used is mainly draws from Langseth & Lindqvist (2003) and Langseth & Lindqvist (2005) present a data study verifying the applicability of their model with the related data. Ferreira et al. (2010) developed a hypothesis test to verify the applicability of the proposed model. Thus, it is here considered that used data can be modeled with the IPRA model considering imperfect repair (the GRP action).

The dataset consists of times between failures of a compression system in an offshore facility. This database is also presented in OREDA (2002). Table 1 shows how data is organized. The column “Time” presents the times between events (PM or critical failures); the column “FM” brings the failure mechanisms and the column “Severity” the failure severity, where critical (C) or degraded/incipient (D).

Table 3. Structure of database used in application.

Time	FM	Severity
220	1.0	D
13	1.0	D
1	1.4	D
…	…	…

The proposed model investigates the Competing Risks between “Severity” field –degraded/incipient and critical situations. It is assumed failure data come from the same failure mechanism. The whole data can be seen in Ferreira et al. (2010).

5.2 Results

Considering the events from Table 1 and whole data in Ferreira et al. (2010), the Weibull distribution is adjusted and the estimated parameters are $\hat{\alpha}$=279.3, $\hat{\beta}$=0.57 and $\hat{q}$=0.0096. The hypothesis test did not reject the applicability of the proposed model to data with $\Delta = 0.1259$ and $P^{(B)}(\Delta = 0.1259 \mid H_0) = 0.455$, according to Ferreira et al. (2010).

The estimates, obtained above through a based MCMC algorithm presented in Yañez et al. (2002), consider the presence of imperfect repair.

According to the IPRA model property, the reliability function can be directly obtained from data. Figure 4 shows the reliability behavior of X considering the estimate of parameters above in CR scenario only. Actually, Figure 4 represents the availability of the system – once occur a failure, the system is repaired and brought to operation again. As said before, one can note that the system always return with reliability 1 to operation which characterize a perfect repair.

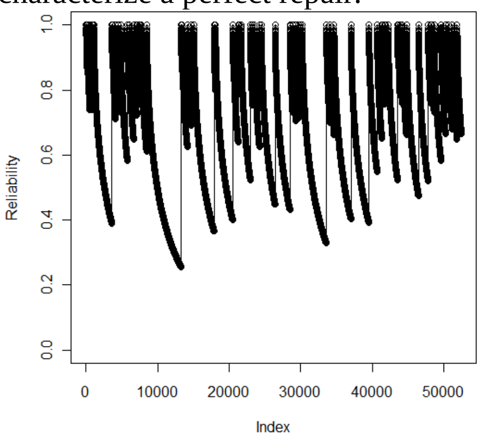

Figure 4. The availability of X (critical failure).

Furthermore, one can analyze the joint probability of occurrence of a critical failure up to a time t and this failure occurs before a degraded one – the subsurvival function of X – as follows in Figure 5.

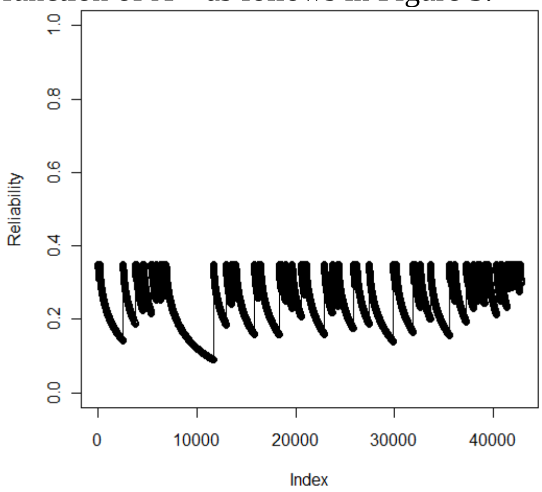

Figure 5. The subsurvival function of risk X measured for 100 failure events.

Again, Figure 5 presents a kind of availability of system considering the subsurvival function after repairs. Since this situation is treated only by CR, one can see that the system always return to a specific value determined by *r* which in this case has an estimate of 0.65.

Finally, one can consider the results concerning the critical failure and the quality of repair.

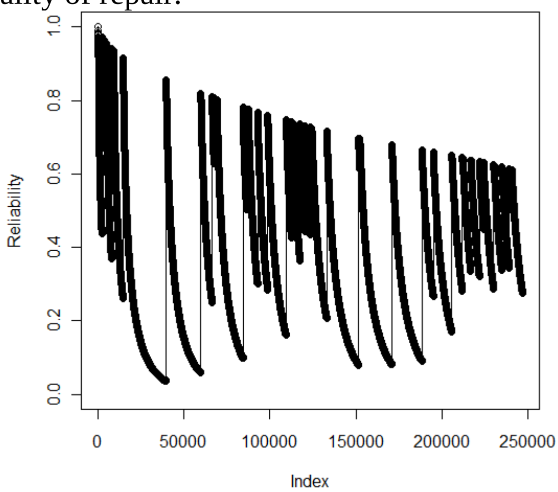

Figure 6. The availability function of X with repair quantified.

Figure 6 shows that, according to the estimates of α, β and q, the availability of

X. It is reasonable to state that, since the value of q is almost 0, the availability suffers minor impact along time.

5.3 Discussion

It is expected the effectiveness of preventive and corrective maintenances are different. Thus, depending on the type of failure observed (degraded/incipient or critical), q has a different value as shown in (1-11).

$$w_{n-1} = \delta t_{X\,n-1} + (1-\delta) t_{Zn-1} \quad (1\text{-}11)$$

Here, δ is an indicator variable that equals 0 or 1 when degraded/incipient or critical failures occur respectively. Also, $t_{n-1} = w_{n-2} + qh_{n-2}$, which represents the Kijima type I model.

Thus, according to (1-7) and (1-8), the value of q differs. From (1-9), a censoring event occurs avoiding a degraded/incipient or a critical event. Since none repair occurs in this situation, when the SC returns to operation, its state is the same as the one before the censoring event (considering only external events). Thus, it is suitable to consider the value of q here as 1.

The Kijima type I model is used here, since the virtual age can be modeled according to the last failure occurred considering the influence of a critical (or degraded) failure in the last failure time.

Since the GRP parameter acts in (1-6), then it is suitable to admit that it jointly captures the effect of preventive and corrective maintenance. In order to provide the correct estimator of q, the corresponding likelihood of the hybrid model must be presented.

According to Yañez et al. (2002), considering the failure-terminated process, the likelihood can be developed as follows.

$$L = f(t_1)\, f(t_2 \mid t_1) \ldots f(t_n \mid t_{n-1})$$

Considering the proposed model, the likelihood can be presented below.

$$L = h(y_1)(1-r)^{1-j} \exp(-H(y_1))^{1-j} r^j Ie(H(y$$
$$\times \prod_{i=2}^{n} \frac{h(y_i)(1-r)^{1-j} \exp(-H(y_i))^{1-j} r^j Ie(H(}{\exp(-H(w_{i-1})) - rH(w_{i-1}) Ie(H(w_{i-1}} \quad (1\text{-}12)$$

Thus, from (1-12), one can develop the Maximum Likehood Estimators for the desired parameters. Actually the parameters set correspond to the used distribution and the GRP parameters given in (1-11). Due to the analytical complexity of this likelihood, the log-likelihood and the derivatives will be not developed from the classical way – as for GRP.

Furthermore, it is suitable to consider, beyond the MLE development, a Bayesian analysis to the estimation process discussed.

6 CONCLUSIONS

This paper has presented a situation where Competing Risks and Generalized Renewal Processes work together in Risk and Reliability analysis. Furthermore, for fields where equipment is expensive, it requires high levels of reliability and knowledge about its failure behavior. In this way, it is possible to know how equipment can fail, what is the probabilistic behavior of each failure mode. An RDB with standard information is used as source, making easier the data collection to estimate the necessary metrics concerning the main functions in reliability field.

Specifically, this paper presents a model where the relationship between critical and degraded/incipient failures is modeled. In this case, the former are treated by a corrective maintenance whereas the latter are handled by preventive actions.

Competing Risks tackles this dependence through the IPRA model and the maintenance effectiveness made is analyzed through the use of Generalized Renewal Processes.

With the example shown, one can see that the situation treated with the traditional modeling can upper or underestimate reliability metrics as the reliability function itself. However, this can be only ensured with the correct use of the estimates considering the proposed model. The proposed hybrid model can represent more realistic the relationship between repair actions. The estimation process can be obtained from the likelihood discussed in Section 4. In

Section 5, a discussion is presented about a more realistic way to represent the impact of maintenances (corrective and preventive). Due to the complexity of presented function, the development of ML estimators of involved parameters has still been treated. Also, a Bayesian analysis is suitable to consider about the estimation process.

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