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Public Goods Provision and Competition Among Districts

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Dissertação apresentada ao Programa de Pós-Graduação em Economia da Universidade Federal de Pernambuco, como requisito parcial para obtenção do título de mestre em Economia. Área de concentração: Teoria Econômica.

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Resumo

As regras de tributação e alocação de recursos públicos variam muito ao redor do mundo, tornando muito importante o estudo das vantagens e desvantagens para a população de cada sistema. Em uma Federação, os distritos podem ter autonomia para escolher políticas e cobrar impostos. Nas últimas decadas, os Estados brasileiros engajaram uma forte competição para atrair empresas. Dessa forma, esse trabalho buscar analisar a provisão de ben públicos para estados e firmas, além de comparar a provisão centralizada e decentralizada dos bens. A competição na provisão de bens públicos é abordada propondo uma economia em que o Governo fornece um bem público aos indivíduos e um fator público às empresas e também mantém parte da receita tributária total como rendas para si. Em seguida, estuda-se a concorrência aplicando essa economia ao Modelo de Hotelling de Diferenciação Espacial. Dois distritos competem no fornecimento de bens públicos, este trabalho constata que não haverá diferenciação de impostos e níveis dos bens ofertados no equilíbrio, no qual ambos distritos terão a mesma renda. Além disso, existem condições sob as quais o governo pode alocar receitas recebidas de indivíduos para beneficiar as empresas. Comparamos os cenários centralizado e descentralizado e constatamos, ao mesmo nível de impostos, o regime centralizado gera mais renda ao poder público.

Palavras-chaves: Competição bens públicos; Hotelling Model; competição fiscal; bens públicos locais; fiscal federalismo.

Abstract

Taxation rules and allocation of public resources vary a lot around the world, making it very important to study the advantages and drawbacks for the population of each system. In a Federation, the districts may have the autonomy to choose policies and collect taxes. In Brazil, states have been competing for firms for the past decades. Thus, this study aims to analyze public goods provision for firms and individuals and compare the provision when the government is centralized and decentralized. The problem of public goods provision competition is approached by modeling an economy in which the Government provides a public good to the individuals and a public factor to the firms and also keeps part of the total tax revenue as rents to the office. Then, the competition is studied by applying this economy to the Hotelling Model of Spatial Differentiation. Two districts engage in competition in the provision of public goods, this works find that there will be no differentiation in taxes and levels provided in the equilibrium, both districts will have the same rent. Moreover, there are conditions under which the government can allocate revenue received from individuals to benefit firms. We compare the centralized and decentralized scenarios and find that at the same level of taxes, centralized regimé yields more rent to the public authority.

Keywords:Public goods competition, tax competition; Public factor; Hotelling Model; fiscal competition; local public goods; fiscal federalism.

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1 Introduction

In a federation, there are many taxation rules and ways to choose how the tax revenue will be allocated among the districts and how it will be spent in the society. The central government can collect all the taxes and provide national public investment, or can allocate the revenue between the districts, among many other tax rules present in the federations. When each district has the possibility of collecting tax in its territory the district competition emerges. They will provide different fiscal conditions to attract firms and individuals and increase their economic activity.

There are several studies and evidence that fiscal competition has been present in Brazil, stating that competition to attract firms has become part of the political agenda in the last decades. The competition increased in part because of changes in the the focus of the central government on national policies, leaving the regional decisions to local authorities, part because of some changes in the laws of tax revenue allocation. This is the subject of Porsse (2005). Versano (1997) analyzes the consequences to the Brazilian states of the strong competition in giving incentives to firms. He finds that sometimes states cannot benefit as expected. Frota, Lima, and Melo (2014) studied the case of Pernambuco, where there have been many fiscal advantages in attracting firms to the state and developing areas far from the capital; they conclude that the tax reduction itself is not enough to achieve the state goals.

These are some examples that in Brazil there have been large effort to provide firms with fiscal incentives to attract them to a place. However, not always it has been shown to be a successful policy. Most of the polices have the objective to attract firms to develop a more intensive economic activity. However, the government has to allocate revenue among incentives to firms or spending the resources favoring individuals with better living and tax conditions. It could also attract people to the state since they would have incentives to live in their state seeking a better quality of life.

Thus, the objective of this work is to study the allocation of public resources among individuals and firms by the local government. Bringing to the analysis the possibility of introducing the individuals as a possible source of income and way to develop the place. More individuals mean more people paying taxes and consuming goods, it could increase the tax revenue for the Government since the expenditures with incentives for firms have not been full success.

In many countries, the functions and attributions of the government role are shared at different levels. It can be composed of central federal and local levels, that are connected and together must work to maximize the welfare of the entire federation. Some decisions that are made by a central authority will be followed by all local districts. However, there are some specific policies that may be more efficient if made by local authorities, due to the characteristics of each jurisdiction. Thus, the allocation of which level of public power would be responsible for which policies are a subject that is present in the literature on Fiscal Federalism and Public Finance. In this scenario, the public goods provision is analyzed, if the goods must be provided and funded by the local or central governments.

Oates (1999) describes the basics of the Fiscal Federalism theory and studies the frameworks for the assignments of functions of the different levels of government and the financial instruments to approach them. In a place where preferences change among districts, the decentralized provision may increase welfare compared to the case the central government chooses a uniform policy for all districts.

The desired level of a public good can vary among places depending on local preferences and the cost of provision. Thus, in some cases, it is difficult for the central authority to deal with local heterogeneity, even if they are willing to be benevolent and make policies to maximize the welfare in each district, instead of providing a uniform level. Oates also states that when costs are the same but demands vary, there is a welfare loss from the central agency provision.

Beyond the demand and costs of each district, there is the taxation discussion. It approaches the study of which level of government must receive the tax revenue. If the central government must collect the total tax, and the fact that the way it is distributed to the lower levels of governance can change and affect the local provision. If the central government allocates an equal amount of the tax revenue to each district but they have different demands for the public good, there is a possibility of inefficient spending.

The public good can be underprovided if the central government allocates less money than it is needed, or there will be excess in a place where people demand less. Alternatively, each district can have a local tax as well. There are many ways to collect taxes, provide public goods and choose policies in an economy made out of districts. In a decentralization model, there is also a new issue, the competition between districts.

Overall, Oates's review of fiscal federalism discusses some conditions under which a centralized system would be more efficient and under other conditions would be better to have a decentralized system. For example, a central government can be responsible for some policies, let's say, macroeconomics policies and national public goods and it is the local government's role to provide some public goods to people in its district. A local authority can choose the level to provide, having more information about costs and particular tastes. Taste for the public good will also be explored.

The public goods provision in districts also can change if individuals are able to move around the federation. Bloch (2014) introduces this feature to the Oates framework, mobility and spillovers may change the provision. In his work, there are two jurisdictions that compete for individuals and have the same characteristics except for the taxation/goods provision package. Their goal is to determine which scenarios would favor the decentralized and which would favor the centralized régime. Household mobility is a parameter that measures the attachment to the place, if individuals are very mobile, for example, they would have an incentive to go to the jurisdiction where more of the good is provided, then, in this case, the jurisdiction wouldn't handle the demand and it may favor the centralized régime. Beyond the attachment parameter, which will be used with the same meaning in this present work, he also considers the fact the jurisdictions only differ in how the taxes and public goods levels are, this is assumed in this work as well.

Centralized and decentralized provision of public goods is also a subject of Besley and Coate (2003). They deal with the presence of spillovers and the type of legislature. They also consider that the place where the persons in charge are from affects the decision on the levels of the public good. If they tend to provide the goods accordingly to the preferences of one district since people from there voted for them. These facts are used to determine the advantages and drawbacks of each régime. Thus, depending on how the legislature behaves, there can be excessive spending or a misallocation of the public good, a district where people demand a lot of the good may not have the good the way they wanted.

Both papers illustrate the importance of the theme and how wide the ways of approaching it are, many features can be added and they will all have implications. In this work, an economy will be constructed with some specific characteristics, one important new feature is that the firm's decision will be added to the district scenario and they will also demand a public good. Thus, instead of considering a district where a public good will be provided to the individuals and the household surplus is maximized, there will be two public goods, the individual's utility and the firm's profits will be maximized after paying the tax.

Another feature that will be introduced into the economy constructed in this work is that the government is not benevolent. As in Wooders and Zissimos (2002), the government maximizes rents to the office. There will be a rent kept by the incumbent, he will work to maximize the individual's utility and the firm's profit. However, a fraction of the total tax revenue will be kept to the government. Gordon Tullock introduced the concept of rent-seeking, it can sometimes be linked to corruption, but it can be interpreted as a social loss due to the use of the government income in a non-productive distributive situation. In this work, it will be a fraction of the total income received that is not allocated to any agent. It is a part of the tax revenue that would be spent on public goods provision if the government didn't keep it for itself.

Once the economy of this work is formalized with the proposed characteristics, it is time to move to the second main objective of this work. Introduce the competition to the model and see the location decisions of firms and individuals. The way it is done is by breaking the economy into two districts, thus there will be two identical districts formed by firms, individuals, and a local government. They are identical at everything but the level of public goods and taxes. Many works have approached the public goods competition by using the Hotelling Model. The model will be used following the approach described in The Theory of Industrial Organization by Jean Tirole, which will be discussed later in this work. The subject of public goods provision by districts that are competing for individuals and taxation competition has been discussed through the Hotelling Model in some works, for example, Önemli(2012), Li-Chen Hsu(2005), Wooders and Zissimos (2002), Hohaus, Konrad, and Thum (1994).

In the traditional basic Hotelling Model, two firms would compete for consumers and they differ in location, however, the location can be interpreted as the heterogeneity of tastes for a good, this interpretation will be detailed in the competition section of this work. Also, when talking about public goods, the two districts aren't necessarily providing different levels of the same good. Actually, they may be providing different types of goods, for instance, different amenities. Hohaus, Konrad, and Thum (1994) approach the competition in this scenario, for example, two cities decide on the traffic infrastructure. A city can be a car city or a pedestrian city, but not both. Thus, the consumer will decide where to live based on his preferences, which city would yield him more satisfaction, and this would be the Hotelling line. This is an example from Hohaus, Konrad, and Thum (1994) to illustrate the kind of good that satisfies the characteristics of the public goods studied in this work.

Wooders and Zissimos (2002) also use the Hotelling Model to study government competition. In their work there are mobile firms and two governments compete for them in the provision of important amenities to the firm's production. They don't consider individuals in their work and focus on discussing the efficiency of competition and the possibility of tax discrimination; however, the way their economy is structured and the features are very similar to those presented in this work. First, they consider that their companies use amenities provided by the government, and it will affect their productivity and profits, and that is exactly the intention of this work when we say there is a public good for the firms.

They also assume that the government is Leviathans, and it is described as having the objective of maximizing rents to offices through amenity provision, and the rent is defined as the difference between taxes and the cost of provision. This government framework is also used here, however, there are two differences. First, in this work, they maximize rents providing public goods for individuals and for firms. Second, rent is also defined as the difference between revenue and costs, but there is a function form for it, it depends on a parameter that is introduced, the fraction of the total tax revenue.

2 The economy with one central Government

The first step to trying to understand how the government chooses the provision of public goods is to describe the agents of the economy and how they interact. Three agents compose the economy proposed in this work: individuals, firms, and the government. There are two public goods provided by the government. One of them is consumed by the individuals and the other one is used in the firm's production. Both individuals and firms pay lump sum taxes, τ_1 and τ_2, respectively. Also, the government keeps its rent from the total tax revenue. Each agent will be described in detail in the following subsections.

2.1 Agents

2.1.1 Individuals

The first group of agents to be described is the individuals. We define individuals as the people who live in the district and maximize utility. The individual's utility is a function of the private good and the public good. The private good can represent how much money he or she can use to buy any good, while the public good is a good that is provided only by the government. It is present in the individual's utility, such as public transportation structure, safety, parks, and nature maintenance, among others.

The public good for the individuals is denoted by g_1 and the preference for the public good is represented by the parameter θ. It shows how much the individual cares about the public good, if the public good is critical to an individual, his or her θ will be higher than the θ of an individual who doesn't care much.

Thus, the utility of individual i can be expressed as the following:

$$u_i(c_i, g_1) = c + \theta_i \ln(g_1)$$

The individual also pays a lump sum tax, τ_1. The high the taxes are, the less the consumers will have available for the private good. Therefore, there is a trade-off between the goods, if taxes are high, the individual can spend less on the other goods. And, the utility would be lower unless, of course, if high taxes implicate more public goods offered, it could compensate for the loss of having to pay high taxes.

We assume that c is the net consumption, in other words, c is the amount available to buy the private good. Therefore, we can subtract the tax paid, τ_1, from his total income, y_1 and define $c = y_1 - \tau_1$. And the individual utility is

$$u_i(g_1) = y_1 - \tau_1 + \theta_i \ln(g_1) \tag{2.1}$$

Since any individual has his own preferences, the θ varies. However, considering single-peaked preferences, the Median Voter Theorem can be applied, and it is the level of θ considered by the government.

Applying The Median Voter Theorem, the subscribed i for individuals is removed, and θ is the parameter of the median voter considered.

$$u(g_1) = y_1 - \tau_1 + \theta \ln(g_1) \tag{2.2}$$

Therefore, the individual pays a lump sum tax and has preferences for a public good that increases utility. The individual behaves to maximize utility.

2.1.2 Firms

In this model, the firms have a Cobb-Douglas production function with two inputs capital and labor, denoted by K and L, respectively, to produce the good, y_2. The cost of labor is w and the cost of capital is r, both of the costs are assumed to be exogenous. The firm's decision for the input labor is not considered in this case, so L is also assumed to be exogenous. So far, the production function is very ordinary.

However, we have introduced a public factor into the function. The public factor is a good used by the firms that are provided by the government. It is also interpreted as amenities for the firms. It can be anything that is provided by the government and affects the firm's productivity. As Wooders and Zissimos (2003) show that an example can be an amenity, firms have diverse technological requirements for levels of amenity provision. They use the legal system of building a new factory and good access to consumers can be amenities that firms would care about.

The public good for the firms is represented by g_2 and introduced into the production function as a "Harrod-neutral"variable. Thus, g_2 is interpreted as a variable that is "capital augmenting,"one unit of capital is more productive when the level of g_2 is higher. The interpretation is analogous to the technology in the Solow Model described in the book Introduction to Economic Growth by Charles Jones. The level of g_2 is exogenous for the firm, it is determined by the government. Firms pay a lump sum tax, denoted by τ_2.

The parameters α and β represent the share in the production of the respective inputs. Introducing g_2 as a "capital augmenting"variable: The Cobb-Douglas production function is written as follows:

$$y_2 = A\,(g_2 K)^{\alpha}\, L^{\beta} \tag{2.3}$$

An interesting point that must be highlighted about the production function is the role of α. The marginal productivity of the capital is $A\alpha K^{1-\alpha}g_2^{\alpha}L^{\beta}$. As expected, the higher the level of public goods, the same level of capital will be more productive. Moreover, the way the g_2 affects the productivity depends on the share of the input capital in the production function, α. So, if α is high, a higher level of the public good would increase the firms' productivity and profits. In other words, α is the parameter representing how the public good affects the firms. Thus, for example, companies with different α would be affected differently by g_2. So, α is a parameter that the government looks at when deciding the public good level. Because it represents how important the public good is for the firms, and his decision is to provide goods for firms and individuals, it is crucial to understand how much individuals and firms care about the good.

Once we defined the production functions, it is time to analyze the firm's profit. Firms maximize profit, defined as the total revenue subtracted of the total costs. There are the lump-sum tax, and the costs of capital and labor, r and w, respectively. And the revenue is the price of the good multiplied by the product, we assume that price levels equal one. This leads to the firm's profit in the equation below:

$$\pi = y_2 - \tau_2 - wL - rK$$

Plugging y_2 in the profit:

$$\pi = Ag_2^{\alpha}K^{\alpha}L^{\beta} - \tau_2 - wL - rK \tag{2.4}$$

Since the firm maximizes profit, and only chooses the capital, the first-order condition for the firm is: $A\alpha K^{\alpha-1}g_2^{\alpha}L^{\beta} - r = 0$. And the level of capital chosen by the firm is a function of the public factor and the parameters of the model.

$$K^* = \left[\frac{Ag_2^{\alpha}L^{\beta}\alpha}{r}\right]^{\dfrac{1}{1-\alpha}} \tag{2.5}$$

Thus, the production function can be rewritten considering the choice for capital, then the product will depend on the firm's parameters and on the level of g_2. The government determines the level of g_2, it is exogenous for the firm.

Therefore, the production function and profit are written as a function of the firm's parameters.

$$y_2 = g_2^{\frac{\alpha}{1-\alpha}}L^{\frac{\beta}{1-\alpha}}A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{r}\right)^{\dfrac{\alpha}{1-\alpha}} \tag{2.6}$$

$$\pi = g_2^{\frac{\alpha}{1-\alpha}}L^{\frac{\beta}{1-\alpha}}A^{\frac{1}{1-\alpha}}\left[\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} - r\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}\right] - \tau_2 - wL \tag{2.7}$$

The profit is written this way because it is a function of exogenous variables and parameters known in the economy, so the government has access to them when deciding the level of public goods.

2.1.3 Government

The third agent in this economy is the government. Its role is to collect taxes and provide public goods. The Government offers goods that aren't possible to be offered by the private sector. It's offered a public good for the individuals, g_1, and a public factor for the firms, g_2. The goods are important for an individual's utility and a firm's profit, so it makes sense to assume that they agree to pay taxes to the government.

Taxes are denoted by τ_1 for the individuals and τ_2 for the firms. However, that is not all, beyond receiving taxes and spending them on the provision of the goods, the government keeps a fraction of the total tax revenue as rent. Thus, the government chooses the level of the public good to maximize consumer's utility and firm's profit, keeping part of the total tax collected.

So, consumers will have the maximum utility possible and firms maximum profits possible, subject to the fact that the government will also care about its own rent. If there was no government, there will be no public goods, but both agents need these goods, so they pay the tax even aware of the fact that not all of it will be allocated to provide the public goods.

Therefore, we can write and detail the government problem. The public authority knows how firms and individuals benefit from each good. It is to say that there is complete information about an individual's utility and firm's profit. Under this knowledge, the government will choose the levels of g_1 and g_2 that maximize the utility and profit functions, subject to the public constraint.

One last step before going into the problem is to understand how the rent is introduced to the model. It's assumed that the rent is a fraction of the total tax revenue, so part of the revenue will be kept for the institution and unspent on the goods provision. Let's define η as the fraction of the total revenue kept. The fraction η is assumed to be greater than zero and less than one. In order to guarantee it, α is assumed to be greater than zero and less the $\frac{1}{2}$. It means that some rent is kept from the total revenue, but not all of it.

Since η is the fraction of the total tax revenue that the government keeps to itself and the total tax revenue is the sum of the lump sum taxes paid by firms and individuals, the rent is written as the following:

$$r = \eta\left(\tau_1 + \tau_2\right) \tag{2.8}$$

Now, the public constraint can be explored in detail. The public constraint must show us how the income collected by the government is allocated. We know that there is only one way by which the government receives revenue, the lump sum taxes. On the other hand, this revenue must be allocated to public goods and the rent kept. Thus, the following equation shows the public constraint, the amount spent to provide both goods and the rent kept must be equal to the total tax revenue.

$$r + g_1 + g_2 = \tau_1 + \tau_2 \tag{2.9}$$

The definition of rent can be plugged into the public constraint to find the constraint that will be used in the problem.

$$\eta(\tau_1 + \tau_2) + g_1 + g_2 = \tau_1 + \tau_2 \tag{2.10}$$

$$g_1 + g_2 = (1 - \eta)(\tau_1 + \tau_2) \tag{2.11}$$

The equation above illustrates what happens with the amount spent to provide the goods and the η of the government. If the η is high, then a smaller part of the revenue will actually be spent on the provision. If there is no rent, $\eta = 0$, then the total tax revenue would be spent exclusively to offer the goods. And, if $\eta = 1$, then there would be no public good.

$$\frac{g_1 + g_2}{1 - \eta} = \tau_1 + \tau_2 \tag{2.12}$$

Equation (12) is just a way to rewrite the public constraint that will be used further.

In this subsection, the role of the government was explored and the details and implications of assumptions about the introduction of the rent in the problem and the public constraint. The next section is dedicated to the government problem. Subject to all the information discussed so far, how are the levels of public goods chosen?

2.1.4 Public Goods Provision

The last section explained the characteristics of individuals and firms and how the public goods affect their utility and profits, respectively. In both cases, the public good is exogenous for the agents, they are located in a place where the provision is determined by the public authority. Now is the moment to analyze how the government chooses this level.

It is the government's role to offer the goods using the tax revenue. As was assumed, the government knows how each good affects individuals and firms and has a rent to keep from the total tax revenue. As a reminder, the goods increase utility and profits and the government is the only supplier; so the government must provide a level that makes the utility and profit as high as possible. So the objective function is the welfare function, the sum of profits and utility. The welfare function can be written as $W = U(g_1) + \pi(g_2)$.

The government will provide the levels that maximize the objective function, but we assume that the government only does this because it is keeping some rent, otherwise, no good would be provided. Therefore, the objective function is maximized subject to the public constraint. To summarize the problem, the government maximizes the sum of utility and profits subject to the constraint that the money spent on the provision and kept as rent is equal to the total tax revenue. Thus, the problem that the government must solve is

$$\max_{g_1, g_2} W(g_1, g_2)$$

s.a.

$$g_1 + g_2 = (1 - \eta)(\tau_1 + \tau_2)$$

The objective function is $W(g_1, g_2) = U(g_1) + \pi(g_2)$, since the utility and the profits are known:

$$W = y_1 - \tau_1 + \theta \ln g_1 + g_2^{\frac{\alpha}{1-\alpha}} L^{\frac{\beta}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r}\right)^{\dfrac{\alpha}{1-\alpha}} - \tau_2 - wL - r\left[\frac{\bar{A} g_2^{\alpha} L^{\beta} \alpha}{r}\right]^{\dfrac{1}{1-\alpha}}$$

$$W = y_1 + \theta \ln g_1 + g_2^{\frac{\alpha}{1-\alpha}} L^{\frac{\beta}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r}\right)^{\dfrac{\alpha}{1-\alpha}} - wL - r\left[\frac{\bar{A} g_2^{\alpha} L^{\beta} \alpha}{r}\right]^{\dfrac{1}{1-\alpha}} - (\tau_1 + \tau_2)$$

Plugging in the utility and profit functions in the equation yields us a welfare function that is a function of the parameters, the public goods, and the taxes. Fortunately, the public constraint appears explicitly in W,

$$\frac{g_1 + g_2}{1 - \eta} = \tau_1 + \tau_2$$

The problem becomes unconstrained. And the function to be maximized with respect to g_1 and g_2 is:

$$W = y_1 + \theta \ln g_1 + g_2^{\frac{\alpha}{1-\alpha}} L^{\frac{\beta}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r}\right)^{\dfrac{\alpha}{1-\alpha}} - wL - r\left[\frac{\bar{A} g_2^{\alpha} L^{\beta} \alpha}{r}\right]^{\dfrac{1}{1-\alpha}} - \left(\frac{g_1 + g_2}{1 - \eta}\right)$$

Solving the first order conditions for each good it is possible to find the level of g_1 and g_2.

Proposition 1 *The public goods levels are*

$$g_1 = (1-\eta)\,\theta$$

and

$$g_2 = \left\{(1-\eta)\frac{\alpha}{1-\alpha}L^{\frac{\beta}{1-\alpha}}A^{\frac{1}{1-\alpha}}\left[(\frac{\alpha}{r})^{\frac{\alpha}{1-\alpha}} - r(\frac{\alpha}{r})^{\frac{1}{1-\alpha}}\right]\right\}^{\frac{1-\alpha}{1-2\alpha}}$$

.

The level of the public goods depends on the parameters of each agent and the η. The higher η is, the lower g_1 and g_2 will be, and the higher would be the government rent.

One may think that if the if the η is very high the levels of the goods will be very low. And since the government wants the maximum rent possible, η could be very high. However, η is also a parameter, the government does not choose it. And, a higher η does not imply a higher rent, there is a η that maximizes rent and it will depend on the parameters α and θ. It it is shown in the appendix.

On the other hand, if the η is close to zero, then the levels of g_1 and g_2 would be similar to a world where there is no rent kept by the public authority.

Proposition 2 *Because α is assumed to be between zero and $\frac{1}{2}$, g_2 will always be positive.*

Proof.

Let's stat with g_2,

$$g_2 = \left\{(1-\eta)\frac{\alpha}{1-\alpha}L^{\frac{\beta}{1-\alpha}}A^{\frac{1}{1-\alpha}}\left[(\frac{\alpha}{r})^{\frac{\alpha}{1-\alpha}} - r(\frac{\alpha}{r})^{\frac{1}{1-\alpha}}\right]\right\}^{\frac{1-\alpha}{1-2\alpha}} \tag{2.13}$$

The expression of g_2 is a multiplication of many terms, if they are all positive, then g_2 is also positive. It is assumed that $\eta < 1$, then$(1-\eta) > 0$. The parameter α is less than one, then $\frac{\alpha}{1-\alpha} > 0$. L and A are positive. Thus, all that is left to check is the signal of $\left[(\frac{\alpha}{r})^{\frac{\alpha}{1-\alpha}} - r(\frac{\alpha}{r})^{\frac{1}{1-\alpha}}\right]$, and it is positive if $\alpha < 1$. Therefore, the condition is for $g_2 > 0$ is $1 > \alpha$. We assume that α is less than $\frac{1}{2}$, it implies that $g_2 > 0$, for any α considered.

■

Now, to analyze g_1, $g_1 > 0$ if $\theta > 0$, which is assumed to be, thus g_1 is also positive.

From the maximization of the objective function of the government, the level o public good is $g_1* = \theta\,(1-\eta)$. Thus, it is possible to relate the individual's preference for the public good with the fraction of total revenues kept by the incumbent, η.

Proof.

It is directly to conclude it from the optimal level of g_1. We know that $g_1 > 1$, but $g_1 = (1-\eta)\,\theta$. Since $\eta < 1$, $(1-\eta)\,\theta > 1 \implies \theta > 1$.

■

Proposition 3 *The rent kept by the government is positive.*

The rent definition and the public constraint can be combined to :

$$r = \frac{\eta}{1-\eta}\,(g_1 + g_2) \tag{2.14}$$

From proposition 2, g_1 and g_2 are positive, thus $(g_1 + g_2) > 0$. $\frac{\eta}{1-\eta}$ is also positive. Therefore, as expected, the way the model is stated allows the government to keep rent.

Propositions 2 to 4 state some characteristics of the model. Their role in this work is to make sure that all the features that were proposed to the economy are satisfied by the framework chosen. We can conclude that the way that consumers, firms, and the government are defined yields us an economy where the government provides two public goods. Both of them are provided at levels that increase an individual's utility and firm's profit, and also yield a positive rent to the government. Thus, the mathematical framework used is consistent with the economic intuition intended.

3 Dividing The Economy in Districts

Up to this point, this work focused on building an economy with some specific characteristics. The discussion approached a situation in which g_1 and g_2 are provided by the central government of an economy. The government provides the goods and keeps rent. Thus, any individual or firm who was born in this economy will need the respective public good and will pay the taxes to this government to benefit from the good, since this government is the only supplier of these goods.

A federation, for instance, can be considered to illustrate the situation where the individuals or firms were born or founded in a place, but there is mobility among the jurisdictions of the federation. Therefore, individuals and firms could decide where to live if there was one district where the utility level or profits would be higher. In other words, the agents could choose where to pay taxes and consume public goods.

The objective of this section is to show a scenario in which two districts together form a Federation. Each of them has the same characteristics as the economy described in this work, they have consumers and firms and a local government that collects taxes, rents, and provides public goods. However, districts are locally administered, each local government is free to choose the level of public goods and taxes that its district will have.

Each district has a level of g_1, g_2, τ_1 and τ_2. Thus, g_1 and τ_1 form the package that consumers will observe in each district and see whether their utility will be higher, in the location where they were born or if it's worth to move to the other district. Each firm makes the same decision by looking at the levels of g_2 and τ_2 in each district and their implications on the profits.

3.1 Competition

Individuals and firms will choose where to live by comparing the package of public good level and taxes that they face in each district. On the government side, so far, the problem was to provide the two public goods subject to a constraint that considers rent kept. However, a new feature is added to this problem when there are two districts and the agents observe the package to decide where to consume it. This new thing is the fact that each district must consider in its problem the levels chosen by the other district. In other words, the competition between firms and individuals is introduced by having two districts.

It is assumed that there are two districts, denoted by A and B. District A will offer a public good for the individuals, g_{1A}, and one for the firms, g_{2A}. Consumers will pay a lump-sum tax of τ_{1A} and firms will also pay one of τ_{2A}. The notation is analogous for district B, g_{1B} and g_{2B} will be provided and taxes τ_{1B} and τ_{2B} will be applied.

To analyze this new feature and approach competition to this model, we will apply the framework of the economy described in this work to the Hotelling model of the of spatial differentiation. In the basic model, consumers are located in different places and must choose where to buy the good that is offered by competing firms. Consumers must go to the firm to purchase the goods, so there is a transportation cost to get from the point in the line the individual is located to the firm's location. The transportation cost and the distance of the individual to the firm represent the utility loss of not being located exactly where the firm is. This is the classic interpretation, but location can also be interpreted as heterogeneous tastes.

The version of the Hotelling Model used in this work is based on The Theory of Industrial Organization by Jean Tirole. As he states the different tastes lie on a continuum, and an example would be the degree of sweetness the consumers prefer most, the transportation cost is the utility loss from consuming the good in a degree of sweetness different from the preferred.

In the basic case of the Hotelling model, there is a linear city of length 1 where consumers are uniformly distributed. Two firms are located at the extremes, one is located at point 0 and the other one at point 1. Consumers living in a point x incur a transportation cost to consume the good at $x = 0$ or at $x = 1$. Thus, if the consumer is located near a firm the total transportation cost to consume the good is small. So, since the consumers are uniformly distributed, some of them are near the extremes, where firms are located, and some of them are closer to the middle.

Individuals who are near the extremes will consume the goods close to the extreme but the ones that are around the middle of the line are the ones that firms want to attract. Thus, the firms will compete for the individuals located around the center of the line, once they are approximately at the same distance from both firms. To do so, the first thing the firm does is to find the consumer who is indifferent between the two firms and use his demand to choose prices that maximize profits.

Hereafter, the main idea of the basic Hotelling model will be used to introduce competition to the economy that was described in this work. The objective now is to analyze the competition between districts A and B for firms and consumers and the Hotelling Model is the framework chosen for doing so. There will be two Hotelling lines, one where consumers are uniformly distributed and another one where firms are uniformly distributed.

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3.1.1 The economy with two districts

As described above district A provides g_{1A} and g_{2A} and has taxes τ_{1A}, τ_{2A} and district B provides g_{1B} and g_{2B} and has taxes τ_{1B} and τ_{2B}.

Districts compete separately for firms and individuals, there are two Hotelling lines. This means that they will look at both the indifferent consumer and the indifferent firm. District A is located at point $x = 0$ and district B at point $x = 1$ in a line of length 1. Consumers are located in one line, and firms are located in the other one.

Individuals choose a district to live in based on their preferences for the public goods and transportation costs. Thus, each individual has a demand for the public goods in each state, if the transportation cost is very high, a person may accept consuming less public good in the closer district. The individual has a decision to make, stay in the district where he or she was born, or move to the other district. The cost of transportation is the cost of moving to another place, it will be denoted as t_c for individuals and t_f for firms. A high cost of transportation means that an individual or firm is less likely to go to a district that is far away.

The individual's decision is made by considering how much of the good is provided in each district, how he cares about the level of the good, measured by θ, the transportation cost, t_c, and the lump-sum tax in the district. The same is valid for the firms, they will make the same decision, stay where they were founded or move to the other district. This decision will consider how much the input capital is relevant to the production function, the cost t_f, the level offered by the districts, and the tax rate.

Firms and individuals incur quadratic transportation costs. Individuals are located at a point in a line of length 1 that starts at 0. A consumer who is at x has a utility loss of $t_c x^2$ if he goes to district A and $t_c (1 - x)^2$ if he chooses B. Firms are located in another line with the same characteristics, a firm that is in y has a transportation cost of ty^2 if he goes to district A and $t_f (1 - y)^2$ if it chooses B.

It's assumed that each individual consumes only one unit of the good, thus, the demand for the good g_{1A} is x, demand for g_{1B} is $1 - x$, demand for g_{2A} is y, and demand for g_{2B} is $(1 - y)$.

When there was only one district, the individual's utility was $u_i(g_1) = y_1 - \tau_1 + \theta \ln(g_1)$. However, the utility function is a little bit different when the two districts compete. The level of the public goods may be different in each district and there is also the moving cost, t, to be considered.

The utility of an individual who consumes the good in A and is located at x is $U_{xA} = y_1 - \tau_{1A} + \theta \ln(g_{1A}) - tx^2$. The difference from the utility in the one district model is that now the level is the one provided by A, and he also considers the cost of moving t_c, and the distance to the most preferred situation and the one in the district

A. Analogously, the utility of the same individual if he or she chooses district B is $U_{xB} = y_1 - \tau_{1B} + \theta \ln(g_{1B}) - t\,(1-x)^2$. Since it is assumed that the individual maximizes utility, an individual i will compare u_{iA} and u_{iB} and choose the place where he gets the highest level of utility.

Thus, the utility of an individual located at x is:

$$U_x = max\,[U_{xA}, U_{xB}]$$

$$U_x = max\left[y_x - \tau_{1A} + \theta \ln g_{1A} - tx^2, y_x - \tau_{1B} + \theta \ln g_{1B} - t\,(1-x)^2\right] \tag{3.1}$$

The districts are trying to attract consumers, there are some individuals who are more likely to go to one state or other. In the traditional Hotelling example, if a consumer is very close to point 0 on the line, he would buy from the firm at 0, if prices of both districts are equal since it would be very costly to go to the other extreme. Thus, the firms care about the consumer who is around the middle because he can go to any extreme depending on the conditions of prices and his preference for the good. That's why firms would look at the indifferent consumer. Therefore, the next step in this model is to look for the indifferent consumer and firm in the model proposed here.

An individual at point x in the line is assumed to be indifferent between both districts. Thus, he would have the same utility living in A or B, from the utility function of individual x:

$$y_x - \tau_{1A} + \theta \ln g_{1A} - t_c x^2 = y_x - \tau_{1B} + \theta \ln g_{1_B} - t_c\,(1-x)^2 \tag{3.2}$$

Solving the condition for the indifferent consumer we can find the demand for each district:

$$D_{A1} = x = \frac{1}{2} + \frac{\tau_{1B} - \tau_{1A} + \theta \ln g_{1A} - \theta \ln g_{1B}}{2t_x} \tag{3.3}$$

$$D_{B1} = 1 - x = \frac{1}{2} + \frac{\tau_{1A} - \tau_{1B} + \theta \ln g_{1B} - \theta \ln g_{1A}}{2t_x} \tag{3.4}$$

Thus, we have the demand of the indifferent consumer for the public goods in each district. As expected, it will depend on the tax difference, the difference in the levels offered in each district, and on the transportation cost. One way to understand x and $1-x$ is to think if $\tau_{1A} = \tau_{1B}$ and $g_{1A} = g_{1B}$, each district would have half of the market, in this case, half of the line.

The competition for firms is analogous. The profit depends on the taxes, level provided, and the cost of the firm to move to the other district.

A firm's profit can be written as the following:

$$\pi = g_2^{\frac{\alpha}{1-\alpha}} L^{\frac{\beta}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left[\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} - r\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} \right] - \tau_2 - wL$$

We are dealing with the indifferent firm and its decision of location, the only changes from one district to the other are the taxes and level of public good. Thus, we can define $P = L^{\frac{\beta}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left[\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} - r\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} \right]$ and the profit can be simplified to

$$\pi = g_2^{\frac{\alpha}{1-\alpha}} P - \tau_2 - wL$$

The firms chooses to be be in district A and B, so it will compare the profit that could be made in both districts and choose locate where profits are higher. Thus, the profit of firm located at y is

$$\pi_y = max\left[\pi_{yA}, \pi_{yB}\right]$$

Plugging the expression of π_y

$$\pi_y = max\left[g_{2A}^{\frac{\alpha}{1-\alpha}} P - \tau_{2A} - wL, g_{2B}^{\frac{\alpha}{1-\alpha}} P - \tau_{2B} - wL\right] \tag{3.5}$$

Districts compete for individuals and for firms. Therefore, he also needs to know the demand of the indifferent firm. The indifferent firm, y, will have the same profit it locates in district A or B. Solving for y and $1 - y$, we find the demands for both districts.

$$D_{A2} = y = \frac{1}{2} + \frac{\tau_{2B} - \tau_{2A} + Pg_{2A}^{\frac{\alpha}{1-\alpha}} - Pg_{2B}^{\frac{\alpha}{1-\alpha}}}{2t} \tag{3.6}$$

$$D_{B2} = 1 - y = \frac{1}{2} + \frac{\tau_{2A} - \tau_{2B} + Pg_{2B}^{\frac{\alpha}{1-\alpha}} - Pg_{2A}^{\frac{\alpha}{1-\alpha}}}{2t} \tag{3.7}$$

The demands are analogous, firms will look at the package taxes/level of g_2 to decide where to locate.

3.1.1.1 Districts' Choice

In the traditional Hotelling Model, each firm will maximize profit by considering the demand of the indifferent individual. In this work, the district will maximize the rent considering the demand of individuals and firms.

It is useful to go back to the public constraint: $r + g_1 + g_2 = \tau_1 + \tau_2$. In the scenario of two districts, each one will have its own constraint and will solve its own maximization problem. The rent can be written as the difference of the tax revenue and money spent to offer the goods : $r = \tau_1 + \tau_2 - g_1 - g_2$. However, the total tax revenue will now depend on how many consumers and firms choose to be in the district. For example, the x consumers who decide to go to A will pay τ_{1A} and the others $1 - x$ will be in B pay τ_{1B}. Also, y firms will be in district A and $1 - y$ in district B. Thus, the total tax revenue of district A is $\tau_{1A}x + \tau_{2A}y$ and of district B is $\tau_{1B}(1 - x) + \tau_{2B}(1 - y)$.

Now the total tax revenue for each district was found. Each district chooses the taxes and levels of public goods to maximize its rent. District A will maximize the rent r_A, where $r_A = \tau_{1A}x + \tau_{2A}y - g_{1A} - g_{2A}$, with respect to $\tau_{1A}, \tau_{2A}, g_{1A}, g_{2A}$ and district B will maximize its rent r_B, where $r_B = \tau_{1B}(1 - x) + \tau_{2B}(1 - y) - g_{1B} - g_{2B}$, with respect to $\tau_{1B}, \tau_{2B}, g_{1B}, g_{2B}$. Some implications of the first-order conditions will be discussed now but they can be found in detail in the appendix.

By manipulating some of the first-order conditions, it is possible to find expressions of the tax rates as functions of the goods level, transportation costs, and the agent's parameter.

$$\tau_{1A} = \frac{t_c}{2} + \frac{\tau_{1B} + \theta \ln g_{1A} - \theta \ln g_{1B}}{2} \tag{3.8}$$

and

$$\tau_{1B} = \frac{t_c}{2} + \frac{\tau_{1A} + \theta \ln g_{1B} - \theta \ln g_{1A}}{2} \tag{3.9}$$

The same conditions are found for the variables related to the firm.

$$\tau_{2A} = \frac{t_f}{2} + \frac{\tau_{2B} + P g_{2A}^{\frac{\alpha}{1-\alpha}} - P g_{2B}^{\frac{\alpha}{1-\alpha}}}{2} \tag{3.10}$$

and

$$\tau_{2B} = \frac{t_f}{2} + \frac{\tau_{2A} + P g_{2B}^{\frac{\alpha}{1-\alpha}} - P g_{2A}^{\frac{\alpha}{1-\alpha}}}{2} \tag{3.11}$$

Combining the expressions of τ_{1A} with τ_{1B} and τ_{2A} with τ_{2B} it is possible of find each tax as a function of the difference of the public good provision in the districts:

$$\tau_{1A} = t_c + \frac{\theta}{3}(\ln g_{1A} - \ln g_{1B}) \tag{3.12}$$

$$\tau_{1B} = t_c + \frac{\theta}{3}(\ln g_{1B} - \ln g_{1A}) \tag{3.13}$$

$$\tau_{2A} = t_f + \frac{P}{3}\left(g_{2A}^{\frac{\alpha}{1-\alpha}} - g_{2B}^{\frac{\alpha}{1-\alpha}}\right) \tag{3.14}$$

$$\tau_{2B} = t_f + \frac{P}{3}\left(g_{2B}^{\frac{\alpha}{1-\alpha}} - g_{2A}^{\frac{\alpha}{1-\alpha}}\right) \tag{3.15}$$

Manipulating the first order condition is possible to find $\tau_{1A} + \tau_{1B} = 2t_c$ and $\tau_{2A} + \tau_{2B} = 2t_f$.

So far, all we know is that the sum of the taxes in each district is equal two times the transportation cost. Thus, from the first order conditions, one district can have higher tax, let's say τ_{1A} higher than τ_{1B}, but they both must sum $2t_c$. However, the districts are competing, thus we have to check their rents to understand their behavior. To illustrate, we can analyze the consumers case, the firm's one is analogous.

Some relations that come from the first order condition that will be used now: $x = \frac{\tau_{1A}}{2t_c}$, $y = \frac{\tau_{2A}}{2t_f}$, $(1-x) = \frac{\tau_{1B}}{2t_c}$, and $(1-y) = \frac{\tau_{2B}}{2t_f}$. Also, the optimal levels provide are: $\theta\left(\frac{\tau_{1A}}{2t_c}\right) = g_{1A}$, $\theta\left(\frac{\tau_{1B}}{2t_c}\right) = g_{1B}$, $g_{2A} = \left[\tau_{2A}\left(\frac{P\alpha}{(1-\alpha)2t_f}\right)\right]^{\frac{1-2\alpha}{1-\alpha}}$, and $g_{2B} = \left[\tau_{2B}\left(\frac{P\alpha}{(1-\alpha)2t_f}\right)\right]^{\frac{1-2\alpha}{1-\alpha}}$.

Using the optimal values we can write the rent as:

$$r_A = \frac{\tau_{1A}^2}{2t_c} + \frac{\tau_{2A}^2}{2t_f} - \frac{\theta\tau_{1A}}{2t_c} - \left[\tau_{2A}\left(\frac{P\alpha}{(1-\alpha)\,2t_f}\right)\right]^{\frac{1-2\alpha}{1-\alpha}}$$

and

$$r_B = \frac{\tau_{1B}^2}{2t_c} + \frac{\tau_{2B}^2}{2t_f} - \frac{\theta\tau_{1B}}{2t_c} - \left[\tau_{2B}\left(\frac{P\alpha}{(1-\alpha)\,2t_f}\right)\right]^{\frac{1-2\alpha}{1-\alpha}}$$

.

A simple example is very useful to see the districts' behavior, let's consider that $g_{2A} = g_{2B}$. From equations (33) e (34), when this is the case, $\tau_{2A} = \tau_{2B}$. Now, the difference between r_A and r_B is:

$$r_A - r_B = \frac{\tau_{1A}^2}{2t_c} - \frac{\theta\tau_{1A}}{2t_c} - \frac{\tau_{1B}^2}{2t_c} + \frac{\theta\tau_{1B}}{2t_c} \tag{3.16}$$

$$r_A - r_B = \frac{\tau_{1A}^2 - \tau_{1B}^2 + \theta\left(\tau_{1B} - \tau_{1A}\right)}{2t_c}$$

If, for example, $\tau_{1A} > \tau_{1B}$, then $\tau_{1A}^2 - \tau_{1B}^2 > 0$ and $(\tau_{1B} - \tau_{1A} < 0)$. Thus, if levels of public goods are different, taxes are different and the rent is different. The districts are competing, so they look have market power to change the prices by changing the level of the goods as equations (29) to (33) tell us.

The first thing to observe is that the tax level depends on the transportation cost and the difference in the level provided by the districts. For example, if district A offers a higher level of g_1 than district B does, the taxes for the consumers in A will be higher too. It makes sense, since the government would need more money to provide more of the goods.

However, if district A is charging over t_c, district B would have a tax lower than t_c, $\tau_{1A} > \tau_{1B}$. The districts are competing so, if this is the case, it may attract consumers to district B, and it could increase the level of g_{1B} which would increase τ_{1B} and it would make τ_{1A} decrease. These movements will happen until they both choose the same level of public good, $g_{1A} = g_{1B}$, this will be the equilibrium. If the levels are different districts will have incentives to change the level, they do so until they have the same one. When the districts choose the same level of the public good, $g_{1A} = g_{1B}$, they will also have the same taxes, $\tau_{1A} = \tau_{1B}$. The districts compete in the same way for the firms, therefore, doing the same analysis, in the equilibrium: $g_{2A} = g_{2B}$ and $\tau_{2A} = \tau_{2B}$.

Proposition 4 *The equilibrium situation is when both districts have the same rent. If they, choose different levels of goods, they will always have incentive to change the provision. The equilibrium is defined by the taxes τ_{1A}, τ_{1B}, τ_{2A} and τ_{2B} such that $\tau_{1A} = \tau_{1B}$ and $\tau_{2A} = \tau_{2B}$. And, the public goods levels g_{1A}, g_{1B}, g_{2A}, g_{2B} such that $g_{1A} = g_{1B}$ and $g_{2A} = g_{2B}$.*

In the Hotelling model of competition this kind of equilibrium has a specific designation. We can say that there is no differentiation in prices and in goods level. In other words, in the equilibrium, both districts choose the same level of goods and the same prices.

A very interesting about the tax rate that comes from the first order condition is its relationship with the transportation cost, it is the key to understand the equilibrium and the districts competition.

We assumed that individuals and firms are located in one line of length 1, district A is point 0 and district B is in point 1. Thus, the consumer has to go to one of the district to live and consume the good. If the individual is located is located in x, he would have the utility loss of $t_c x^2$ to go to A and utility loss of $t_c (1 - x) x^2$ to go to B. For example, if individual x' is located near point 0, he would have a smaller utility loss if he chooses district A instead of B. This example works exactly the same way for the firms.

If the transportation cost of individuals t_c is low and the tax in district A is very high, than individual x' who is very near district A, can consider move to district B, if τ_{1A} is much higher than τ_{1B} and t_c. That's how the competition exists in the model. So, if transportation costs are high, an individual or firm is less likely to choose the district that is far away, he tends to stay in the one that is closer, thus districts have a high market power. That's why, if transportation costs are high, they will both choose higher levels of taxes, and it will be equal the transportation cost, because if it is not, the other district will change the level of the good and it will affect the taxes. They will change the level of the good and taxes until they are equal.

Therefore, there will be no differentiation in prices and in the goods level.

One last characteristic of the equilibrium is the rent of each district. Before start it is useful to write here some relations that come from the first order condition that will be used now: $x = \frac{\tau_{1A}}{2t_c}$, $y = \frac{\tau_{2A}}{2t_f}$, $(1-x) = \frac{\tau_{1B}}{2t_c}$, and $(1-y) = \frac{\tau_{2B}}{2t_f}$.

The rent of district A is:

$$r_A = \tau_{1A}x + \tau_{2A}y - g_{1A} - g_{2A}$$

From the first order condition we know the values of x and $1-x$, which can be plugged in the rent:

$$r_A = \tau_{1A}\frac{\tau_{1A}}{2t_c} + \tau_{2A}\frac{\tau_{2A}}{2t_f} - g_{1A} - g_{2A}$$

$$r_A = \frac{\tau_{1A}^2}{2t_c} + \frac{\tau_{2A}^2}{2t_f} - g_{1A} - g_{2A}$$

In the equilibrium, we have that $\tau_{1A} = t_c$ and $\tau_{2A} = t_f$.

$$r_A = \frac{\tau_{1_A}}{2} + \frac{\tau_{2A}}{2} - g_{1A} - g_{2A} \tag{3.17}$$

or

$$r_A = \frac{t_c}{2} + \frac{t_f}{2} - g_{1A} - g_{2A} \tag{3.18}$$

The same process can be done to find the rent of district B:

$$r_B = \frac{\tau_{1_B}}{2} + \frac{\tau_{2B}}{2} - g_{1B} - g_{2B} \tag{3.19}$$

or

$$r_B = \frac{t_c}{2} + \frac{t_f}{2} - g_{1B} - g_{2B} \tag{3.20}$$

Once we know r_A and r_B in the equilibrium, it is just subtract one from the other to check that they are equal.

$$r_A - r_B = \frac{t_c}{2} + \frac{t_f}{2} - g_{1A} - g_{2A} - \left[\frac{t_c}{2} + \frac{t_f}{2} - g_{1B} - g_{2B}\right] \tag{3.21}$$

Since we are analysing the equilibrium situation, we have that $g_{1A} = g_{1B}$ and $g_{2A} = g_{2B}$.

$$r_A - r_B = 0$$

$$r_A = r_B \tag{3.22}$$

The conclusion here is that in the equilibrium there is no differentiation in the level of the public goods provided for firms and individuals by both districts. And, no differentiation in the taxes charged by both districts.

Both districts will have the same rent. If rents were different, one district would change the provision of one good, which would change the prices and bring the economy back to the equilibrium, then, rents would be equal.

Finally, the public goods levels depends only on the parameters of the model. Thus, the governments will choose based on the parameter individuals and firms have. In equilibrium, we have $g_{1A} = g_{1B} = \frac{\theta}{2}$ and $g_{2A} = g_{2B} = \left[\frac{\alpha P}{2(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}$.

3.1.1.2 Tax revenue allocation among the goods in one district

An interesting point is that the rent kept by district A is equal the rent kept by district B, they are equal and positive in the equilibrium $\tau_{1A}x + \tau_{2A}y - g_{1A} - g_{2A} > 0$ and $\tau_{1B}(1-x) + \tau_{2B}(1-y) - g_{1B} - g_{2B} > 0$. The rent represents the fact part of tax revenue will not be spent to provide the public goods, thus the residual money after discounting the rent is destined to the goods. We defined the rent as a fraction of the total tax revenue, in this case, $r_A = \eta[\tau_{1A}x + \tau_{2A}y]$ and $r_B = \eta[\tau_{1B}(1-x) + \tau_{2B}(1-y)]$.

Therefore, the government A, for example, receives the total tax revenue, $\tau_{1A}x+\tau_{2A}y$, keeps $\eta[\tau_{1A}x + \tau_{2A}y]$ for itself and spend the rest in the public goods provision.

$$r_A = \eta[\tau_{1A}x + \tau_{2A}y] = \tau_{1A}x + \tau_{2A}y - g_{1A} - g_{2A} \tag{3.23}$$

$$(1-\eta)[\tau_{1A}x + \tau_{2A}y] = g_{1A} + g_{2A} \tag{3.24}$$

The equation above states that the net tax revenue must be equal to the total cost of providing public goods. To emphasize, it doesn't state anything about how the resource is allocated among the two goods, the government can spend exactly $(1-\eta)\tau_{1A}x$ to provide the public good of the individuals, g_{1A}, and exactly $(1-\eta)\tau_{2A}y$ to provide the public factor of the firms, g_{2A}. However, this is not the only possible scenario. The government can use part of the money that was paid by the consumers to provide more public goods for the firms and the opposite. Thus, there is the scenario in which the individual subsidizes firms, the one where firms subsidize individuals, and the one in which there is no subsidy. These scenarios will be discussed in detail. Since we are dealing with the revenue allocation among the goods in the district, we will focus in district A, but results are analogous to district B.

The situation first analyzed is the one in which the government uses the net revenue received from the consumers to provide g_1 and the net revenue received from the firms to provide g_2.

$$(1-\eta)\,\tau_{1A}x + (1-\eta)\,\tau_{2A}y - g_{1A} - g_{2A} = 0 \tag{3.25}$$

Plugging the values of x and $1-x$ from the first order conditions in the equilibrium:

$$(1-\eta)\,\frac{\tau_{1_A}}{2} + (1-\eta)\,\frac{\tau_{2A}}{2} - g_{1A} - g_{2A} = 0 \tag{3.26}$$

The fact that no agent is subsidizing the other implies:

$$(1-\eta)\,\frac{\tau_{1_A}}{2} - g_{1A} = 0 \tag{3.27}$$

and

$$(1-\eta)\,\frac{\tau_{2A}}{2} - g_{2A} = 0 \tag{3.28}$$

Combining both equations:

$$\frac{g_{2A}}{\tau_{2A}} = \frac{g_{1A}}{\tau_{1_A}} \tag{3.29}$$

or,

$$\frac{\tau_{1A}}{\tau_{2A}} = \frac{t_c}{t_f} = \frac{g_{1A}}{g_{2_A}} \tag{3.30}$$

The last equation relates the cost of transportation with the level of taxes and goods provided. If individuals have a high transportation cost, then the district will have a high tax and will provide more of the good, comparing to the firms. Same for the firms, the higher the taxes to be paid by one agent, the higher the good for this agent will be. Thus, each tax is allocated to the specific agent who paid it. In other words, the level of the good is positively related to the tax paid.

From the first order conditions, we know the optimal value of $g_{1A} = \frac{\theta}{2}$ and $g_{2A} = \left[\frac{\alpha P}{2(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}$.

$$\frac{\tau_{1A}}{\tau_{2A}} = \frac{t_c}{t_f} = \frac{g_{1A}}{g_{2A}} = \frac{\frac{\theta}{2}}{\left[\frac{\alpha P}{2(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}} \tag{3.31}$$

Therefore, if the ratio of the transportation cost is equal to the ratio of the public goods provided by the district, then there will be no subsidy. This relationship is determined by the individual and firm characteristics, the parameters θ, and α.

$$\frac{t_c}{t_f} = \frac{\frac{\theta}{2}}{\left[\frac{\alpha P}{2(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}} \tag{3.32}$$

If individuals, for example, have a high transportation cost, they will pay higher taxes, but if they also have a high θ, more of the good must be provided to them.

However, this may not be true in all scenarios, maybe a high tax for the individuals may not imply a high level of g_1, and the government may choose to use part of this revenue to provide g_2, and it leads us to the second scenario. Individuals are subsidizing firms. This scenario happens when part of the income that was supposed to be spent on the public good for the individuals is spent to provide the public factor for the firms. Thus, $(1-\eta)\,\frac{\tau_{1_A}}{2} > g_{1A}$, and the surplus is spent on g_{2A}.

$$(1-\eta)\,\frac{\tau_{1_A}}{2} - g_{1A} > 0$$

and

$$(1-\eta)\,\frac{\tau_{2A}}{2} - g_{2A} < 0$$

Combining both equations:

$$\frac{g_{1A}}{\tau_{1_A}} < \frac{(1-\eta)}{2} < \frac{g_{2A}}{\tau_{2_A}}$$

$$\frac{g_{1A}}{\tau_{1_A}} < \frac{g_{2A}}{\tau_{2_A}}$$

$$\frac{g_{1A}}{g_{2A}} < \frac{\tau_{1_A}}{\tau_{2_A}} = \frac{t_c}{t_f}$$

Thus, the relationships above describe the condition for the government allocates an agent's revenue to provide goods to the other agent, the ratio of taxes or transportation cost is different from the ratio of the public good. A high tax τ_{1A}, for example, does not imply that g_{1A} is also high, consumers may have been paying a lot and receiving little, because the government is favoring the firms. The tax τ_{1A} is high because the transportation cost is high. It will depends on the individual's and firm's parameters, plugging the value of the public goods i, the relation above $g_{1A} = \frac{\theta}{2}$ and $g_{2A} = \left[\frac{\alpha P}{2(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}$. If taxes for individuals are high because their cost of transportation is high and they don't care a lot about the public good, θ is low, the government may use part of the revenue to provide goods to the firms. Thus, we can rewrite the condition as:

$$\frac{\frac{\theta}{2}}{\left[\frac{\alpha P}{2(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}} < \frac{t_c}{t_f}$$

For example, if part of revenue of individuals is spent on g_2, , let's suppose that t_c is high, thus $\frac{t_c}{t_c}$ is high. Since the money is going to g_2, it makes the $\frac{\frac{\theta}{2}}{\left[\frac{\alpha P}{2(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}}$ be lower.

Therefore, public goods and taxes will always depend on the parameters. Depending on the relationship between θ and α, the government will allocate resources differently among g_{1A} and g_{2A}.

The case where firms are subsidizing individuals is analogous and yields the following condition:

$$\frac{\frac{\theta}{2}}{\left[\frac{\alpha P}{2(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}} > \frac{t_c}{t_f}.$$

The three scenarios conclude that the government will look at the individuals and firms and decide what to do in other to obtain a positive rent. The government will analyze the individual's parameter of preference for the public good, θ; analyze how important the public good is for the firms, by looking at α, and then will compare it to the costs of transportation of each agent. The analysis is made based on the relationship discussed, if this is the case that $\frac{\frac{\theta}{2}}{\left[\frac{\alpha P}{2(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}} < \frac{t_c}{t_f}$, then the government will take part of the revenue paid by the consumers to provide more g_2 and attract more firms. When the parameters yields the opposite : $\frac{\frac{\theta}{2}}{\left[\frac{\alpha P}{2(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}} > \frac{t_c}{t_f}$, then firms would be subsidizing individual's public good. And if this condition is equality, there will be no subside.

Let's take as an example an economy where individuals highly value the public good g_1, this is represented in the model by a very high θ, Also, they value the good much more than the firms value g_2, if firms are more intensive in Labor, increase the productivity of capital may not be a priority. Let's now assume that individuals also have a lower transportation cost than firms have. Then this would be in one possible case where the government would have an incentive to allocate revenue that came from the firms to benefit individuals since they appreciate much more the good, and won't pay high taxes, because a low t_c implies a low τ_1. It will all depends on the parameters θ, α, t_c and t_f.

4 Centralized and decentralized government systems

This work first defined one economy in which a central government provides public goods to firms and individuals. Then, this economy was divided into two districts that compete against themselves. We will call the first scenario the centralized regimé and the second one the decentralized regimé.

It is natural to wonder which situation provides a higher level of public goods, and if the government rent changes in both scenarios. It is important to state that the population is the same in both scenarios. We considered 1 as the population in the first economy. With competition, x individuals would go to A and $1-x$ to B, so the total is still 1. The total number of individuals and firms does not change if it system is centralized or decentralized.

When there is no competition, the central authority chooses the level of g_1 and g_2 for all the citizens maximizing the welfare function, the levels offered were found in section 3. In the centralized regimé, taxes are τ_1 and τ_2, and the levels provided are:

$$g_1 = (1-\eta)\,\theta$$

and

$$g_2 = \left[(1-\eta)\,\frac{\alpha P}{1-\alpha}\right]^{\frac{1-\alpha}{1-2\alpha}}$$

With competition the levels and taxes, in equilibrium, the optimal values are:

$$\tau_{1A} = \tau_{1B} = t_c$$

$$\tau_{2A} = \tau_{2B} = t_f$$

$$g_{1A} = g_{1B} = \frac{\theta}{2}$$

$$g_{2A} = g_{2B} = \left[\frac{\alpha P}{2\,(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}$$

Now, since the values are all known it is possible to make some comparisons and see the differences between both systems in this economy.

We are studying the same economy, in two different scenarios. When it is centralized, the total level of g_1 is $(1-\eta)\,\theta$. Under the decentralized regimé, the total level of g_1 provided by both districts, in the equilibrium, is $g_{1A}+g_{1B}=\frac{\theta}{2}+\frac{\theta}{2}$, then $g_{1A}+g_{1B}=\theta$. Since $\eta<1$, we have $g_1<g_{1A}+g_{1B}$. Thus, under the centralized system, a lower level of g_1 is provided to individuals than it is provided by the two districts together in the decentralized system.

The same can be done for the firms, $g_{2A}+g_{2B}=\left[\frac{\alpha P}{(1-\alpha)}\right]^{\frac{1-\alpha}{1-2\alpha}}$ is the total level of the public good provided by districts A and B in the equilibrium when they compete. And, $g_2=\left[(1-\eta)\,\frac{\alpha P}{1-\alpha}\right]^{\frac{1-\alpha}{1-2\alpha}}$ is the amount provided when there is no competition and the central authority chooses the levels. Again, since $\eta>0$, the level provided by the central authority is lower than the level provided by the two districts together.

Proposition 5 *The total level of the public goods for firms and individuals provided by districts A and B together in the decentralized regimé is higher than the level provided by only one centralized government.*

It may be immediate to think that it happens because there is no η in the equation of the levels of the good under competition, and then no rent. But this is not true, the rent is positive in both scenarios, they all keep a fraction of η of the total tax revenue, and the difference can be in the taxes. To explore more this topic, it is useful to bring back the districts rent in the equilibrium situation:

$$r_A+r_B=\frac{\tau_{1_A}}{2}+\frac{\tau_{2A}}{2}-g_{1A}-g_{2A}+\frac{\tau_{1_B}}{2}+\frac{\tau_{2B}}{2}-g_{1B}-g_{2B} \tag{4.1}$$

In the equilibrium, individuals in different districts pay the same tax level, same for the firms.

$$r_A+r_B=\tau_{1_A}+\tau_{2A}-(g_{1A}+g_{1B})-(g_{2A}+g_{2B}) \tag{4.2}$$

To avoid confusion with the variables, in this section let's define $\tau_{1_A}=\tau_{1_B}=\tau_1'$, and $\tau_{2_A}=\tau_{2_B}=\tau_2'$. Thus, in the equilibrium under competition the tax level for the individuals is τ_1' and for the firms is τ_2'.

And the sums of the level of public goods are: $G_1=g_{1A}+g_{1B}$ and $G_2=g_{2A}+g_{2B}$. We can now rewrite the sum r_A+r_B.

$$r_A+r_B=\tau_1'+\tau_2'-G_1-G_2 \tag{4.3}$$

The rent in the centralized system is :

$$r=\tau_1+\tau_2-g_1-g_2 \tag{4.4}$$

Using the fact that rent is defined as a fraction of total tax revenue:

$$r_A + r_B = \eta\left(\tau_1' + \tau_2'\right) = \tau_1' + \tau_2' - G_1 - G_2 \tag{4.5}$$

and

$$r = \eta\left(\tau_1 + \tau_2\right) = \tau_1 + \tau_2 - g_1 - g_2 \tag{4.6}$$

Thus, the total amount available to provide the goods satisfies the following conditions, for both scenarios.

$$(1-\eta)\left(\tau_1' + \tau_2'\right) = G_1 + G_2 \tag{4.7}$$

$$(1-\eta)\left(\tau_1 + \tau_2\right) = g_1 + g_2 \tag{4.8}$$

Both equations above have $1-\eta$, so we can compare them.

$$(1-\eta) = \frac{G_1 + G_2}{\tau_1' + \tau_2'} = \frac{g_1 + g_2}{\tau_1 + \tau_2} \tag{4.9}$$

Since we know that $G_1 + G_2 > g_1 + g_2$, then the we have that $\tau_1' + \tau_2' > \tau_1 + \tau_2$.

Thus, one explanation for the fact that more public goods are being offered when there are two districts, is higher total tax revenue. They can tax firms and individuals more because now they have a transportation cost. In the centralized system, they didn't have what to choose from, they just had to pay the tax and receive the level provided by the central government. It leads to the following proposition.

Proposition 6 *Under the decentralized system, the districts together provide a higher level of each public good and also tax firms and individuals more.*

However, to say that the districts provide a higher level of a public good because taxes are higher and they receive more tax revenue is only possible if they can have higher taxes. An interesting result appears when it is assumed the taxes are the same in centralized and decentralized systems.

Proposition 7 *If the total tax revenue is the same in the centralized and decentralized regimé, the centralized government has a higher rent the districts A and B have together in the decentralized system.*

Assuming that $\tau_1' + \tau_2' = \tau_1 + \tau_2$. We can subtract r from $r_A + r_B$:

$$r_A + r_B = \tau_1' + \tau_2' - G_1 - G_2$$

$$r = \tau_1 + \tau_2 - g_1 - g_2$$

$$r_A + r_B - r = \tau_1' + \tau_2' - G_1 - G_2 - (\tau_1 + \tau_2 - g_1 - g_2)$$

$$r_A + r_B - r = -G_1 - G_2 + g_1 + g_2$$

Since $G_1 + G_2 > g_1 + g_2$, we know that the rent is higher in the centralized regimé.

$$r_A + r_B - r = -G_1 - G_2 + g_1 + g_2 < 0$$

$$r_A + r_B < r$$

This makes sense because if more of the good is been offered, and the tax revenue cannot increase, less rent will be kept, since η is the same. There are, therefore, some advantages and drawbacks to the government systems. The optimal level of the public good that maximizes utility and profit is offered in the decentralized system, however, taxes are higher than they would be if only a central government is running the whole territory.

5 Conclusion

This work analyzes the provision of public goods to individuals and firms in a government system in which the government keeps rent to itself. Moreover, two scenarios of the government system are considered, the first one where a central authority collects taxes and provides the goods, and the second one where the economy is divided into two districts that locally provide the public goods and decide the tax levels.

When there is only one central government there will always be a public good for the firms and individuals provided. The parameter of public good preferences of the individuals and the share the capital in the production function, the public good for the firms increase capital's productivity, are crucial in determining the level of public goods. The level of the goods depends on the parameter η, the fraction of the total tax revenue. It reduces the level provided of the goods. Thus, given the parameters of the model the government provides both goods, and will always keep a positive rent.

The Hotelling model is used to approach the competition between the two districts for firms and individuals. When they compete, there will be no differentiation in the level of public goods and taxes in the equilibrium. The reason for that is the fact that the level provided in one district affects the tax level in the other. So, if a district increases the goods level, its taxes will be higher and the taxes from the competitor are lower. So, there is no differentiation in the level and the rent. Both districts keep the same rent from the total tax revenue.

These results are consistent with Hotelling's (1929) principle of minimum differentiation. Also, Bolko Hohaus, Kai A. Konrad, Marcel Thum" (1993) find that there is a tendency to no differentiation because the communities are too similar, this is also something present in this model. However, Lu-Chen Hs (2005) and Wooders and Zissimos (2003) find the opposite result, both works find that the Hotelling's (1929) principle of minimum differentiation is not valid for their model. Actually, maximum differentiation is found.

Although Wooders and Zissimos (2002), use a very similar structure of the economy, there are a few differences. They discuss tax discrimination but also consider the uniform tax. They found that there could be a possibility in which one district provides all of the goods and the other provides nothing but also keeps rent. This cannot happen in this model because of the function form of utility and production function, the assumption of the parameter θ and α implicate is a positive value for both of them. So, districts A and B must provide both goods, otherwise, they will not maximize their own rent.

This work has some different results from the ones that used the Hotelling Model to study competition among local governments to provide public goods, but it also has some differences in the economy that is applied to the Hotelling Model. This work considers consumers and firms in the district, so the government provides two different goods instead of only one in the papers analyzed. There is also a positive rent to the government for any level of public good provided, and each government provides both goods, in all scenarios.

The existence of the two goods affects the government's choice because they have to allocate the total tax revenue received from consumers and firms among the goods. They have to decide how much to provide considering the parameters of firms, α, and individuals, θ. Since there are two public goods, the constraint proposed here is what connects the agents to the government. The constraint is responsible to make sure that both goods will be provided, rent will be kept. The public constraint gives the government to power to choose how to allocate the rent. He can look at the consumers and firms, and provide the goods to the ones who value it more.

Also, the way the parameters of firms and individuals are related to the costs of transportation will determine if the government will use part of the revenue collection from individuals, for example, to benefit the firms. Thus, depending on the economy's characteristics, the government can favor firms, favor consumers or simply spend money from individuals with individuals and money from firms with firms.

The way the economy was described allowed us to compare the situation of no competition to the economic scenario of the Hotelling Model. Thus, the last result is about the differences between the scenario of one central government and the scenario of the economy divided into two districts. The sum of the level of each good provided by the two districts in the decentralized regimé is higher than the level provided in the centralized regimé. However, taxes are also higher. If the districts cannot raise the taxes, the rent of the centralized regimé is higher than the total rent of the two districts together.

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Appendix

A.1 First order conditions of each district

A.1.1 District A

$$r_A + g_{1A} + g_{2A} = \tau_{1A}x + \tau_{2A}y \tag{7.1}$$

$$r_A = \tau_{1A}x + \tau_{2A}y - g_{1A} - g_{2A} \tag{7.2}$$

$$\frac{\partial r_a}{\partial \tau_{1A}} = x + \tau_{1A}\frac{\partial x}{\partial \tau_{1A}} = 0 \tag{7.3}$$

$$\frac{\partial r_a}{\partial \tau_{2A}} = y + \tau_{2A}\frac{\partial y}{\partial \tau_{2A}} = 0 \tag{7.4}$$

$$\frac{\partial r_a}{\partial \tau_{g1A}} = \tau_{1A}\frac{\partial x}{\partial g_{1A}} - 1 = 0 \tag{7.5}$$

$$\frac{\partial r_a}{\partial \tau_{g2A}} = \tau_{2A}\frac{\partial y}{\partial g_{2A}} - 1 = 0 \tag{7.6}$$

$$\frac{\partial r_a}{\partial \tau_{1A}} = x + \tau_{1A}\left(\frac{-1}{2t}\right) = 0 \tag{7.7}$$

$$\frac{\partial r_a}{\partial \tau_{2A}} = y + \tau_{2A}\left(\frac{-1}{2t}\right) = 0 \tag{7.8}$$

$$\frac{\partial r_a}{\partial g_{1A}} = \tau_{1A}\left(\frac{\theta}{g_{1A}2t}\right) - 1 = 0 \tag{7.9}$$

$$\frac{\partial r_a}{\partial g_{2A}} = \tau_{2A}\left(\frac{P\alpha}{(1-\alpha)\,2t}\right) g_{2A}^{\frac{2\alpha-1}{1-\alpha}} - 1 = 0 \tag{7.10}$$

$$x = \frac{\tau_{1A}}{2t} \tag{7.11}$$

$$y = \frac{\tau_{2A}}{2t} \tag{7.12}$$

$$\theta\left(\frac{\tau_{1A}}{2t}\right) = g_{1A} \tag{7.13}$$

$$\tau_{2A}\left(\frac{P\alpha}{(1-\alpha)\,2t}\right) = g_{2A}^{\frac{1-2\alpha}{1-\alpha}} \tag{7.14}$$

A.1.2 District B

Since they choose prices simultaneously, District B also solves its problem of rent maximization.

$$r_B = \tau_{1B}\,(1-x) + \tau_{2B}\,(1-y) - g_{1B} - g_{2B} \tag{7.15}$$

$$\frac{\partial r_B}{\partial \tau_{1B}} = (1-x) + \tau_{1B}\frac{\partial\,(1-x)}{\partial \tau_{1B}} = 0 \tag{7.16}$$

$$\frac{\partial r_B}{\partial \tau_{2B}} = (1-y) + \tau_{2B}\frac{\partial\,(1-y)}{\partial \tau_{2B}} = 0 \tag{7.17}$$

$$\frac{\partial r_B}{\partial g_{1B}} = \tau_{1B}\frac{\partial\,(1-x)}{\partial g_{1B}} - 1 = 0 \tag{7.18}$$

$$\frac{\partial r_B}{\partial g_{2B}} = \tau_{2B}\frac{\partial\,(1-y)}{\partial g_{2B}} - 1 = 0 \tag{7.19}$$

$$\frac{\partial r_B}{\partial \tau_{1B}} = (1-x) + \tau_{1B}\left(\frac{-1}{2t}\right) = 0 \tag{7.20}$$

$$\frac{\partial r_B}{\partial \tau_{2B}} = (1-y) + \tau_{2B}\left(\frac{-1}{2t}\right) = 0 \tag{7.21}$$

$$\frac{\partial r_B}{\partial g_{1B}} = \tau_{1B}\left(\frac{\theta}{g_{1B}2t}\right) - 1 = 0 \tag{7.22}$$

$$\frac{\partial r_B}{\partial g_{2B}} = \tau_{2B}\left(\frac{P\alpha}{(1-\alpha)\,2t}\right) g_{2B}^{\frac{2\alpha-1}{1-\alpha}} - 1 = 0 \tag{7.23}$$

$$(1-x) = \frac{\tau_{1B}}{2t} \tag{7.24}$$

$$(1-y) = \frac{\tau_{2B}}{2t} \tag{7.25}$$

$$\theta\left(\frac{\tau_{1B}}{2t}\right) = g_{1B} \tag{7.26}$$

$$\tau_{2B}\left(\frac{P\alpha}{(1-\alpha)\,2t}\right) = g_{2B}^{\frac{1-2\alpha}{1-\alpha}} \tag{7.27}$$

A.2 Public good levels

$$\theta\left(\frac{\tau_{1A}}{2t}\right) = g_{1A} \tag{7.28}$$

$$\theta\left(\frac{\tau_{1B}}{2t}\right) = g_{1B} \tag{7.29}$$

$$\tau_{2A}\left(\frac{P\alpha}{(1-\alpha)\,2t}\right) = g_{2A}^{\frac{1-2\alpha}{1-\alpha}} \tag{7.30}$$

$$\tau_{2B}\left(\frac{P\alpha}{(1-\alpha)\,2t}\right) = g_{2B}^{\frac{1-2\alpha}{1-\alpha}} \tag{7.31}$$

A.2.1 Levels in the equilibrium

$\tau_{1A} = \tau_{1B} = t_c$, $\tau_{2A} = \tau_{2B} = t_f$, $g_{1A} = g_{1B}$, $g_{2A} = g_{2B}$

$$\frac{\theta}{2} = g_{1A} \tag{7.32}$$

$$\frac{\theta}{2} = g_{2A} \tag{7.33}$$

$$\left(\frac{P\alpha}{(1-\alpha)\,2}\right)^{\frac{1-\alpha}{1-2\alpha}} = g_{2A} \tag{7.34}$$

$$\left(\frac{P\alpha}{(1-\alpha)\,2}\right)^{\frac{1-\alpha}{1-2\alpha}} = g_{2B} \tag{7.35}$$

A.3 There is η that maximizes the rent

The rent definition and the public constraint can be combined to :

$$r = \frac{\eta}{1-\eta}\left(g_1 + g_2\right) \tag{7.36}$$

Plugging the optimal values of g_1 and g_2:

$$r = \eta\theta + \eta\left(1-\eta\right)^{\frac{\alpha}{1-2\alpha}} \left\{\frac{\alpha}{1-\alpha} L^{\frac{\beta}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left[\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} - r\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}\right]\right\}^{\frac{1-\alpha}{1-2\alpha}} \tag{7.37}$$

Defining $M = \left\{\frac{\alpha}{1-\alpha} L^{\frac{\beta}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left[\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} - r\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}\right]\right\}^{\frac{1-\alpha}{1-2\alpha}}$, the rent can be written as $r = \eta\theta + \eta\left(1-\eta\right)^{\frac{\alpha}{1-2\alpha}} M$. Since it would be useful to check if a very high η would actually maximize the rent, the first order condition can be checked.

The first order condition is:

$$\frac{\partial r}{\partial \eta} = \theta + \left(1-\eta\right)^{\frac{\alpha}{1-2\alpha}} M + \eta M \left(\frac{\alpha}{1-2\alpha}\right)\left(1-\eta\right)^{\frac{-1+3\alpha}{1-2\alpha}}(-1) = 0 \tag{7.38}$$

It is quite complicated to solve the equation for η, but with a few manipulations it is possible to get some insights and find a bound for it. Rewriting the FOC:

$$M\left[\left(1-\eta\right)^{\frac{\alpha}{1-2\alpha}} - \eta\left(\frac{\alpha}{1-2\alpha}\right)\left(1-\eta\right)^{\frac{-1+3\alpha}{1-2\alpha}}\right] = -\theta \tag{7.39}$$

For this relation I consider the parameter area where $g_1 > 1$ and $\theta > 1$. Therefore, the right side of the equation is negative.

Now, it is time to analyse the sign of M. Since, α is assumed to be greater than zero and less than 1, $\frac{\alpha}{1-\alpha}$ is greater than zero. The same holds for the term $\left[\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} - r\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}\right]$, which would be positive, when $\alpha < 1$, as it is proved as the condition for g_2 be positive. A and L are also positives, so M is greater than zero.

$$\alpha < 1 \implies \frac{\alpha}{1-\alpha} > 0$$

$$\alpha < 1 \implies \left[\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} - r\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}\right] > 0$$

Therefore,

$$M = \left\{\frac{\alpha}{1-\alpha} L^{\frac{\beta}{1-\alpha}} A^{\frac{1}{1-\alpha}} \left[\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} - r\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}\right]\right\}^{\frac{1-2\alpha}{1-\alpha}} > 0$$

From the first order condition, the multiplication of M and the term between brackets yields a negative result, $-\theta$.

In order to the equation be true, the term: $\left[\left(1-\eta\right)^{\frac{\alpha}{1-2\alpha}} - \eta\left(\frac{\alpha}{1-2\alpha}\right)\left(1-\eta\right)^{\frac{-1+3\alpha}{1-2\alpha}}\right]$ must be negative. This condition leads to the following relationship:

$$1 < \eta \frac{(1-\alpha)}{(1-2\alpha)} \tag{7.40}$$

There are two cases when solving it for η because the sign of $(1-2\alpha)$ depends on the value of α. $(1-2\alpha) > 0$ if $\alpha < \left(\frac{1}{2}\right)$ and $(1-2\alpha) < 0$ if $\alpha > \left(\frac{1}{2}\right)$. Then, when we multiply both sides by $(1-2\alpha)$, the inequality sign will change if $\alpha > \left(\frac{1}{2}\right)$.

Solving for η:

i) $\alpha > \left(\frac{1}{2}\right) \implies \frac{1-2\alpha}{1-\alpha} > \eta$

ii) $\alpha < \left(\frac{1}{2}\right) \implies \frac{1-2\alpha}{1-\alpha} < \eta$

In situation i), $\frac{1}{2} < \alpha < 1$. For any value of α in this range, the value of $\frac{1-2\alpha}{1-\alpha}$ would be less than zero. It means that η is also a negative number, which does not make sense for this analysis.

In situation ii), $0 < \alpha < \frac{1}{2}$. When α is close to zero, then η is near to 1, and as α gets closer to $\frac{1}{2}$, η tends to zero. Thus, for any value of α between 0 and $\frac{1}{2}$ yields a value of η between 0 and 1 and higher than $\frac{1-2\alpha}{1-\alpha}$. We have that $0 < \alpha < \frac{1}{2}$, then There is an inferior bound η: $\frac{1-2\alpha}{1-\alpha} < \eta$.

From the maximization of the objective function of the government, the level o public good is $g_1* = \theta(1-\eta)$. Thus, it is possible to relate the individual's preference for the public good with the fraction of total revenues kept by the incumbent, η. Since, $g_1 > 1$ and $\theta > 1$, there is a superior bound for η: $\eta < \frac{\theta-1}{\theta}$

If $g_1 > 1$ and the optimal level g_1 is $g_1* = \theta(1-\eta)$, then:

$$g_1 > 1 \implies \theta(1-\eta) > 1 \tag{7.41}$$

The last equation can be solved for η.

$$\theta > \frac{1}{1-\eta} \tag{7.42}$$

$$\eta < \frac{\theta - 1}{\theta} \tag{7.43}$$

There is an η that is greater than zero and less than one that maximizes the government rent. To prove it we will use some of the results found so far. To do so it is useful to summarize them:

i) $\alpha < \left(\frac{1}{2}\right) \implies \frac{1-2\alpha}{1-\alpha} < \eta$

ii) If $0 < \alpha < \frac{1}{2} \implies 0 < \eta < 1$

iii) $g_1 > 1$

iv) $\theta > \frac{1}{1-\eta}$

v) $\eta < \frac{\theta-1}{\theta}$

We are considering that the increases in the capital marginal productivity are diminishing, thus $\alpha < \frac{1}{2}$, and the relation $i)$ is valid. Since $\alpha < \frac{1}{2}$, then $\frac{1-2\alpha}{1-\alpha} > 0$ always. Then, we add information to relation $i)$ and write relation $i')$: $0 < \frac{1-2\alpha}{1-\alpha} < \eta$.

We also found that θ is greater than one, thus $\frac{\theta-1}{\theta}$ will always be less than 1, for any theta greater than one, $\theta > 1$, $\frac{\theta-1}{\theta} < 1, \forall\theta > 1$. Therefore relation $v')$ cam be written as : $\eta < \frac{\theta-1}{\theta} < 1$. Combining relations $i')$ and $v')$:

$$0 < \frac{1-2\alpha}{1-\alpha} < \eta < \frac{\theta-1}{\theta} < 1$$

$$0 < \eta < 1 \tag{7.44}$$

Fortunately the first order conditions and the relations involving α and θ give us more information about η. And, in fact, is possible to find the range that maximizes the rent. This range is in the last proof, when relations $i')$ and $v')$ are combined.

$$\frac{1-2\alpha}{1-\alpha} < \eta < \frac{\theta-1}{\theta} \tag{7.45}$$