UNIVERSIDADE FEDERAL DE PERNAMBUCO
PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA DE
PRODUÇÃO

A STRONG ARGUMENT FOR USING
NON-COMMODITIES TO GENERATE
ELECTRICITY

DISSERTAÇÃO SUBMETIDA À UFPE
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KATARINA TATIANA MARQUES SANTIAGO

**"A STRONG ARGUMENT FOR USING NON-COMMODITIES TO GENERATE
ELECTRICITY"**

ÁREA DE CONCENTRAÇÃO: PESQUISA OPERACIONAL

A comissão examinadora, composta pelos professores abaixo, sob a presidência do(a) primeiro(a), considera a candidata KATARINA TATIANA MARQUES SANTIAGO **APROVADA**.

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“...when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the state of Science, whatever the matter may be.” [Lord Kelvin PLA, vol. 1, Electrical Units of Measurement, 1883-05-03]

“Dedico esta dissertação à minha família.”

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RESUMO

Usa-se controle ótimo para analisar o *trade-off* entre o uso das fontes primárias consideradas commodities como combustíveis fósseis, biomassa e água para gerar eletricidade versus outros usos econômicos (indústria petroquímica, produção de combustíveis, agricultura, siderurgia, entre outros). Para isso, um modelo dinâmico que estabelece relações entre crescimento econômico, o setor de combustíveis fósseis, o setor da água, o setor da biomassa e políticas energéticas, a partir da aplicação do Princípio do Máximo de Pontryagin, é apresentado. Entre outros resultados, a análise estabelece que, na trajetória ótima, o preço das commodities para usos não energéticos deve ser o dobro do preço dos bens energéticos, indicando a necessidade do uso de fontes que não são commodities para gerar energia elétrica, tais como: energia solar, energia eólica, energia geotérmica, entre outras.

Palavras-Chave: Política energética, Recursos exauríveis, Commodities, Crescimento econômico ótimo, Usos dos combustíveis fósseis, Usos da água, Usos da biomassa.

ABSTRACT

An optimal control approach towards generating electricity is used to analyze the trade-off between the use of primary sources which are regarded as commodities, such as fossil fuels, biomass and water, and their other economic uses (for example, in the petrochemical industry, in the production of fuels, in agriculture, in steelmaking, and so forth). In order to do so, a dynamic model is presented which establishes relationships between economic growth, the fossil fuel, water and biomass sectors, and energy policies, based on the application of the Pontryagin Maximum Principle. Among other results, the analysis establishes that, in the optimal path, the price of commodities for non-energy uses should be twice the price of the energy assets, which indicates the need to use sources which are not commodities such as solar energy, wind energy, and geothermal energy, to generate electricity.

Keywords: Energy policy, Exhaustible resources, Commodities, Optimal economic growth, Uses of fossil fuels, Water uses, Biomass uses.

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List of Abbreviations

BEM Brazilian Energy Matrix.

BTE Board of Trade Electricity.

CAISO California Independent System Operator.

CALPX California Power Exchange.

CEGB Central Electricity Generating Board.

ERC Energy Research Company.

CPUC California Public Utility Commission.

CTC Competition Transition Charge .

DES Domestic Energy Supply.

DGES Director General of Electricity Supply.

EPACT Energy Police Act.

ESI Electric Sector Industry.

ESP Electricity Service Provider.

EWG Exempt Wholesale Generator

FEC Final Energy Consumption.

FERC Federal Energy Regulatory Commission.

FHE Free Hiring Environment.

GGR Global Guarantee Reserve.

IOU Investor Owned Utilities.

MCEI Monitoring Commission os the Electrical Industry.

MME Ministry of Mines and Energy .

NDWE National Department of Water and Elecricity.

NEB National Energy Balance.

NEEA National Electric Energy Agency.

NETA New Electricity Trade Agreement.

NEWD National Energy and Water Department.

NGC National Grid Company.

NIB Nuclear Industry in Brazil.

NSO National System Operator .

OFFER Office of Electricity Regulation.

OFGEM Office of Gas and Electricity Markets.

OPEC Organization of the Petroleum Exporting Countries.

PGE Pacific Gas and Electric.

PRE Procurement Regulated Environment.

PSA Pooling and Settlement Agreement.

PUHCA Public Holding Company.

PURPA Public Utility Regulatory Policies Act.

REC Regional Electricity Companies.

RTO Regional Transmission Organization.

ROR Rate of Return.

SCE South California Edison.

SDE San Diego Gas and Electric.

SEWH Solar Energy for Heating Water.

UDC Utility Distribution Companies.

WME Wholesale Market of Electricity.

1 INTRODUCTION

The classic definition of economics is that it is the study of the allocation of scarce resources between competing uses. Scarcity is a characteristic of exhaustible resources. Therefore, economists, by definition, must be concerned with them.

Hotteling's classic article, *The Economics of Exhaustible Resources* is the most important contribution to understanding the role of these resources in economic growth. Hotteling's purpose was to find an extraction rate that ensured maximum growth continued. Although it was published in 1931, Hotteling's work only received its due recognition in the 70s during the two petroleum crises which showed the dependence on this resource and the danger to sustained economic growth brought about when supplies of oil are scarce.

The idea that when resources are exhaustible, there are limits to economic growth were first mentioned by Thomas Robert Malthus, in his book *Essay on the Principle of Population* (1798). He was the first to announce some stagnation in growth because of the scarcity of the resource of land. He believed that as the population grew geometrically and the land factor remained steady, there would be a lack of food in the future, and thus the need for a policy of population control. What Malthus did not predict was that technological development would be able to raise labor productivity and thus correct the problem. This was what the Industrial Revolution brought about.

The technological development that came with the Industrial Revolution, later added to by other advances, created industries that were heavily dependent on energy and a society with new habits of consumption, especially with regard to energy. Since then meeting energy requirements has become one of the main inputs for production in any economy in the world. In fact, the global demand for primary energy compared to 2005 will have grown by 50% by the year 2030, according to *International Energy Outlook*, 2008.¹

Most of the world's energy sources today are exhaustible. Oil comes first in the global energy matrix followed by coal and natural gas. These sources together account for approximately 80 % of the world's energy supply(MME, 2008). Coal is the source

¹available at <www.eia.doe.gov/oiaf/ieo/world.html>

most used to generate electricity. It accounts for generating 41% of the world's electricity supply. The United States and China are examples of countries that are highly dependent on this resource. In Brazil, water is the resource most used to generate electricity and is followed by biomass. These resources, although renewable, are limited.

What oil, coal, natural gas, water and biomass have in common is that they are conventional. Conventional resources, as argued by (Campello de Souza, 2005) represent stored energy; are found in specific and unchangeable locations; and offer a limited supply. Their scarcity and high demand creates a capitalizable commodity and an avid and impatient market, i.e., these sources are goods with high marketability in the international market and thus subject to price variations.

The issue is that these natural resources are allocated to both electricity production and the production of non-energy goods. Therefore, they are used as an input for production and as an input to produce a different kind of input, namely power.

So, what is the best use of these resources? Should oil be used in the petrochemical industry, in the plastics industry or for electricity generation? Similarly, should water be used in agriculture or in hydroelectric? Why? Similarly what alternatives are there to the use of coal, natural gas and biomass in generating power?

1.1 Objectives

In order to study these problems, some objectives have been set.

- Main Objective
 - To analyze the economic implications of the trade-off between the multiple uses of commodities, such as fossil fuels (oil, coal and natural gas), water with head and biomass.
- Specific Objectives
 - To discuss the classification of energy sources, the availability of these sources in Brazil and worldwide, non-conventional sources of energy and the factors that influence energy consumption.

- To depict the process of regulation and deregulation of the electric sector in the U.S.A., the UK and Brazil and how energy has become a commodity.
- To elaborate and develop an economic growth model, with energy resources, based on the model by Stamford da Silva e Campello de Souza (2008), that can give support to decisions in planning for the electricity sector.

The Trade-off of uranium will not be studied because its non-energy use is very small. It is used primarily to generate electricity in the world.

1.2 Dissertation Organization

- Chapter 2 shows the process of regulation and deregulation of the electric sector in the U.S.A., the UK and Brazil and how energy has become a commodity.
- Chapter 3 discusses the classification of energy sources, the availability of these sources in Brazil and worldwide, non-conventional sources of energy and the factors that influence energy consumption.
- Chapter 4 presents the model proposed.
- Finally, Chapter 5 draws conclusions and makes suggestions for future studies.

2 Deregulation and Regulatory Reform in the Electric Power Sector:U.S, U.K and Brazil cases

“The difficulty lies, not in the new ideas, but in escaping from the old ones, which ramify, for those brought up as most of us have been, into every corner of our minds”

John Maynard Keynes

2.1 Introduction

Prior to 1970 the Electric Sector Industry (ESI) was structured under the model of a regulated natural monopoly. According to this model companies were merged (generation), verticalized (generation and transmission) and there was price regulation which made use of a rate-of-return (ROR), which determined that the prices charged by companies had to generate sufficient income to cover their operational and capital costs. This structure allowed cost reductions and improved the quality of service . Lower tariffs and better services induced a growth in consumption that opened up opportunities for economies of scale and scope. The significant gains in productivity created the perception that the ESI was inducing a strong economic progress and social welfare.

However, the oil crisis changed the global economic background. Companies' operating costs increased because of the dependence on that source and this led to an increase in tariffs. Raising tariffs promoted the conservation of energy, co-generation and a consequent decrease in demand. On the supply side, companies had already planned their expansion based on historical demand, but when the new plants started to operate, there was no consumption for their electricity. This idle capacity also put pressure on costs. This increase in costs was explained by the monopolistic situation of companies which set their prices based on the ROR. Regulation was causing economic inefficiency. Reforms which would introduce competition in the sector were necessary.

Structural and regulatory reform of the ESI around the world followed the basic model applied to networked industries such as those for telecommunications and natural gas. Potentially competitive segments (generation) were separated structurally or functionally

from the natural monopoly segments (transmission and distribution). Entry to and exit from the competitive segment were deregulated and consumers could choose among competing suppliers. Services provided by the natural monopoly segments were unbundled from the supply of competitive services, and non-discriminatory access to mandatory network facilities and the prices for using these facilities were determined by new regulatory mechanisms. The following sections discuss the reforms in the electricity sector that have been taking place in the United States, the UK and Brazil.

2.2 The American Experience

In USA, Electric Utility Companies were forming and expanding during the early 1900s, and by the 1920s they controlled much of the industry. Privately owned utilities were by then providing most of the total generation. These utilities formed a natural monopoly and operated in designated exclusive franchise areas which were usually municipalities. These companies were unregulated so they abused their power over their subsidiaries. Sometimes, the result was increased prices paid by consumers of electricity.

Since individual States could not regulate an interstate holding company, Roosevelt edited the Public Utility Holding Company Act (PUHCA) of 1935. Under the provisions of the Act, holding companies were to be regulated by the Securities and Exchange Commission and utilities involved in the interstate wholesale marketing or transmission of electric power became regulated by the Federal Power Commission, which, in 1977, became the Federal Energy Regulatory Commission (FERC).

The legislation ensured the concessionaires a monopoly of power. However, this was limited to their geographical area of expertise and regulatory mechanisms were established to prevent the use of monopoly power to the detriment of consumers. The regulatory agents began to monitor especially the expansion of supply and tariffs. The PUHCA thus allowed the operation of the ESI by private capital, although it was strongly regulated.

The model worked well until the 70s. However, due to the OPEC oil crisis, concessionaires' costs increased significantly. In 1978, concern over the supply of energy supply led the Carter government to introduce the Public Utility Regulatory Policies Act (PURPA) with the goal of creating favorable economic conditions for the spread of co-generation

and the use of alternative sources of oil. Therefore, so-called qualified facilities (QFs) were created which guaranteed the purchase of energy surpluses by concessionaires in their geographical area at a rate equivalent to that requested by the local concessionaire in their projects for expansion (the concessionaire's marginal costs in the long-term or cost avoidance).

However, this payment system was extremely profitable for qualified facilities because these generators were not subject to PUHCA legislation, and thus their costs were not controlled by regulators. On the other hand, their charges were set at the level of cost avoidance. In doing so, all cost reductions achieved by these generators increased their profitability and the consequent expansion of these companies. This was not the case for established companies because they were regulated by the rate-of-return (ROR). Therefore, any reductions in costs were passed on to consumers by way of lower charges. There was no incentive for such companies to lower their costs.

This situation increased criticism of this model. Concessionaires' costs increased with respect to qualified facilities. Therefore, the need arose to open the market to independent generators and promote competition by expanding capacity.

In 1992, President George Bush signed *the Energy Policy Act (EPACT)*. The Act substantially reformed PUHCA and made it even easier for non-utility generators to enter the wholesale market for electricity by exempting them from PUHCA constraints. The law created a new category of power producers, called Exempt Wholesale Generators (EWGs). This eliminated a major barrier for affiliated and non-affiliated power producers who wished make competitive bids for to building new power plants. EWGs differ from PURPA QFs in two ways. First, they are not required to meet PURPA's co-generation or limitations on renewable fuels. Second, utilities are not required to purchase power from EWGs. The marketing of EWG power has come to be facilitated by transmission provisions that gave FERC the authority to order utilities to provide access to their transmission systems (D. Liggett & S. Hankey, 2000). Thus, the concessionaires were forced to compete with independent generators over the supply of electricity.

The main consequence of EPACT was to increase the independent supply of electricity by companies whose costs were always lower than those of established concessionaires,

especially in states where energy charges were higher. Gradually, a deregulated energy market was created in which prices were governed by the independent generators.

As a result, verticalized concessionaires which for historical reasons (rate-of-return regulation) maintained high costs compared to those of independent generators began to face financial crisis since many of their investments were in sunk costs. The attempt to pass on these costs created a conflict of interest between the concessionaires and their customers. The former claimed that they had made legal investments, under the regulator, and losses were not acceptable. The major consumers did not want higher charges in order not to compromise their competitiveness and small consumers did not want to pay for the concessionaires' sunk costs. In fact, as (Joskow, 1997) argues, the primary stimulus for reform of the U.S electricity sector was the gap that existed in some parts of the United States between the implicit price of generation services embedded in regulation through bundled electricity prices, and the "unbundled" price of generation services that would be available on the wholesale market if consumers could buy electricity directly, and only pay the local utility for transmission and distribution costs.

In 1996, FERC issued a *Order 888*, which promoted wholesale competition through open-access, non-discriminatory transmission services by public utilities; and the recovery of stranded costs by public utilities and transmission utilities. The goal was to promote the flow of energy from low-cost to high-cost regions in order to reduce prices in the latter regions.

The reactions of more aggressive reform came from states with higher prices, such as California, for example. The reform process in California was different from other American states with respect to the structure of its wholesale electricity market. Because of this, California's reform will be described later.

In 1999, FERC issued the *Order 2000* which formatted the Regional Transmission Organizations (RTOs). The regulations require that each public utility that owns, operates, or controls facilities for the transmission of electric energy is free of any discriminatory practices in interstate commerce and meets certain conditions in order to form and participate in an RTO. The Commission also codifies the minimum characteristics and functions that a transmission entity must satisfy in order to be considered an RTO. The goal was to

promote efficiency in wholesale electricity markets and to ensure that electricity consumers pay the lowest price possible for reliable service.

Another highlights also in the process of reform was *The Energy Policy Act of 2005* (EPAct) that introduced significant new responsibilities for the FERC which include¹:

- overseeing the reliability of the nation's electricity transmission grid;
- implementing new tools, including the authority to exact penalties, in order to prevent market manipulation;
- providing rate incentives to promote investment in the transmission of electricity;
- supplementing state efforts to transmit electricity, when in the national interest, by creating electricity transmission corridors;
- reviewing certain holding company mergers and acquisitions involving electric utility facilities, as well as certain public utility acquisitions of generating facilities.

Therefore the American Electric Industry was characterized by a large number of economic entities of diverse types active in the sector and a large number and diversity of regulations and regulators.

2.2.1 The California Crisis

According to (Rezende & Bruginski de Paula, 1997), prior to the reform, most generation capacity in California was owned or controlled by three integrated investor-owned utilities (IOUs): Pacific Gas and Electric (PGE), Southern California Edison (SCE) and San Diego Gas and Electric (SDGE), which delivered power via their transmission and distribution (TD) networks. The IOUs' prices, costs and services were regulated by the California Public Utility Commission (CPUC). The IOUs imported energy from Mexico and Canada and exported to the northeast and southwest of the country.

This Electricity service was reliable and of good quality but had the highest retail electric rates in the United States. In 1993, the price was 9.7 cents per kilowatt-hour while

¹<available in www.ferc.gov>

the national average was of 6.9 cents. As (Borenstein, 2002) comments, this difference was, for the most part, due to stranded costs that were generated by IOUs regulated by rate-of-return and vertically integrated. Because of this, many consumers, above all major industries, started to demand reforms in the electricity system hitherto in force. In the same year, CPUC started a review of the structure and performance of this market.

At the beginning of the 1996, the Assembly Bill 1890, legislation which regulated the power sector so far, was revised. It was the intent of the Legislature to ensure that California's transition to a more competitive electricity market structure allows its citizens and businesses to achieve the economic benefits of restructuring the industry at the earliest possible date; creates a new market structure that provides a competitive, low cost and reliable electric service; provides assurances that electricity customers in the new market will have sufficient information and protection; and preserves California's commitment to developing diverse, environmentally sensitive electricity resources. In accordance with the 1996 revision of the the A.B. of 1890: ²

- Retail consumers could choose an Electricity Service Provider (ESP) to provide them and the IOUs became Utility Distribution Companies (UDCs);
- The IOUs were de-verticalized. They had to assure free access of their transmission lines and distribution to supply the competitive market with prices determined by FERC and CPUC;
- Incentives were created for the IOUs to divest themselves of their fossil-fuel generation units;
- The Competition Transition Charge (CTC) was created. This fixed the retail price for electricity at the 1996 level so that the difference between the average embedded cost for generation and the wholesale market price, which was expected to be lower than the fixed prices, would help pay for the IOUs stranded generation costs;
- Residential and small commercial ratepayers had a rate reduction of 10 % that was financed through the issue of rate reduction bonds to be repaid by a charge on retail

²available at<http://www.leginfo.ca.gov/pub/95-96/bill/asm/ab_1851-1900/ab_1890_bill_960924_chaptered.pdf>

consumption;

- The California Power Exchange (CALPX) was created to operate the wholesale electricity. It was in charge of the electricity market of the day-ahead and hour ahead;
- The California Independent System Operator (CAISO) was created to manage and operate the California grid. It was regulated by FERC.

As a result of Assembly Bill 1890 (AB 1890) and California Public Utilities Commission (CPUC) decisions, California's investor owned-utilities (IOUs) and their customers were to undergo a multi-year transition to a more competitive market for electricity (Arikawa, 1998).

In fact, the CTC helped the SDE to end its stranded costs, at least until the summer of 2000 when the wholesale prices spiked, while the other two utilities were still under the CTC. This was a result of the volatility of prices in the market. As (Borenstein, 2002) argues, for the first two years, prices fluctuated substantially within a month and even within a day. On a few days, the market registered severe shortages, and the independent system operator's real-time market price shot up to its price cap, which was 250/*MWh*, until October 1, 1999, when it was raised to 750/*MWh*. Still, the average wholesale price was never greater than 50/*MWh* in any month. Then, in June 2000, the precarious balance that the market had maintained fell apart. Wholesale prices increased dramatically, the independent system operator found itself unable to purchase as much power as it needed through its real-time market, and the utilities were paying wholesale prices that vastly exceeded the retail prices they were allowed to charge. Several of the state's electric utilities declared bankruptcy. It was the start of the crisis.

In May 2001, there was a black-out for two days due to the incapacity of the UDCs to provide electricity. The main factors responsible for the crisis were: poor market design, market power, sustained growth in demand and the Permits to emit nitrogen oxide (NOx).

The Californian wholesale electricity structure was very controversial. The CAISO was set up to operate under the bilateral model, but the CALPX was created to run a day-ahead market as a pool. For the first few years, all three utilities, which had the most

consumers and a large share of the production, were required to transact all their business in the CALPX. There were no long-term contracts. Therefore, all trading was centralized in the CALPX and this market did not achieve sufficient volume to be considered reliable.

With regard to Market Power, FERC argued that a firm with a market share below 20% could not exercise significant power but this analysis ignored the fact that a firm with a small market share could exercise market power on a day when the demand was high, if the product had an inelastic demand.

The Permits to emit NO_x were credits purchased by generators to cover their emissions. These permits could be negotiated between the firms. This increased their costs (direct variable cost and opportunity cost) considerably.

Although the California experience is unique to the United States, the experience highlighted two issues. Retail tariffs must recover all costs of supply and should be flexible enough to allow consumers to respond to changes in prices at the wholesale level. Market power needs to be monitored closely and mechanisms established to minimize the potential for any electricity supplier to exercise a monopoly (Choynowski, 2004).

2.3 The Experience of the United Kingdom

Prior to reform, the British Electricity Supply Industry (ESI), consisted of the Central Electricity Generating Board (CEGB), which held a monopoly of electricity generation and transmission and supplied more than 95% of total electricity, and 12 independent Area Electricity Boards, each in charge for operating distribution networks in their territories. The CEGB supplied energy to the Area Boards in the form of a single supply tariff. These areas summed their costs and formulated proposals for tariffs to final consumers. These proposals were to be assessed by the Department of Energy in the Ministry of Trade and Industry. There were also a few large consumers who bought energy directly from CEGB who paid a supply tariff.

The CEGB and Area Boards were under the control of the government through the Electricity Council, which was responsible for all general policy decisions.

However, this model for operating the electricity sector was much criticized since: (Gorini de Oliveira & Tiomno Tolmasquim, 2004)

- The structure was rigid, bureaucratic, inefficient and hard to reorganize due to its political clout;
- There were few and inadequate tools that were able to avoid requests for new funding and tariff increases;
- Supplies were at risk because of strike threats and fuel crises.

Due to this, Thatcher, by the Electricity Act of 1983, opened up the transmission system so that private producers, independent of CEGB, could send energy into the system. This was designed to encourage the growth of independent power producers and meant to remove barriers to entry by non-utility generators and to provide independent producers of electricity open access to the national grid. Later, in the political campaign for her re-election, she proposed the privatization of ESI as part of her program to reduce the role of the state in the economy, to introduce a business mentality into services and to democratize the capital market.

After her victory, Thatcher introduced The Electricity Act of 1989 that re-structured the electricity industry. This re-structuring included de-verticalization, privatization and the introduction of competition in the electricity sector. Privatization began with the distributors in 1990 and in 1991 the central generator was privatized.

The 12 regional area boards were privatized and renamed Regional Electricity Companies (RECs). The CEGB was separated into three generation utilities (National Power, PowerGen and Nuclear Electric), which were constrained to compete among themselves, and one Transmission utility, the National Grid Company (NGC), in charge of the high-tension distribution network to each one of the 12 Regional Electricity Companies (RECs).

National Power was assigned 46% of all generation capacity in England and Wales, while PowerGen received around 28%. Both used coal to generate energy. Almost 17% consisted of nuclear power (transferred to Nuclear Electric, which remained a public company until 1996), while just 1% was generated by independent producers (IP), and the remainder consisted of other sources, including imports from France and Scotland (Gorini de Oliveira & Tiomno Tolmasquim, 2004).

The supply segment was gradually deregulated. From 1998 onwards, all consumers

have been able to choose their supplies to buy energy and the RECs can purchase electricity from any plants, including imports from France and Scotland. Transmission remained a natural monopoly, although privatized.

To make all this restructuring possible, it was necessary to have a regulatory scheme requiring the participants to respect the operational rules of the electricity market to ensure the stability of the system. Therefore, the Electricity Act 1989 created the post of Director General of Electricity Supply, the DGES, to regulate the natural monopoly of the transmission line business of the National Grid Company (NGC) and the Regional Electricity Companies, and to set price caps, which would be reset at periodic reviews, i.e. every 4-5 years. He had a duty to ensure that reasonable demands for electricity were met, that licence holders were able to finance their activities, to promote competition in generation and supply, to protect customer interests, and to promote efficiency. The Office of Electricity Regulation, OFFER, was set up by the Government as an independent body under the Electricity Act, headed by the DGES. The DGES has extensive powers to ensure compliance with the rules, and the other participants may appeal against his decisions in the courts or to the Competition Commission.

Commercial relations among the power generations utilities, NGC and the RECs were enforced by bilateral contracts, which aimed to reduce risks and facilitate investment in the long term, under the wholesale market called Pool. The Pool has been operated by the NGC Settlements Ltd which was a subsidiary of the transmission company, by means of a computer scheduling programme (GOAL) as follows, according (Wolfram, 1999): every day is divided into 48 half-hour periods and a single price covers all purchases and sales in each half hour. Pool prices are based on bid schedules submitted daily by generators, in which they specify the prices at which they would be willing to supply power from each of the plants they own. The bids are ranked from lowest to highest and are used, together with the capacity offered by each plant, to construct a supply curve that indicates the least expensive way to meet a given level of demand. Using demand forecasts for the following day, the administrator determines a “system marginal price” for each half-hour period based on the highest bid that must be accepted to meet forecast demand.

All the utilities placed under these constraints were paid based on the marginal price

of the system. The price paid to generators (Pool Purchase Price-PPP) was given by, according to (Rezende & Bruginski de Paula, 1997):

$$PPP = SMP + VOLL * LOLP$$

Where:

PPP = Pool Purchase Price;

SMP = System Marginal Price;

$LOLP$ = Loss of Load Probability;

$VOLL$ = Value of Lost Load.

Thus, the PPP included a financial incentive for maintaining some additional (peak load) generation capacity in the event that demand exceeds consumption forecasts. This capacity payment equals the value of lost load (VOLL) times the loss of load probability (LOLP). The VOLL attempted to measure the system cost of not producing enough electricity to meet peak load. Another way of looking at VOLL is that it attempted to measure the extent to which generators are prepared to invest in additional capacity in excess of the actual maximum on the system. The LOLP simply measures the probability that supply will be insufficient to meet demand at a particular point in time.

The LOLP changed over the course of the year and the course of the day. The closer demand is to scheduled supply, the higher the LOLP and therefore the higher the capacity payment.

The distributors and consumers bought energy directly from Pool and paid the Pool Selling Price (PSP) which was given by:

$$PSP = PPP + CA$$

where:

PSP = Price of Pool for distributors and Major Consumers;

PPP = Price of Pool for the generators;

CA = cost of maintaining the stability of the system (the part that the company pays for the services of transmission of maintaining the stability of the network).

Consumers and distributors should also have to pay toll for the displacement of energy through the transport network to the final consumer. That cost was set by the regulatory body, by encouraging a price cap scheme, in which the readjustment of the transport

charge was made through an index of X below the inflation rate which reflected gains in productivity by the carrier company. Thus, it reduced costs in the period when tariffs were revised to increase its profitability, a fact that benefited consumers by a subsequent decline in real rates.

However, during 1993 and early 1994, price peaks, attributed to a failure of the Pool pricing software, occurred. Since then, several criticisms have been made of the pool and the operation of the English electricity market as a whole. The most serious criticism of the performance of the electricity market was that the 1990 restructuring created an effective duopoly, in which National Power and PowerGen set the price more than 90% of the time (M. David, 1998). In other words, these companies had exercised market power, which continued even after several interventions by the regulator in order to increase the number of generators. In September 1999, electricity pool prices spiked again, increasing to as much as 80% above those of the previous September (Hadjilambrinos, 2005).

As a result of the continuing problems, the new Director General and the Office of Gas and Electricity Markets (OFGEM), which replaced OFFER, undertook a review of trading rules which led to a proposal for the abolition of the Pool and its replacement by new market mechanisms. The Utilities Bill (2000) constituted the New Electricity Trade Agreement (NETA) and some new regulations which increased the power of the state to intervene in the market (through the Secretary of State) and in the licenses of the utilities.

NETA replaced the PSA (Pooling and Settlement Agreement) with a Balancing and Settlement Code which had a well-defined method of making modifications, and this gave OFGEM more influence in the process. The Pool ceased to exist. Electricity was now to be traded in four voluntary, overlapping and interdependent markets operating over different time scales. Bilateral contract markets cover the medium and long term, while forward markets offer standard contracts (base-load, peak hours) for periods of up to several years ahead. A short-term “prompt bilateral market” operating from at least 24 hours to Gate Closure, allowed parties to adjust their portfolio of contracts to match their predicted physical positions. This short-term market would yield information to construct a spot price for each half-hour (Newbery, 2004).

2.4 The Brazilian Experience

2.4.1 A brief history of the Brazilian Electric Power Sector

The Brazilian Electric Power sector has been modified over the years. The first ventures began towards the end of the 19th century when D. Pedro II allowed Thomas Edison to introduce the equipment and processes of his invention for using electricity in public lighting, into Brazil. The first permanent electric lighting facility was inaugurated in the Central Station of the D. Pedro II Railroad, the current Central do Brasil. In 1883, the first hydroelectric power plant in the country, located in the city of Diamantina entered operation, and D. Pedro II inaugurated the first municipal public service of electric lighting of Brazil and South America, in the city of Campos. In 1889, in the city of Marmelos, Minas Gerais, the first medium-sized hydroelectric power plant (375 KW) was inaugurated.

Some international companies got concession contracts to exploit electricity services in the cities. Such contracts had the “golden clauses” which allowed foreign companies to revise their tariffs in accordance with the exchange variation. In 1899, *The Brazilian Traction Light and Power* (Light) company obtained a concession to supply energy to São Paulo and Rio de Janeiro and in 1927, the *American Foreign and Power Co. (Amforp)* entered the sector in order to supply energy to the interior of the state of São Paulo and to the capitals of Rio Grande do Sul, Bahia, Rio Grande do Norte, Pernambuco, Espírito Santo and the interior of the state of Rio de Janeiro. Later, this was extended to the states of Paraná and Minas Gerais. Thus there were, for practical purposes, two private companies in the sector (Lessa, 2001a).

During the 1930s the Great Depression changed the world economic scene. The shock to the structures of liberal economics, a consequence of the crisis for capitalism, led to the regulation of the US and European economies. The *New Deal*, which was adopted by Roosevelt, proposed intervention by the state in all sectors of the economy. In the US electric power sector, several state-owned hydroelectric power stations were built.

In the Brazil, President Getúlio Vargas followed the trend and annulled the “golden clause” in 1933 and, in 1934, created o Código de Águas - literally, the Water Code - that

regulated the ownership of water and its use, especially in the electricity sector. The rates were based on the historical cost of production, service at cost and the limited profits of companies.

Due to the growth of industrialization in Brazil after the Second World War, the demand for electricity increased considerably. However, foreign companies have decreased their investments because it was no longer advantageous for them to invest in the sector, due to the annulment of the “golden clause”. This situation resulted in the first energy crisis with rationing in several parts of the country which reinforced the need for increasing the role of the state in the electric energy sector. Some of the larger state-owned companies of the Brazilian electricity sector were created in this period, such as: CHESF (1945), to supply the northeast region; CEMIG (1952), to supply Minas Gerais and Furnas (1957) to supply the Southeast.

In 1960, some Public Administration reforms that aimed at enlarging the action of the state system were implemented. As a result of this, the number of companies in the production sector grew. In the Electric Power Sector, the Juscelino government created the Ministry of Mines and Energy (MME), which was responsible for policy and regulatory actions for the energy sector and, in 1965, it created the National Energy and Water Department (NEWD), a normative body that monitored the sector.

On June 11, 1962, during the João Goulart government, Eletrobras (Brazilian Central Electric, A.S) was created. It was a holding company for the state companies (Furnas and Chesf) and its mission was to coordinate bodies and to finance ventures connected to the electricity sector, instead of the development bank BNDES, and it fell to it to administer not only plans for expansion but also non-tariff resources. It expanded further after the construction, in 1968, of ELETROSUL, its subsidiary serving the south of Brazil, and of ELETRONORTE, another subsidiary for the North.

The centralized model of the Brazilian electric sector worked well until the 70s. In 1973, the world economy went into recession which caused changes in financial and trade transactions as a result of the crisis in oil supply. Brazil suffered a strong negative impact on its external trading accounts, especially due to its imports of oil. President Geisel, in the II National Economic Developing Plan, to solve this problem, decided to maintain fast

economic development with an emphasis on substituting imports. He thought that the strong negative balance in overseas trade would be offset by overseas credit operations, with flexible interest rates, using the private banking system.

In the same year, Brazil and Paraguay signed a deal to build the Itaipu plant on the border between the two countries. Meanwhile the Group Coordinator of Interconnected Operations (GCOI in Portuguese) was created. This group aimed at the rational use of the premises of generation and transmission systems which could interconnect the South and Southeast regions.

Law decree no. 1.383/74. adopted an equalization tariff in order to sustain a policy of equal treatment for all consumers in the national territory. According to this policy, the remuneration of the companies was adjusted through subsidy transfers from companies that were profitable because they had access to hydro-electric energy to companies in deficit because they had to use electricity generated by fossil fuels. Because of the creation of the Global Guarantee Reserve (GGR), in 1975, all concessionaires began to contribute 2% on equity detained. Thus, at the expense of profitable companies, a minimum return of 10% and maximum of 12% on the assets of all enterprises was ensured, whatever the result of individual income less expenditure. The consequence of this policy was administrative inefficiency and companies being negligent with regard to their costs.

In addition to this, there was a drastic reduction in charges for energy in the post-oil crisis. According to (Dias Leite, 1997), between the maximum in 1972 and minimum in 1986, the reduction in charges was 44%. At the same time, major construction works were launched to increase installed capacity, including a nuclear power station.

The second oil shock, which occurred in 1979, worsened the crisis in foreign markets. Because of the rise in interest rates on the international market, the decrease in the grace period and an increase in demand for the release of bank loans, the sector experienced heavy restrictions on its main sources of financing. This situation increased dependence on resources and a shrinking of treasury profits generated internally. Thus began the process of the sector's increasing de-capitalization due to a combination of flattening prices, high debt, difficulties in attracting new loans and constraints in obtaining contributions from the government (Rezende & Bruginski de Paula, 1997).

This situation also generated an institutional crisis since the state's concessionaire enterprises and the Eletrobras enterprises started to quarrel over resources for the expansion of their systems. Thus the Brazilian electric sector entered the 90s in disarray in terms of its organizational and financial structure. A reform was necessary to ensure the expansion of this sector which is essential to any economy.

2.4.2 First phase of the process of reform and restructuring

Following the trend of the privatization of enterprises that occurred in the world due to the inefficiencies generated by state control, Brazil began to make reforms in the electricity sector. According to (Lessa, 2001b), the basic principles of these reforms were in:

- Removing the verticalization structures of companies which operated under the monopoly and were remunerated based on the costs of the service;
- Splitting the large companies in the sector into smaller companies so that competing units could be created, and when necessary, separating the activities of generation, transmission and distribution;
- Maintaining the transmission systems of energy (transmission and distribution) operated under a monopoly with another kind of remuneration;
- Ensuring the generators and consumers free access to such networks, through a transportation charge fixed by the government;
- introducing competition among generators;
- privatizing state assets.

In 1993, Law 8.631/93 was promulgated. It eliminated the system of equalization rates and guaranteed remuneration; created the obligation for supply contracts to be signed between energy generating and distributing companies; and promoted a settlement of accounts between debtors and creditors. The way to set charges was based on the cost structure of the enterprises.

The formal initiation of the restructuring process was set out in the General Law of Concessions (Law 8.987/95) that provided rules for bids, established the rights and obligations of the concessionaires and acknowledged the creation of a regulatory and charge system that would regulate and guarantee the economic and financial equilibrium of the concession.

Law 9.074/95, which regulated the previous one, created the legal figure of the independent producer of electric power and established the possibility that consumers were free to contract, initially, independent energy producers and, after five years, any concessionaire or producer of energy.

In 1996 a British consulting firm, Coopers and Lybrand, was contracted by Eletrobras to assist in drawing up a new model for regulating the energy sector. As (Lessa, 2001c) says, its report is based on the following assumptions:

- Transformation of the cooperative model, which is the conceptual basis of the Brazilian hydropower system, into a competitive model, in which there may be alternatives offered by the market for the supply of electricity to consumers;
- Division of the large companies in the sector into smaller companies so that competing units might be created and thereby separating the functions of generation, transmission and distribution;
- Creation of a wholesale energy market where companies can buy and sell energy. This would be a kind of stock exchange where the prices would not be regulated by the government. It was expected that the wholesale energy market would act as a regulator, in lieu of the government;
- Expansion of supply guaranteed by the operation of the market described above.

These premises were the basis for Law 9.648/98, which defined, among other things, the rules of entry, charges and market structure.

According to this new model, the government would act as an advisor and supervisory agent of the Department of Energy provided to the population, so it should be responsible for formulating energy policies and regulating the economic sector while the private sector operated the energy systems and made investments.

De-verticalization divided the energy system into four independent and autonomous activities: generation, distribution, transmission and marketing. The goal was to privatize the areas of generation and marketing in order to introduce the competitive model of free competition while the sectors of distribution and transmission continued as natural monopolies subjected to regulated charges.

To ensure the functioning of the new structure, the Wholesale Market of Electricity (WME), the National System Operator (NSO) and the National Electric Energy Agency (NEEA) were created to replace the National Department of Water and Electricity (NDWE).

The proposal for the operation of model was a bilateral long-term contract system and a Pool. Most of the electricity supply was to be negotiated between the agents by bilateral contracts traded freely. These contracts were financial instruments which specify a contract price for a fixed volume of energy. The difference between contracted and effectively consumed energy was to be settled by the Wholesale Market of Electricity (WME) which was responsible for the generation of the energy spot price. According to (Maceira & Melo, 2002), the process was as follows: if the generator produced more than the contract it would be a creditor, otherwise it would be a debtor of WME. On the other hand, if a distributor consumed more than the contract, it would be a debtor or a creditor, if it consumed less.³The main aims of the WME were: to set a price which reflects, in each period, the marginal cost of energy to the system and this price should support long term contracts and create a multilateral environment to support the competition under which a retailer may buy from any generator and a generator may sell to any retailer.

The operation of WME was to be the responsibility of the National System Operator (NSO) which, based on the technical data received (water levels in the reservoirs, rate of inflow, etc), established a generation schedule which describes what generation plants should be dispatched and the associated generation target in order to achieve least cost from the whole system. This schedule was obtained through a chain of optimization models that also calculated the value of water. This formed the basis for determining the

³The WME was created by a multilateral agreement which is compulsory for all generators with installed capacity greater than 50MW and all distribution and retail companies with consumption greater than 100GWh per annum. Consumers with demand above 10MW can choose to become WME members (Maceira & Melo, 2002).

MAE price in each period. Who decides on the use of a transmission system and ancillary services would be calculated ex-post and assigned to agents as per their participation.

The NEEA was created by Law No. 9,427 in 1996 and has the attribute of regulating and monitoring the generation, transmission, distribution and marketing of electricity; mediating the conflicts of interests among energy sector agents and between them and consumers; granting, permitting and authorizing energy facilities and services; ensuring fair rates; ensuring the quality of service, requiring investments, stimulating competition between operators and ensuring services are universally available.

However, the investments in the private sector were not the ones expected. The prospectuses for buying plants through privatizing generation and the uncertainties of the regulatory framework discouraged private enterprise (Tiomno Tolmasquim, 2000). Moreover, there was strong political pressure for generation to continue under government control.

On the other hand, the investments made by state enterprises were blocked since under the second agreement with the International Monetary Fund (IMF), the country was obliged to maintain a positive balance on the current account and such investments, though profitable, were counted as deficits.

In the absence of alternatives and to avoid a collapse, the government tried to attract private investment and on February 24, 2000 launched the Priority Program for thermoelectricity, based on building thermoelectric plants using gas from Bolivia as fuel. This program was a failure because of the environmental requirements and the saturation of the market by turbines discouraged investors.

The situation was not worse because Petrobras had invested in more thermoelectric plants. This complete lack of investments, especially between 1996 and 2000, led the country in 2001 to the crisis which became known as the “blackout and” thus there was rationing. It should be emphasized that, as (Tiomno Tolmasquim, 2000) comments, Brazil had time to invest but did not. Furnas, for example, was extremely lucrative on account of its low costs and its plants had already discharged. The explanation from the government for the crisis was the lack of rain. There was a need to change the strategy.

2.4.3 The second process of restructuring

The crisis which the sector underwent, combined with WME's lack of credibility with respect to the liquidation results which in the period showed a company as a creditor and debtor during the same period led to the need to change the model as it had been hitherto. In 2003, the Lula government drew up new proposals to ensure private investment in the sector and economic growth. The main objectives of these proposals were:

- A new mode for setting tariffs (lower tariffs and efficient recruitment);
- Security in the supply of energy;
- stability of the regulatory framework(to attract private capital).

Marketing energy under the new model was made in two different environments: the Procurement Regulated Environment (PRE) and the Free Hiring Environment (FHE). The PRE is a Pool where the buyers are the distributors, while, in the FHE, buyers are free consumers and commercial agents. For both environments, sellers are generating, independent producers and marketing companies. The rules for accounting and settlement remained the same. The order continues with the ONS.

The PRE is organized by the Board of Trade of Electricity (BTE). The BTE was created by Law 10.848 on March 15, 2004 and regulated by Decree No 5177 of August 12, 2004. As a non-profit making private business and under the regulation and supervision of the National Electric Energy Agency (NEEA), the BTE aims to facilitate the marketing of electric energy in the National Interconnected System (NIS). The BTE replaced the Wholesale Market of Electricity - (WME).

To ensure supply, the Energy Research Company (CER) (Law 10.847/2004) was created to promote studies that help the planning of the energy sector in the long term and the Monitoring Commission of the Electrical Industry (MCEI) was also created to continually assess the supply of electric power.

2.5 Conclusions

Reforms in the electricity market must take into account particular characteristics inherent in the system. For instance, the storage of electricity is costly and there is a capacity restriction on generating electricity. Thus demand and supply must always be in balance at any point within a transmission system. In addition, prices are very volatile as a result of inelastic demand and inelastic supply (when output nears capacity). Therefore, even small changes will lead to a price boom or bust because there is no elasticity on the supply or demand side that allows the market to adjust to such a mismatch. Therefore, an effective System Operator is needed who can deal with such uncertainties.

Another point is the operation of the transmission system. Unlike other transport systems, which can pinpoint the journey made from the seller to the buyer, energy flows in freely alternating chains throughout the entire transmission system according to specific laws of physics. Because of this, it is impossible to assign output from a specific plant to a particular customer and an imbalance at any one location on an electricity grid can threaten the stability of the entire grid. Investments should be encouraged to reduce congestion on the network and forecast a reliable system.

The reforms should also stimulate effective competition. (Borenstein, 2002) comments that markets are not fully competitive and a tight supply puts even a fairly small seller in a strong position to exercise market power unilaterally, because there is very little elasticity of demand and other suppliers are unable to increase their output. The crisis in California demonstrated this. The power market, left to itself, can reduce operational efficiency, limit customer choice and discourage new entrants into the market. Introducing competition can decrease prices and attract the private investments needed to expand the sector.

Finally, the use of appropriate regulatory instruments is required such as: controlling entry to and exit from the market, where appropriate; regulating the conduct of companies, the target being to curb anti-competitive practices and the abuse of economic power; promoting competition, whenever appropriate; applying criteria to determine tariffs in order to encourage production efficiency and to allow the appropriation, by consumers, of some of the productivity gains, and to supervise that the concessionaire fully meets the

terms of the contract.

The effectiveness of these instruments depends mainly on the independence of sectoral agencies. The agency should be independent, both in relation to the government as well as to the other players in the sector, and the regulation should be de-coupled from short-term economic policies and be based on criteria of economic rationality and efficiency.

The objective of reforms in the electricity sector was the creation of a competitive market for electricity. Due to this, one can think that energy is no longer a public service and has become a commodity. Indeed, this is generated on account of the shortage of the main natural resources used in the world for generating electricity. This will be explained more completely in subsequent chapters.

3 *Energy and the Economy*

“...The most fundamental definition of money is that it is a mechanism to allow the exchange and allocation of different forms of energy. The economy is energy.”

Michael Lardelli

3.1 Introduction

Energy is an essential input for economic growth. Amongst other uses, it is used to operate machinery and equipment; for lighting; to fuel the transport of goods and people; and to meet domestic needs in people’s homes. The growth potential of a country is linked to its supply of energy sources.

Energy sources are the states in which energy is found in nature and can be classified in many ways. A frequently used classification is to distinguish between primary energy sources, which are those in their crude form (oil, natural gas, coal); secondary sources, which will be stored or distributed (gasoline or electricity, for example); and final use, which is primary energy converted into the secondary energy available to consumers, after having discounted losses in storage and distribution.

Therefore the energy system is divided into three stages: production and conversion of primary energy sources into secondary ones; storage and distribution; and final consumption.

Sources also can be renewable (naturally replenished) or non-renewable (natural resource that cannot be produced, re-grown, regenerated, or reused on a scale which can sustain its consumption rate). However, as argue (Jannuzzi Martino & Swisher N. P., 1997), this classification is controversial because, in principle, any source can be considered inexhaustible. Even so, according to these authors, sources are considered renewable if their use by humanity does not cause a significant variation in their potential and if it is relatively certain that they can be replaced in the short run. Similarly, sources are non-renewable if it would take many centuries or millennia to replace them under many individual conditions (oil, for example), or if to replace them artificially is totally unviable

because this would involve processes the expenditure on which would be greater than or equal to the value of the amount of energy obtained.

One can distinguish between primary energy sources that are commodities and those that are not commodities, i.e. those goods which may or may not add economic value over time and are highly tradable on the international market, so the following sources are considered commodities:

- Fossil Fuels (Oil, Natural Gas, Coal);
- Biomass(Sugar Cane, Barons (water hyacinth), Firewood);
- Radioactive Materials (mainly Uranium);
- Water with a head;
- Naturally heated water;
- Hydrogen.

The main sources that are not considered commodities:

- Solar Energy;
- Wind (Eolic);
- Ocean Energy (Wave energy, Tidal power);
- Geothermal Energy.

Moreover, energy resources may be restricted or not restricted and have a conventional use or unconventional use. The latter classification relates to technology. Conventional resources are those for which the technology is already widely available. Unconventional resources are those for which there is a proven technology is proven but for economic reasons or by social custom have not gained wide acceptance.

Making distinctions between the resources will be determined by their technological aspects, their limitations and their economic value as follows:

- Restricted Conventional Energy Resources (Fossil Fuels and Uranium);

- Restricted Conventional Renewable Energy Resources (hydroelectric energy and Biomass);
- Unlimited Non Conventional Energy Resources (Solar Energy, Wind, Geothermal Energy and Ocean Energy).

In the first and second, the resources are commodities; in the third, they are not. The sections that follow will examine the energy supply in Brazil and the world and the factors which interfere in the energy consumption of any economy.

3.2 The Energy Supply in the World

Oil is the most used raw material in the world. It accounts for 35% of the world energy supply matrix. Coal (25.3%) ranks second followed by natural gas (20.7%)(MME, 2008).

With regard to generating electricity, according to the International Energy Agency (IEA,2007), coal is the most used resource, accounting for 41% of total energy production. Approximately 78% of electricity in China is powered the thermal coal. Another country heavily dependent on coal is the United States.

In fact, according to the Energy Information Administration (EIA) ¹the three primary energy sources for generating electric power in the United States are coal (48,5%), natural gas (21,6%), and nuclear energy (19,4%). These three sources consistently provided between 84.6% and 89.5% of total net generation during the period 1997 through 2007. Petroleum's relative share of total net generation was unchanged in 2007 from 2006 at 1.6%. Conventional hydroelectric power continues to decline as a share of total net generation. In 2007, conventional hydroelectric generating capacity accounted for 6.0% of total net generation, as compared to 10.2% in 1997. Renewable energy sources, excluding conventional hydroelectric generation, contributed 2.5% of total net electricity generation in 2007. This marks the fourth consecutive year in which renewables'share of total net generation has increased.

¹Electric Power Industry 2007: Year in Review available at <www.eia.doe.gov/cneaf/electricity/epa/epasumhtml>

3.3 Brazilian Energy Matrix

The Ministry of Mines and Energy (MME) promotes various studies and analyses that underpin the formulation of energy policies and guides planning the sectors of the economy. One of the most important research centers is the National Energy Balance (NEB), which documents data on consumption, production and marketing of various energy resources in Brazil annually. The data in this section are from NEB 2008 for which the base year is 2007.

The domestic energy supply (DES)² reached 238.3 million toe (tons of oil equivalent), which was 5.5% higher than the demand in 2006 and equivalent to about 2% of world energy. The final energy consumption (FEC) reached 215.6 million toe in 2007, a total that was 6.2% higher than in 2006, and which also exceeding DES growth in 2007.

The growth in total energy demand grew almost at the same rate as economic growth in 2007, which was 5.4%. The structure of the DES by energy source (the Brazilian Energy Matrix-BEM) is shown in Table 3.1.

Table 3.1: Domestic Energy Supply.

SPECIALIZATION	MILL TOE		07/06%	STRUCTURE %	
	2006	2007		2006	2007
NON-RENEWABLE	124.464	129.102	3.7	55.0	54.1
PETROLEUM AND DERIVATIVES	85.545	89.239	4.3	37.8	37.4
NATURAL GAS	21.716	22.199	2.2	9.6	9.3
MINERAL COAL AND DERIVATIVES	13.576	14.356	6.1	6.0	6.0
URANIUM(U308) AND DERIVATIVES	3.667	3.309	-9.8	1.6	1.4
RENEWABLE	101.88	109.656	7.6	45	45.9
HYDRAULIC AND ELECTRICITY	33.537	35.505	5.9	14.8	14.9
FIREWOOD AND CHARCOAL	28.589	28.628	0.1	12.6	12
SUGARCANE DERIVATIVES	32.999	37.847	14.7	14.6	15.9
OTHER RENEWABLES	6.754	7.676	13.7	3	3.2
TOTAL	226.344	238.758	5.5	100.00	100

Source: BEN

One notes that 109.656 million toe (or 45.9%) corresponds to the renewable energy domestic supply. This ratio is one of the largest in the world and contrasts with the global average of 12%.

For the first time the participation of "hydraulic and electricity" energy was supplanted by "derived from sugar" in the BEM. The derivatives from sugar cane, which represent

²In NEB, as in energy balances from other countries, the domestic energy supply-DES, also known as total energy demand, is the sum of final energy consumption, loss of distribution and storage and losses in the process of transformation.

15.7% of BEM and 34.3% from renewable sources, supplant, respectively, 14.9% and 32.5% of hydraulic and electricity. Biomass consumption (8.4%) was also higher than that of Hydroelectricity (5.7%) due, mainly, to the use of mulch for heating in the sugar and alcohol industry.

However, the production of energy from non-renewable resources is still greater in the DES (54.2%), with the emphasis on oil and its derivatives (37.4%). In recent years, the constant increases in the production of this resource and natural gas in Brazil are allowing successive reductions in depending on energy sources from overseas: 12.9% in 2004, 10.2% in 2005 and 8.3% in 2006. In 2007, this figure was 9.5% due to the increase in imports of coal.

In fact, in Brazil and abroad, ever since the oil crisis in the 70s, a concern has been to reduce dependence on fossil fuel due to its scarcity and the contribution to global warming from CO₂ emissions. Investment in non-conventional energy has grown considerably in Brazil with regard to energy generation as a whole, but is still not expressive in the Electric Power Matrix, as will be shown in the sections that follow.

3.4 Brazilian Electricity Supply Matrix

In 2007, the supply of electricity in Brazil supply grew 5.0%, less than the economy, and reached 483.4 TWh. The increase in the use of biomass (21.0%) was higher than that of hydraulic means (7.2%) and of oil (7.8%). However, hydraulic power remains supreme in the matrix with 85.4%, including imports. Next come biomass (3.7%) and gas (3.2%). Wind energy ³ had a sharp increase of 236 GWh in 2006 to 559 GWh in 2007. According to Table 3.2:

Brazilian Electricity Supply Matrix makes the country's dependence on its hydraulic facilities clear. The preference for Hydroelectricity is explained by natural conditions (the abundance of water in rivers, lakes and basins) and the operational cost of the plants that is negligible compared to producing thermoelectricity from fossil fuels.

Power plants cannot be built simply anywhere and with the power that one wants. The geographical provision is fundamental to decisions on "where and how much". In

³Biomass included 559GWh of Wind Energy em 2007

Table 3.2: Brazilian Electricity Supply Matrix.

Especialização	GWH		07/06%	ESTRUTURA %	
	2006	2007		2006	2007
HYDRO	348.805	374.015	7,2	75,7	77,4
NUCLEAR	13.754	12.350	-10,2	3	2,6
NATURAL GAS	18.258	15.497	-15,1	4	3,2
MINERAL COAL	7.222	6.792	-6	1,6	1,4
PETROLEUM AND DERIVATIVES	12.374	13.333	7,8	2,7	2,8
BIOMASS	14.959	18.104	21	3,2	3,7
INDUSTRIAL GAS	3.964	4.492	13,3	0,9	0,9
IMPORT	41.164	38.832	-5,7	8,9	8
TOTAL	460.000	483.415	5	100	100

Source: BEN

addition, as argues (Shrivastava, 2006), hydroelectric generation is not the cleanest of all energy sources. Its reservoirs have serious environmental impacts, such as:

1. Greenhouse gas emissions;
2. Disruption of the ecosystem;
3. Destruction of fisheries;
4. The depletion of wildlife and
5. Soil loss and vulnerability to droughts.

And social impacts such as:

1. Displacement of the human population;
2. High installation costs and
3. Loss of land to reservoirs.

Biomass comes second in the national electricity supply matrix. It makes a contribution of 3.7%. In terms of energy, biomass is a renewable resource from animal or vegetable organic matter which can be used as a source of energy.

The condition for its production is a strong agribusiness with large plantations, such as soybean, rice, corn or sugar cane. Biomass is obtained by processing the waste of these crops. Thus, the cob, stem, and leaf from corn can be used as raw material for energy. So too can the straw from corn, soybean and rice, gleaned from stubble in the fields, and likewise, the chaff, straw and vinasse from sugar cane.

The condition is necessary because the procedures for obtaining energy are not efficient, i.e., one needs a large volume of raw material to produce small quantities of energy. Therefore, the largest potential suppliers of these products are those countries with large cultivated or arable land and extensive agribusiness development.

In fact, Brazil is the second largest producer of ethanol, which, obtained from sugar cane, has similar potential energy and costs much less than the ethanol from countries such as the United States and regions such as the European Union. According to NEB (MME, 2008), Brazilian production in 2007 reached 8.612 million toe (tons of oil equivalent) against 6.395 million toe in 2006, representing an increase of 34.7%.

The production of bio-diesel is also growing, partly intended to supply domestic needs, and partly for export to developed countries, such as members of the European Union.

Indeed there is a tendency for the production of fuels derived from biomass to grow, in Brazil and abroad, mainly to reduce dependence on oil derivatives such as diesel and gasoline.

One of the main advantages of biomass is that their use can be made directly, through combustion in furnaces, boilers etc. Moreover, it has low cost, is renewable, allows the reuse of waste and the sugarcane harvest coincides with the drought of major river basins of the Brazilian hydroelectric park.

The disadvantages are the low thermodynamic efficiency of the plants and the relatively high costs of production and transportation. With respect to environmental and social aspects, there is a need for better management of the use and occupation land, due to the lack of regularity in supply (seasonal production), the creation of monocultures, the loss of biodiversity, the intensive use of agricultural etc. The burning of biomass causes the release of carbon dioxide into the atmosphere, but as this compound has previously been absorbed by plants that led to the fuel, the balance of CO₂ emissions is zero.

With regard to uranium, its production in 2007, according to the National Energy Balance (MME, 2008) fell to 10.2%. In 2007, nuclear energy accounted for 2.6% of the electricity generated in Brazil through Angra I and Angra II.

The largest use of uranium is in thermal power plants to generate electricity. In thermonuclear plants, the nucleus of the atom undergoes a process of fission (division) to

generate energy. This energy heats the water inside the reactor in order to produce the steam that drives the turbines.

Uranium found in nature cannot be used directly to generate heat. It is enriched through a process known as the uranium fuel cycle. Only nine countries, including Brazil, have the technology for the enrichment stage, although, in Brazil, this is not yet used on a commercial scale. This step is performed in Holland and Germany.

Only the nuclear industries in Brazil (NIB) are authorized by the Federal Government to extract and process the uranium and other radioactive minerals.

According to the Electric Energy of Brazil Atlas (Kelman, 2008), currently, only 25% of the country has been prospected for ore. Even so, the country is ranked 7th in known reserves. Brazil is estimated to have 278.7 mil tons which corresponds to about 6% of the world total. The deposits are located mainly in Bahia, Ceara, Minas Gerais and Paraná. The main one, in Caetité, Bahia, has 100 mil tons, enough to supply the nuclear complex of Angra I, II and III for 100 years.

New investments have been announced to expand the number of uranium mines in Brazil. The country has a competitive advantage because it has uranium reserves and the enrichment technology. Recently, the federal government announced the construction of Angra III which should come into operation in 2014 and a partnership with Argentina for the construction of a bi-national company with uranium enrichment activities, applications for agriculture and the production of radiopharmaceuticals, etc. The export of enriched uranium is also in the expansion plan.

The main advantage of operating with nuclear energy is the emission of low volumes of carbon dioxide (CO₂), which is mainly responsible for the greenhouse effect, and, consequently, for global warming. Furthermore there are abundant reserves of uranium on the planet - which, in the medium and long term, ensures the security of supply.

The main disadvantages are the difficulties of storing radioactive waste and the risk of accidents in nuclear power stations.

3.5 Multiple Uses

The issue is that the natural resources considered commodities (coal, natural gas, oil, water and biomass), which is widely used in the electricity production in Brazil and in the world, are allocated to both electricity production and the production of non-energy goods. Therefore, they are used as an input for production and as an input to produce a different kind of input, namely power. For example:

- Coal is used for electricity generation by thermoelectric power plants and in the industry for the heat generation (thermal energy) required for production processes such as drying products, ceramics and glass manufacture. Moreover, it is widely used in steelmaking;
- Natural gas is used for electricity generation by thermoelectric and in the production of electrical energy to produce heat and steam used in industrial processes;
- Oil is used by thermoelectric and in the petrochemical industry, the production of plastics, fuels, etc;
- Water, which is a renewable resource, yet a limited one, in stretches of rivers before they reach the hydroelectric plants is used in agriculture and for generate power), ;
- Biomass is used in the production of electric energy, bio-fuels, in the paper and cardboard industry, in the manufacture of a type of wood, in civil construction, as feed for livestock (the chaff of sugar cane mixed with urea serves as food for cattle in the semi-arid region of Brazil).

Non-conventional resources are used just to generate electricity. However, the participation of those resources is still very small in Brazil and worldwide. Section 3.6 is a brief summary about the most important non conventional resources and their use around the world.

3.6 Non Conventional Resources

The volatility of oil prices, concerns about the environment and the scarcity of this resource has led countries around the world to replace fossil sources with other cleaner resources and which are not marketable on the international market. This process was speeded up in the late 90s when several nations pledged themselves to decrease their emissions of carbon dioxide (CO_2) when they signed the Kyoto Protocol.

In 2007, according to the Renewable Energy Network for the 21st Century (REN21)⁴ there were more than 4 million people who consumed the energy produced by "green plants" (which are supplied by solar energy, wind energy, ocean Energy and geothermal Energy) in Europe, the United States, Canada, Australia and Japan.

In Brazil, according to the Bank of Information on Generation (BIG), the National Electric Energy Agency (Aneel), at January 21, 2009, 26 wind power plants and 01 photovoltaic venture are in operation in Brazil.

Wind Power

Wind Power is the conversion of wind energy (kinetic energy) into a useful form using wind turbines. Initially, it was used in agriculture for pumping water and grinding grain. The use of such energy for electricity generation began in the 20th century, and in the 70s, due to the oil crisis, it became feasible for some nations.

In 2007, worldwide capacity of wind-powered generators grew by 27% and reached 94.1 gigawatts. Although wind produces about 1% of electricity used world-wide, it accounts for approximately 19% of electricity production in Denmark, 9% in Spain and Portugal, and 6% in Germany and the Republic of Ireland. Globally, wind power generation increased more than fivefold between 2000 and 2007.⁵

In Brazil, the domestic supply of wind power has increased from 236 GWh to 559 GWh, a variation of 136.9%, according to the National Energy Balance (MME, 2008). In accordance with the Bank of Information for Generation (BIG)(ANEEL, 2009), there are today in the country, 26 wind power plants which have an installed capacity of 359 MW.

⁴available in <www.ren21.net>

⁵available in <www.gwec.net>

In Brazil the presence of winds is twice the world average with a volatility of 5% (oscillation speed), which gives greater predictability to the volume being produced. (Kelman, 2008). The main obstacles to Brazilian wind power are significant import duties and taxes which make projects less profitable.

In accordance with (Tiomno Tolmasquim, 2003), a wind power system consists of the following components:

1. A rotor, which transforms kinetic energy into mechanical energy through the axis of rotation;
2. A Multiplier Box, which transmits the mechanical energy of the rotor shaft to the generator;
3. An Electric Generator, which converts mechanical energy into electrical energy;
4. A Tower, which maintains and positions the rotor at the appropriate high.

The advantages of wind power are:

1. It does not use water as a source or coolant, and when the area is cleared of the wind farm, it can be used for other purposes;
2. The speed of the winds tends to be higher during periods of drought, so it is possible to operate wind power plants in a system that is complementary to hydroelectric power plants in order to preserve water in the reservoirs at times of low rainfall. Its operation would, therefore, be the "storage" of electric energy;
3. The cost of obtaining the supply is zero;
4. The supply is perennial and is readily available.

The disadvantages are the visual impact, noise emission, impact on wildlife (death of birds), and the cost is still high when compared with other sources.

Solar Energy

Almost all energy sources - hydro, biomass, wind, fossil fuels and ocean energy - are indirect forms of solar energy. The practical applications of this kind of energy can be

divided into: active solar energy and passive solar energy; the latter is used for heating and lighting environments by using the penetration and absorption of solar radiation; the former is used to convert solar energy into electric energy (from photovoltaic cells, for example) or into heat energy (by means of collectors and concentrators).

The contribution of solar energy matrix in the world energy matrix is negligible. Nevertheless, it grew more than 2,000% between 1996 and 2006. According to the New Energy Atlas (Kelman, 2008), Germany is the largest producer, with 49% of the total power installed world-wide. Moreover, together, Germany, Japan, the United States and Spain concentrated 84% of world capacity as at 2007.

Brazil can take advantage of using such energy, since the sun provides it with more than 2,200 solar hour equivalents per year with a total solar insolation of 15 trillion MWh throughout almost all the country. This corresponds to 50 thousand times the national electricity consumption (Rodrigues & Matajs, 2005).

The National Energy Plan 2030⁶ reproduces data from the Solarimetric Atlas of Brazil and records that this radiation varies from 8 to 22 MJ (megajoules) per square meter (m^2) during daylight hours, with minor variations occurring from May to July, when it ranges from 8 to 18 MJ/ m^2 . According to the study, the Northeast has radiation comparable to the best variable in the world, which is in the desert town of Dongola, Sudan, and the region of Dagget in the Mojave Desert, California. Figure 3.1 shows the incidence of hours of sunshine per day in Brazil.⁷

The use of solar energy for heating water (SEHW) in Brazil, by installing collectors, represents a viable economic solution for a technique to reduce the consumption of power in the Brazilian residential sector such as replacing the electricity used in showers⁸. According to (PROCEL, 2007), the use of electric showers represents around 6% of Brazilian electricity consumption and around 18% at the time of peak demand, and is the principal reason for residential electricity demand in the country, as shown in Figure 3.2⁹

A system of solar water heating is composed of plates of collecting energy, heat water

⁶The National Energy Plan is a study developed by School of Energy Research. Is available at: <www.epe.gov.br>

⁷Source: National Energy Plan 2030

⁸For details about economic feasibility of using solar energy for water heating, see Walley e Campello de Souza, 1990 and (Stamford da Silva & Campello de Souza, 1999)

⁹Source: PROCEL 2007

Figure 3.1: Map of Solar Incidence.

tank, hoses for connecting and double pipes to lead the water to its place of use. These systems are called natural circulation or thermosiphon. The operation is simple and based on a system which uses the force of gravity in the transport of heated water. The demand for hot water determines the quantity of collectors and the size of the reservoir.

In Brazil, the use of solar energy for heating is already quite widespread. However, investment in solar energy to generate electricity is still very small in the country. According to the National Electric Energy Agency NEEA, there is only one Central generating Photovoltaic Solar in operation for generation of 20 KW.

According to Campello de Souza¹⁰, the disadvantage of using SEHW are uncertainties such as:

1. Stochasticity of discontinuity and solar flow;
2. The need for storage at night because there is no solar flow;
3. The use of the facility does not mean savings, unless the use of the source is maximized in order to optimize the return but in most cases this is not possible;
4. the investment is very risky because it depends on other markets such as future prices and savings being made on conventional energy;

¹⁰in (Stamford da Silva & Campello de Souza, 1999)

Figure 3.2: The average percentage share of domestic appliances in the consumption of energy in households.

5. low liquidity;
6. The long time taken until there is a return on the investment such that the buyer is subject to uncertainties and stuck with a technology that can be changed at any time.

On the other hand, there are many advantages to using solar energy to heat water solar such as:

1. The demand can be continuously monitored because the technology is modular and individual;
2. The investment is gradual;
3. The independence of the prices of fossil fuels;
4. The source is inexhaustible;
5. The generation of energy is clean;
6. the period of maturation is distributed, i.e. as soon as collectors are installed, energy is used, unlike hydroelectric power which has a maturity period of 15 years;
7. The creation of a new activity in the economy, with permanent jobs;

8. Solar energy, because it is abundant, is not prone to cartels or monopolies being formed;
9. Long useful life;
10. Its cost of operation is zero;
11. Making use of solar energy does not contribute to overheating the planet;
12. It is distributive;
13. It helps to reduce demand during times of peak, so that generates externality for the electric power industry and avoids unnecessary investment in electricity.

3.7 Energy consumption and Economic Growth

According Edmonds and Reilly (1985)¹¹, the main influences in the use of energy are:

- demography;
- productivity of work;
- yield;
- energy productivity;
- energy price;
- uncertainty.

Demography influences the energy demand both because of the number of individuals who consume energy as well as because of those who use energy to produce goods and for its transport. Thus, the residential and commercial sectors, and the transport and industrial sectors are influenced by population growth.

Productivity can be defined as a ratio of physical output to physical input. Thus the work productivity is defined as $WP = \frac{GDP}{L}$, where GDP is the gross domestic product and L is the labor force, i.e, the productivity of work is the gross domestic product *per*

¹¹apud(Stamford da Silva & Campello de Souza, 1999)

worker. From the above definition, we can say that labor productivity multiplied by labor force is equal to the *GDP*, which is indicative of the level of income and production, so the higher rates of productivity growth are associated with high rates for growth

The energy supply is a function of *GDP*. Theoretically, the more a country grows, the more energy it needs. However, economies are heterogeneous and therefore there is no proportional change in *GDP* and the consumption of energy. The percentage change in the use of energy due to the percentage change in income is measured by the income elasticity of demand for energy. Speaking slowly, the income elasticity of demand for electric power shows how much power is required to support each 1% more than the *GDP*.

The price-elasticity of energy measures the change of ratio in energy use for each percentage change in energy prices, *ceteris paribus*. As a price increase encourages the conservation of energy, this measure is negative.

The productivity of energy is given by $\frac{GDP}{E}$, i.e, it is the level of production obtained per unit of energy. This measure captures technological and managerial changes.

Uncertainty is present in all such factors. As a result, random patterns should be considered when planning energy policies.

As mentioned above, it is very common to link economic growth with an increase in energy consumption. The ratio between these two variables given by $\frac{E}{GDP}$. *GDP* is called the energy intensity (*EI*). The *IE* measures the energy required to produce one unit of *GDP*, and thus provides indications of the energy efficiency of a country and helps it to make decisions on energy policy. The levels of *EI* may be influenced by several factors such as the state of technology, the price of energy, environmental restrictions, the level of activities in energy-intensive sectors, the composition of *GDP* and sociological and demographic factors (Edmonds and Reilly 1985)¹².

In Brazil, according to BEN 2008, the energy intensity of *GDP*, measured in toe/thousand U.S., although higher than in 2006 (0.5%), is the same as was observed in 1990: 0,182 toe / thousand U.S. This is observed despite the per capita demand for energy having increased by 4.4% in 2007, and thus having reached 1,3 toe / inhabitant. Indeed, countries

¹²apud(Stamford da Silva & Campello de Souza, 1999)

with high per capita *GDP* have a lower level of energy intensity.

3.8 Conclusions

Since economic growth demands more energy, what would be the best decision to be taken by the planner for expanding the electricity sector in order to sustain growth? In what source would it be best to invest?

The primary sources for generating electricity, in Brazil or anywhere else in the world, such as fossil fuels (coal, oil and natural gas), biomass, and water are limited and have multiple uses, i.e. they are used to generate electricity and also play a major part in the production of non-energy goods.

To support decisions, there is a need to assess the impact on production of the multiple uses of these resources. This is what will be done in the next chapter.

4 The Model

“All models are wrong, but some are useful.”

George Box

4.1 Introduction

An economic growth model is presented which aims to support decisions by the planner when drafting a policy for expanding the electricity supply. The model is based on the economic growth model proposed by (Stamford da Silva & Campello de Souza, 2008) which just dealt with the trade-off between using water to generate energy and in production. Unlike that analysis, this paper studies not only the water sector but also the fossil fuel and biomass sectors because these resources have multiple uses, that is, analyses are made of the interaction between the energy sector’s use of water, fossil fuels and biomass and the economy as a whole. Before explaining the model proposed, some considerations need to be made about the importance of mathematical models for policy formulation and the guidance of agents. Moreover, it is important to present a fuller explanation of the tools used in dynamic optimization models. This is what will be done in the next section. Section 4.3 will discuss economic growth further. In section 4.4, the new model will be presented and in section 4.5 its main results will be investigated.

4.2 Mathematics and Economy

Models are simplifications of reality. They are representations of the "real world" used in order to capture the main properties of a system. They need to be complex enough to capture these properties and sufficiently simple so that they may be handled mathematically. Therefore, as (Campello de Souza, 2005) argues, there is a compromise between precision and complexity on the one hand, and on the other, between the approach and reality. The approaches use assumptions that limit the type and range of their results.

With this in mind, the researcher needs to develop models of systems that are easy to

manipulate, which can accommodate a large number of variables and interrelationships and have some security in the representation of the real system, in addition to a reasonable degree of approximation (Campello de Souza, 2005).

In Modern Economic Theory, models are used to represent the behavior of agents. In models with exhaustible resources, one applies dynamic tools. As (Hotelling, 1931) argues, the static balance in economic theory has already been much developed. However, it does not help very much when the central issue is a potential reserve that is being consumed at a determined rate. At this point several questions are raised that can only be answered with dynamic modeling. There are three forms of modeling for dynamic optimization problems: dynamic programming, the calculus of variations and optimal control.

The concepts of the next subsections are founded mainly on: Chiang (1992), Intriligator (1971), Pontryagin (1962) and Campello de Souza (2007).

4.2.1 Dynamic Programming

The Dynamic Programming was developed by R.E Bellman in the 50s. Its simplest use is in problems of discrete time. In the continuous case, the method implies solving partial differential equations, which frequently do not have analytical solutions

The technique is based on the principle of optimality by Bellman who states the following: “An optimal policy has the property that, whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision”.

From that principle one obtains a relation of fundamental recurrence, which is a partial differential equation, known as Bellman’s equation. Once resolved, this furnishes the solution for a large class of optimization problems. Intriligator (1971) gives a general description of the assumptions of this technique:

Supposing that there exists a solution to the general problem of control, let $J^*(\mathbf{x}, t)$ be the maximized performance function, i.e., the maximized value of the objective functional of the problem, starting from the initial state x at time t .

The optimal value of the functional objective for the problem of the optimal control is $J^* = J^*(\mathbf{x}_0, t_0)$.

According to the optimality principle, if $J^*(\mathbf{x}, t)$ is the optimal performance function for the problem starting at the state x and at time t , then $J^*(\mathbf{x} + \Delta\mathbf{x}, t + \Delta t)$ is the optimal performance function for the second portion of the trajectory, started at the state $\mathbf{x} + \Delta\mathbf{x}$ and at the time $t + \Delta t$. Thus, it can be said that the performance function optimal starting at the time X can be equal to the sum of the contributions of the two portions at the time interval, then:

$$J^*(\mathbf{x}, t) = \max_{\{\mathbf{u}(t)\}} [I(\mathbf{x}, \mathbf{u}, t)\Delta t + J^*(\mathbf{x} + \Delta\mathbf{x}, t + \Delta t)] \quad (4.2.1)$$

constitutes a fundamental recurrence relation.

By hypothesis $J^*(\mathbf{x}, t)$ is continuous and differentiable. Using a Taylor's series to represent $J^*(\mathbf{x} + \Delta\mathbf{x}, t + \Delta t)$ at the point (x, t) one comes to Bellman's equation:

$$-\frac{\partial J^*}{\partial t} = \max_{\{u(t)\}} [I(\mathbf{x}, \mathbf{u}, t) + \frac{\partial J^*}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}, \mathbf{u}, t)] \quad (4.2.2)$$

It can be seen that what is being sought is the solution of a sequence of problems simple than the original problem.

Dynamic programming suffers from what Bellman defined as “The curse of dimensionality” which makes the problem practically impossible to be solved when the dimension of the system becomes very large.

4.2.2 Calculus of Variation

The calculus of variations had its origins with the famous problem of the brachistochrone set by Johan Bernoulli in 1696 which consisted in determining the trajectory by which a heavy body goes down more rapidly, from one point to another not directly under it, sliding along a surface with no attrition. From then on, there were several contributions.

In the 20th century, three that stand out are: by Hotelling (*The Economics of Exhaustible Resources*) in 1931, with regard to the economics of exhaustible resources; by Bellman (*The structure of dynamic programming processes*) in 1957, which was already been mentioned; and by Pontryagin (*The Mathematical Theory of Optimal Processes*) in

1961, which will be explained further below. Dynamic optimization developed at that time especially because of the space race between the USA and the Soviet Union.

It is a classical method that is still much used but, like regular calculus, it can only be applied if the functions that enter the problem can be differentiated in addition to which, only interior solutions can be dealt with. A more modern development that can work with non-classical characteristics such as the corner solution, is found in the theory of optimal control (Chiang, 1992).

4.2.3 Optimal Control

The formulation via optimal control of the problem of dynamic optimization focuses one or more variables that serve as instruments of optimization. However, unlike the calculus of variations, where the objective is to find the optimal temporary direction to a variable state Y , the main goal of optimal control theory is to determine the optimal direction to the variable of control u . Certainly, as soon as the direction of optimal control, $u^*(t)$, is found, we can also find the direction of optimal state, $x^*(t)$, which corresponds to it. In fact, the directions $u^*(t)$ and $x^*(t)$ are usually found in the same process. But the presence of a control variable at a central stage changes the basic orientation of the problem of dynamic optimization (Chiang, 1992).

In general lines, the technique implies choosing a trajectory of certain control variables from an admissible set, in order to obtain, via a set of differential equations (movement equations) a trajectory of the state variables that describe the system and maximize a given functional objective. The mathematical formulation of this problem is:

$$\max_{\{\mathbf{u}(t)\}} J = J\{\mathbf{u}(t)\} = \int_{t_0}^{t_1} I(\mathbf{x}(t), \mathbf{u}(t), t) dt + F(\mathbf{x}_1, t_1) \quad (4.2.3)$$

subject to :

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \quad (4.2.4)$$

$$t_0 \quad \text{and} \quad \mathbf{x}(t_0) = \mathbf{x}_0 \quad \text{given} \quad (4.2.5)$$

$$(\mathbf{x}(t), t) \in T \quad \text{where} \quad t = t_1 \quad (4.2.6)$$

$$\{\mathbf{u}(t)\} \in U \quad (4.2.7)$$

The main elements are:

- State variable $x(t)$: a continuous function of time that characterizes the state of the system at any instant t inside the specified interval $[t_0, t_1]$;
- Trajectory of state: $\{x\} = [x : -\infty, +\infty] \rightarrow R^n$. geometrically can be interpreted as a direction of points at E^n , starting from the initial state $\mathbf{x}(t_0) = \mathbf{x}_0$ and ending at terminal state $\mathbf{x}(t_1) = \mathbf{x}_1$;
- Control variables $u(t)$: They are values that characterize the choices (decisions) made at any time t of the specified interval;
- Trajectory of control : $\{\mathbf{u}(t)\} = \{\mathbf{u}(t) \in E^n \mid t_0 \leq t \leq t_1\}$. Geometrically represents a direction of points at E^n ;
- Control Set U : this is the set of all admissible control trajectories. Any optimal trajectory of control must belong to this set;
- Movement equations $\dot{\mathbf{x}}(t)$: a set of n differential equations that, by supplying the rate of change at the time of each state variable as a function of the other state variables, control and time variables, characterizes the trajectory of the state $\{\mathbf{x}(t)\}$;
- Terminal time , t_1 : $(\mathbf{x}(t), t) \in T$ em $t = t_1$ where T is a given sub-set of E^{n+1} , named terminal surface;
- Functional Objective 4.2.3: this is the map the control trajectories to points in the real line. The value that must be maximized, where $I(\dots)$ is the so-called intermediate function that must characterize the dependence of the functional at the time interval in relation to the state and control variables and the time itself. The second $F(\dots)$, called the final function relates the functional to the state and the terminal time.

Two basic classifications of the Control Systems (CS) are: the closed loop control system, also called the feedback control system, where the trajectory of optimal control is determined from the present state variables and of the time $\mathbf{u}^*(\mathbf{x}(t), t)$, that is, the decisions are revised based on new information added to the present state variables (this problem being known as synthesis). The other type of control system is open loop control system, where the trajectory of control is determined only in view of the time $\{\mathbf{u}^*(t)\}$, it being possible to specify it completely at an initial time t_0 , and thus to obtain the trajectory of the state $\{\mathbf{x}(t)\}$ by integrating of movement equations.

Other classifications of control systems are relation among variables (whether linear or not) and to the relation of the constants of the system (whether varying in time or not). One can also classify by analyzing the way the time is considered (discrete or continuous), besides the classification of deterministic or stochastic.

The theory of optimal control contributes so strongly to economic theory that “capital theory, then dealt with by the calculus of variations, was transformed so deeply that it was agreed to rename it the theory of growth, the maximum principle being the fundamental theorem of the theory of optimal control”(Dorfman, 1969).

4.2.4 The Maximum Principle

The maximum principle was developed by the mathematician Lev Pontryagin (The Mathematical Theory Optimal of Processes) in 1962. It took the opposite approach to the then known methods that made control and the states discrete in order to transform the problem to be solved by non-linear programming.

That principle can be considered an extension of the Lagrange multipliers of static optimization to problems of dynamic optimization and therefore involves adding Pontryagin multiplier to the problem of n variables $\mathbf{y}(t)$. Thus, the necessary conditions for a local maximum are summarized below.

$$\max_{\mathbf{u} \in \Omega} H(\mathbf{x}, \mathbf{u}, \mathbf{y}, t) \quad \text{para todo } t, \quad t_0 \leq t \leq t_1 \quad (4.2.8)$$

$$\dot{\mathbf{x}} = \frac{\partial H}{\partial \mathbf{y}}, \quad \mathbf{x}(t_0) = \mathbf{x}_0 \quad (4.2.9)$$

$$\dot{\mathbf{y}} = -\frac{\partial H}{\partial \mathbf{x}}, \quad \mathbf{y}(t_1) = \frac{\partial F}{\partial \mathbf{x}_1}. \quad (4.2.10)$$

Where $H(\mathbf{x}, \mathbf{u}, \mathbf{y}, t)$ is the Hamiltonian defined by:

$$H(\mathbf{x}, \mathbf{u}, \mathbf{y}, t) = I(\mathbf{x}, \mathbf{u}, t) + \mathbf{y}\mathbf{f}(\mathbf{x}, \mathbf{u}, t) \quad (4.2.11)$$

The conditions of the dynamic programming (Bellman's equation and its contour conditions) imply the PMP (Pontryagin Maximum Principle) but the PMP is best fitted for theoretical developments and the BOP (Bellman Optimality Principle), for the dynamic programming, is better suited for the numerical applications.

4.3 Economic Growth

The achievement of sustainable economic growth, in terms of an increase in national income, full employment or potential production, is one of the main goals of economic policy in most countries. Economic growth means the rate at which national income is growing and it is measured, usually, by the annual percentage rate change in a nation's gross domestic product (*GDP*), which is the market value of all goods and services produced within an economic area over a given period of time, i.e, it indicates the volume of economic activity within a nation's borders.

In order to simplify a comparison of one country's economy with that of another country, the per capita *per capita GDP* is used. As countries have different currencies, *GDP* is multiplied by a conversion factor, called the exchange rate adjusted for parity of purchasing power, so an attempt can be made to assess the real value of a currency in terms of its ability to buy similar products. The *per capita GDP* is usually measured in dollars.

Some economists argue that the per capita growth rate is not a measure of social welfare because it does not take into account quality of life indicators such as those for

infant mortality and life expectancy. However, it is not our intention to discuss this. The *per capita* growth rate is highly correlated with these indicators.

Economic growth can be understood as a sustained rise in a country's production of goods and services. It results from investments in human and physical capital, research and development, technological change, and improved institutional arrangements and incentives.

Modern studies of this issue are said to have begun in 1956 with the article *The Contribution to the Theory of Economic Growth* by Solow. Solow's model has shown the role played by the accumulation of physical capital and highlighted technical progress as fundamental for economic growth. In 1992 David Romer, Gregory Mankiw and David Weil published *The Contribution to the Empirics of Economic Growth*, a model which included the human capital in the Solow's model. For more details of these models consult (Jones, 2000) and (Romer, 2006).

4.3.1 Models with Exhaustible Resources

As mentioned the Hottelling's article, *The Economics of Exhaustible Resources*, published in the Journal of Political Economy in 1931, is the most important contribution to understanding the role of these resources in the development of the economy. is the earliest and still a most important contribution to understanding the role these resources play in developing a country's economy. In order to deal with such resources he used the only tool then available: the calculus of variations, as explained in the previous section. On considering the limitations of time, and to simplify his model, he considered that demand is in a linear form; a natural resource is homogenous; the expectation is of equal prices for all firms, and of a constant marginal extraction cost independent of remaining stock and fixed-extracting technology. The objective was to find an extraction rate that would ensure maximum growth continued.

One important result from the model is Hotteling's rule which requires the return of a non-renewable asset to consist entirely of observation of its opportunity cost and that market balance requires that the opportunity cost increase at the market interest rate.

The importance of Hotteling's article was not recognized until 1970s when the oil

shocks showed the world's dependence on this resource. Thus, the rapid changes in oil prices, coupled with emerging concern about the long-term sustainability of patterns of economic growth and development, focused economists' attention and many academics began to introduce exhaustible resources into their growth models.

The Hotelling's article is the progenitor of all subsequent work. Many other authors constructed models adding limited resources. For these models, consult (Heal, 1997), (Stamford da Silva, 2008), (Campello de Souza, 2005) and (Stamford da Silva & Campello de Souza, 2008).

4.4 The Model Proposed

The model is formulated as a problem of optimal control and represents an evolving economy. The objective is to maximize an inter-temporal social welfare function, subject to constraints defined by the identity of income, the identity of investment, production technologies, the dynamics of consumption of the reserves, the dynamic evolution of the labor force, energy balance and labor balance all of which will be explained later.

Maximization is achieved by the choice of investments in each sector; the consumption rates of energy resources; the non-energy consumption rates for water, fossil fuels and biomass; the imported energy consumption rate; the energy goods consumption rate per capita and the growth rate of the labor force.

The application of the Pontryagin Maximum Principle results in optimal conditions that represent theoretical conclusions about the tradeoffs water-energy, fossil fuels-energy and biomass-energy.

4.4.1 Assumptions

In the same way as (Stamford da Silva & Campello de Souza, 2008) are, the new model is deterministic, i.e, it does not address the random characteristics involved in the system, such as droughts or discoveries of new fields. In addition to that it does not analyze monetary matters; instead value relations are considered. Consumption distribution among the labor force is not modeled: an average individual is used. The structure is also

considered to be utilitarian, i.e. all individual generations have the same preferences.

Exports of energy are not taken into account while non-conventional energy technologies such as the solar, Aeolian and nuclear ones are not presumed as backstop technologies. It is assumed these technologies are ready for use and can replace conventional technologies.

Moreover the production function is assumed to be concave and it is further supposed that there is no technological progress of extraction. Therefore the decrease in storage is offset by an increase of more capital and labor.

4.4.2 Notation

- J =intertemporal welfare function;
- δ =social rate of discount;
- L =total labor force;
- u =per capita instantaneous utility function;
- c =per capita consumption of non-energy goods;
- t =time;
- α =per capita consumption of energy goods.
- β =labor force growth rate;
- F =production function of non-energy goods(except the availability of water, oil and biomass for productive ends);
- E =aggregate non-energy consumption rate of energy resources (except the energy consumption by the labor force);
- W =annual consumption or extraction rate of non-energy water(which affects the generation of electric energy produced by hydropower utilities, and when not used for the generation of that energy, can produce non-energy goods),in equivalent energy units, that is, W is measured in the same unit as E ;

- O = annual consumption or extraction rate of non-energy fossil fuels-oil, coal and natural gas- (which affects the generation of electric energy produced by thermoelectric plants, and when not used for the generation of that energy, can produce non-energy goods), in equivalent energy units, that is, O is measured in the same unit as E ;
- B = annual consumption or extraction rate of non-energy Biomass (which affects the generation of electric energy produced by thermoelectric plant, and when not used for the generation of that energy, can produce non-energy goods), in equivalent energy units, that is, B is measured in the same unit as E ;
- $s(E_e)$ =expenses for acquisition of imported energy resources;
- $D_{h,w}$ =water stock for any use (corresponding to the reservoirs of the hydropower utilities; water with hydraulic head);
- $D_{to,o}$ = fossil fuel stock (corresponding to the fossil fuel reserves for thermoelectric plants and other uses);
- $D_{tb,b}$ = biomass stock (corresponding to the biomass reserves for thermoelectric plants and other uses).

The subscripts used in the remaining variables have the following meanings:

- 0 (zero)- refers to the non-energy goods, except non-energy water, non-energy biomass and non-energy oil;
- w - refers to the non-energy water;
- h - refers to energy water (electricity);
- b - refers to the non-energy Biomass;
- tb - refers to energy Biomass;
- o - refers to non-energy Fossil Fuel;
- to - refers to energy Fossil Fuel;

- *nm*- refers to resources used only to generate energy, such as Uranium¹, Sun and wind.

The constraints are:

4.4.3 The Income Identity

Income identity, which represents the products of the economy in aggregate form, is given by:

$$F(K_0, L_0, E, W, O, B) = I_0 + I_w + I_h + I_o + I_{to} + I_b + I_{tb} + I_{nm} + s(E_e) + Lc \quad (4.4.1)$$

that is the income identity verified in the basic neoclassical growth model. The difference lies in the use of expenses for the purchase of imported energy resources, a fact that classifies the model as a model open to energy resources. As mentioned earlier exports of energy were not modeled. It is supposed that the market is in equilibrium and that all output is either consumed or invested.

It is assumed that the Production Function has a constant return to scale in its two arguments. That is, doubling the quantities of capital and labor doubles the amount of goods produced.

It is also assumed that the production function is invariant over time and twice differentiable for all positive factor inputs, (Intriligator, 1971):

$$\frac{\partial F}{\partial K_0}(K_0, L_0, E, W, O, B) > 0, \quad \frac{\partial^2 F}{\partial^2 K_0^2}(K_0, L_0, E, W, O, B) < 0 \quad (4.4.2)$$

$$\frac{\partial F}{\partial L_0}(K_0, L_0, E, W, O, B) > 0, \quad \frac{\partial^2 F}{\partial^2 L_0^2}(K_0, L_0, E, W, O, B) < 0 \quad (4.4.3)$$

So this implies that the marginal product of capital is positive, but that it declines as capital rises. Similarly, the marginal product of labor is positive, but that it declines as labor rises.

¹The uranium is used in the production of drugs and mini-conductors. However, the amount of resource used is negligible when compared to that used for production of electricity.

As to limits:

$$\lim_{k \rightarrow 0} \frac{\partial F}{\partial K_0}(K_O, L_O, E, W, O, B) = \infty, \quad \lim_{k \rightarrow \infty} \frac{\partial F}{\partial K_0}(K_O, L_O, E, W, O, B) = 0 \quad (4.4.4)$$

$$\lim_{L \rightarrow 0} \frac{\partial F}{\partial L_0}(K_O, L_O, E, W, O, B) = \infty, \quad \lim_{L \rightarrow \infty} \frac{\partial F}{\partial L_0}(K_O, L_O, E, W, O, B) = 0 \quad (4.4.5)$$

These conditions state that the marginal product of capital is very large when the capital stock is sufficiently small and that it becomes very small as the capital stock becomes large; their role is to ensure that the path of the economy does not diverge (Romer, 2006).

4.4.4 The Investment Identity

The gross investment identities are usually given by:

$$\dot{K}_i = -\mu_i k_i + I_i; \quad i = 0, w, h, o, to, b, tb, nm \quad (4.4.6)$$

and represent the fact that investments are employed both to increase the stock of capital and to replace depreciated capital.

4.4.5 Production Technologies

As in the Stamford and Campello model. the Hicks' neutrality² will be assumed in the production technologies to separate the extraction effort from the extraction technology.

The supposition is made that the markets for each kind of energy - water, fossil fuels and biomass - are in equilibrium. Furthermore, it is assumed that all energy extracted from water, fossil fuels and biomass is consumed in the production of non-energy goods.

²A Hicks-neutral technological progress makes the marginal rate of substitution technical unchanged to the same capital-labor relationship $\frac{K}{L}$ (Chiang, 1992).

The production technology equation for energy i is given by:

$$E_i = F_i(K_i, L_i)G_i(D_i, j), \quad i = h, to, tb, nm \quad e \quad j = w, o, b \quad (4.4.7)$$

A similar function, but including a production function namely *learning by doing* by Arrow, is exposed in (Campello de Souza, 2005).

The G functions are the weighing function which depend on the type of energy resource that is being produced. For all restrained, limited resources, they generally decrease with D . The weighing function of all resources is assumed monotonically increasing so they diminish returns when the reserve is consumed and it is necessary allocate more capital and more labor for extraction. One assumes that (Campello de Souza, 2005):

$$\lim_{D_i \rightarrow 0} G_i(D_i) = 0, \quad \lim_{D_i \rightarrow \infty} G_i(D_i) = 1 \quad (4.4.8)$$

$$\lim_{D_i \rightarrow 0} \frac{dG_i(D_i)}{dD_i} = \infty, \quad \lim_{D_i \rightarrow \infty} \frac{dG_i(D_i)}{dD_i} = 0 \quad (4.4.9)$$

One notes that the resources only cease to exist in the infinity. So, along the path, the resource can be treated as unlimited.

For any exhaustible resource, when D decrease due to other uses, the amount of resource used for extraction should be greater in order to compensate for this loss, and thus, G will decrease.

The equation for water extraction production technology used for the production of non-energy goods is given by:

$$W = F_w(K_w, L_w)G_w(D_{h,w}) \quad (4.4.10)$$

and represents the water production function for irrigation processes, water for fish or shrimp farms; and for cooling processes in industries, etc.

The equation for fossil fuel production technology used for the production of non-energy goods is given by:

$$O = F_o(K_o, L_o)G_o(D_{to,o}) \quad (4.4.11)$$

and represents the fossil fuels production function of the petrochemical industry, in the production of plastics, in the production of fuels, in steelmaking, etc.

The equation for biomass extraction production technology used for the production of non-energy goods is given by:

$$B = F_b(K_b, L_b)G_b(D_{tb,b}) \quad (4.4.12)$$

and represents the biomass production function mainly by the bio-fuel, paper and cardboard industry, in the manufacture of agglomerates, in civil construction and for livestock

The functions $F_i(K_i, L_i)$, with $i = h, w, o, to, b, tb, nm$, are also assumed twice differentiable and for all positive inputs.

4.4.6 Reserve Consumption Dynamics

The consumption rate of the reserve D_{nm} of resources used only to generate energy is usually given by:

$$\dot{D}_{nm} = -(E_{nm}) \quad (4.4.13)$$

On the other hand, the consumption rate of the reserves $D_{h,w}$, $D_{to,o}$ and $D_{tb,b}$ will be give, respectively, by:

$$\dot{D}_{h,w} = -(E_h + W) \quad (4.4.14)$$

$$\dot{D}_{to,o} = -(E_{to} + O) \quad (4.4.15)$$

$$\dot{D}_{tb,b} = -(E_{tb} + B) \quad (4.4.16)$$

Since the water, the fossil fuels and the biomass are used for two ends - generating electricity and producing non-energy goods - any amount of these resources used to produce non-energy goods will affect the production of hydroelectric and thermoelectric energy.

4.4.7 Energy Balance Equation

This equation represents the aggregated consumption rate of global energy resources (except for the labor force consumption) and is expressed by:

$$E = E_h + E_{to} + E_{tb} + E_{nm} + E_e - W - O - B - L\alpha \quad (4.4.17)$$

Where W means how much hydraulic energy would be generated if a portion of water used for this purpose were allocated for other uses. Analogously O and B represent how much energy from the thermoelectric stations would be subtracted from the total energy if some part of fossil fuels or biomass were allocated for other uses. The α denotes energetic resources consumption *per capita*.

4.4.8 Labor Force Evolution Dynamics

The growth rate of labor force will be described by the following equation:

$$\dot{L} = \beta L \quad (4.4.18)$$

and the total labor force will be:

$$L = L_0 + L_h + L_w + L_o + L_{to} + L_b + L_{tb} + L_{nm} \quad (4.4.19)$$

which is the sum of the labor force in each sector.

4.4.9 objective function

According to the hypothesis, the structure utilitarian will be expressed by the follow objective functional

$$\bullet J = \int_0^\infty e^{-\delta t} Lu(c, \alpha, \beta).$$

Where u is the intertemporal utility as a function of the consumption per worker, c , the consumption per worker of energetic goods, α , and the population growth rate, β . It is

assumed that the rate of discount, δ is constant and non-negative. It is further assumed about u that:

- It is continuous in $\mathbb{R}^+$;
- It is homogenous of the 1st degree;
- It is strictly concave in $\mathbb{R}^+$;
- It is of class C_2 in $\mathbb{R}^+$;

These hypothesis are basic. (Intriligator, 1971)and (Campello de Souza, 2005)make use of them.

4.4.10 State Variables

The state variables are:

- $K_i, i = 0, w, h, o, to, b, tb, nm$;
- $D_{h,W}, D_{to,o}, D_{tb,b}, D_{nm}$; and
- L .

4.4.11 Control Variables

The control variables are:

- $I_i, i = 0, w, h, o, to, b, tb, nm$;
- $E_i, i = h, to, tb, nm$;
- $E_e, W, B, O, \alpha, \beta$

4.4.12 Problem Synthesis

The problem is:

$$\max \int_0^\infty e^{-\delta t} Lu(c, \alpha, \beta) dt \quad (4.4.20)$$

subject to:

$$F(K_0, L_0, E, W, O, B) = I_0 + I_w + I_h + I_o + I_{to} + I_b + I_{tb} + I_{nm} + s(E_E) + Lc \quad (4.4.21)$$

$$\dot{K} = -\mu_i k_i + I_i, \quad i = 0, w, h, o, to, b, tb, nm \quad (4.4.22)$$

$$E = E_h + E_{to} + E_{tb} + E_{nm} + E_e - W - O - B - L\alpha \quad (4.4.23)$$

$$E_h = F_h(K_h, L_h)G_h(D_{h,w}) \quad (4.4.24)$$

$$W = F_w(K_w, L_w)G_w(D_{h,w}) \quad (4.4.25)$$

$$\dot{D}_{h,w} = -(E_h + W) \quad (4.4.26)$$

$$E_{to} = F_{to}(K_{to}, L_{to})G_{to}(D_{to,o}) \quad (4.4.27)$$

$$O = F_o(K_o, L_o)G_o(D_{to,o}) \quad (4.4.28)$$

$$\dot{D}_{to,o} = -(E_{to} + O) \quad (4.4.29)$$

$$E_{tb} = F_{tb}(K_{tb}, L_{tb})G_{tb}(D_{tb,b}) \quad (4.4.30)$$

$$B = F_b(K_b, L_b)G_b(D_{tb,b}) \quad (4.4.31)$$

$$\dot{D}_{tb,b} = -(E_{tb} + B) \quad (4.4.32)$$

$$E_{nm} = F_{nm}(K_{nm}, L_{nm})G_{nm}(D_{nm}) \quad (4.4.33)$$

$$\dot{D}_{nm} = -(E_{nm}) \quad (4.4.34)$$

$$L = L_0 + L_h + L_w + L_o + L_{to} + L_b + L_{tb} + L_{nm} \quad (4.4.35)$$

$$\dot{L} = \beta L \quad (4.4.36)$$

4.4.13 The Hamiltonian

The Hamiltonian is given by:

$$H = e^{-\delta t} \{ Lu(c, \alpha, \beta) + \Sigma q_i (-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - p_{D_{to,o}}(E_o + O) - p_{D_{tb,b}}(E_b + B) - p_{D_{nm}}(E_{nm}) + q_L \beta L \}$$

4.5 Mainly Results

From the application of the Pontryagin Maximum Principle (details in Appendix) the following optimality conditions can be obtained:

$$q_i = \frac{\partial u}{\partial c} \quad (i = 0, w, h, o, to, b, tb, nm) \quad (4.5.1)$$

The marginal value of capital K_i must be equal to the marginal utility regarding the non-energy goods consumption per-worker. In other words, in the optimal path, the additional consumption of more non-energy goods must be equal to the additional investment in capital goods.

$$\frac{\partial F}{\partial E} = \frac{\partial s}{\partial E_e} \quad (4.5.2)$$

Domestic and imported energy goods contribute equally to the yield, i.e, the internal price of energy should be the same for both. Furthermore:

$$\frac{\partial F}{\partial E_i} = \frac{\partial F}{\partial E} = \frac{ds}{dE_e} \quad (4.5.3)$$

In the optimal course there should be a single price for all primary energy sources.

$$q_L = -\frac{\partial u}{\partial \beta} \quad (4.5.4)$$

The shadow price of the labor force in the economy must be equal to the negative marginal utility regarding the labor force growth rate.

$$\frac{\partial F}{\partial E} = \frac{\frac{\partial u}{\partial \alpha}}{\frac{\partial u}{\partial c}} = \frac{p}{q} = \frac{\partial s}{\partial E_e} \quad (4.5.5)$$

The price of energy resources must be equal to the substitution rate between non-energy goods and energy goods or the substitution rate between non-energy goods and energy goods must be equal to the rate between the marginal value of the energy resource in the reserve and the marginal value of capital which must be equal to the contribution of imported energy to the yield.

$$\frac{\dot{p}}{p} = \delta \quad (4.5.6)$$

This is the Hotelling rule which is an expected result when extraction rates are employed as control forces. It means that in the situ stock value should grow at the interest rate, i.e, energy resources must be treated like any capital good.

$$\mu_h = \mu_w = \mu_{tb} = \mu_b = \mu_{to} = \mu_o = \mu_{nm} = \mu \quad (4.5.7)$$

The capital goods for all the sectors must be homogenous, i.e., the depreciation rates should be the same.

$$\frac{\dot{q}_h}{q_h} = \frac{\dot{q}_w}{q_w} = \frac{\dot{q}_{to}}{q_{to}} = \frac{\dot{q}_o}{q_o} = \frac{\dot{q}_{tb}}{q_{tb}} = \frac{\dot{q}_{nm}}{q_{nm}} = \delta - \mu \quad (4.5.8)$$

The capital accumulation rate should be smaller than the interest rate.

$$\frac{\partial F}{\partial K_0} = (\mu - \mu_0) \quad (4.5.9)$$

Since the depreciation rates μ and μ_0 are assumed constant, this means that the contribution of non-energy capital in producing non-energy goods is constant.

$$\frac{\partial F}{\partial W} = 2 \frac{\partial F}{\partial E} \quad (4.5.10)$$

This is the most important result of the Stamford and Campello's model. It means that the contribution of water to the production of non-energy goods has to be twice the contribution of the energy goods for the same production. In other words, the marginal productivity of water should be equal to twice the internal price of energy.

However, the inclusion of the other two sectors- Fossil Fuels and Biomass- both with multiple uses lead to the following new results:

$$\frac{\partial F}{\partial O} = 2 \frac{\partial F}{\partial E} \quad (4.5.11)$$

And

$$\frac{\partial F}{\partial B} = 2 \frac{\partial F}{\partial E} \quad (4.5.12)$$

So, the contribution of fossil fuels, biomass or any resource that has multiple use to the production of non-energy goods also has to be twice the contribution of the energy goods to the same production.

$$p_{D_{h,w}} = p_{D_{to,o}} = p_{D_{tb,b}} = p_{D_{nm}} = \frac{\partial u}{\partial \alpha} \quad (4.5.13)$$

The marginal utility of consuming energy should equal the in situ stock value.

5 *Concluding Remarks and Suggestions*

5.1 Concluding Remarks

All the results from Stamford and Campello's model were preserved in the model proposed, including classic ones, such as:

- the marginal utility of consuming non-energy goods should be equal the capital accumulation rate;
- the marginal utility of consuming energy should be equal the in situ stock value and;
- The Hotelling's Rule.

These act as arguments for the validity of the model.

However, the most important result of that model - the fact that in the optimal path, water contributes twice to producing non-energy goods than to generating energy - can be expanded to all energy resources which have multiple uses. The economic value of biomass and fossil fuels in producing energy goods is also double that of generating energy. So when these resources are used to produce energy, one imputes to them half of their values. These results also reflect the scarcity of the resources, which leads to their price being doubled in a competitive market.

In view of this, it is not, in economic terms, rational to use commodities to generate electricity. The suggestion therefore arises that in order to generate energy, these resources be replaced by others which are not scarce, and, consequently, do not accumulate value over time. Furthermore, they need to be distributed and clean in order to reduce emissions of CO_2 into the environment. Non-conventional sources such as the sun, wind, the oceans and geysers have these characteristics.

It is known that current resources will not be largely replaced in the short term. In the case of water in Brazil, this is all the more difficult because water represents most of the energy matrix of the country and investments made in hydroelectric power are difficult to reverse.

In principle, one can think, therefore, in terms of the partial substitution of hydro-electric energy by a clean energy source, which is unlimited, distributed and can be used in homes, industries and commercial establishments. One that has these characteristics is solar energy.

As already mentioned, using solar energy for heating water is a viable economic solution and technique for reducing the consumption of power in the Brazilian homes starting with replacing electrical showers. This represents a positive externality for hydroelectric plants at times of peak demand. Brazil, especially in the northeast of the country, has taken advantage of using such energy, since it enjoys more than 2,200 solar hour equivalents per year, total solar insolation amounting to 15 trillion MWh almost all over the country.

In fact, solar energy is an abundance of energy in the form of radiation which cannot be monopolized. Solar radiation is not in itself a scarce and endangered commodity, and therefore is not subject to market forces and the market forces and foreign policy. However, the solar market will not grow from the tasks necessary to provide energy as a commodity, but from the production and marketing of equipment which can convert solar radiation into usable energy (Campello de Souza, 2005).

Wind energy could also be an option, since Brazil also has advantages with respect to the presence of winds which tend to be higher during periods of drought, and, therefore, would permit the “storage of electric energy”.

Uranium is scarce. Furthermore, most countries do not yet have the technology for enriching uranium. Brazil does, although it is not used on a large scale. In this study, the multiple uses of uranium were not considered, since its main use today is to generate electricity. However, by the time that its use for production is significant, that is, when investments in these sources allow greater marketing of the product in the international market, it will not be rational to use it for energy production.

Investment in non-conventional energy generates employment in the economy and creates a free competitive market.

As (Campello de Souza, 2005) argues, the choice of an energy system, i.e. the choice between primary energy resources and the technologies needed to use them, is a complex

problem and, of course, full of social and political aspects. It is not clear or explicit which considerations act as a basis for arguing for or against the various energy options. In fact, this was not the objective of this study. It is simply to give guidelines for planning in the electricity sector based on an analysis of the trade-off between the multiple uses of resources when they are considered commodities.

5.2 Suggestions

Since water is needed to generate energy coming from conventional resources and energy is needed to deliver water, one source limits the other. It would be interesting to include water in the production function of non-energy goods from fossil fuels and biomass and to analyze the trade-off between the multiple uses.

Another suggestion is to use a stochastic dynamic approach to make the model closer to the “real world”. One might think of a model that takes into account the uncertainties involved in the Brazilian energy matrix such as:

- demand;
- institutional instability in the country;
- the historical series of energy flows;
- discovery of new deposits and
- variations in the Exchange rate.

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A Appendix- Obtaining Analytical Results

A.1 Optimal Control Problem

$$\max \int_0^\infty e^{-\delta t} Lu(c, \alpha, \beta) dt \quad (\text{A.1.1})$$

subject to:

$$F(K_0, L_0, E, W, O, B) = I_0 + I_w + I_h + I_o + I_{to} + I_b + I_{tb} + I_{nm} + s(E_E) + Lc \quad (\text{A.1.2})$$

$$\dot{K} = -\mu_i k_i + I_i, i = 0, w, h, o, to, b, tb, nm \quad (\text{A.1.3})$$

$$E = E_h + E_{to} + E_{tb} + E_{nm} + E_e - W - O - B - L\alpha \quad (\text{A.1.4})$$

$$E_h = F_h(K_h, L_h)G_h(D_{h,w}) \quad (\text{A.1.5})$$

$$W = F_w(K_w, L_w)G_w(D_{h,w}) \quad (\text{A.1.6})$$

$$\dot{D}_{h,w} = -(E_h + W) \quad (\text{A.1.7})$$

$$E_{to} = F_{to}(K_{to}, L_{to})G_{to}(D_{to,o}) \quad (\text{A.1.8})$$

$$O = F_o(K_o, L_o)G_o(D_{to,o}) \quad (\text{A.1.9})$$

$$\dot{D}_{to,o} = -(E_{to} + O) \quad (\text{A.1.10})$$

$$E_{tb} = F_{tb}(K_{tb}, L_{tb})G_{tb}(D_{tb,b}) \quad (\text{A.1.11})$$

$$B = F_b(K_b, L_b)G_b(D_{tb,b}) \quad (\text{A.1.12})$$

$$\dot{D}_{tb,b} = -(E_{tb} + B) \quad (\text{A.1.13})$$

$$E_{nm} = F_{nm}(K_{nm}, L_{nm})G_{nm}(D_{nm}) \quad (\text{A.1.14})$$

$$\dot{D}_{nm} = -(E_{nm}) \quad (\text{A.1.15})$$

$$L = L_0 + L_h + L_w + L_o + L_{to} + L_b + L_{tb} + L_{nm} \quad (\text{A.1.16})$$

$$\dot{L} = \beta L \quad (\text{A.1.17})$$

A.2 Hamiltonian

$$H = e^{-\delta t} \{ Lu(c, \alpha, \beta) + \Sigma q_i (-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - \\ p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_b + B) - p_{D_{nm}}(E_{nm}) + q_L \beta L \}$$

From expression A.1.2 the variable c can be represented as:

$$c = \frac{1}{L} [F(K_0, L_0, E, W, O, B) - I_0 - I_w - I_h - I_o - I_{to} - I_b - I_{tb} - I_{nm} - s(E_E)]$$

So, one obtains:

$$\frac{\partial c}{\partial F} = \frac{1}{L}; \quad \frac{\partial c}{\partial I_i} = -\frac{1}{L} \quad (i = 0, w, h, o, to, b, tb, nm); \quad \frac{\partial c}{\partial s} = -\frac{1}{L} \quad (\text{A.2.1})$$

From $E = E_h + E_{to} + E_{tb} + E_{nm} + E_e - W - O - B - L\alpha$ one gets:

$$\frac{\partial E}{\partial E_i} = 1 \quad (i = 0, w, h, o, to, b, tb, nm); \quad \frac{\partial E}{\partial W} = -1; \quad \frac{\partial E}{\partial O} = -1; \quad \frac{\partial E}{\partial B} = -1; \quad \frac{\partial E}{\partial \alpha} = -L \quad (\text{A.2.2})$$

A.3 Control Variables

- $I_i, i = 0, w, h, o, to, b, tb, nm;$
- $E_i, i = h, to, tb, nm;$
- $E_e, W, B, O, \alpha, \beta$

To maximize the Hamiltonian:

$$\frac{\partial H}{\partial I_i} = 0 \quad (i = 0, w, h, o, to, b, tb, nm) \quad (\text{A.3.1})$$

$$\begin{aligned} \frac{\partial H}{\partial I_i} &= e^{-\delta t} \frac{\partial}{\partial I_i} \{ Lu(c(I_i)) + \Sigma q_i (-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - \\ &\quad p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_b + B) - p_{D_{nm}}(E_{nm}) + q_L \beta L \} = 0 \\ &= e^{-\delta t} \left\{ L \frac{\partial u}{\partial c} \frac{\partial c}{\partial I_i} + q_i \right\} = 0 \\ &= L \frac{\partial u}{\partial c} \left(-\frac{1}{L} \right) + q_i = 0 \end{aligned}$$

Thus,

$$q_i = \frac{\partial u}{\partial c} \quad (i = 0, w, h, o, to, b, tb, nm) \quad (\text{A.3.2})$$

$$\frac{\partial H}{\partial E_{nm}} = 0 \quad (\text{A.3.3})$$

$$\begin{aligned}
\frac{\partial H}{\partial E_{nm}} &= e^{-\delta t} \frac{\partial}{\partial E_{nm}} \{Lu(c(F(E(E_{nm})))) + \Sigma q_i(-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - \\
&\quad p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_{tb} + B) - p_{D_{nm}}(E_{nm}) + q_L \beta L\} = 0 \\
&= e^{-\delta t} \left\{ L \frac{\partial u}{\partial c} \frac{\partial c}{\partial F} \frac{\partial F}{\partial E} \frac{\partial E}{\partial E_{nm}} - p_{D_{nm}} \right\} = 0 \\
&= L \frac{\partial u}{\partial c} \frac{1}{L} \frac{\partial F}{\partial E} 1 - p_{D_{nm}} = 0
\end{aligned}$$

So,

$$\frac{\partial u}{\partial c} \frac{\partial F}{\partial E} = p_{D_{nm}} \quad (\text{A.3.4})$$

From A.3.2 and assuming $\frac{\partial u}{\partial c} > 0$, one gets:

$$\frac{p_{D_{nm}}}{q} = \frac{\partial F}{\partial E} \quad (\text{A.3.5})$$

$$\frac{\partial H}{\partial E_h} = 0 \quad (\text{A.3.6})$$

$$\begin{aligned}
\frac{\partial H}{\partial E_h} &= e^{-\delta t} \frac{\partial}{\partial E_h} \{Lu(c(F(E(E_h)))) + \Sigma q_i(-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - \\
&\quad p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_b + B) - p_{D_{nm}}(E_n m) + q_L \beta L\} \\
&= e^{-\delta t} \left\{ L \frac{\partial u}{\partial c} \frac{\partial c}{\partial F} \frac{\partial F}{\partial E} \frac{\partial E}{\partial H} - p_{D_{h,w}} \right\} = 0 \\
&= L \frac{\partial u}{\partial c} \frac{1}{L} \frac{\partial F}{\partial E} 1 - p_{D_{h,w}} = 0 \\
&= \frac{\partial u}{\partial c} \frac{\partial F}{\partial E} = p_{D_{h,w}}
\end{aligned}$$

From A.3.2 and assuming $\frac{\partial u}{\partial c} > 0$, one gets:

$$\frac{p_{D_{h,w}}}{q} = \frac{\partial F}{\partial E} \quad (\text{A.3.7})$$

Similarly, for other resources one obtains:

$$\frac{\partial u}{\partial c} \frac{\partial F}{\partial E} = p_{D_{to,o}} \quad \frac{\partial u}{\partial c} \frac{\partial F}{\partial E} = p_{D_{tb,b}} \quad (\text{A.3.8})$$

Thus,

$$p_{D_{h,w}} = p_{D_{to,o}} = p_{D_{tb,b}} = p_{D_{nm}} \quad (\text{A.3.9})$$

$$\frac{\partial H}{\partial W} = 0 \quad (\text{A.3.10})$$

$$\begin{aligned} \frac{\partial H}{\partial W} &= e^{-\delta t} \frac{\partial}{\partial W} \{ Lu(c(F(E(W)))) + \Sigma q_i(-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - \\ &\quad p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_b + B) - p_{D_{nm}}(E_{nm}) + q_L \beta L \} \\ &= e^{-\delta t} \left\{ L \left[\frac{\partial u}{\partial c} \frac{\partial c}{\partial F} \left(\frac{\partial F}{\partial E} \frac{\partial E}{\partial W} + \frac{\partial F}{\partial W} \right) \right] - p_{D_{h,w}} \right\} = 0 \\ &= \left\{ L \left[\frac{\partial u}{\partial c} \frac{1}{L} \left(\frac{\partial F}{\partial E}(-1) + \frac{\partial F}{\partial W} \right) \right] - p_{D_{h,w}} \right\} = 0 \\ &= \frac{\partial u}{\partial c} \left(-\frac{\partial F}{\partial E} + \frac{\partial F}{\partial W} \right) - p_{D_{h,w}} = 0 \\ &\quad \frac{\partial u}{\partial c} \left(-\frac{\partial F}{\partial E} + \frac{\partial F}{\partial W} \right) = p_{D_{h,w}} \end{aligned}$$

Thus,

$$p_{D_{h,w}} - \frac{\partial u}{\partial c} \left(-\frac{\partial F}{\partial E} + \frac{\partial F}{\partial W} \right) = 0 \quad (\text{A.3.11})$$

From $\frac{\partial u}{\partial c} \frac{\partial F}{\partial E} = p_{D_{h,w}}$ and A.3.11 one gets:

$$\frac{\partial F}{\partial W} = 2 \frac{\partial F}{\partial E} \quad (\text{A.3.12})$$

Similarly, for the resources with multiple uses one obtains:

$$\frac{\partial F}{\partial O} = 2 \frac{\partial F}{\partial E} \quad (\text{A.3.13})$$

And

$$\frac{\partial F}{\partial B} = 2 \frac{\partial F}{\partial E} \quad (\text{A.3.14})$$

$$\frac{\partial H}{\partial E_e} = 0 \quad (\text{A.3.15})$$

$$\begin{aligned} \frac{\partial H}{\partial E_e} &= e^{-\delta t} \frac{\partial}{\partial E_e} \{ Lu(c(F(E(E_e)), s(E_e))) + \Sigma q_i(-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - \\ &\quad p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_{tb} + B) - p_{D_{nm}}(E_{nm}) + q_L \beta L \} \\ &= e^{-\delta t} \left\{ L \frac{\partial u}{\partial c} \left(\frac{\partial c}{\partial F} \frac{\partial F}{\partial E} \frac{\partial E}{\partial E_e} + \frac{\partial c}{\partial s} \frac{ds}{dE_e} \right) \right\} = 0 \\ &= \left\{ L \frac{\partial u}{\partial c} \left(\frac{1}{L} \frac{\partial F}{\partial E} (1) + \left(-\frac{1}{L} \right) \frac{ds}{dE_e} \right) \right\} = 0 \\ &\quad \frac{\partial u}{\partial c} \left(\frac{\partial F}{\partial E} - \frac{ds}{dE_e} \right) = 0 \end{aligned}$$

Thus

$$\frac{\partial F}{\partial E} = \frac{ds}{dE_e} \quad (\text{A.3.16})$$

Since $\frac{\partial E}{\partial E_i} = 1 \quad i = h, to, tb, nm, e$.

$$\frac{\partial F}{\partial E_i} = \frac{\partial F}{\partial E} = \frac{ds}{dE_e} \quad (\text{A.3.17})$$

$$\frac{\partial H}{\partial \alpha} = 0 \quad (\text{A.3.18})$$

$$\begin{aligned}
\frac{\partial H}{\partial \alpha} &= e^{-\delta t} \frac{\partial}{\partial \alpha} \{ Lu(c(F(E(\alpha)), \alpha) + \Sigma q_i(-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - \\
&\quad p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_{tb} + B) - p_{D_{nm}}(E_{nm}) + q_L \beta L \} \\
&= e^{-\delta t} \left\{ L \left[\frac{\partial u}{\partial c} \left(\frac{\partial c}{\partial F} \frac{\partial F}{\partial E} \frac{\partial E}{\partial \alpha} \right) + \frac{\partial u}{\partial \alpha} \right] \right\} = 0 \\
&= L \left(\frac{\partial u}{\partial c} \frac{1}{L} \frac{\partial F}{\partial E} (-L) \right) + L \left(\frac{\partial u}{\partial \alpha} \right) = 0 \\
&\quad L \left(-\frac{\partial u}{\partial c} \frac{\partial F}{\partial E} + \frac{\partial u}{\partial \alpha} \right) = 0
\end{aligned}$$

$$\frac{\partial u}{\partial \alpha} = \frac{\partial u}{\partial c} \frac{\partial F}{\partial E} \quad (\text{A.3.19})$$

Or

$$\frac{\partial F}{\partial E} = \frac{\frac{\partial u}{\partial \alpha}}{\frac{\partial u}{\partial c}} \quad (\text{A.3.20})$$

$$\frac{\partial H}{\partial \beta} = 0 \quad (\text{A.3.21})$$

$$\begin{aligned}
\frac{\partial H}{\partial \beta} &= e^{-\delta t} \frac{\partial}{\partial \beta} \{ Lu(\beta) + \Sigma q_i(-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - \\
&\quad p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_{tb} + B) - p_{D_{nm}}(E_{nm}) + q_L \beta L \} \\
&= e^{-\delta t} \left\{ L \frac{\partial u}{\partial \beta} + q_L L \right\} = 0
\end{aligned}$$

$$q_L = -\frac{\partial u}{\partial \beta} \quad (\text{A.3.22})$$

For the co-state variables:

$$\frac{de^{-\delta t} \cdot p_{D_{nm}}}{dt} = -\frac{\partial H}{\partial D_{nm}} \quad (\text{A.3.23})$$

$$\begin{aligned}
e^{-\delta t}(\dot{p}_{nm} - \delta p_{nm}) &= -e^{-\delta t} \frac{\partial}{\partial D_{nm}} \{Lu(c(F(E(E_{nm}(D_{nm})))))) + \Sigma q_i(-\mu_i K_i + I_i) - \\
&\quad p_{D_{h,w}}(E_h + W) - p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_{tb} + B) - \\
&\quad p_{D_{nm}}(E_{nm}) + q_L \beta L\} \\
(\dot{p}_{D_{nm}} - \delta p_{D_{nm}}) &= - \left\{ L \left(\frac{\partial u}{\partial c} \frac{\partial c}{\partial F} \frac{\partial F}{\partial E} \frac{\partial E}{\partial E_{nm}} \frac{\partial E_{nm}}{\partial D_{nm}} \right) - p_{D_{nm}} \frac{\partial E_{nm}}{\partial D_{nm}} \right\} \\
\dot{p}_{D_{nm}} &= p_{D_{nm}} \delta + \left(p_{D_{nm}} - \frac{\partial u}{\partial c} \frac{\partial F}{\partial E} \right) \frac{\partial E_{nm}}{\partial D_{nm}} \tag{A.3.24}
\end{aligned}$$

From A.3.9, A.3.11 and A.3.12 one gets:

$$\frac{p_{D_{nm}}}{p_{D_{nm}}} = \delta \tag{A.3.25}$$

$$\frac{de^{-\delta t} \cdot p_{D_{h,w}}}{dt} = - \frac{\partial H}{\partial D_{h,w}} \tag{A.3.26}$$

$$\begin{aligned}
e^{-\delta t}(\dot{p}_{D_{h,w}} - \delta p_{D_{h,w}}) &= -e^{-\delta t} \frac{\partial}{\partial D_{h,w}} \{Lu(c(F(E(E_h(D_{h,w}), W(D_{h,w}), W(D_{h,w})))) + \\
&\quad \Sigma q_i(-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - p_{D_{to,o}}(E_{to} + O) - \\
&\quad p_{D_{tb,b}}(E_{tb} + B) - p_{D_{nm}}(E_{nm}) + q_L \beta L \\
(\dot{p}_{D_{h,w}} - \delta p_{D_{h,w}}) &= - \left\{ L \frac{\partial u}{\partial c} \frac{\partial c}{\partial F} \left[\frac{\partial F}{\partial E} \left(\frac{\partial E}{\partial E_h} \frac{\partial E_h}{\partial D_{h,w}} + \frac{\partial E}{\partial W} \frac{\partial W}{\partial D_{h,w}} \right) + \right. \right. \\
&\quad \left. \left. \frac{\partial F}{\partial W} \frac{\partial W}{\partial D_{h,w}} \right] - p_{D_{h,w}} \left(\frac{\partial E_h}{\partial D_{h,w}} + \frac{\partial W}{\partial D_{h,w}} \right) \right\}
\end{aligned}$$

$$\dot{p}_{D_{h,w}} = p_{D_{h,w}} \delta + (p_{D_{h,w}} - \frac{\partial u}{\partial c} \frac{\partial F}{\partial E}) \frac{\partial E_h}{\partial D_{h,w}} + (p_{D_{h,w}} + \frac{\partial u}{\partial c} \frac{\partial F}{\partial E} - \frac{\partial u}{\partial c} \frac{\partial F}{\partial W}) \frac{\partial W}{\partial D_{h,w}} \tag{A.3.27}$$

From $\frac{\partial u}{\partial c} \frac{\partial F}{\partial E} = p_{D_{h,w}}$ and A.3.12 one obtains:

$$\frac{\dot{p}_{D_{h,w}}}{p_{D_{h,w}}} = \delta \quad (\text{A.3.28})$$

$$\frac{de^{-\delta t} \cdot q_{nm}}{dt} = -\frac{\partial H}{\partial K_{nm}} \quad (\text{A.3.29})$$

$$\begin{aligned} e^{-\delta t}(\dot{q}_{nm} - \delta q_{nm}) &= -e^{-\delta t} \frac{\partial}{\partial K_{nm}} \{Lu(c(F(E(E_{nm}(K_{nm})))))) + \Sigma q_i(-\mu_i K_i + I_i) - \\ &\quad p_{D_{h,w}}(E_h + W) - p_{D_{to,o}}(E_{to} + O) - \\ &\quad p_{D_{tb,b}}(E_{tb} + B) - p_{D_{nm}}(E_{nm}) + q_L \beta L\} \\ (\dot{q}_{nm} - \delta q_{nm}) &= -\left\{ L \frac{\partial u}{\partial c} \frac{\partial c}{\partial F} \frac{\partial F}{\partial E} \frac{\partial E}{\partial E_{nm}} \frac{\partial E_{nm}}{\partial K_{nm}} - q_{nm} \mu_{nm} - p_{D_{nm}} \left(\frac{\partial E_{nm}}{\partial K_{nm}} \right) \right\} \\ \dot{q}_{nm} &= q_{nm}(\delta - \mu_{nm}) + \frac{\partial E_{nm}}{\partial K_{nm}} \left(p_{D_{nm}} - \frac{\partial u}{\partial c} \frac{\partial F}{\partial E} \right) \end{aligned} \quad (\text{A.3.30})$$

From A.3.4 one obtains:

$$\frac{\dot{q}_{nm}}{q_{nm}} = \delta - \mu_{nm} \quad (\text{A.3.31})$$

$$\frac{de^{-\delta t} \cdot q_h}{dt} = -\frac{\partial H}{\partial K_h} \quad (\text{A.3.32})$$

$$\begin{aligned} e^{-\delta t}(\dot{q}_h - \delta q_h) &= -e^{-\delta t} \frac{\partial}{\partial K_h} \{Lu(c(F(E(E_h(K_h)))))) + \Sigma q_i(-\mu_i K_i + I_i) - \\ &\quad p_{D_{h,w}}(E_h + W) - p_{D_{to,o}}(E_{to} + O) - \\ &\quad p_{D_{tb,b}}(E_{tb} + B) - p_{D_{nm}}(E_{nm}) + q_L \beta L\} \\ (\dot{q}_h - \delta q_h) &= -\left\{ L \frac{\partial u}{\partial c} \frac{\partial c}{\partial F} \frac{\partial F}{\partial E} \frac{\partial E}{\partial E_h} \frac{\partial E_h}{\partial K_h} - q_h \mu_h - p_{D_{h,w}} \left(\frac{\partial E_h}{\partial K_h} \right) \right\} \\ \dot{q}_h &= q_h(\delta - \mu_h) + \frac{\partial E_h}{\partial K_h} \left(p_{D_{h,w}} - \frac{\partial u}{\partial c} \frac{\partial F}{\partial E} \right) \end{aligned} \quad (\text{A.3.33})$$

From $\frac{\partial u}{\partial c} \frac{\partial F}{\partial E} = p_{D_{h,w}}$ one gets then:

$$\frac{\dot{q}_h}{q_h} = \delta - \mu_h \quad (\text{A.3.34})$$

$$\frac{de^{-\delta t} \cdot q_w}{dt} = -\frac{\partial H}{\partial K_w} \quad (\text{A.3.35})$$

$$\begin{aligned} e^{-\delta t}(\dot{q}_w - \delta q_w) &= -e^{-\delta t} \frac{\partial}{\partial K_w} \{ Lu(c(F(E(W(K_w))), W(K_w))) + \Sigma q_i(-\mu_i K_i + I_i) - \\ &\quad p_{D_{h,w}}(E_h + W) - p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_{tb} + B) - \\ &\quad p_{D_{nm}}(E_{nm}) + q_L \beta L \} \\ \dot{q}_w - \delta q_w &= - \left\{ L \frac{\partial u}{\partial c} \frac{\partial c}{\partial F} \left(\frac{\partial F}{\partial E} \frac{\partial E}{\partial W} \frac{\partial W}{\partial K_w} + \frac{\partial F}{\partial W} \frac{\partial W}{\partial K_w} \right) - \right. \\ &\quad \left. q_w \mu_w - p_{D_{h,w}} \left(\frac{\partial W}{\partial K_w} \right) \right\} \\ \dot{q}_w &= q_w(\delta - \mu_w) + \left(p_{D_{h,w}} - \frac{\partial u}{\partial c} \left[\frac{\partial F}{\partial W} - \frac{\partial F}{\partial E} \right] \right) \frac{\partial W}{\partial K_w} \end{aligned} \quad (\text{A.3.36})$$

From $\frac{\partial u}{\partial c} \frac{\partial F}{\partial E} = p_{D_{h,w}}$ and A.3.12 one gets:

$$\frac{\dot{q}_w}{q_w} = \delta - \mu_w \quad (\text{A.3.37})$$

Similarly, for the energy and non-energy resources one obtains:

$$\frac{\dot{q}_o}{q_o} = \delta - \mu_o; \quad \frac{\dot{q}_{to}}{q_{to}} = \delta - \mu_{to}; \quad (\text{A.3.38})$$

$$\frac{\dot{q}_b}{q_b} = \delta - \mu_b; \quad \frac{\dot{q}_{tb}}{q_{tb}} = \delta - \mu_{tb}; \quad (\text{A.3.39})$$

$$\frac{\dot{q}_{nm}}{q_{nm}} = \delta - \mu_{nm} \quad (\text{A.3.40})$$

From A.3.2, A.3.38, A.3.39 and A.3.40 one obtains:

$$\mu_h = \mu_w = \mu_{tb} = \mu_b = \mu_{to} = \mu_o = \mu_{nm} = \mu \quad (\text{A.3.41})$$

And

$$\frac{\dot{q}_h}{q_h} = \frac{\dot{q}_w}{q_w} = \frac{\dot{q}_{to}}{q_{to}} = \frac{\dot{q}_o}{q_o} = \frac{\dot{q}_{tb}}{q_{tb}} = \frac{\dot{q}_{nm}}{q_{nm}} = \delta - \mu \quad (\text{A.3.42})$$

$$\frac{de^{-\delta t} \cdot q_0}{dt} = -\frac{\partial H}{\partial K_0} \quad (\text{A.3.43})$$

$$\begin{aligned} e^{-\delta t}(\dot{q}_0 - \delta q_0) &= -e^{-\delta t} \frac{\partial}{\partial K_0} \{Lu(c(F(K_0))) + \Sigma q_i(-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - \\ &\quad - p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_{tb} + B) - \\ &\quad p_{D_{nm}}(E_{nm}) + q_L \beta L\} \\ (\dot{q}_0 - \delta q_0) &= -L \left(\frac{\partial u}{\partial c} \frac{\partial c}{\partial F} \frac{\partial F}{\partial K_0} \right) - q_0 \mu_0 \end{aligned}$$

$$\dot{q}_0 = q_0(\delta - \mu_0) - \frac{\partial u}{\partial c} \frac{\partial F}{\partial K_0} \quad (\text{A.3.44})$$

From A.3.2 one gets:

$$\frac{\dot{q}_0}{q_0} = (\delta - \mu_0) - \frac{\partial F}{\partial K_0} \quad (\text{A.3.45})$$

From A.3.42 one obtains:

$$\frac{\partial F}{\partial K_0} = (\mu - \mu_0) \quad (\text{A.3.46})$$

$$\frac{de^{-\delta t} \cdot q_L}{dt} = -\frac{\partial H}{\partial L} \quad (\text{A.3.47})$$

$$\begin{aligned}
e^{-\delta t}(\dot{q}_L - \delta q_L) &= -e^{-\delta t} \frac{\partial}{\partial L} \{ Lu(c(L, F(E(L)))) + \Sigma q_i(-\mu_i K_i + I_i) - p_{D_{h,w}}(E_h + W) - \\
&\quad p_{D_{to,o}}(E_{to} + O) - p_{D_{tb,b}}(E_{tb} + B) - \\
&\quad p_{D_{nm}}(E_{nm}) + q_L \beta L \} \\
(\dot{q}_L - \delta q_L) &= - \left\{ L \frac{\partial u}{\partial c} \left(\frac{\partial c}{\partial L} + \frac{\partial c}{\partial F} \frac{\partial F}{\partial E} \frac{\partial E}{\partial L} \right) + u + q_L \beta \right\} \\
\\
\dot{q}_L &= q_L(\delta - \beta) + \frac{\partial u}{\partial c} \left(c + \alpha \frac{\partial F}{\partial E} \right) - u \tag{A.3.48}
\end{aligned}$$