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Entrepreneurship, Life Cycle of Firms and Resource Misallocation in Brazil

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Dissertação de Mestrado apresentada ao Programa de Pós-Graduação em Ciências Econômicas (área de concentração: Teoria Econômica), como parte dos requisitos necessários para a obtenção do Título de Mestre em Economia.

Orientador: Rafael da Silva Vasconcelos

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Resumo

Este artigo investiga a relação entre o ciclo de vida das firmas e a má alocação de recursos no Brasil. Usando bancos de dados administrativos no nível da firma, apresentamos novos fatos estilizados e quantificamos o grau de má alocação de recursos, descobrimos que (i) a eficiência de alocação está próxima a 20% nas indústrias brasileiras; (ii) Os *input wedges* são altos e há um sobre-investimento em capital; (iii) O *Capital Wedge* é quase 20% maior em firmas com menos de 10 anos do que em firmas com 40 anos e o *labor wedge* em firmas com 40 anos é 2 vezes inferior quando comparada com firmas com menos de 10 anos; (iv) As firmas com menos de 10 anos são 65% mais produtivas do que as firmas com mais de 40 anos. Este padrão de alto grau de má alocação e ineficiência é persistente ao longo do tempo. Para explicar esses novos resultados, construímos um modelo teórico de *search* com micro-distorções no mercado de insumos. Demonstramos que a persistência do alto nível de ineficiência de alocação e baixo nível de produtividade é potencializada por fricções nas trocas. Isso sugere que altos *input wedges*, altos custos de busca e entrada inibem a entrada de novas firmas e a saída de firmas antigas.

Palavras-chaves: Ciclo de Vida. Má Alocação. Empreendedorismo. Modelo de Search and Matching.

Abstract

This paper investigates the relation between life cycle of firms and the resource misallocation in Brazil. Using plant-level administrative databases, we find that (i) allocation efficiency is near 20% in the Brazilian industries; (ii) The input wedges are high and there is an over-investment in capital; (iii) Capital wedge is near 20% higher in under 10 years old firms than 40 years old firms and the labor wedge in 40 years old firms is 2 times inferior when compared with under 10 years old firms; (iv) Under 10 years old firms are 65% more productive than firms over 40 years old. This pattern of high misallocation degree and inefficiency is persistent over time. To explain these new results, we built a Search-theoretic model with micro-distortions in the input market. We demonstrate that the persistence of the high level of allocation inefficiency and low level of productivity is enhanced by trading frictions. Our theoretical results demonstrate that high input wedges, high search and entry costs inhibit the entry of new firms and the exit of old firms.

Keywords: Life Cycle. Misallocation. Entrepreneurship. Search and Matching model.

Sumário

1 Introduction

Macroeconomics literature suggests technological progress is the main source to the differences in economic growth path across countries (Syverson, 2011; Restuccia and Rogerson, 2017). With that in mind our research has the objective to identify the extent of firms life cycle over the resource misallocation in a large market with frictions.

We want to understand how the dynamic of entry and exit of firms from the market impacts on the firm's productivity and the degree of resource misallocation in Brazil. The present work follows two complementary strands. From the point of view of the quantitative set, we set a framework model that will provide the influence of firm's age over productivity and misallocation based on their revenues similar as seen in Hsieh and Klenow (2014). According to the open public data base provided by the Brazilian Internal Revenue Service, the youngest firms with 5-9 years old comprehend over 20% of the quantity of total firms in Industrial sector and Services sector in 2019. However, according to the Brazilian Annual Survey of Industry (*PIA*, in portuguese acronym), provided by the Brazilian Institute of Geography and Statistics, industrial firms over 40 years old, which comprehend less than 5% of the total of firms, have shares of output over 40%, employment over 30% and investment over 57% by age group. In terms of labor productivity, we find that firms over 40 years old firms are twice more productive than under 10 years old Industrial firms and three times more productive than 10 years old Services firms.

We use two groups micro-data sets which provide us important features to better understand the role of life cycle of firms on the dynamics of the efficiency of allocation and productivity in Brazil. One is the Administrative Data of the Brazilian Industry (Manufacturing and Extraction) and the Administrative Data of the Services (Construction, Commerce and Services). These data contain detailed and identified plant-level information about revenue, employment, investment and other variables. Another data set is the public open data base from the Brazilian Internal Revenue Service. This data set provides a rich information about identified firms, such as register day, the age of each firm, their legal code and the corporate structure partners.[1] For now, our analysis focus are the private owned profit-maximizing firms in the period between 1996 to 2017.

This work seeks contribute to the literature by documenting misallocation of resources in the Brazilian economy and the role of the life cycle component over the misallocation. We find that (i) allocation efficiency is near 20% in Brazil; (ii) The input wedges are high and there is an over-investment in capital; (iii) Capital wedge is near 20%

1 This information was not available in other researches which aimed the firm's life cycle component as an important variable for firm dynamics analysis in Brazil (Assunção, 2018)

higher in under 10 years old firms than 40 years old firms and the labor wedge in 40 years old firms is 2 times inferior when compared with under 10 years old firms; (iv) Under 10 years old firms are 65% more productive than firms over 40 years old after Levinsohn and Petrin (2003) estimation and ACF correction (Ackerberg et al., 2015).

We observe from the quantitative results that besides the high misallocation, there is a high persistence of the misallocation level through years. We set a theoretical framework with entry and exit of firms in an economy with trading friction on the input market inspired by Hopenhayn (1992) to understand the misallocation dynamics and persistence. We define trading frictions as input market failures which there is no certain double coincidence of the agents and exists non-productive costs of trade. The model simulations show that (i) an increase in micro-distortions would decrease wages and earnings of capital by 0.03% and 0.07% for old and young firms, would increase the Reserve Productivity of labor and capital in 40% and 50% for old firms and 27% and 31% for young firms, and would increase the disincentive to invest specially for old firms; (ii) an increase in search costs would decrease the job and the capital creation by 13% for old firms and 22% for young firms; (iii) The entry cost increase only affects the Job and Capital creation of the young firms and an increase in 10% in Entry cost would affect near 3.5%.

The model contributes to a literature that seeks to explain these persistence of resource misallocation in development countries. For example, Yang (2016) described a fully-specified model with Indonesian plant-level micro-data aiming the impact of inefficient firms that still survives and efficient firms that are forced to exit due to the micro-distortions in the economy. It finds that removing the distortions allowing the survival of efficient firms and the exit of inefficient firms could increase aggregate TFP by over 40%. Baqaee and Farhi (2020) described how frictions on input market impact on the productivity measurement and resource misallocation in a general equilibrium framework. Eliminating the misallocation resulting from the large and dispersed markups estimated in the data would raise aggregate TFP in 20%. Understanding why the misallocation degree is persistent is one of the challenges of the literature (Eslava and Haltiwanger (2020), Banerjee and Moll (2010)). Adding the life cycle determinants in an environment with such trading frictions is what we would like to contribute.

Also related to the subject, Restuccia and Rogerson (2017) suggested that the misallocation theme research is pointing to a direct approach, i.e, to an investigation of what mechanisms are contributing for the inefficient allocate of inputs as in Restuccia and Rogerson (2013), Buera and Shin (2013) and Ponticelli and Alencar (2016). They argue that follow the direct approach are more likely to reach concrete, persuasive, and specific conclusions of practical policy relevance. From this point of view, we discuss a potential institutional change that may have had an impact over the entrepreneurship in Brazil. However, the indirect approach can be useful to identify which sector or which factor the inefficency of allocation can be more significant.

So, the work is organized as follows. Section 2 is a Data section which provides a explanation of what we can collect from the sets and also some stylized facts about misallocation and firm's life cycle in Brazil. Section 3 provides the quantitative framework we used and the importance of life cycle component in it. Section 4 presents results using the quantitative setting. In Section 5 we present our trading friction built model to understand patterns in the dynamics of misallocation. Section 6 we express the results from the theoretical setting. Section 7 shows the empirical strategy that we aim in the near future. Finally, Section 8 are the Final Remarks.

2 Data and Stylized facts

In this section, we present the plant-level data used to measure productivity and resource misallocation in the Brazilian firms. We also introduce the public open data base from the Brazilian Internal Revenue Services. We present new stylized facts on the life cycle of firms in Brazil. Further details on the datasets are contained in Appendix A.1 and A.2.

2.1 Firm-level data

We use plant-level administrative data from four different surveys constructed by the Brazilian Institute of Geography and Statistics (IBGE, at Portuguese acronym) between 1996 and 2017. All the data contains information on the economic situation at the plant-level of the sectors of Industry (Manufacturing and Extracting) and Services (Construction, Commerce and other private services).[1] According to the IBGE, Services sector comprehended over 60% of the GDP in the first three months of 2020 and the Industrial sector comprehended approximately 17% of the GDP. Thus our research comprehend almost 80% of the Brazilian production. The choice for firm-level data despite plant-level data as Hsieh and Klenow (2014) was motivated by the impossibility of construct a plant-level capital series which is consistent through the data sets.[2]

The manufacturing and extraction firms are inserted in the Brazilian Annual Survey of Industry (PIA, at Portuguese acronym). The database is an unbalanced panel of 40,000 firms (or approximately 300,000 plants) per year on average for each year and only comprehend firms with more than 30 employees and over R$ 14.5 million of annual revenue. Services firms are distributed across three different data bases. First, Brazilian Annual Survey of Construction Industry (PAIC, at Portuguese acronym) reunites relevant information about firms which the main activity is construction. Second is the Brazilian Annual Survey of Services (PAS, at Portuguese acronym) gathers data from firms which the main activity is contained as follows: Sanitation, Transportation, Accommodation and Food Service, Communication, Real State, Professional Guidance, sports and cultural activities and others services. Third, the Brazilian Annual Survey of Commerce (PAC, at Portuguese acronym) collects data from firms which the main activity are Commerce and

[1] Unfortunately, there are not enough data from the Brazilian Agricultural production to measure firm-level productivity and resource misallocation with the same accuracy that the proposed work aims

[2] In the National Survey of Industries data dues to the absence of composition from the different types of capital. For the Services data, beyond what was quoted, there is also an absence of compatible identification for all plants (For instance, in the questionnaires of services firms, the manager reports only the total of investment from all the plants in a state by year).

Auto maintenance. Using these data, the database of services is an an unbalanced panel of 2,300,000 plants on average per year. PAC and PAS contain the largest portion of firms in Brazil. Due to recent adjustments to the data bases, it became possible to properly measure the productivity and resources misallocation of the Services sector in Brazil.

These administrative data include employees (blue and white collars); wages and salaries; revenues (gross, net, retail, stocks, financial incomes) costs and expenses (operational and non-operational); investment (machinery, equipments, buildings, office material, computers and others); depreciation for each type of capital; output and intermediate consumption.

2.2 Entrepreneur register data

We use the public data base of the Brazilian plants provided by the Brazilian Internal Revenue Services. This data base provides, from each plant of Brazil, information about: their legal code; beginning date of activities; ending date of activities, the reason of the exit, and the situation; location, size, and sector. Besides, it provides information about the 44 million legal entities registered in Brazil: Identification of the entrepreneurs; entry date; it provides the occupied position in the entrepreneurship and the legal representative identification from each entrepreneur.

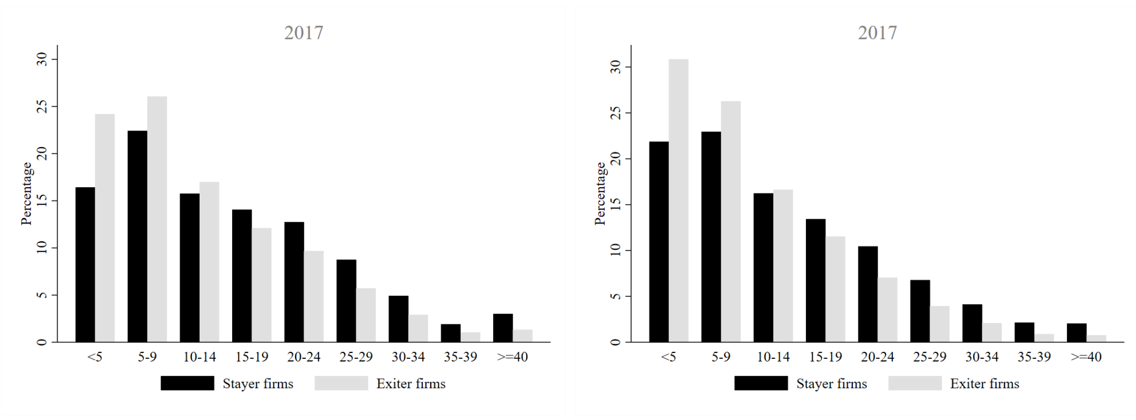
Merging the firm-level data and the entrepreneur register, we can quantify the dynamics of productivity and misallocation across life cycle of the Brazilian firms. Furthermore, we can exploit the firm's entry or exit month date, which may avoid potential mismeasurements and bias. We can distinguish the ownership and the object of the firm, i.e, if a firm is from a state ownership category, or if it is a non-profit organization. Those concerns were issues in previous works (Hsieh and Klenow (2009) and Hsieh and Klenow (2014)). A firm in our analysis must be a profit-maximizing agent, to capture a more quantitative-based dispersion across the distribution of firms.

2.3 Stylized facts on Life Cycle of the Brazilian Firms

We present new stylized facts about the life cycle of the Brazilian firms. These new facts are important as motivation for the following exercises. Figure 1 shows the distribution of Stayer and Exiter firms by age group in the industry and services sectors in 2017. Stayer firms under 15 years old represent about 55% of the industrial sector and 60% of the services sector, while the 40 years old or more firms are less than 5% of the set for both sectors in 2017. Comparing to Hsieh and Klenow (2014), the number of Brazilian firms by age group profile get closer to the Mexican profile. This distribution did not change much in another years as we can see in appendix A.2, however we can observe a decrease in the rate of Exiter firms under 15 years old and an increase in the exit rate of firms over 30 years old since 2002. Firms under 15 years old comprehend over 65% and 70% of the Exiter firms in the Industry and Services sectors, respectively, getting closer to the Exiter profile rate distribution of USA and Mexico and very distinct of the Indian profile.

We observe in figure 2 panel (a) that 40 years old or more industrial firms comprise over 55% of the investment in the sector, 50% of the output, and employs about 35% of the labor force, getting closer to what we observe in the US employment share (Hsieh and Klenow, 2014) and Colombian employment share (Eslava and Haltiwanger, 2020). On the other hand, firms under 15 years old have together about 15% of the investment and output and about 20% of the labor force. Figure 2 panel (b) shows that 40 years old or more services firms comprise almost 30% of the investment, over 30% of the output and over 20% of the labor force in the sector. Firms under 15 years old have together about over 10% of the investment, near 15% of the output and over 20% of the labor force. Although, there is a bias which most of the younger firms die earlier and the firms with less than 30 employees are not available in our data.

Figura 1 – Distribution of firms in Brazil



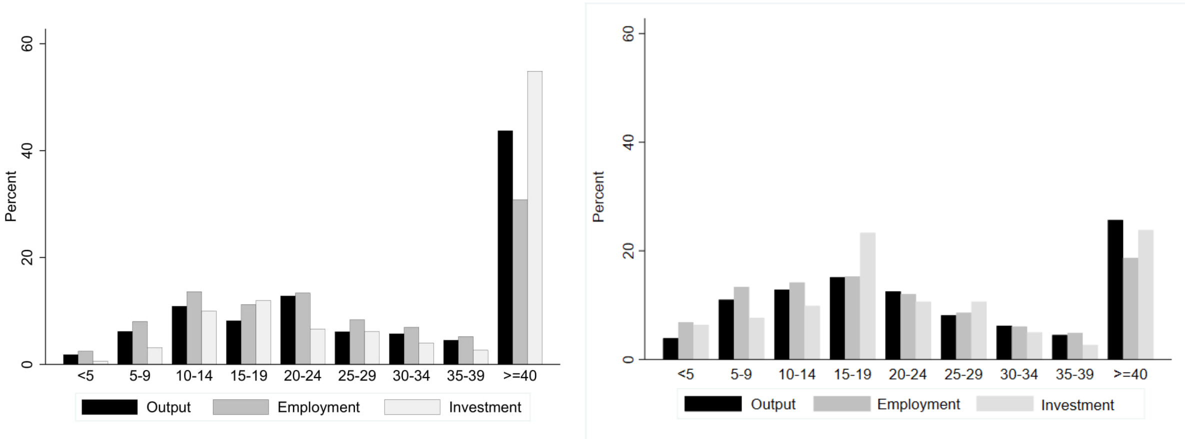
(a) Industry (b) Services

Note: Stayer Firms are firms that remain in the economy. Exiter firms are firms that exit the economy. Stayer firms comprehend over ten times the quantity of exiter firms. Data collected from the Entrepreneur Register Data provided by the Brazilian IR system.

Figure 3 shows the usual labor productivity measure by age group.[3] Comparing both sectors, we observe that the labor productivity for the services sectors are higher than the industrial sector in real terms (detailed in A.2.5), although we observe that there is a consistent productivity gap between the age group of firms in the Brazilian Industry and Services sector. Firms over 40 years old are over 100% more productive than firms under 10 years old in Industry and near 170% in the services sector. We also observe a

[3] Further detail see A.1

Figura 2 – Distribution of firms' outcomes in Brazil

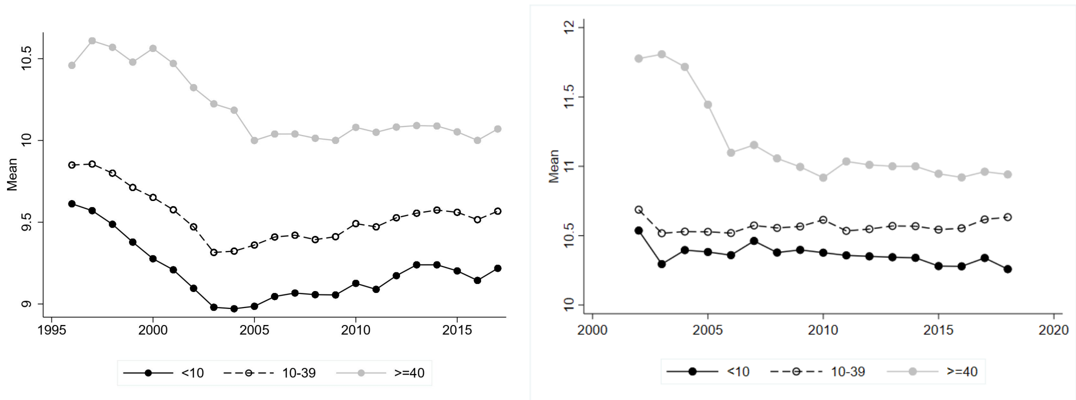


(a) Industry (b) Services

Note: Only profit-maximizing firms in the Brazilian Industry (excluding state owned firms). Data collected from Brazilian Institute of Geography and Statistics.

persistence of low labor productivity over the years since 2005. In section 5 we set a trade friction model to better discuss this persistence.

Figura 3 – Dynamic of Labor productivity by age group



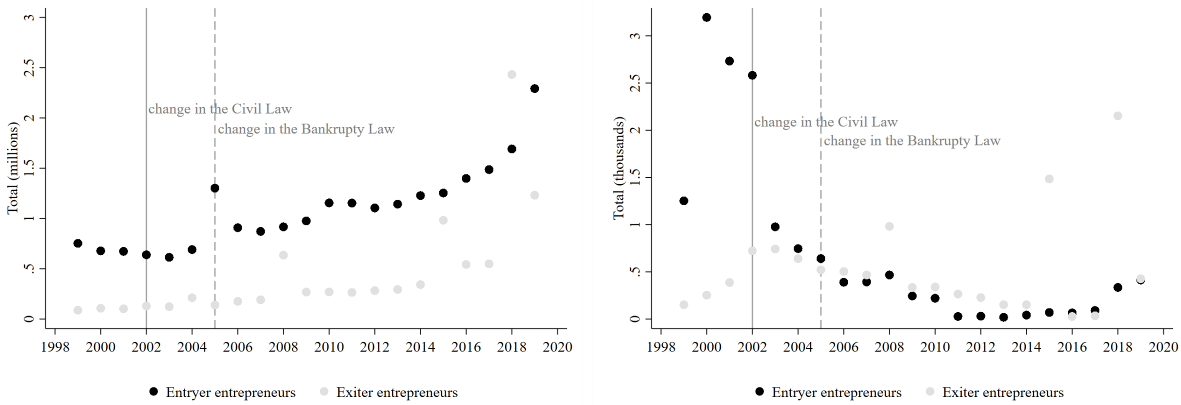
(a) Industry (b) Services

Note: Mean by group. Only profit-maximizing and Manufacturing firms (excluding state owned firms). Data collected from Brazilian Institute of Geography and Statistics.

In Figure 4 we present the entry and exit of domestic and foreign entrepreneurs since 1998 in Brazil. It shows that either entry and exit of domestic and foreign entrepreneurs reached very different profiles over the years. While the number of new domestic entrepreneurs have increased year by year since 2005, new foreign entrepreneurs decreased since 2000. This fact could be related to the persistence and gaps of figure 3. The impact of this dynamic we discuss more in section 7.

Overall, these new stylized facts presented suggest that resource allocation and distribution change significantly between the age groups. Firms under 10 years old com-

Figura 4 – Entry of Domestic and Foreign Entrepreneurs



(a) Domestic Entrepreneurs (b) Foreign Entrepreneurs

Note: Number of New Entryer and Exiter entrepreneurs each year since 1998 by nationality. Data collected form the Entrepreneur Register Data provided by the Brazilian IR system.

prehend about 55% of Industry and 60% of Services, but the over 40 years old firms concentrate 55% of the investment, 50% of the output and employs about 35% of the labor force in Industry and 30% of the investment, 30% of the output and 20% of the labor force in Services. Also, the trajectory of the labor productivity as seen in figure 3 is uneven between the age groups of firms. Firms over 40 years old are 100% more productive in Industry and 170% more productive in Services sector than firms under 10 years old. Then, we observe that the quantity of new foreign entrepreneurs did not increase through the years.

3 Quantitative Setting

In this section, we present the framework utilized to measure resource misallocation and TFP. It is a simple framework constructed by Hsieh and Klenow (2009), as in Hsieh and Klenow (2014) which has brought a life cycle measure to the discussion. From the control functions methods to estimate productivity, we show how the age of firm may reduce the mismeasurement of capital stock.

3.1 Measurements of Resource Misallocation

Environment. Suppose an environment at infinite and discrete time of a closed-economy without government. Then, consider that aggregate output is given by a CES aggregate of the output of individual plants:

$$Y_t = \prod_{s=1}^{S} \left(\left(\sum_{i=1}^{I_s} Y_{ist}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right)^{\theta_s} \tag{3.1}$$

where i indexes the firm and s indexes the sector, Y_{ist} is value added of the establishment, $\sigma \in (0, \infty)$ is the elasticity of substitution between sectors, and $\theta_s \in (0, 1)$ is the weight of each sector in the final good and $\sum \theta_s = 1$. The production function of each plant is given by:

$$Y_{ist} = A_{ist} K_{ist}^{\alpha_s} N_{ist}^{1-\alpha_s}, \tag{3.2}$$

where A_{ist} is the firm-specific productivity which represents the process efficiency here for realism, but it is also an equivalent to firm-specific quality or sector under certain assumptions, following Hsieh and Klenow (2009); N_{ist} is the firm's labor input and K_{ist} is the firm's capital stock; P_{is} is the firm-specific output price and w and R are typical costs of labor and capital, respectively. We assume that each intermediate firm is a monopolistic competitor selecting its labor and capital inputs (and therefore its output and price) to maximize current profits. There are input distortions where $\tau_{Yis} \in (0, 1)$ denotes an plant-specific revenue distortion and $\tau_{Kis} \in (0, 1)$ denotes the capital distortion. In these wedges are incorporated overheads or adjustment costs, which could be for technological reasons or related to policies.

Static Equilibrium. Consider that allocative choice depends only on technological level. However, inputs can have their values distorted. Thereby, resource allocation would result from differences between the input marginal revenue among firms. The marginal revenue product of labor (MRPL) is proportional to revenue per worker. The marginal revenue product of capital (MRPK) is proportional to the revenue-capital ratio. Thus, firms equalize the marginal revenue of inputs after distortion.

Following Hsieh and Klenow (2009) among others, the aggregate TFP is a function of misallocation of resources at the firm-level. Our focus analysis is the firm productivity A_{is}, and average revenue product τ_{Yis} and τ_{Kis} with age. Solving the allocative equilibrium between firms and using the equations of the Marginal Revenue Product of the factors, we obtain TFP at plant-level as

$$TFPR_{ist} = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{R_t}{\alpha_s}\right)^{\alpha_s}\left(\frac{w_{ist}}{1-\alpha_s}\right)^{1-\alpha_s}\frac{(1+\tau_{Kist})^{\alpha_s}}{1-\tau_{Yist}} \tag{3.3}$$

and TPF at sector-level as

$$TFP_{st} = \left[\sum_{i\in I_s}\left(A_{ist}\frac{\overline{TFPR}_{st}}{TFPR_{ist}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}, \tag{3.4}$$

where $\overline{TFPR}_{st}$ is a geometric average between the revenue of inputs in each sector. Equations 3.3 and 3.4 suggest that firm-level distortions affect the plant productivity and the sector productivity.

Define the nominal TFP is given by

$$TFPQ_{st} \equiv P_{st}TFP_{st}$$

and assume that the firm-level productivity and the distortions has a multivariate lognormal distribution

$$\begin{aligned}\log TFP_{st} \quad &= \quad \frac{1}{1-\sigma}\log E_t\left\{(TFPQ_{st})^{\sigma-1}\right\} \\ &\quad -\frac{\sigma}{2}\phi_Y^2-\left(\frac{\alpha_s+\alpha_s^2(\sigma-1)}{2}\right)\phi_K^2+\sigma\alpha_s\phi_{YK},\end{aligned} \tag{3.5}$$

where ϕ_Y and ϕ_K are standard deviations; ϕ_{YK} is covariance. Equations 3.5 suggests that more distortion means that productivity of the firms get more distant from the sectoral productivity. Thus, the dispersion exposes the allocation differences that are not related to productivity, but to misallocation extent.

Optimal allocation. Another measure of resource misallocation is the degree of misallocation, or in the other words, how far the current product is from the product implied by optimal allocation. Similarly to Hsieh and Klenow (2009), we can obtain the optimal allocation of inputs that maximizes the sector's product subject to the availability of inputs. In each time period, then optimal final good is

$$Y_t^E = \prod_{s\in S}\left[\sum_{i\in Is}\left(A_{it}\left(\frac{\alpha_s}{\alpha^E}K_{st}\right)^{\alpha_s}\left(\frac{1-\alpha_s}{1-\alpha^E}N_{st}\right)^{1-\alpha_s}\right)^{\sigma-1}\right]^{\frac{\theta_s}{\sigma-1}}, \tag{3.6}$$

where $\alpha^E = \frac{\partial lnY_t^E}{\partial lnK_{st}}$ and $1-\alpha^E = \frac{\partial lnY_t^E}{\partial lnN_{st}}$ such that Y_t^E is the optimal final good implied by efficient allocation in sectors of the economy.

We assume now that α can change among all firms. Therefore, Oberfield (2013) and Yang (2016) suggest that misallocation could occur in each sector. Otherwise, there exists unmeasurable technological heterogeneity at the firm-level, and this would imply an inefficiency on the aggregate product. Thus, the allocation of inputs that maximizes the product at the sector-level would not necessarily be the same allocation of inputs that maximizes the aggregate product. Given the availability of inputs in the economy, we can obtain the optimal allocation of these factors to maximize the final good. In this situation, the optimal final good is

$$Y_t^M = \prod_{s\in S}\left[\sum_{i\in Is}\left(A_{ist}\left(\frac{\alpha_i}{\alpha^M}\theta_s K_t\right)^{\alpha_i}\left(\frac{1-\alpha_i}{1-\alpha^M}\theta_s N_t\right)^{1-\alpha_i}\right)^{\sigma-1}\right]^{\frac{\theta_s}{\sigma-1}}, \tag{3.7}$$

where $\alpha^M = \frac{\partial lnY_t^M}{\partial lnK_t}$ and $1-\alpha^M = \frac{\partial lnY_t^M}{\partial lnN_t}$ such that Y_t^M is the optimal final good given by efficient allocation within sectors. From equations 3.1 and 3.6 we can determine the degree of misallocation within-sectors $MW = \frac{Y_t}{Y_t^E}$. From equations 3.6 and 3.7 we can determine the degree of misallocation between-sector as $MB = \frac{Y^E}{Y^M}$. Furthermore, the local effect of misallocation is given by the relationship between two measurements,

$$MW \times MB = \frac{Y}{Y^M} \tag{3.8}$$

Now we have a measure of the degree of misallocation of resources. For a given aggregate of input in each time period, the maximum output is the efficient output, and the greater the degree of misallocation resources.

Input Wedges. Computing α^E and α^M is one of the difficulties in this work. Given this, if the inputs' availability is time-variant, then the parameters α^E and α^M can change over time. If the resources are efficiently allocated in each sector, we can obtain that

$$\frac{\alpha^E P_{ist}^E X_{ist}^E}{K_{ist}^E} = \frac{\alpha_s P_{st} X_{st}}{K_{st}}$$

for each firm and time period. Identical to Vasconcelos (2017) and Oberfield (2013), we define the capital wedge of each firm as parameters of deviation in relation to the efficient case (within-industry)

$$TK_{ist} = \frac{P_{ist}X_{ist}/K_{ist}}{P_{ist}^E X_{ist}^E/K_{ist}^E} \tag{3.9}$$

and the labor wedge is given by

$$TL_{ist} = \frac{P_{ist}X_{ist}/N_{ist}}{P_{ist}^E X_{ist}^E/N_{ist}^E} \tag{3.10}$$

the firm's input wedges describe the relationship between the output at firm-level and the misallocation of resources. The greater the input wedge, the greater dispersion of the output (equations 3.3 and 3.4) among firms and, consequently, the greater the input distortion.

Then equation 3.4, 3.8, 3.9 and 3.10 define the key variables of resource misallocation that we focus- TFP dispersion at firm-level and input wedges. As the productivity distribution is time-variable, the life cycle of the firms should be correlated to the misallocation degree over time.

3.2 Productivity measurements and the role of Life Cycle of plants

We now have a question to attend: the life cycle of the firms contributes in the Brazilian's aggregate productivity? Is there any contribution of the distortions over the life cycle? In the plant-level available data, there is a capital bias specially from the older firms and this feature generate an underestimate of their productivity and, consequently, a poor measurement of the aggregated dispersion and productivity.

For estimate the productivity empirically, Levinsohn and Petrin (2003) added to the methods for conditioning out serially correlated unobserved shocks to the production technology. They build the idea based on Olley and Pakes (1996). They used investment to control for correlation between input levels and the unobserved firm-specif productivity process. What the Levinsohn-Petrin method shows is that intermediate inputs can solve this simultaneity problem.

Thus, we use the semi-parametric method as Levinsohn and Petrin (2003) and estimate:

$$\log Y_{ist} = \beta_0 + \beta_N \log N_{ist} + \beta_K \log K_{ist} + \omega_{ist} + \eta_{ist} \tag{3.11}$$

where N_{ist} is the free variable, K_{ist} is the state variable. $\omega_i st$ represents the intermediate inputs, as raw materials and electrical energy, and $\eta_i st$ is either measurement error or a shock to productivity which is not forecastable during the period in which labor can be adjusted and ω and η are unobserved. Additionally we make a correction to conditional the intermediate input function to the labor input (Ackerberg et al., 2015).

It is worth mentioning that Levinsohn-Petrin method brings a improvement compared with Olley-Packes method when adding Capital as a State variable, but also creates the possibility of capital bias from older firms. For example, imagine an old firm with several plants and workers around the world. This company allocates capital for great investments every 20 years. It is possible that the database had begun between this time period and could not capture the previous investment made. This mismeasurement would influence the construction of capital stock of old firms. For this we estimate:

$$\log Y_{ist} = \beta_0 + \beta_N \log N_{ist} + \beta_K \log K_{ist} + \beta_I \log I_{ist} + \beta_a \log a_t + \omega_{ist} + \eta_i st \tag{3.12}$$

where I_t is a second freely variable input, which they call the intermediate input. Additionally to the equation 3.11, we add another state variable a_t which represents the Age of the firm. Given that, we seek to reduce the bias generated from the firm's capital measurement.

4 Quantitative Results

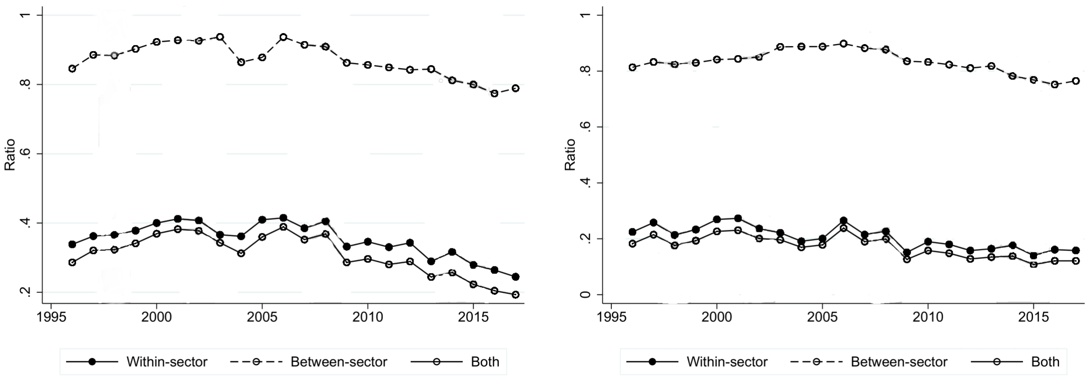
Figures 5 to 8 show the quantitative results. In this draft, we present results for the brazilian industry. Figure 5 shows the degree of resource misallocation in the brazilian industry. The line labeled Within-Industry Only shows actual output divided the output that could be attained if resources were allocated optimally within industry. The line labeled Between-Industry Only shows the output ratio that could be attained if resources were allocated optimally across all firms in each industry. The line labeled Both shows actual output divided by the output that could be attained if resources were allocated across all firms. Due to the entrepreneur register data, we could eliminate from the set all the non-profit maximizer firms. We also added the extraction industry to the analysis. These features were not available in similar exercises made by Vasconcelos (2017).

According to this figure, the gap implied by the within-industry effect varies more than implied by the between-industry effect. Therefore, the misallocation reflects the dynamic effect of the within-industry allocation confirming that the smaller the dispersion of TFP, the lower the misallocation. The lower the misallocation, the smaller the gap is to the effective product. What we observe, according to the figure is that, the degree of resource misallocation in the Brazilian Industry is persistent even though the reduction of efficiency since 2005. Furthermore, panel (a) of figure 5 presents misallocation degree using U.S Industry shares. We assume that factor intensities are the same as those of corresponding U.S. industries and that these U.S. industries are on average undistorted.[1] Figure 5 Panel (b) proceeds under the assumptions that all long-run differences in factor expenditure shares reflect differences in underlying technology rather than distortions. Brazilian Firm-Specific shares assumes that while a firm may face a distortion in particular years, it is, on average, undistorted. To this end, we compute the parameters of a single firm's production as follows: in each year, we compute the log the of the ratio of nominal expenditure on capital to nominal expenditure on labor. Under the assumption that, for each firm, the median of this quantity over all the years that the firm is in the sample reflects an undistorted choice of inputs, the parameters of the production function can be backed out accordingly. In both exercises, the result appears the same: the allocation efficiency is 20% if there was not misallocation. That means that removing misallocation the productivity may have increased in 5 times.

What can explain this upward trend of the last 12 years? Figure 3 also indicated such trend. We try to achieve some explanation through the life cycle optical, conducting

[1] We use expenditure data from the INDSTAT to compute the cost shares for the relevant industries in the U.S for 2000. We also impute these parameters based on other years. However, the dynamics and the results extension do not change significantly. We also use the German manufacturing sector as a reference and, again, the results did not change significantly.

Figura 5 – Allocation efficiency in the Brazilian industry



(a) U.S. industry Share (b) Brazilian Firm-Specific Share

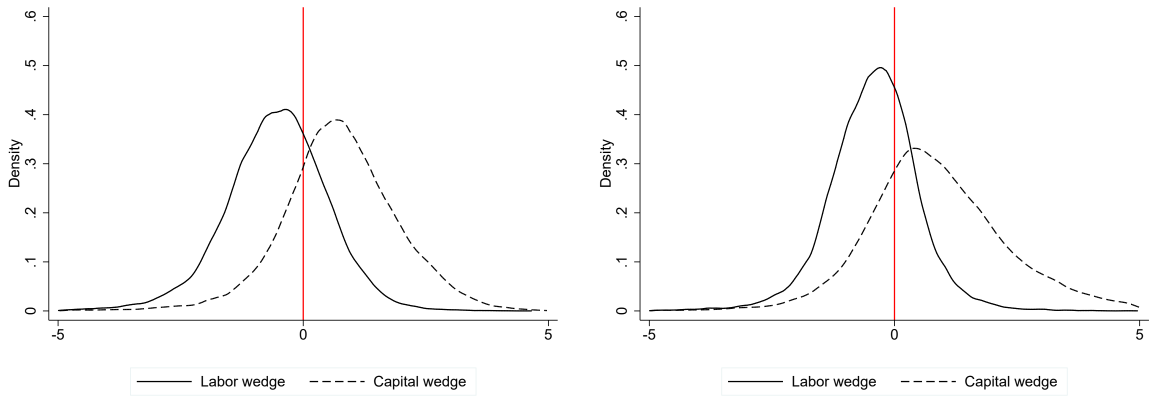
Note: The figure shows the evolution of allocation efficiency over time. The line labeled Within-Sector shows actual output divided the output that could be attained if resources were allocated optimally within industry. The line labeled Between-Sector shows the ratio of the output that could be attained if resources were allocated optimally in each industry and across all firms, respectively. The line labeled Both shows actual output divided by the output that could be attained if resources were allocated across all firms.

exercises to understand the misallocation persistence. Figure 6 shows the (log) distribution of capital and labor wedges in the Brazilian industry in 2017. We observe that there is high deviation as a result of a high extension misallocation. The mean of the capital being positive suggests a capital over-investment on the firms, making that the firms have more fixed capital they should have as we have shown in figure 2; or the capital bias from older firms (which hold the great share of capital and employment). In Brazil, firms of manufacturing of sugar and ethyl alcohol as firms of mechanical forming sector are the main sources of the high capital wedges we observe. In table A.2.1 we detailed the input wedges for the Brazilian industry by sector.

Figure 7 presents the dynamic of capital and labor wedge by age group in the Brazilian industries. Under 10 years old firms face capital wedges 20% higher than 40 years old firms capital wedges. Labor wedge in 40 years old firms is 2 times inferior when than under 10 years old firms labor wedge. However, over the years, the capital wedge is getting inferior for the older industries and, apparently, better for younger industries. It is important mentioning that, this result can be a consequence of younger firms moving to the older groups. We detailed in A.2.6 these results year by year.

We applied the Control function method developed by Levinsohn and Petrin (2003) in profit-maximizing industries, using age as a state variable (see equation 3.12). In our work applying such method reduces the capital omitted variable bias. We refer to much older industries which are in the data, for example. Then, we compute the input wedges as the major indicative of resource misallocation at disaggregated level, according to Hsieh and Klenow (2009) and Oberfield (2013). A relatively high input wedge indicates a firm

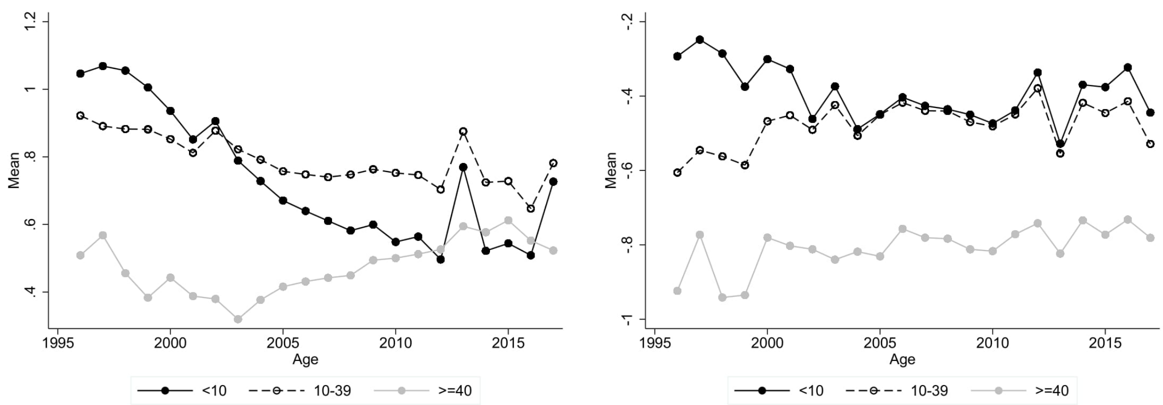
Figura 6 – Distribution of input wedges in the Brazilian industry, 2017



(a) U.S. industry Share (b) Brazilian Firm-Specific Share

Note: Firm-level log input wedges distribution in 2017. Data collected from Brazilian Institute of Geography and Statistics

Figura 7 – Dynamic of Input wedges by age group



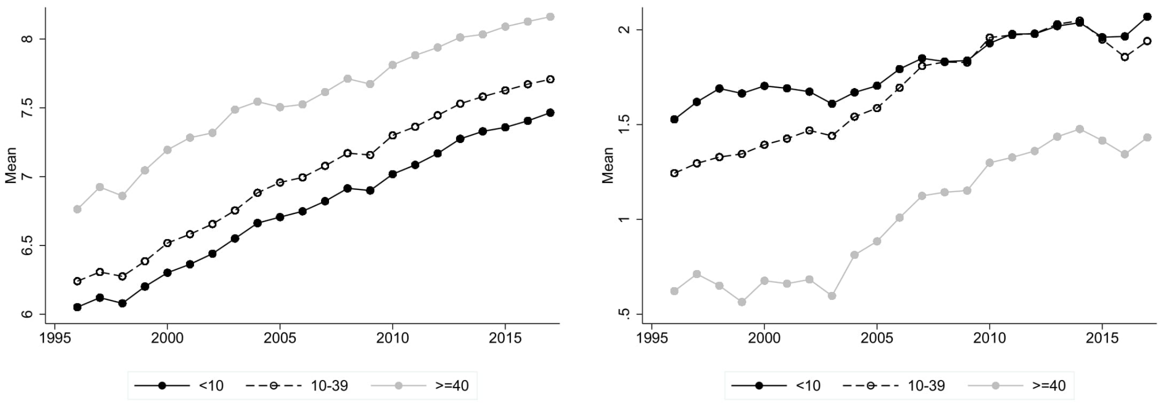
(a) Capital wedge (b) Labor wedge

Note: Labor and Capital Wedges Dynamic by age, 1996-2017. Data collected from Brazilian Institute of Geography and Statistics

does not have input enough, whereas a relatively low capital/labor wedge indicates the opposite. Figure 8 presents the measurement of productivity by firm's age group using the Control Function approach. We observe that after the reduction of capital omitted variable bias the order of the curves changes, indicating that firms under 10 years old are 65% more productive than 40 years old firms.

Overall, the results presented above are consistent with the misallocation evidences of a developing country as Brazil. (i) TFPR is scattered; (ii) Misallocation degree is persistent; (iii) Within-sectors misallocation are the major source . Besides, we showed that this differences varies with the age of firms, what indicates that the life cycle may have a relevant role to explain the previous evidences. Some exercises are made using price

Figura 8 – Dynamic of Productivity by age group



(a) TFPR (b) Levinsohn and Petrin (2003)

Note: Productivity measurements by age, 1996-2017. Data collected from Brazilian Institute of Geography and Statistics

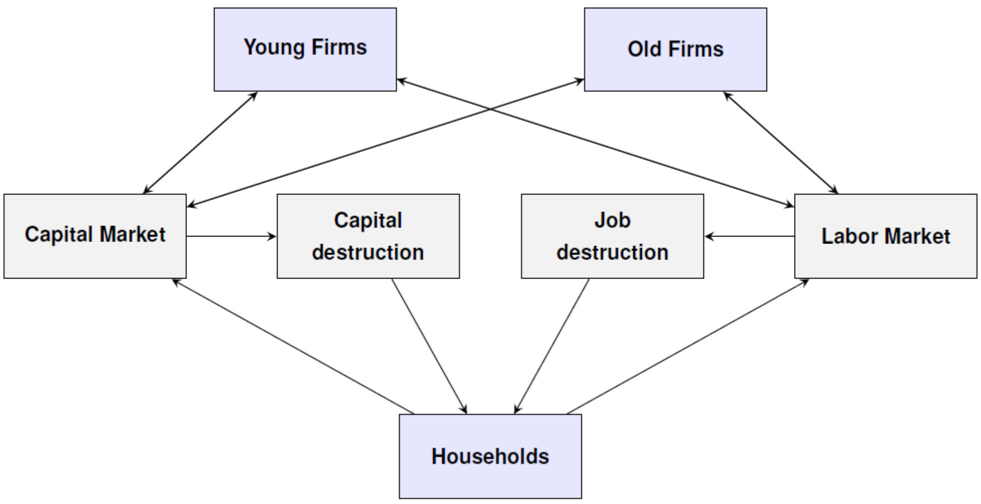
and quantity of firms besides using firms revenue, as in Eslava and Haltiwanger (2020). As we have only revenue of firms information, these exercises are not possible in our case. However, the set for Brazilian firms that we obtain is much representative than the set for Colombian firms they use.

5 Theoretical Setting

The elevated and persistent misallocation in Brazil that we observed in section 4, it is observed in others developing countries as India and China (Hsieh and Klenow, 2009), Chile (Oberfield, 2013) and Indonesia (Yang, 2016). For younger firms, micro-distortions can be an obstacle to their entry and growth. For older firms, micro-distortions can be the reason for their survival. We exploit an explanation for this persitence and show how this is connected to the life of cycle of firms, building a trading fricction theroretical model relating both stylized facts and Quantitative Results. As in the text books, we define trading frictions as market failures due the absence of double coincidence of the agents.

We set a model with labor and financial frictions using main features inspired by Hopenhayn (1992) and Hsieh and Klenow (2009). In resume, we develop a model with Random search, Endogenous input destruction, Price bargain, Entry barriers and input cost distortions. Figure 9 can illustrate the environment of our model. What we would like to explore is that inside a environment with severe market failures as Brazil, the trading frictions tend to create a high persistence to misallocation degree. Finally, we show as exogenous changes over the parameters which determine the life cycle of firms could influence on aggregate productivity and the misallocation degree of an economy.

Figura 9 – Representation of theoretical model



5.1 Environment

Households. Assume a continuum of different Households and time discrete. Each member has a labor endowment with different productivity $x \in (0, 1)$, with a nondegenerated distribution G and a wealth quantity to be allocated in free risk assets that pays $r \in [0,1]$ or in an investment project with profitability $i(\mathtt{x})$ where $\mathtt{x} \in (0,1)$ is the capital productivity of the investment. Assume that $i(\mathtt{x}) > r$. Households do not differ between Young and Old firms, and their only significant payoff is the factor remuneration.

Firms. Assume a continuum of firms with search externalities, a fixed labor search cost $\kappa_n \geq 0$ and a fixed operational cost $c \geq 0$ in initial time. There are two profiles of firms in this economy indexed by $j = o, y$: Old and Young firms, respectively, where $\varphi \in (0,1)$ is the share of new entry firms. Each firm has a level of productivity $p^o \neq p^y$ and young firms face an entry cost that we call $T > 0$. If the younger firm do not match in both markets, it cannot produce. If the older firm do not match in both markets, it keeps with the input quantity of last period after depreciation and exogenous separation.

Labor market. We consider N as the total of workers in the economy, where u is the unemployment rate and v is the vacant jobs rate. Once employed a worker do not return to search for a new job. Still, the total Matching is given by $m(uN, vN)$, where $m(\cdot,\cdot)$ is concave and increases in both arguments. Let $\theta^j = v^j/u$ be a non-negative measure of Labor-market tightness, then $m(\cdot) \equiv q(\theta^j)$ and $q(\theta^j)$ represents the probability of a firm making a match with a worker in the labor market. When there is a matching, the rate which an unemployed worker gets a job is $\theta^j q(\theta^j)$.

Firms maximizes their operational profits $(1-\tau_y)p^j x - w^j(x) - i(\mathtt{x}) - c$, where $p^j > 0$ is the productivity level of the firm; $w^j(x) > 0$ is the cost of production per hired worker. The decision to hire depends on the labor productivity $x \in (0,1]$ of the worker. There is a distortion in production $\tau_y \in [0,1]$. Firms can hire workers with productivity $R_N^{\prime j} \in (0,1)$, where $R_N^{\prime j} \leq R_N$ and R_N is the productivity in the absence of micro-distortion. This last extension conditioned to the matching is a new ingredient to understand the persistence of labor misallocation. If $\tau_y > 0$, then a low productivity workers $(R_N' \leq x \leq R_N)$ workers will remain employed.

Then, we can show that the Labor Value for a firm when enters a labor market is:

$$rV^o = -\kappa_n + q(\theta^o)(J^o(x) - V^o), \tag{5.1a}$$

$$rV^y = -\kappa_n - T + q(\theta^y)(J^y(x) - V^y), \tag{5.1b}$$

where V is the firm earnings under current prices when a job vacancy is open; and $J^j(x)$ is the firm earnings under current prices when the job vacancy is occupied by a worker with productivity x. Furthermore, the value function which represents the Labor Value for a firm that is already in the labor market, can be expressed by,

$$\begin{aligned} rJ^o(x) &= (1-\tau_y)p^o x - w^o(x) - i(\mathtt{x}) - c \\ &\quad -\lambda\left(J^o(x) - \int_{R_N'^o}^1 J^o(s')dG(s') - G(R_N^o)V\right), \qquad (5.2a) \\ rJ^y(x) &= (1-\tau_y)p^y x - w^y(x) - i(\mathtt{x}) - c \\ &\quad -\lambda\left(J^y(x) - \int_{R_N'^y}^1 J^y(s')dG(s') - G(R_N^y)V\right) \qquad (5.2b) \end{aligned}$$

where $\lambda \in (0,1)$ represents the separation rate. The firm pays a wage $w(x)$ for the worker with productivity x. The value functions for Households are similar to the previous firms Value Functions. We can express the opportunity cost to remain unemployed of a worker with productivity x as:

$$rU = b + \theta^o q(\theta^o)\left(W^o(x) - U\right) + \theta^y q(\theta^y)\left(W^y(x) - U\right), \qquad (5.3)$$

where U represents the worker earnings under current prices when unemployed; b is the unemployment earnings; W represents the worker earnings under current prices when employed. Now, the value of the employment for a worker with productivity x is:

$$rW^o(x) = w^o(x) - \lambda\left(W^o(x) - \int_{R_N'^o}^1 \max\left\{W^o(s'), W^y(s')\right\} dG(s') - G(R_N^o)U\right), \qquad (5.4a)$$

$$rW^y(x) = w^y(x) - \lambda\left(W^y(x) - \int_{R_N'^y}^1 \max\left\{W^o(s'), W^y(s')\right\} dG(s') - G(R_N^y)U\right) \qquad (5.4b)$$

Then, the aggregated labor input is given by:

$$N_t^o + N_t^y \leq N_t, \qquad (5.5)$$

where N_t is the total of employed workers in the economy.

Capital market. We consider I as the total of loanable funds in the economy, where $\mathtt{u}$ is the rate of Households providing funds and $\mathtt{v}$ is the rate of firms seeking funds. Still, the total Matching is given by $m(\mathtt{u}I, \mathtt{v}I)$, where $m(\cdot,\cdot)$ is concave and increases in both arguments. Moreover, it is still a homogeneous function with one degree of freedom. Let $\theta^j = v^j/\mathtt{u}$ be a measure of Capital-market tightness ($\theta^j > 0$), then $m(\mathtt{u}/\mathtt{v}^j, 1) \equiv q(\theta^j)$. So $q(\theta^j)$ represents the probability of a firm making a match with the loan in the Capital Market. When there is a matching, the rate which a household provides a loanable fund and makes a match is $\theta^j q(\theta^j)$. Then, $\mathtt{u}I$ and $\mathtt{v}I$ are the total of funds provided and firms seeking these funds, respectively. Moreover, once loaned there is not a search for a new loan.

Firms maximizes their operational profits $(1-\tau_y)p^j\mathtt{x} - i(\mathtt{x}) - c$; $i(\mathtt{x}) > 0$ is the cost of production per borrowed fund. The decision to get a loan depends on the capital productivity $\mathtt{x} \in (0,1]$. We considered a distortion in production $\tau_y \in [0.1]$. A firm borrows capital with productivity R_K^j, where $R_K \leq R_K$ and R_K is the productivity in the absence of distortion. If $\tau_y \geq 0$, then a group of low productivity capital ($R'_K \leq \mathtt{x} \leq R_K$) will remain allocated in the market.

The value functions for firms are basically the same as seen in the labor market. The Investment Value for a firm enters is given by:

$$r\mathcal{V}^o = -\kappa_k + q(\theta^o)(J^o(\mathtt{x}) - \mathcal{V}^o), \tag{5.6a}$$

$$r\mathcal{V}^y = -\kappa_k - T + q(\theta^y)(J^y(\mathtt{x}) - \mathcal{V}^y) \tag{5.6b}$$

where κ_k is an administrative fee; $\mathcal{V}$ is the firm earnings under current prices when it is in the market; $J(\mathtt{x})$ is the firm earning under current prices when the loan is obtained. The Investment Value for a firm that is already in the market is similar to 5.2a and 5.2b and is given by:

$$\begin{aligned} rJ^o(\mathtt{x}) &= (1-\tau_y)p^o\mathtt{x} - i(\mathtt{x}) - c \\ &\quad -\lambda\left(J^o(\mathtt{x}) - \int_{R'^o_K}^1 J^o(s')dG(s') - G(R^o_K)V\right), \end{aligned} \tag{5.7a}$$

$$\begin{aligned} rJ^y(\mathtt{x}) &= (1-\tau_y)p^y\mathtt{x} - i(\mathtt{x}) - c \\ &\quad -\lambda\left(J^y(\mathtt{x}) - \int_{R'^y_K}^1 J^y(s')dG(s') - G(R^y_K)V\right) \end{aligned} \tag{5.7b}$$

The value functions for Households are similar to the shown in the labor market. We express the value of the non-allocated capital for a lender with productivity $\mathtt{x}$ as

$$r\mathcal{U} = \mathtt{b} + \theta^o q(\theta^o)(I^o(\mathtt{x}) - \mathcal{U}) + \theta^y q(\theta^y)(I^y(\mathtt{x}) - \mathcal{U}) \tag{5.8}$$

where $\mathcal{U}$ represents the lender earnings under current prices when the capital is not allocated; $\mathtt{b}$ represents the earnings of the lender when the capital is not allocated, as a free risk asset; I represents the lender earnings under current prices when the capital is allocated. We can express the value of the allocated capital for a lender with productivity $\mathtt{x}$ is:

$$rI^o(\mathtt{x}) = i^o(\mathtt{x}) - \lambda\left(I^o(\mathtt{x}) - \int_{R'^o_K}^1 \max\left\{I^o(s'), I^y(s')\right\} dG(s') - G(R^o_K)\mathcal{U}\right), \tag{5.9a}$$

$$rI^y(\mathtt{x}) = i^y(\mathtt{x}) - \lambda\left(I^y(\mathtt{x}) - \int_{R'^y_K}^1 \max\left\{I^o(s'), I^y(s')\right\} dG(s') - G(R^y_K)\mathcal{U}\right) \tag{5.9b}$$

where $i^o(\mathtt{x}) \geq r$ and $i^y(\mathtt{x}) \geq r$.

Then, the investment defines the firms' capital such that:

$$K^o_t + K^y_t \leq K_t \tag{5.10}$$

where K_t is the total of capital allocated in investment in this economy.

5.2 Equilibrium

Labor Market equilibrium. We have to resolve a Nash Bargain problem given the surplus generated for each worker subject to equations 5.2a or 5.2b, 5.4a or 5.4b, and total surplus and entry condition ($J(R) = 0$). Define $\Theta \in (0, 1)$ as the bargain power of worker.

Assume that the Entry Condition for both firms is ($V = 0$). In order to facilitate, assume that $\theta^o = \theta^y$. The fact of a firm being older or younger impacts on the value functions J^o and J^y. The worker could work in both types of firms and the wage bargain equilibrium. We can define the equilibrium condition for old firms as:

$$\pi^o_N(1, R) + \zeta(\theta)(1 - R)\Big[\Theta\Omega(\theta)((i(1) - i(R)) - (1 - \tau_y))p^o - \eta(\theta)p^y\Big] = \frac{\kappa_n}{q(\theta)}(r + \lambda), \tag{5.11a}$$

$$\begin{aligned} R'^o_N = & \frac{1}{(1 - (\eta(\theta))^2)\left[(1 - \Phi(\theta))(1 - \tau_y)p^o + \Phi(\theta)(i(R) + c)p^o + \eta(\theta)p^y\right]} \\ & \times \Big[b(r + \lambda)^2(1 - \Theta)\Omega(\theta) + (1 - (\eta(\theta)^2)(i(R) - c) - \frac{\lambda}{r + \lambda}(1 - (\eta(\theta))^2) \\ & \times \Big((1 - \tau_y)p^o \int_R^1 (s - R) - (i(s) + i(R))dG(s) - \zeta\eta(\theta)p^y \int_R^1 (s - R)dG(s) \\ & + \Phi p^o \int_R^1 (i(s) - i(R))(s - R) - (s - R)dG(s)\Big)\Big] \end{aligned} \tag{5.11b}$$

where

$$\Omega(\theta) = \Theta(1 - (\theta q(\theta))^2) + (1 - \Theta)[(r + \lambda)[(r + \lambda) + \theta q(\theta)]] > 0$$
$$\eta(\theta) = (1 - \Theta)(r + \lambda)(\theta q(\theta)) > 0,$$
$$\pi_N(1, R) = (1 - \tau_y)p^o(1 - R) - (i(1) - i(R)) \geq 0$$
$$\zeta(\theta) = \frac{(1 - (\theta q(\theta))^2)}{1 - (\eta(\theta))^2} \in (0, 1)$$
$$\Phi(\theta) = \zeta(\theta)\Omega(\theta)\Theta > 0$$

The Equilibrium condition for young firms can be written as:

$$\pi^y_N(1, R) + \zeta(\theta)(1 - R)\Big[\Theta\Omega(\theta)((i(1) - i(R)) - (1 - \tau_y))p^y - \eta(\theta)p^o\Big] = \frac{\kappa_n + T}{q(\theta)}(r + \lambda), \tag{5.12a}$$

$$R'^{y}_{N} = \frac{1}{(1-(\eta(\theta))^2)\left[(1-\Phi(\theta))(1-\tau_y)p^y + \Phi(\theta)(i(R)+c)p^o + \eta(\theta)p^o\right]}$$
$$\times \Bigg[b(r+\lambda)^2(1-\Theta)\Omega(\theta) + (1-(\eta(\theta)^2)(i(R)-c) - \frac{\lambda}{r+\lambda}(1-(\eta(\theta))^2)$$
$$\times \Bigg((1-\tau_y)p^y \int_R^1 (s-R) - (i(s)+i(R))dG(s) - \zeta\eta(\theta)p^o \int_R^1 (s-R)dG(s)$$
$$+ \Phi p^y \int_R^1 (i(s)-i(R))(s-R) - (s-R)dG(s)\Bigg)\Bigg] \tag{5.12b}$$

Capital market equilibrium. With the same assumptions made in the labor market case, the equilibrium of the capital market is similar to Old firms equations 5.11a and 5.11b:

$$\pi^o_K(1,R) - \zeta(\theta)(1-R)\Big[\Theta\Omega(\theta)((1-\tau_y))p^o + \eta(\theta)p^y\Big] = \frac{\kappa_k}{q(\theta)}(r+\lambda), \tag{5.13a}$$

$$R'^{o}_{K} = \frac{1}{(1-(\eta(\theta))^2)\left[(1-\Phi(\theta))(1-\tau_y)p^o + \Phi(\theta)cp^o + \eta(\theta)p^y\right]}$$
$$\times \Bigg[\mathtt{b}(r+\lambda)^2(1-\Theta)\Omega(\theta) - (1-(\eta(\theta)^2)c - \frac{\lambda}{r+\lambda}(1-(\eta(\theta))^2)$$
$$\times \Bigg((1-\tau_y) - \Phi)p^o - \zeta\eta(\theta)p^y \int_R^1 (s-R)dG(s)\Bigg)\Bigg] \tag{5.13b}$$

where

$$\pi_K(1,R) = (1-\tau_y)p^o(1-R) \geq 0$$

and for Young firms similar to 5.12a and 5.12b:

$$\pi^y_K(1,R) - \zeta(\theta)(1-R)\Big[\Theta\Omega(\theta)((1-\tau_y))p^y + \eta(\theta)p^o\Big] = \frac{\kappa_k + T}{q(\theta)}(r+\lambda), \tag{5.14a}$$

$$R'^{y}_{K} = \frac{1}{(1-(\eta(\theta))^2)\left[(1-\Phi(\theta))(1-\tau_y)p^y + \Phi(\theta)cp^y + \eta(\theta)p^o\right]}$$
$$\times \Bigg[\mathtt{b}(r+\lambda)^2(1-\Theta)\Omega(\theta) - (1-(\eta(\theta)^2)c - \frac{\lambda}{r+\lambda}(1-(\eta(\theta))^2)$$
$$\times \Bigg((1-\tau_y) - \Phi)p^y - \zeta\eta(\theta)p^o \int_R^1 (s-R)dG(s)\Bigg)\Bigg] \tag{5.14b}$$

6 Theoretical Results

Given our not closed-form expressions we found in our model, in this section we made simulations of how our model behaves in significant parameters changes. We seek mostly to understand the mechanisms behind the resource misallocation persistence. In this draft, we do not fit our simulation results to real data. In Table 1 we set values for the initial guesses of the parameters.

Tabela 1 – Parameters

Description	Parameter	Value
Distortion in the production	τ_y	0.1
Earnings of the household when the inputs are not allocated	$b, \mathtt{b}$	3
Free risk asset return	r	0.01
Separation rate	λ	0.2
Bargain Power	Θ	0.5
Search cost of labor	κ_n	0.15
Search cost of capital	κ_k	0.25
Entry cost	T	0.05
Operational cost	c	0.05

Note: Initial guesses for the parameters of the model.

We set different values and grids for θ^o, θ^y and then the matching probability for old and young firms are different. We also set $p^y > p^o$. This assumption is befitting with the result we obtained in figure 8. To do the simulation, we aim a contraction mapping of the equilibrium equations we found. Then, in tables 2 to 4 we see the fixed point for those parameters and we alter them.

In table 2 we see the impact of a change in τ_y. The wage and earnings paid by the firms decrease is very subtle for both types of firms: 0.03% and 0.07% every +0.2 in τ_y, respectively to old and young firms. The Reserve Productivity of labor increases over 40% for old firms and 27% for young firms when adding +0.4 in τ_y. The increase in the Reserve Productivity of capital is even higher: 50% for old firms and 31% for young firms. These latest results suggests that a worker or capital allocated by a firm, when τ_y increases, must be more productive to a firm survival as the results from Yang (2016) suggested. The value of the Job allocated decrease is also subtle for both profiles, however the value of the Capital of old firms returns negative values when increasing the input wedge. This suggests to us that in certain point there is a disincentive to investment by the old firms.

Table 3 we simulate how the model reacts to a change in the search costs. As the only equations that express κ_l, κ_k are the Job and Capital Creation curves, then they will

Tabela 2 – Change in input wedge

2*Description	2*Variable	2*Benchmark	τ_y	
			+0.2	+0.4
Panel A: Old firms				
Wage paid to the worker with productivity 1	$w^o(1)$	0.02822	0.02812	0.02802
Earnings paid to the lender with productivity 1	$i^o(1)$	0.02844	0.02835	0.02826
Reserve Productivity of labor	${R'_N}^o$	0.5375	0.6107	0.7539
Reserve Productivity of capital	${R'_K}^o$	0.6559	0.7673	0.9847
Job Creation	$J^o_N(1)$	0.1778	0.1734	0.1689
Capital Creation	$J^o_K(1)$	0.0153	-0.0024	-0.0202
Panel B: Young firms				
Wage paid to the worker with productivity 1	$w^y(1)$	0.04552	0.04516	0.04479
Earnings paid to the lender with productivity 1	$i^y(1)$	0.03719	0.03686	0.03653
Reserve Productivity of labor	${R'_N}^y$	0.4628	0.5075	0.5883
Reserve Productivity of capital	${R'_K}^y$	0.5689	0.6326	0.7463
Job Creation	$J^y_N(1)$	0.1517	0.1455	0.1392
Capital Creation	$J^y_K(1)$	0.0955	0.0597	0.0239

Note: Simulations based on parameters of table 1.

Tabela 3 – Change in search cost

2*Description	2*Variable	2*Benchmark	κ_n, κ_k	
			+0.05	+0.10
Panel A: Old firms				
Wage paid to the worker with productivity 1	$w^o(1)$	0.02822	0.02822	0.02822
Earnings paid to the lender with productivity 1	$i^o(1)$	0.02844	0.02844	0.02844
Reserve Productivity of labor	${R'_N}^o$	0.5375	0.5375	0.5375
Reserve Productivity of capital	$R'N^o$	0.6559	0.6559	0.6559
Job Creation	$J^o_N(1)$	0.17787	0.16601	0.1542
Capital Creation	$J^o_K(1)$	0.01537	0.00352	-0.00832
Panel B: Young firms				
Wage paid to the worker with productivity 1	$w^y(1)$	0.04552	0.04552	0.04552
Earnings paid to the lender with productivity 1	$i^y(1)$	0.03719	0.03719	0.03719
Reserve Productivity of labor	${R'_N}^y$	0.4628	0.4628	0.4628
Reserve Productivity of capital	${R'_K}^y$	0.5689	0.5689	0.5689
Job Creation	$J^y_N(1)$	0.15179	0.13451	0.11723
Capital Creation	$J^y_K(1)$	0.0955	0.0850	0.0745

Note: Simulations based on parameters of table 1.

be the only results to change. When adding +0.10 in the benchmark of $\kappa_{l,k}$ the value of the Job allocated for old firms decreases 13% and for Young firms decreases 22%. For the value of the Capital, old firms returns negative values when adding +0.10 in the search cost and decreases 22% for the values of young firms. These results suggest that changes

Tabela 4 – Change in entry cost for young firms

2*Description	2*Variable	2*Benchmark	T	
			+0.05	+0.10
Young firms				
Wage paid to the worker with productivity 1	$w^y(1)$	0.04552	0.04552	0.04552
Earnings paid to the lender with productivity 1	$i^y(1)$	0.03719	0.03719	0.03719
Reserve Productivity of labor	${R'_N}^y$	0.4628	0.4628	0.4628
Reserve Productivity of capital	${R'_K}^y$	0.5689	0.5689	0.5689
Job Creation	$J_N^y(1)$	0.15179	0.0999	0.0826
Capital Creation	$J_K^y(1)$	0.0955	0.0627	0.0509

Note: Simulations based on parameters of table 1.

in the search costs do not affect the inputs distribution.

Then, in table 4 we see the impact of a change in the entry cost of the firms. Our assumption is that only Young firms pay that Entry cost, then they will be the only affected by a change in T. Our results suggest that an increase of +0.10 will decrease the value of the Job allocated in 45% and the value of the Capital in almost 47%.

Overall these theoretical simulations suggest that (i) an increase in micro-distortions would decrease wages and earnings of capital by 0.03% and 0.07% for old and young firms, would increase the Reserve Productivity of labor and capital in 40% and 50% for old firms and 27% and 31% for young firms, and would increase the disincentive to invest specially for old firms; (ii) an increase in search costs would decrease the value of the job and the capital allocated by 13% for old firms and 22% for young firms; (iii) The entry cost increase only affects the value of the Job and Capital of the young firms and an increase in 10% in Entry cost would affect near 3.5%. Moreover, these distortions reverberate the misallocation through years and these costs could be an explanation for the high persistence of low productivity.

7 A Future Empirical check: Negative shock on entrepeneurship

In this section, we exploit how institutional changes over the entry incentives of entrepreneurs could impact productivity and misallocation. We discuss the main changes to Brazilian Civil Law code introduced in 2002 as a direct approach to capture the causes of resource misallocation in Brazil[1]. The code replaced the 1916s Civil code and incorporated several other codes to condense them and summarize the extensive list of codes and acts. We will focus on the changes associated with the formation and participation of foreign entrepreneurs and companies on Brazilian economy.

In the early 2000s, Brazilian economy have experienced a major economic progress, which had mainly increased the incomes of the population. In theory, that environment could have become an attraction for new businesses, including foreign organizations, to operate in the country. In fact, the number of legal entities in Brazil have substantially increased in the period (see 4 panel (a)), even considering that the Legal Entity National Register (CNPJ, at Portuguese acronym) started, in the current form as known, around 1998.

However, something happened in the meanwhile, as we can observe in 4 panel (b). The number of foreign partners in Brazilian's firms have decreased abruptly. With this in mind, we exploit the result by the institutional optical. It may have been a change which induced the foreign entrepreneur and firm to not enter in the country. Given this, we considered the Civil Code change as a good explanation. In particular, the change turned an obligation to a foreign company and investment group, the need of a federal government authorization to operate in Brazil. The act is valid to operational companies, i.e., that have the will to produce in Brazil, and investment groups which want to be part of the share capital of a Brazilian firm.

Indeed, the code incorporated the Corporate act (Law nº 2.627). The article which we refer is the art.64 that expressed the need of authorization for Inc. and private firms[2]. Our argument then is when the code was implemented, the need of authorization dismissed the Incs. which may have influenced a shift to this group of companies. However, even when dismissing the Incs., we observe an exit flow of foreign investors. That leads us to two possible explanations: First, the non-national now can arbitrate if it is gainful to remain in a Brazilian Inc. or acquire an asset from any other company in another country. Second, the law change may had created a disturb for participation in private firms. In fact, there

[1] Law act nº 10.406 available in link.
[2] Law act nº 2.627 available in link.

were several cases which companies had legal issues because of foreign participation in private firms.

In the next drafts, our strategy is to determinate the impact of the change of Brazilian civil law on the exit of foreigners. Based in the theoretical simulations, reducing the entry of new foreign entrepreneurs would impact the availability of resources in the national economy. Given that, this institutional change should affect the productivity distribution, i.e, resource misallocation.

8 Final Remarks

This work discussed about the role of the life cycle of firms in the resource misallocation in Brazilian firms. Our results suggest that the life cycle component has a significant impact in the misallocation analysis. Hsieh and Klenow (2014) shows that manufacturing plants in the US grow with age more than plants in India and Mexico. This pattern of low growth with age with age is also observed in the Brazilian firms. Also the input wedges are concentrated in firms under 10 years old.

We also discussed with a trade friction model why the misallocation degree remains so persistent over the years. This persistence is not an exclusivity of Brazil, but also of many developing countries. The model contributes showing that distortions, as well as search and entry costs for new firms, can impact negatively in the entry of new and productive firms and propitiate the survival of old firms in the economy.

An important question we left for the future we briefly discussed in section 7. Institutional issues are largely defended in the literature as an direct approach source for resource misallocation and low efficiency and productivity of firms. We provided suggestive facts on how a change in the Civil Code law could impact in the entry of new foreign entrepreneurs. This might help to understand the mechanism behind productivity issues in the Brazilian case.

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A Supplementery Appendix

A.1 Data details

Age of firms. We used the year of the surveys and subtracted the year of firm's start activity.

Employment. We first calculate the average annual employment for each company. Each survey has its particular method to capture this variable in questionnaire. PAC survey reports the Total of Busy Staff every trimester. The Total of Busy Staff comprehends the paid and non-paid staff, and the owners as well. We sum the four total of busy staff in the year and divide the result by 4. PAIC reports the Total of Busy Staff every month. We sum all 12 months of busy staff in the year and divide the result by 12. PAS reports the Total of Busy Staff every trimester. We sum the four total of busy staff in the year and divide the result by 4. PIA already reports the average annual employment and we only sum the busy staff connected to industrial activity, to non-industrial activity and owners. Then, we sum the average annual employment of each survey by firm's age group. We use the average annual employment because we mitigate the effects of cyclical employment through the year.

Investment. We first calculate the annual investment of each firm. PAC reports the Total of Acquisitions, Own production and Improvements in the year. This total comprehends acquisition and improvements on lands, edifications, machines, computers, transportation facilities and others. We subtract from this variable the Total of write-offs of fixed assets in the year, which comprehends the write-offs on lands, edifications, machines, computers, transportation facilities and others. PAIC reports the Total of Acquisitions, Own Production and Improvements, separately. Each total comprehends the same assets: Land, Edifications, Machines and equipments, Transportation Facilities and others. We sum these Totals and subtract the Total of write-offs of assets. PAS reports the Total of Acquisitions, Own Production and Improvements in the year. This total comprehends acquisition and improvements on lands, edifications, machines, computers, transportation facilities and others. We subtract from this variable the Total of write-offs of fixed assets in the year, which comprehends the write-offs on lands, edifications, machines, computers, transportation facilities and others. In PIA, we sum the value of acquisitions, own production and improvements of lands, edifications, machines, transportation facilities and others. We subtract from this result the write-offs of lands, edifications, machines, trasportation facilities and others. Then, we sum the annual investment of each survey by firm's age group.

Output. We first calculate the annual production value for each firm. PAC reports the net revenue, which is the revenue from the sales, commissions over sales of commercial representative, sales of own production products, services of maintenance, franchise royalties and from other activities; subtracted the deductions of cancelled sales and taxes over the sales. We subtract from the net revenue, other operational and non-operational incomes which comprehend financial income, rental income, participation in others entrepreneurships and others. PAIC reports the net revenue, which is the revenue from construction services, incorporation of buildings, office and technical services, construction material sales, sales of buildings, labor rental and other activities income; subtracted the deductions of cancelled sales and taxes. We subtract from the net revenue, other operational and non-operational incomes which comprehend financial income, rental income, participation in others entrepreneurships and others. PAS reports the net revenue, which is the revenue from services provision, incomes of edition of books, magazines and others graphic products, products sales, industrial services and others; subtracted the deductions of cancelled sales and taxes. We subtract form the net revenue, other operational and non-operational incomes which comprehend financial income, rental income, subsidies, participation in others entrepreneurships and others. PIA reports the gross value of industrial production that we subtract other operational and non-operational incomes, to capture the output for industrial sector. Then, we sum the annual production value of each survey by firm's age group and we obtain the Output.

Capital stock. On the derivative variables, the greatest challenge in this kind of work is the construction of firm-level capital variables. The database we own, collects information regarding the investment in machinery, vehicles, buildings and land. We use the perpetual inventory method to estimate gross fixed capital stock at the firms. Therefore, we assume the depreciation rate is 5%, 10% and 20%, respectively, for machinery, land and buildings and vehicles, identical to Oberfield (2013) and Vasconcelos (2017).

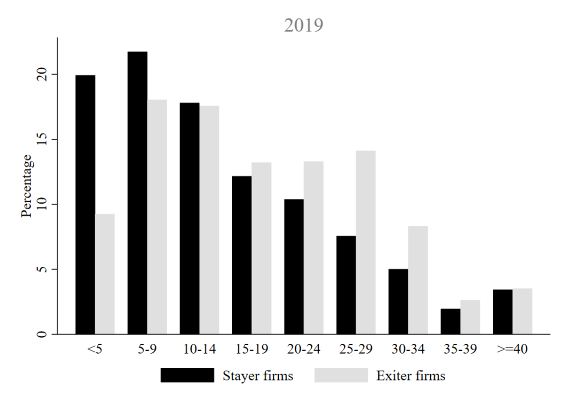
Wage. The labor remuneration is annual wages paid to all workers associated with the firm. For comparison, we used the Brazilian National Consumer price index to deflate the wages, as the deduction of the industrial operating cost from the industrial gross values, that we used for defining the value added.

Labor Productivity. To measure the labor productivity, we use the total output of each sector by age group and divided by the quantity of workers. We deflated the prices using the Brazilian General price index (IGP).

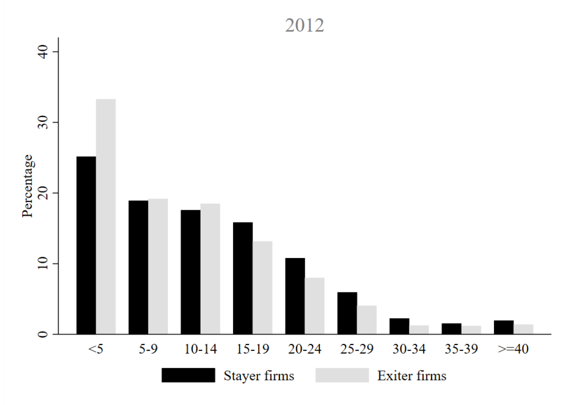
A.2 Additional figures and tables

What is expressed as Industry contains: Extracting industries, Manufacturing industries; Services: Construction, Commerce, Transportation, Accommodation and Food, Information and Communication, Financial and Insurance, Real estate, Science and Technology, Administrative activities, Education, Health and social services, Arts, Culture, Sports and Leisure, Other services.

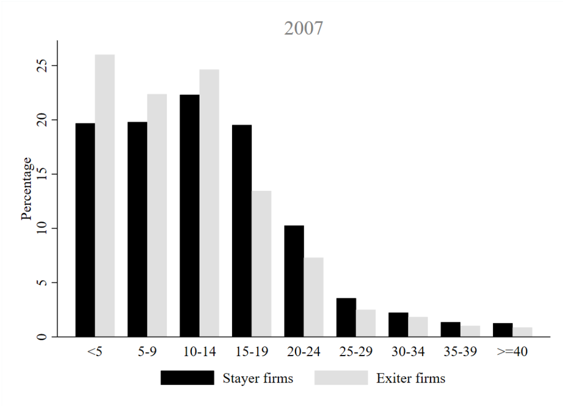
Figura A.2.1 – Distribution of Industry firms in Brazil



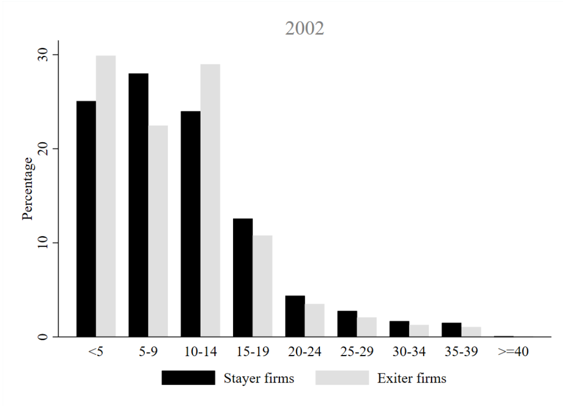
(a) 2019



(b) 2012



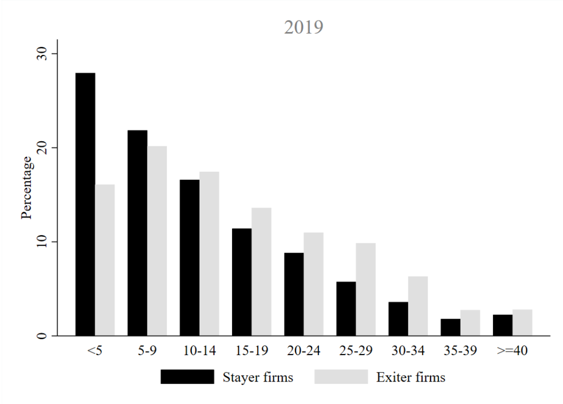
(c) 2007



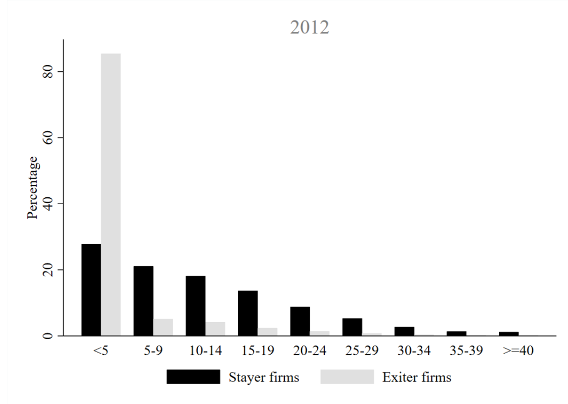
(d) 2002

Note: Stayer Firms are firms that remain in the economy. Exiter firms are firms that exit the economy. Stayer firms comprehend over ten times the quantity of exiter firms. Data collected from the Entrepreneur Register Data provided by the Brazilian IR system.

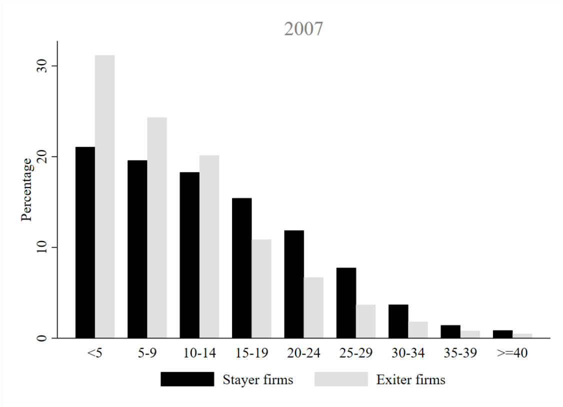
Figura A.2.2 – Distribution of Services firms in Brazil



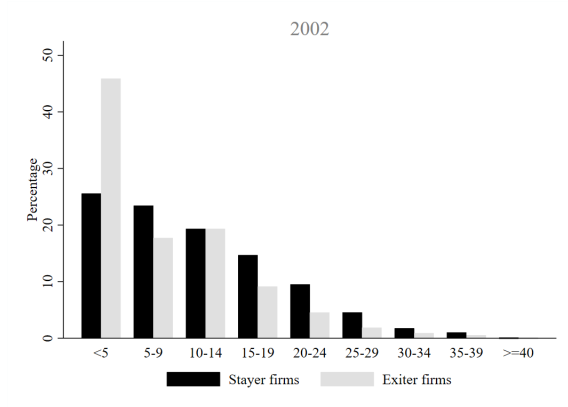
(a) 2019



(b) 2012



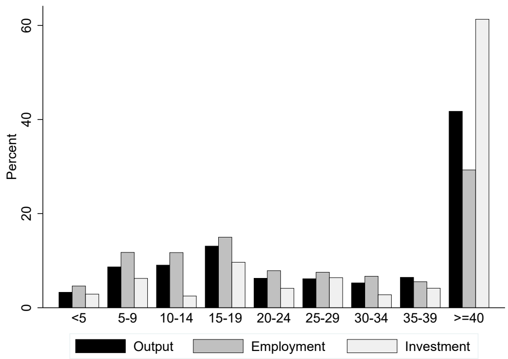
(c) 2007



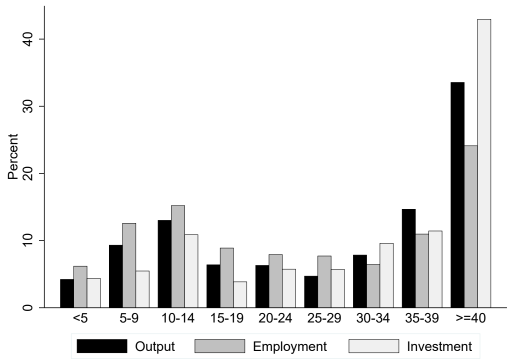
(d) 2002

Note: Stayer Firms are firms that remain in the economy. Exiter firms are firms that exit the economy. Stayer firms comprehend over ten times the quantity of exiter firms. Data collected from the Entrepreneur Register Data provided by the Brazilian IR system.

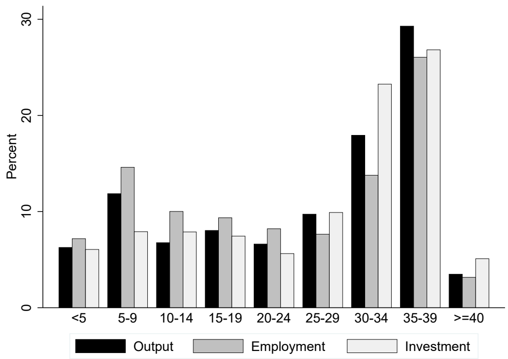
Figura A.2.3 – Other share of firms' outcomes by age group



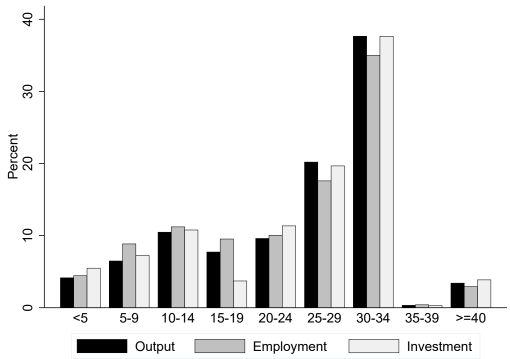
(a) 2012



(b) 2007



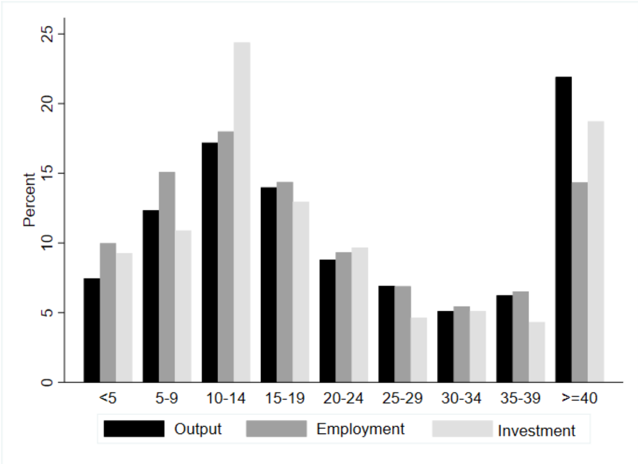
(c) 2002



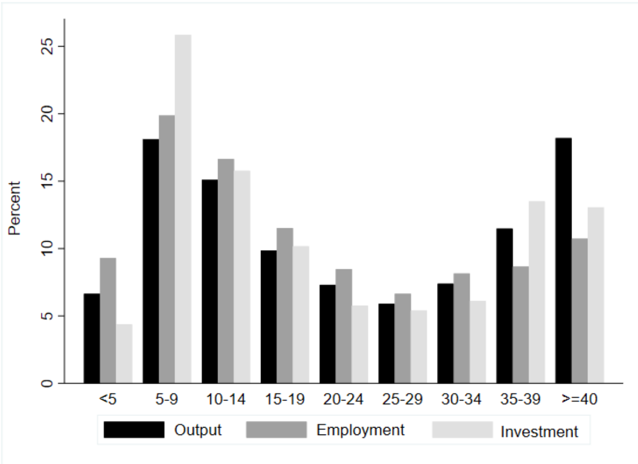
(d) 1997*

Note: Only profit-maximizing firms in the Brazilian Industry (excluding state owned firms). Data collected from Brazilian Institute of Geography and Statistics.

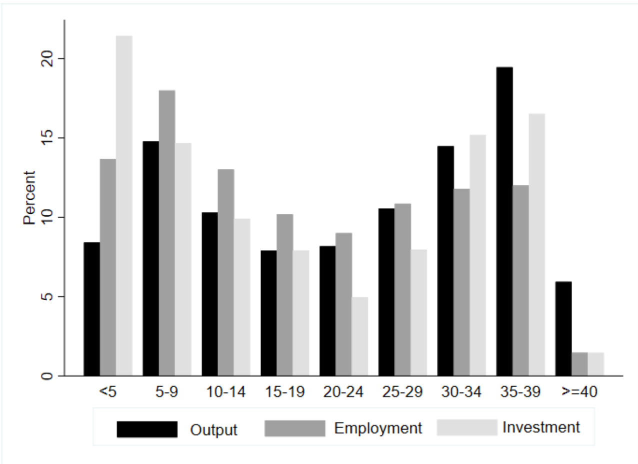
Figura A.2.4 – Other share of services firms' outcomes by age group



(a) 2012



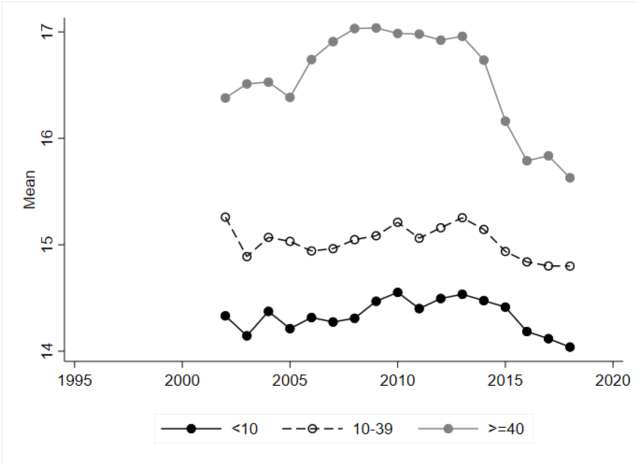
(b) 2007



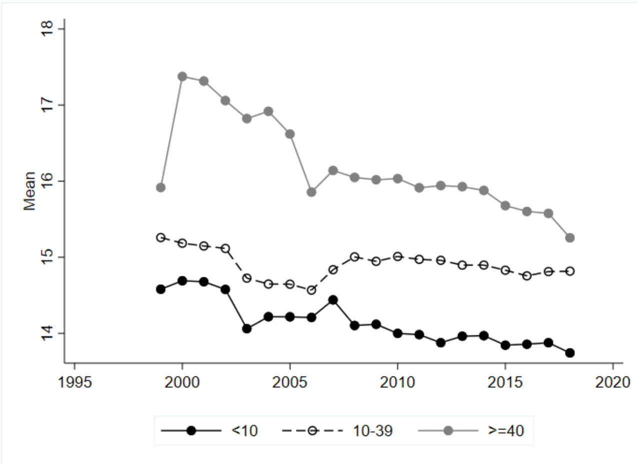
(c) 2002

Note: Data collected from Brazilian Institute of Geography and Statistics.

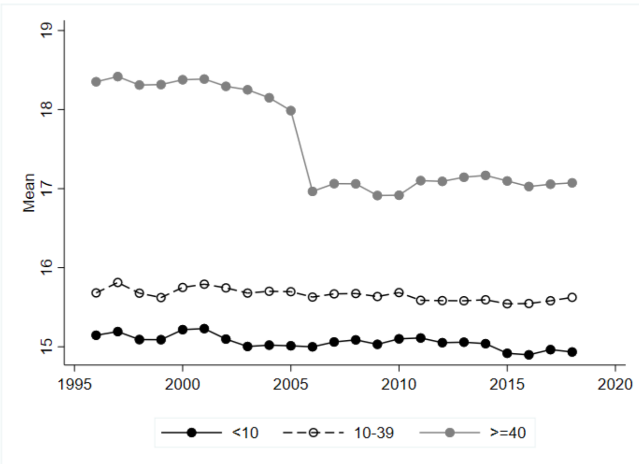
Figura A.2.5 – Dynamic of Labor Productivity of Services sector by age group



(a) Construction



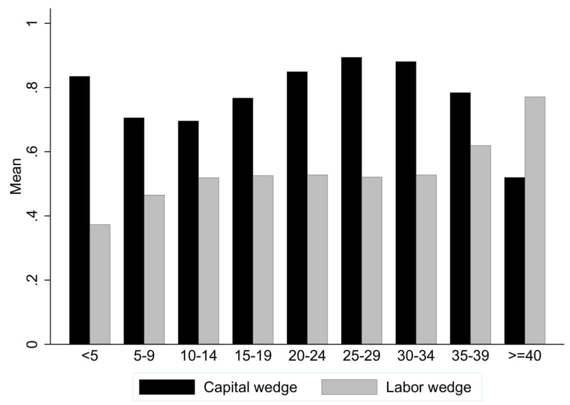
(b) Services



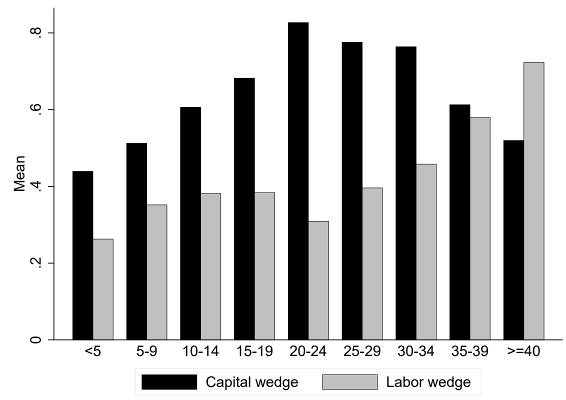
(c) Commerce

Note: Only profit-maximizing firms in the Brazilian Services Sector (excluding state owned firms). PAIC and PAS data are available since 2002 and 1998, respectively. Data collected from Brazilian Institute of Geography and Statistics.

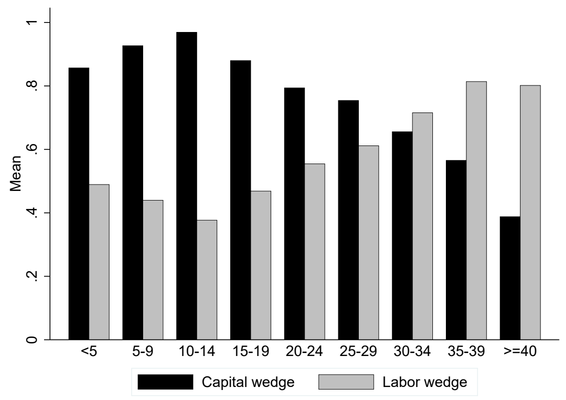
Figura A.2.6 – Input wedges by age group



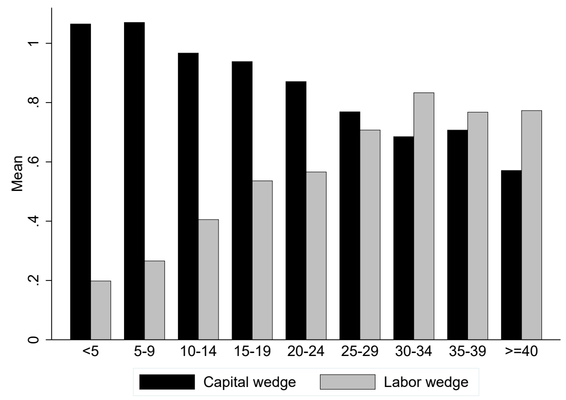
(a) 2017



(b) 2012



(c) 2002



(d) 1997

Note: Firm-level input wedges by age group. Mean by group. Only profit-maximizing and Manufacturing firms (excluding state owned firms). Data collected from Brazilian Institute of Geography and Statistics

Tabela A.2.1 – Input wedges in the Brazilian industry (2017)

Sector (isic31)	Capital wedge	Labor wedge	Scala wedge	Total of firms	Age (years)	Sector (isic31)	Capital wedge	Labor wedge	Scala wedge	Total of firms	Age (years)
Mining and quarrying						Manufacturing					
1030	1.3	1.4	1.5	18	36.5	2519	2.6	1.0	1.9	279	26.5
1120	2.3	1.3	1.5	101	15.4	2520	2.6	1.2	1.9	2153	20.3
1310	4.7	1.2	3.8	48	24.0	2610	4.5	0.9	2.2	347	19.2
1320	7.4	1.2	5.0	100	22.0	2691	3.6	1.3	2.0	89	25.1
1410	3.8	0.6	2.0	799	26.8	2692	3.0	1.3	2.2	43	29.0
1421	1.9	0.7	1.5	20	34.8	2693	3.0	3.1	1.7	1141	25.5
1422	1.0	1.1	1.0	39	28.8	2694	5.9	0.9	4.0	97	29.0
1429	3.9	0.8	2.1	182	24.1	2695	2.8	0.8	1.6	729	21.4
						2696	3.0	1.1	1.8	332	22.5
Manufacturing						2710	10.3	1.0	5.4	345	26.0
1511	3.2	0.8	1.9	1010	19.8	2720	8.8	0.8	4.6	278	21.6
1512	4.2	0.5	2.0	164	18.6	2731	3.3	1.1	2.1	222	24.8
1513	2.2	0.9	1.6	309	20.9	2732	5.2	1.0	2.4	119	23.3
1514	6.0	0.4	2.7	129	24.3	2811	5.0	1.0	2.1	845	19.3
1520	2.6	0.9	1.9	811	23.1	2812	1.9	1.5	1.7	126	22.2
1531	2.3	0.9	1.7	437	27.5	2813	2.4	1.3	1.7	107	15.4
1532	3.5	0.7	2.4	52	22.4	2891	2.2	3.0	2.5	217	21.9
1533	2.5	0.8	1.8	306	20.0	2892	4.8	0.8	2.2	484	19.8
1541	2.4	0.9	1.4	1296	20.9	2893	2.7	1.1	1.8	227	26.1
1542	10.7	0.4	4.1	149	34.7	2899	2.8	3.1	1.9	1116	24.6
1543	2.5	0.6	1.7	113	28.8	2911	1.6	1.9	1.7	24	29.7
1544	1.9	1.4	1.7	220	22.3	2912	695.9	0.5	2.4	299	26.7
1549	2.7	1.0	1.9	713	24.1	2913	16.2	0.9	4.0	89	30.8
1551	28.8	0.8	5.8	248	27.3	2914	2.0	1.1	1.7	71	26.2
1552	2.9	0.9	1.8	82	32.5	2915	4.8	1.4	3.4	182	25.6
1553	4.5	0.7	2.4	105	15.6	2919	2.8	1.0	1.8	232	23.8
1554	3.5	0.7	1.9	645	21.6	2921	2.4	1.0	1.9	420	25.8
1600	6.2	1.0	2.3	152	23.1	2922	3.2	0.8	1.8	232	27.1
1711	4.3	1.9	3.0	208	23.9	2923	2.4	0.8	1.5	59	20.2
1712	2.3	0.9	1.4	371	20.9	2924	1.9	1.2	1.6	150	24.6
1721	3.3	0.9	1.8	570	24.7	2925	2.6	0.9	1.7	177	27.6
1722	4.9	1.3	2.2	44	18.1	2926	2.4	1.0	1.8	55	29.4
1723	5.2	0.7	2.6	54	25.4	2927	2.0	1.5	1.9	10	34.4
1729	3.5	0.9	2.0	347	23.2	2929	3.3	0.8	2.0	482	24.1
1730	3.3	1.0	2.0	356	24.1	2930	3.6	1.0	2.1	172	23.0
1810	15.6	8.3	6.7	4321	17.9	3000	2.6	4.2	2.2	159	16.9
1911	3.0	0.8	2.0	206	25.2	3110	4.0	1.1	2.4	223	21.7
1912	4.5	1.4	2.2	258	20.1	3120	2.6	1.4	1.7	274	22.6
1920	2.1	1.0	1.4	1479	16.8	3140	3.0	0.9	1.6	84	21.3
2010	3.6	1.2	2.1	596	21.4	3150	3.9	1.2	2.2	166	21.9
2021	3.9	0.9	2.2	217	22.5	3190	2.9	1.0	1.8	462	21.4
2022	3.4	1.2	2.0	255	25.3	3210	3.2	0.6	1.6	207	18.4
2023	2.0	1.0	1.4	127	19.7	3220	1.2	1.6	1.4	134	19.5
2029	3.0	1.0	1.8	152	21.2	3230	3.1	1.0	1.8	143	19.6
2101	7.8	1.5	5.0	165	28.2	3311	6.7	1.4	2.9	96	22.3
2102	2.9	1.5	2.1	525	23.4	3313	2.9	1.1	2.0	235	24.7
2109	4.4	0.9	2.3	250	19.2	3320	2.6	1.1	1.8	951	20.0
2219	2.8	1.2	1.8	219	21.3	3330	1.1	0.9	1.0	12	28.1
2221	2.9	1.0	1.9	807	21.9	3410	2.2	1.5	1.8	270	25.6
2222	2.4	0.9	1.4	376	17.7	3420	4.9	1.2	2.4	304	23.8
2230	2.1	1.2	1.6	32	16.1	3430	2.7	0.9	1.9	327	27.4
2310	1.1	0.5	0.9	4	18.5	3511	9.2	2.2	5.0	91	20.5
2320	8.1	1.7	5.1	137	24.0	3512	11.7	1.4	4.0	64	15.9
2411	3.0	1.2	2.4	300	25.8	3520	5.8	1.2	3.0	63	21.6
2412	2.9	1.3	2.5	230	21.1	3530	2.9	1.7	2.4	49	19.4
2413	3.1	0.9	2.5	186	22.6	3591	2.9	1.0	1.8	78	21.0
2421	4.1	1.2	3.1	82	26.4	3599	1.5	1.2	1.2	58	25.1
2422	2.6	0.9	1.8	294	23.2	3610	2.9	1.0	1.8	2132	21.9
2423	3.7	1.0	1.7	366	30.4	3691	1.6	0.8	0.9	182	19.8
2424	2.5	2.3	2.4	696	23.5	3692	7.2	0.9	2.3	54	23.8
2429	3.3	1.0	2.4	382	24.7	3693	3.4	1.1	1.7	120	19.3
2430	8.8	0.7	2.7	34	17.1	3694	4.5	0.7	1.8	238	21.3
2511	7.8	1.0	2.8	283	22.6	3699	3.6	1.3	2.0	1524	22.2

Note: Capital wedge is the mean of the calculated capital wedge from each sector; Labor wedge is the mean of the calculated wedge from each sector; Age is the average age of each sector. Results constructed based on identified microdata collected at PIA. To maintain the confidentiality, sectors with less than 3 firms were excluded.

A.3 Mathematical Appendix

A.3.1 Reaching the equilibrium equations

Labor Market Equations. To reach the labor market equilibrium equations we set from the equations of the firms environment. Let $V = 0$ (entry condition). Then, equations 5.1a and 5.1b can be written as:

$$J^o(x) = \frac{\kappa_n}{q(\theta^o)}, \tag{A.3.1a}$$

$$J^y(x) = \frac{\kappa_n + T}{q(\theta^y)} \tag{A.3.1b}$$

And assume now that $x = 1$. Then equations 5.2a and 5.2b can be expressed as:

$$J^o(1) = \frac{(1-\tau_y)p^o \cdot 1 - w^o(1) - i(1) - c}{(r+\lambda)}, \tag{A.3.2a}$$

$$J^y(1) = \frac{(1-\tau_y)p^y \cdot 1 - w^y(1) - i(1) - c}{(r+\lambda)} \tag{A.3.2b}$$

And then we merge equations A.3.1a to A.3.2a and equations A.3.1b to A.3.2b to find:

$$\frac{\kappa_n}{q(\theta^o)} = \frac{(1-\tau_y)p^o \cdot 1 - w^o(1) - i(1) - c}{(r+\lambda)}, \tag{A.3.3a}$$

$$\frac{\kappa_n + T}{q(\theta^y)} = \frac{(1-\tau_y)p^y \cdot 1 - w^y(1) - i(1) - c}{(r+\lambda)} \tag{A.3.3b}$$

For the Households equations, we subtract equation 5.4a and 5.4b by 5.3 with $x = 1$,

$$W^o(1) - U = \frac{w^o(1) - b}{r + \lambda + \theta^o q(\theta^o)} - \frac{\theta^y q(\theta^y)(W^y(1) - U)}{r + \lambda + \theta^o q(\theta^o)}, \tag{A.3.4a}$$

$$W^y(1) - U = \frac{w^y(1) - b}{r + \lambda + \theta^y q(\theta^y)} - \frac{\theta^o q(\theta^o)(W^o(1) - U)}{r + \lambda + \theta^y q(\theta^y)} \tag{A.3.4b}$$

Assuming $\theta^o = \theta^y$, we can achieve:

$$W^o(1) - U = \frac{(r+\lambda)(w^o(1) - b) + \theta q(\theta)(w^o(1) - w^y(1))}{1 - (\theta q(\theta))^2}, \tag{A.3.5a}$$

$$W^y(1) - U = \frac{(r+\lambda)(w^y(1) - b) + \theta q(\theta)(w^y(1) - w^o(1))}{1 - (\theta q(\theta))^2} \tag{A.3.5b}$$

To reach equations of equilibrium in section 5, we resolve the Nash-Bargain problem, optimizing the Surplus equation:

$$w^j(x) = arg\max(W^j(x) - U)^{\Theta}(J^j(x) - V)^{(1-\Theta)} \tag{A.3.6}$$

subject to

$$J(R^j) = 0, \tag{A.3.7a}$$

$$rJ^j(x) = (1-\tau_y)p^j x - w^j(x) - i(\mathtt{x}) - c$$
$$-\lambda\left(J^j(x) - \int_{R'^j}^1 \max\{J^o(s'), J^y(s')\}\, dG(s') - G(R^j)V\right), \tag{A.3.7b}$$

$$rW^j(x) = w^j(x) - \lambda\left(W^j(x) - \int_{R'^j}^1 \max\{W^o(s'), W^y(s')\}\, dG(s') - G(R^j)U\right), \tag{A.3.7c}$$

$$S(x) = W^j(x) - U + J^j(x) - V^j \tag{A.3.7d}$$

Then, for $W^j(x)$ and assuming $x = 1$ and the Entry Condition $V = 0$:

$$\frac{\Theta J(1)}{1-\Theta} = W^j(1) - U \tag{A.3.8}$$

Using the relations found in A.3.1a/A.3.1b and A.3.5a/A.3.5b we get,

$$\Theta\frac{\kappa_n}{q(\theta)}[1-(\theta q(\theta))^2] = [1-\Theta][(r+\lambda)(w^o(1)-b) + \theta q(\theta)(w^o(1) - w^y(1)],$$
$$\Theta\frac{\kappa_n + T}{q(\theta)}[1-(\theta q(\theta))^2] = [1-\Theta][(r+\lambda)(w^y(1)-b) + \theta q(\theta)(w^y(1) - w^o(1)]$$

and using the relations we found in A.3.3a/A.3.3b we reach:

$$w^o(1) = \frac{[1-\Theta][r+\lambda][(r+\lambda)(-b) + \theta q(\theta)(-w^y(1)] - \Theta[(1-\tau_y)p^o \cdot 1 - i(1) - c][1-(\theta q(\theta))^2]}{\Theta[(\theta q(\theta))^2 - 1] - [1-\Theta][(r+\lambda)[\theta q(\theta) + (r+\lambda)]]},$$
$$w^y(1) = \frac{[1-\Theta][r+\lambda][(r+\lambda)(-b) + \theta q(\theta)(-w^o(1)] - \Theta[(1-\tau_y)p^y \cdot 1 - i(1) - c][1-(\theta q(\theta))^2]}{\Theta[(\theta q(\theta))^2 - 1] - [1-\Theta][(r+\lambda)[\theta q(\theta) + (r+\lambda)]]}$$

At last we replace the equations above in each other:

$$\begin{aligned} w^o(1) = {} & \frac{1}{1-(1-\Theta)^2(r+\lambda)^2(\theta q(\theta))^2} \\ & \cdot \Big[b(r+\lambda)^2(1-\Theta)[\Theta(1-(\theta q(\theta))^2 + (1-\Theta)(r+\lambda)[(r+\lambda) + 2\theta q(\theta)]] \\ & + (1-(\theta q(\theta))^2)[\Theta((1-\tau_y) - i(\mathtt{x}) - c)(\Theta(1-(\theta q(\theta))^2) \\ & + (1-\Theta)[(r+\lambda)[(r+\lambda) + \theta q(\theta)]]p^o \cdot 1 \\ & + (1-\Theta)(r+\lambda)(\theta q(\theta))p^y \cdot 1 \Big], \end{aligned} \tag{C.9a}$$

$$\begin{aligned} w^y(1) = {} & \frac{1}{1-(1-\Theta)^2(r+\lambda)^2(\theta q(\theta))^2} \\ & \cdot \Big[b(r+\lambda)^2(1-\Theta)[\Theta(1-(\theta q(\theta))^2 + (1-\Theta)(r+\lambda)[(r+\lambda) + 2\theta q(\theta)]] \\ & + (1-(\theta q(\theta))^2)[\Theta((1-\tau_y) - i(\mathtt{x}) - c)(\Theta(1-(\theta q(\theta))^2) \\ & + (1-\Theta)[(r+\lambda)[(r+\lambda) + \theta q(\theta)]]p^y \cdot 1 \\ & + (1-\Theta)(r+\lambda)(\theta q(\theta))p^o \cdot 1 \Big] \end{aligned} \tag{C.9b}$$

Where equations C.9a and C.9b are the wage of equilibrium for Old and Young firms respectively.

Without loss of generality set $x \geq R$. Then, merging equations C.9a/C.9b with equations 5.2a/5.2b and setting $x = R \Rightarrow J^j(R) = V = 0$ (point of indifference), we get:

$$-\lambda(J^o(x) - \int_{R'^o_N}^{1} \max\{J^o(s'), J^y(s')\}\, dG(s') = (1-\tau_y)p^o R - w^o(R) - i(R) - c, \tag{A.3.10a}$$

$$-\lambda(J^y(x) - \int_{R'^y_N}^{1} \max\{J^o(s'), J^y(s')\}\, dG(s') = (1-\tau_y)p^y R - w^y(R) - i(R) - c \tag{A.3.10b}$$

Finally, to reach Job Creation Curve of the Old and Young Firms, we impute the relations A.3.10a/A.3.10b on:

$$(r+\lambda)J^j(x) = (1-\tau_y)p^j x - w^j(x) - i(\mathtt{x}) - c - (1-\tau_y)p^j R + w^j(R) + i(R) + c \tag{A.3.11}$$

Setting $x = 1$ and solving for $J^j(1)$ we reach the Job Creation Curves 5.11a and 5.12a.

Then, we set the follow relation:

$$\begin{aligned}\int_{R'_N{}^j}^{1} J^j(s')dG(s') = \int_{R'_N{}^j}^{1} \frac{1}{r+\lambda} \cdot \Big[(1-\tau_y)p^j(s-R) - i(s) + i(R) \\ + \frac{1}{\Theta[(\theta q(\theta))^2 - 1] - [1-\Theta][(r+\lambda)[\theta q(\theta) + (r+\lambda)]} \\ \cdot \Big[[1-\Theta][r+\lambda][\theta q(\theta)][w^j(s) - w^j(R)] \\ + \Theta[1 - (\theta q(\theta))^2][(1-\tau_y)p^j(s-R) - i(s) + i(R)]\Theta[(\theta q(\theta))^2 - 1] \\ - [1-\Theta][(r+\lambda)[\theta q(\theta) + (r+\lambda)]\Big]\Big]\end{aligned} \tag{A.3.12}$$

We use this last relation on $(r+\lambda)J^j(x) = (1-\tau_y)p^j x - w^j(x) - i(\mathtt{x}) - c - \lambda(J^j(x) - \int_{R'^o_N}^{1} \max\{J^o(s'), J^y(s')\}\, dG(s') - G(R'_N{}^j)V$ and set $x = R'$. Solving for R we reach our Job Destruction Curves 5.11b and 5.12b.

Capital Market Equations. To reach the capital market equilibrium equations we set from the equations of the firms environment. Let $V = 0$ (entry condition). Then, equations 5.7a and 5.7b can be written as:

$$J^o(\mathtt{x}) = \frac{\kappa_k}{q(\theta^o)}, \tag{A.3.13a}$$

$$J^y(\mathtt{x}) = \frac{\kappa_k + T}{q(\theta^y)} \tag{A.3.13b}$$

And assume now that $x = 1$. Then equations 5.7a and 5.7b can be expressed as:

$$J^o(1) = \frac{(1-\tau_y)p^o \cdot 1 - i^o(1) - c}{(r+\lambda)}, \tag{A.3.14a}$$

$$J^y(1) = \frac{(1-\tau_y)p^y \cdot 1 - i^y(1) - c}{(r+\lambda)} \tag{A.3.14b}$$

And then we merge equations A.3.13a to A.3.14a and equations A.3.13b to A.3.14b to find:

$$\frac{\kappa_k}{q(\theta^o)} = \frac{(1-\tau_y)p^o \cdot 1 - i^o(1) - c}{(r+\lambda)}, \tag{A.3.15a}$$

$$\frac{\kappa_k + T}{q(\theta^y)} = \frac{(1-\tau_y)p^y \cdot 1 - i^y(1) - c}{(r+\lambda)} \tag{A.3.15b}$$

For the Households equations, we subtract equation 5.9a and 5.9b by 5.8 with $\mathtt{x} = 1$,

$$I^o(1) - \mathcal{U} = \frac{i^o(1) - \mathtt{b}}{r + \lambda + \theta^o q(\theta^o)} - \frac{\theta^y q(\theta^y)(I^y(1) - U)}{r + \lambda + \theta^o q(\theta^o)}, \tag{A.3.16a}$$

$$I^y(1) - \mathcal{U} = \frac{i^y(1) - \mathtt{b}}{r + \lambda + \theta^y q(\theta^y)} - \frac{\theta^o q(\theta^o)(I^o(1) - U)}{r + \lambda + \theta^y q(\theta^y)} \tag{A.3.16b}$$

Assuming $\theta^o = \theta^y$, we can achieve:

$$I^o(1) - \mathcal{U} = \frac{(r+\lambda)(i^o(1) - \mathtt{b}) + \theta q(\theta)(i^o(1) - i^y(1))}{1 - (\theta q(\theta))^2}, \tag{A.3.17a}$$

$$I^y(1) - \mathcal{U} = \frac{(r+\lambda)(i^y(1) - \mathtt{b}) + \theta q(\theta)(i^y(1) - i^o(1))}{1 - (\theta q(\theta))^2} \tag{A.3.17b}$$

To reach equations of equilibrium in section 5, we resolve the Nash-Bargain problem, optimizing the Surplus equation:

$$i^j(x) = arg\max(I^j(\mathtt{x}) - \mathcal{U})^{\Theta}(J^j(\mathtt{x}) - \mathcal{V})^{(1-\Theta)} \tag{A.3.18}$$

subject to

$$J(R^j) = 0, \tag{A.3.19a}$$

$$rJ^j(\mathtt{x}) = (1-\tau_y)p^j x - i^j(\mathtt{x}) - c$$
$$-\lambda\left(J^j(\mathtt{x}) - \int_{R'^j}^{1} \max\{J^o(s'), J^y(s')\}\, dG(s') - G(R^j_K)V\right), \tag{A.3.19b}$$

$$rI^j(\mathtt{x}) = i^j(x) - \lambda\left(I^j(x) - \int_{R'^j}^{1} \max\{I^o(s'), I^y(s')\}\, dG(s') - G(R^j_K)U\right), \tag{A.3.19c}$$

$$S(\mathtt{x}) = I^j(\mathtt{x}) - \mathcal{U} + J^j(\mathtt{x}) - \mathcal{V}^j \tag{A.3.19d}$$

Then, for $W^j(x)$ and assuming $x = 1$ and the Entry Condition $V = 0$:

$$\frac{\Theta J(1)}{1-\Theta} = I^j(1) - \mathcal{U} \tag{A.3.20}$$

Using the relations found in A.3.13a/A.3.13b and A.3.17a/A.3.17b we get,

$$\Theta\frac{\kappa_k}{q(\theta)}[1-(\theta q(\theta))^2] = [1-\Theta][(r+\lambda)(i^o(1)-\mathtt{b})+\theta q(\theta)(i^o(1)-i^y(1)],$$
$$\Theta\frac{\kappa_k+T}{q(\theta)}[1-(\theta q(\theta))^2] = [1-\Theta][(r+\lambda)(i^y(1)-\mathtt{b})+\theta q(\theta)(i^y(1)-i^o(1)]$$

and using the relations we found in A.3.3a/A.3.3b we reach:

$$i^o(1) = \frac{[1-\Theta][r+\lambda][(r+\lambda)(-\mathtt{b})+\theta q(\theta)(-i^y(1)]-\Theta[(1-\tau_y)p^o\cdot 1-c][1-(\theta q(\theta))^2]}{\Theta[(\theta q(\theta))^2-1]-[1-\Theta][(r+\lambda)[\theta q(\theta)+(r+\lambda)]]},$$
$$i^y(1) = \frac{[1-\Theta][r+\lambda][(r+\lambda)(-\mathtt{b})+\theta q(\theta)(-i^o(1)]-\Theta[(1-\tau_y)p^y\cdot 1-i(1)-c][1-(\theta q(\theta))^2]}{\Theta[(\theta q(\theta))^2-1]-[1-\Theta][(r+\lambda)[\theta q(\theta)+(r+\lambda)]]}$$

At last we replace the equations above in each other:

$$\begin{aligned} i^o(1) = &\frac{1}{1-(1-\Theta)^2(r+\lambda)^2(\theta q(\theta))^2} \\ &\cdot\Big[\mathtt{b}(r+\lambda)^2(1-\Theta)[\Theta(1-(\theta q(\theta))^2+(1-\Theta)(r+\lambda)[(r+\lambda)+2\theta q(\theta)]] \\ &+(1-(\theta q(\theta))^2)[\Theta((1-\tau_y)-c)(\Theta(1-(\theta q(\theta))^2) \\ &+(1-\Theta)[(r+\lambda)[(r+\lambda)+\theta q(\theta)]]p^o\cdot 1 \\ &+(1-\Theta)(r+\lambda)(\theta q(\theta))p^y\cdot 1\Big], \end{aligned} \tag{C.21a}$$

$$\begin{aligned} i^y(1) = &\frac{1}{1-(1-\Theta)^2(r+\lambda)^2(\theta q(\theta))^2} \\ &\cdot\Big[\mathtt{b}(r+\lambda)^2(1-\Theta)[\Theta(1-(\theta q(\theta))^2+(1-\Theta)(r+\lambda)[(r+\lambda)+2\theta q(\theta)]] \\ &+(1-(\theta q(\theta))^2)[\Theta((1-\tau_y)-c)(\Theta(1-(\theta q(\theta))^2) \\ &+(1-\Theta)[(r+\lambda)[(r+\lambda)+\theta q(\theta)]]p^y\cdot 1 \\ &+(1-\Theta)(r+\lambda)(\theta q(\theta))p^o\cdot 1\Big] \end{aligned} \tag{C.21b}$$

Where equations C.21a and C.21b are the wage of equilibrium for Old and Young firms respectively.

Without loss of generality set $x \geq R$. Then, merging equations C.21a/C.21b with equations 5.7a/5.7b and setting $\mathtt{x} = R \Rightarrow J^j(R) = V = 0$ (point of indifference), we get:

$$-\lambda(J^o(\mathtt{x}) - \int_{R'^o_K}^1 \max\{J^o(s'), J^y(s')\}\, dG(s') = (1-\tau_y)p^o R i^o(R) - c, \tag{A.3.22a}$$
$$-\lambda(J^y(\mathtt{x}) - \int_{R'^y_K}^1 \max\{J^o(s'), J^y(s')\}\, dG(s') = (1-\tau_y)p^y R - i^y(R) - c \tag{A.3.22b}$$

Finally, to reach Capital Creation Curve of the Old and Young Firms, we impute the relations A.3.22a/A.3.22b on:

$$(r+\lambda)J^j(\mathtt{x}) = (1-\tau_y)p^j\mathtt{x} - i(\mathtt{x}) - c - (1-\tau_y)p^j R + i^j(R) + c \tag{A.3.23}$$

Setting $\mathtt{x} = 1$ and solving for $J^j(1)$ we reach the Capital Creation Curves 5.13a and 5.14a.

Then, we set the follow relation:

$$\int_{R'_K{}^j}^{1} J^j(s')dG(s') = \int_{R'_K{}^j}^{1} \frac{1}{r+\lambda} \cdot \Bigg[(1-\tau_y)p^j(s-R) \\ + \frac{1}{\Theta[(\theta q(\theta))^2 - 1] - [1-\Theta][(r+\lambda)[\theta q(\theta) + (r+\lambda)]} \\ \cdot \Bigg[[1-\Theta][r+\lambda][\theta q(\theta)][i^j(s) - i^j(R)] \\ + \Theta[1-(\theta q(\theta))^2][(1-\tau_y)p^j(s-R)]\Theta[(\theta q(\theta))^2 - 1] \\ - [1-\Theta][(r+\lambda)[\theta q(\theta) + (r+\lambda)]\Bigg]\Bigg] \tag{A.3.24}$$

We use this last relation on $(r+\lambda)J^j(\mathtt{x}) = (1-\tau_y)p^j x - i^j(\mathtt{x}) - c - \lambda(J^j(x) - \int_{R'^o_K}^{1} \max\{J^o(s'), J^y(s')\} dG(s') - G(R'_K{}^j)V$ and set $x = R'$. Solving for R_K we reach our Capital Destruction Curves 5.13b and 5.14b.

A.3.2 Mathematical Issues

We need to prove the bounds of some equations of the section 5.

$$\Omega(\theta) = \Theta(1-(\theta q(\theta))^2) + (1-\Theta)[(r+\lambda)[(r+\lambda) + \theta q(\theta)]] > 0$$
$$\Theta(1-(\theta q(\theta))^2) + (1-\Theta)[(r+\lambda)[(r+\lambda) + \theta q(\theta)]] > 0$$
$$\Theta(1-(\theta q(\theta))^2) > -(1-\Theta)[(r+\lambda)[(r+\lambda) + \theta q(\theta)]]$$
$$\underbrace{\Theta(1-(\theta q(\theta))^2)}_{\in(0,1)} > -\underbrace{\underbrace{(1-\Theta)}_{\in(0,1)} \cdot \underbrace{[(r+\lambda)^2 + (r+\lambda)\theta q(\theta)]}_{\in(0,6)}}_{\in(0,6)}$$

□

$$\eta(\theta) = (1-\Theta)(r+\lambda)(\theta q(\theta)) > 0$$
$$\underbrace{(1-\Theta)}_{\in(0,1)}\underbrace{(r+\lambda)}_{\in(0,2)}\underbrace{(\theta q(\theta))}_{\in(0,1)} > 0$$

□

$$\zeta(\theta) = \frac{(1-(\theta q(\theta))^2)}{1-(\eta(\theta))^2} \in (0,1)$$

$$\frac{1-(\theta q(\theta))^2}{1-(\eta(\theta))^2} < 1$$

$$-(\theta q(\theta))^2 < -(1-\Theta)^2(r+\lambda)^2(1-\theta q(\theta))^2$$

$$-(\theta q(\theta))^2 < -(1-\Theta)^2(1-2(\theta q(\theta)) + (\theta q(\theta))^2)(r+\lambda)^2$$

$$-(\theta q(\theta))^2 < -(\theta q(\theta))^2(1-\Theta)^2(r+\lambda)^2 - (1-\Theta)^2(r+\lambda)^2(1-2\theta q(\theta))$$

$$(\theta q(\theta))^2(-1+(1-\Theta)^2(r+\lambda)^2) < -(1-\Theta)^2(r+\lambda)^2(1-2\theta q(\theta))$$

$$(\theta q(\theta))^2(1-(1-\Theta)^2(r+\lambda)^2) > (1-\Theta)^2(r+\lambda)^2(1-2\theta q(\theta))$$

$$\underbrace{\left(\frac{\theta q(\theta)}{(1-\Theta)(r+\lambda)}\right)^2}_{>0} > \frac{1-2\theta q(\theta)}{1-(1-\Theta)^2(r+\lambda)^2}$$

$$\zeta(\theta) < 1$$

$$\frac{1-(\theta q(\theta))^2}{1-(\eta(\theta))^2} > 0$$

$$1-(\theta q(\theta))^2 > 0$$

$$\zeta(\theta) > 0$$

□