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UNCERTAINTY SHOCKS AND BUSINESS CYCLES IN BRAZIL: A DSGE APPROACH

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Master's thesis presented to the Programa de Pós-Graduação em Economia (Pimes) of Department of Economics of Universidade Federal de Pernambuco as a partial requirement for obtaining a Master's Degree in Economics.

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ABSTRACT

In this present work I analyze the impact of uncertainty shocks on the Brazilian economy. I employ a Dynamic Stochastic General Equilibrium model to assess the transmission channels of uncertainty shocks. could be identified and I solve the model using a third order approximation for the policy functions since lower order approximations are not able to capture the effects of volatility shocks. Examining the channels through which shocks of uncertainty on TFP and interest rates may affect Brazilian economy, the impulse-response functions suggest that an increase in domestic and external volatility usually leads to a drop in consumption, investment, and output, and an increase in labor supply and external debt. Through the variance decomposition uncertainty shocks are relevant to explain the dynamics of economic activity in Brazil. Additionally I perform a robustness analysis and showed that the results are maintained in response to different calibrations.

Keywords: Uncertainty. General Equilibrium. Brazilian Economy. Business Cycle.

JEL Classifications: C32, E32, F41, E37.

RESUMO

Neste trabalho é analisado o impacto dos choques de incerteza na economia brasileira. Foi empregado um modelo de equilíbrio geral estocástico dinâmico para que os canais de transmissão de choques de incerteza pudessem ser identificados e o modelo é resolvido usando uma aproximação de terceira ordem para as funções de política, uma vez que aproximações de ordem inferior não são capazes de capturar os efeitos dos choques de volatilidade. Examinando os canais através dos quais os choques de incerteza sobre a PTF e as taxas de juros podem afetar a economia brasileira, as funções impulso-resposta sugerem que um aumento na volatilidade interna e externa geralmente leva a uma queda no consumo, investimento e produção e um aumento na oferta mão de obra e dívida externa. Por meio da decomposição da variância, os choques de incerteza são relevantes para explicar a dinâmica da atividade econômica no Brasil. Além disso, é executada uma análise de robustez e é mostrado que os resultados são mantidos em resposta a diferentes calibrações.

Palavras-Chave: Incerteza. Equilíbrio Geral. Economia Brasileira. Ciclos de Negócios.

Classificação JEL: C32, E32, F41, E37.

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1. Introduction

The global financial crisis and the recession unleashed in 2008 have (and still have) economic consequences worldwide. One of the channels by which these events have spread throughout the world is the increase of uncertainty (volatility). The basic argument for this channel is that higher uncertainty triggers precautionary responses, such as postponing consumption or investment decisions, leading to recessions even in countries that do not have a direct link with the countries in which the crisis originated (IMF, 2009). This has been particularly the case for many small open economies that suffered the effects of these global events without being exposed to the kind of fragility that triggered the crisis in the developed world (for example, in most Latin American countries there was no real estate bubble, financial institutions were not exposed to mortgage markets and governments were not facing a fragile fiscal situation). In this way, my main objective in this present work is to analyse the effects of changes in different sources of uncertainty in which an economy is exposed (García-Cicco et al., 2010).

We focus the analysis on the Brazilian economy, an emerging small open economy. For this particular economy, the uncertainty has been a hallmark of the country's economic dynamics since at least the 2014 presidential elections. In 2015 and 2016, two consecutive records of the highest annual uncertainty average of the whole series the historical index of the index constructed by Baker et al. (2016)[1]. The growing uncertainty, however, has not been a Brazilian exclusivity. Several countries have been through turbulent moments, with unexpected changes and unpredictable consequences. In this sense, the Brexit in the United Kingdom and the election of Donald Trump in the United States are representative events. Figure 3 (Appendix) relates the evolution of this uncertainty indicator since 1996 for Brazil and to the rest of the world with the evolution of Brazilian real GDP. To do this, I extracted the cyclic component through the Hodrick-Prescott filter of the three series, resulting in a correlation coefficient of -0.41 between the cyclical component of real GDP and the specific uncertainty index of Brazil, and -0.33 between real GDP and the

1 The Economic Policy Uncertainty Index is constructed for Brazil following the methods of ”Measuring Economic Policy Uncertainty”. The authors use text files from *Folha de São Paulo* newspaper from 1991 onwards. Each month counts the number of articles containing the terms ”uncertain” or ”uncertainty”, ”economic” or ”economy”, and one or more of the following terms relevant to the policy: adjustment, deficit, budget, tax, central bank , dawn, plateau, congress, senate, chamber of deputies, legislation, law, tariff.

global uncertainty index.

My approach follows Fernández-Villaverde et al. (2011) and is based on a dynamic stochastic general equilibrium model that allows us to capture the channels that are conceptually relevant to explain the transmission of volatility shocks (for example real options or precautionary savings). Despite the model used in this present work has many elements in common with that used in Fernández-Villaverde et al. (2011), my analysis differs mainly by the introduction of volatility shocks on TFP. I include this feature by the fact that the economic evidence on the aggregate effects of uncertainty on TFP still inconclusive. Empirical studies using different proxies and identification schemes to uncover the effects of uncertainty have produced a variety of results. One group of studies like Alexopoulos and Cohen (2009) and Bloom et al. (2013) reports an important impact of uncertainty about productivity on real aggregate variables like GDP and employment. In contrast, a second group of studies led by Bekaert et al. (2010), Popescu and Smets (2010) and Bachmann and Bayer (2011) reports little or no impact at all.

Moreover, capturing time-varying volatility creates a computational challenge. As one is interested in the implications of an increase of volatility keeping the level of the variable constant, it is necessary to consider a third (or higher) order Taylor expansion for the solution of the model. In a first order approximation, stochastic volatility would not even have a role, since the rules of the representative agent follow a certainty equivalence principle. In a second-order approximation, only the product of innovations at the level and volatility of real interest rates appears in the policy rules. Only in a third order approximation to the innovations volatility plays a role by themselves.

This present work contributes to the existing literature by providing some insights to better understand consumption/savings and labor decisions under uncertainty. In addition, it helps to inquire about the impact of uncertainty on the TFP and to highlight the economic behavior behind the positive correlation between spreads and volatility, namely the fact that increases in volatility may induce an increase in external debt.

We find that an unexpected increase in volatility triggers a drop in consumption and an increase in labor supply. A potential driver of these effects is the countercyclical fluctuations of precautionary savings. This occurs because of agent's risk aversion in times of recession: agents tend to be more careful and raise their savings levels, thus amplifying the effects of volatility shocks. In addition, uncertainty shocks negatively affect investment because the returns on projects become riskier. As shown in the results section, my results are in line with what is found in the existing literature.

The remainder of this present work proceeds as follows. Section 2 provides a brief review of the literature in which this present work is related. Section 3 describes the small open economy DSGE model and its resolution. Section 4 presents the econometric strategy, the solution method, data, calibration and estimation. Section 5 discusses the results, explaining the transmission channels of volatility shocks and their effects through impulse response functions and the importance of uncertainty to explain the aggregate fluctuations of the Brazilian economy through the variance decomposition. Section 6 presents a robustness analysis, showing that the model is stable when calibrated with different parameters. Finally, section 7 concludes.

2. Related Literature

This present work is related to the literature that studies the role and importance of volatility shocks over the business cycle such as the seminal analysis of Bloom (2009) that considers uncertainty in a firm-level model and shows that unanticipated uncertainty shocks decreases investment. More specifically, the author shows that higher uncertainty expands an internal region, that is, the degree of inaction of firms, giving rise to the value of the real option of waiting; thus, firms make a ”delay” in their investment and contracting decisions, resulting in negative effects on the allocation of inputs.

Fernández-Villaverde and Rubio-Ramírez (2007) and Justiniano and Primiceri (2008) shows that time-varying volatility helps to explain the Great Moderation between 1984 and 2007. Bachmann et al. (2013) analyse the role of uncertainty in Germany and the United States. They find that increasing uncertainty induces, in both countries, a drop in industrial output, hours worked and employment. Leduc and Liu (2013) study the macroeconomic effects of uncertainty shocks in a general equilibrium model model with frictions in the demand for labor and sticky prices. The authors show that uncertainty shocks act as aggregate demand shocks as they increase unemployment and reduce inflation. Born and Pfeifer (2014) through a general equilibrium model with nominal frictions find that uncertainty shocks over monetary policy and government spending are an important factor in explaining business cycles in the U.S.

Alexopoulos and Cohen (2015) concludes that uncertainty shocks explain a significant part of the variance of aggregate variables, such as output, consumption, and investment in the U.S. economy. Jurado et al. (2015) argues that uncertainty shocks are probably the largest source of U.S. economic fluctuations. Basu and Bundick (2017) indicates that the introduction of price rigidity is a determining factor in reproducing the effects of uncertainty. In a model with flexible prices, an uncertainty shock stimulates a precautionary reaction that leads the households to increase labor supply. Therefore, increasing uncertainty implies a counter intuitive result, since it reduces consumption but increases production. In contrast, when price adjustment is slow, output is determined by demand. Because firms are not able to freely adjust their own prices, they need to reduce their output to meet demand. This mechanism induces a decrease in consumption, investment, production and employment.

Specifically for Brazil, Costa (2014) shows a negative effect on economic activity. Industrial production, consumer confidence and unemployment fall by more than 1% after a uncertainty shock. Barbosa (2018) finds that uncertainty shocks reduce fiscal revenue and cause a deficit in the primary result of the government. Barboza and Zilberman (2018) find significant contractionary effects of uncertainty about the activity, particularly on investment. In addition, estimates indicate that the effects of domestic uncertainty are more pronounced than those of external uncertainty[2].

3. Theoretical Model

3.1. Environment

In this section I present the model to evaluate the effects of volatility shocks. The structure of the model is similar to the one used by Mendoza (1991), Neumeyer and Perri (2005) and Uribe and Yue (2006). Households maximise a utility function with two arguments (consumption and labor) over an infinity horizon, have access to international assets markets where they trade a non-contingent asset under full commitment and accumulate capital. Firms operate in a competitive market and produce a final good required by households. For this, they hire labor and capital, remunerating these factors with wages and rents, respectively. I now turn to a detailed description of the economic

2 All three analyses use VAR models.

environment.

3.2. Households

The small open economy is populated by a representative household whose preferences are captured by the utility function given by

$$U(C_t, L_t) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left(C_t - \frac{L_t^{\eta}}{\eta} \right)^{1-\sigma}, \tag{1}$$

in which E_0 is the conditional expectation operator, C_t denotes consumption, L_t denotes hours worked, and $\beta \in (0,1)$ corresponds to the intertemporal discount factor. The choice of the preferences of Greenwood et al. (1988) (GHH) follows Correia et al. (1995) that shows this utility function is best suited to match the second moments of small open economies. The main characteristic of the GHH preferences is the lack of income effect in the decision of the labor supply. In this way, labor supply depends only on real wages, and the model is capable of generating a contraction in consumption and output after a positive shock in the interest rate.

Households can invest in two types of assets: physical capital, K_t, and an internationally traded security, D_t. Thus, the budget constraint of households is described by:

$$\frac{D_{t+1}}{1+r_t} = D_t - W_t L_t - R_t K_t + C_t + I_t + \frac{\Phi_D}{2}(D_{t+1} - D)^2, \tag{2}$$

in which W_t and R_t represent the real wage and rental rate of capital, I_t denotes the domestic investment, $\Phi_D > 0$ is a parameter that controls the costs of holding a net asset abroad, and D is the level of debt at steady state. The cost Φ_D is paid to some foreign international institution (for example, an investment bank that manages the issuance of securities for the representative household). The representative agent is also subject to the No-Ponzi-Game condition, defined by:

$$\lim_{t \to \infty} \frac{D_t}{\prod_{t}^{\infty} r_{t+1}} = 0 \tag{3}$$

3.3. Firms

Firms use capital and labor from households to produce a final good in a competitive market according to the production function defined by:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{4}$$

in which A_t corresponds to the total factor productivity that follows an autoregressive process $AR(1)$

$$A_t = (1-\rho_a)A + \rho_a A_{t-1} + \sigma_{a,t}\varepsilon_{a,t}, \tag{5}$$

in which $\varepsilon_{a,t} \sim iid\ \mathcal{N}(0,1)$.

Firms maximize their profits by equating wages and interest rates with marginal productivity of labor and capital, respectively. The stock of capital evolves according to:

$$K_{t+1} = (1-\delta)K_t + \left(1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t \tag{6}$$

in which $\delta \in (0,1)$ is the capital depreciation rate and $\phi > 0$ determines the magnitude of the cost of adjustment. The introduction of these adjustment costs is common in business cycle models of small open economies because they are a convenient way of avoiding excessive investment volatility in response to changes in the real interest rate (Fernández-Villaverde and Rubio-Ramírez, 2007).

3.4. Law of Motion of Exogenous Shocks

To model uncertainty shocks, I use the stochastic volatility approach, assuming time volatility of the innovation of TFP, international real interest rate and country-specific spreads. An uncertainty shock is a second-moment shock that affects the shape of the distribution, increasing its variance, but keeping its mean unchanged[3].

The stochastic volatility approach ensures that the dispersion of level shocks varies

3 A first moment shock (level shock) is a shock that changes the level of the variable in question, keeping its distribution unchanged.

over time, so there are shocks of high and low magnitude. As defined in the previous section, the TFP follows a $AR(1)$ process with variable volatility given by:

$$A_t = (1-\rho_a)A + \rho_a A_{t-1} + \sigma_{a,t}\varepsilon_{a,t}, \tag{7}$$

in which $\varepsilon_{a,t} \sim iid\ \mathcal{N}(0,1)$ and the coefficient $\rho_a \in (-1,1)$ determines the persistence of TFP level shock and A represents the steady state level of TFP. In addition, the standard deviation of innovations, σ_t^a, evolves according to the following stationary process:

$$\sigma_{a,t} = (1-\rho_{\sigma_a})\sigma_a + \rho_{\sigma_a}\sigma_{a,t-1} + \eta_a u_t^{\sigma_a}, \tag{8}$$

in which $\rho_{\sigma_a} \in (-1,1)$ determines the persistence of the uncertainty shock and σ_a represents the steady-state level of $\sigma_{a,t}$ and η_a is the standard deviation of the uncertainty shock on TFP, $u_t^{\sigma_a}$. For the interest rate, I decompose the real interest rate of a country at time t into the average real interest rate of the country over time, r^*, a time-demeaned international risk-free rate, i_t, and a time-demeaned country spread component, s_t. Specifically, I have

$$r_t = r^* + i_t + s_t, \tag{9}$$

The laws of motion for i_t and s_t are given by

$$i_t = \rho_i i_{t-1} + \sigma_{i,t}\varepsilon_{i,t} \tag{10}$$

$$s_t = \rho_s s_{t-1} + \sigma_{s,t}\varepsilon_{s,t} \tag{11}$$

$$\sigma_{i,t} = (1-\rho_{\sigma_i})\sigma_i + \rho_{\sigma_i}\sigma_{i,t-1} + \eta_i u_t^{\sigma_i} \tag{12}$$

$$\sigma_{s,t} = (1-\rho_{\sigma_s})\sigma_s + \rho_{\sigma_s}\sigma_{s,t-1} + \eta_r u_t^{\sigma_s} \tag{13}$$

in which the innovations $\varepsilon_{n,t}$, $u_t^{\sigma_n}$, $n \in \{i,s\}$, are $iid\ \mathcal{N}(0,1)$ processes. Equations (11) and (13) describe the process for country spreads, while equations (10) and (12) describe the process for the real international risk-free rate.

3.5. Equilibrium

The first order conditions that characterize $C_t, D_{t+1}, K_{t+1}, H_t$ and I_t are given below:

$$\left[C_t - \frac{L_t^\eta}{\eta}\right]^{-\sigma} = \lambda_t \tag{14}$$

$$\frac{\lambda_t}{1+r_t} = \lambda_t \Phi_D(D_{t+1} - D) + \beta E_t \lambda_{t+1} \tag{15}$$

$$-\varphi_t + \beta E_t \left[(1-\delta)\mu_{t+1} + \alpha \frac{Y_{t+1}}{K_{t+1}} \lambda_{t+1}\right] = 0 \tag{16}$$

$$L_t^\eta = (1-\alpha) Y_t \tag{17}$$

$$\varphi_t \left[1 - \frac{\phi}{2}\left(\frac{I_t - I_{t-1}}{I_{t-1}}\right)^2 - \frac{\phi I_t}{I_{t-1}}\left(\frac{I_t - I_{t-1}}{I_{t-1}}\right)\right] \; + \; \beta E_t \left[\varphi_{t+1}\phi\left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_t - I_{t-1}}{I_{t-1}}\right)\right] \tag{18}$$

together with the resource constraint, the law of motion of capital, the production function, and the process for the interest rate. The Lagrangian λ_t is associated with the debit level and the Lagrangian φ_t with the physical capital. The deterministic steady state is given by the solution of the following equations:

$$\left[C - \frac{L^\eta}{\omega}\right]^{-\sigma} = \lambda \tag{19}$$

$$\beta\left[(1-\delta)\varphi + \alpha \frac{Y}{K}\lambda\right] = \varphi \tag{20}$$

$$L^{\eta-1}\left[C - \frac{L^\eta}{\omega}\right]^{-\sigma} = (1-\alpha)\lambda \frac{Y}{K} \tag{21}$$

$$\lambda = \varphi \tag{22}$$

$$\frac{D}{1+r} = D - Y + C + I \tag{23}$$

$$Y = K^{\alpha} L^{1-\alpha} \tag{24}$$

$$I = \delta K \tag{25}$$

Thus, I find the system of 7 equations for 7 unknowns: C, L, λ, φ, K, I e Y. Next, I present the econometric methodology.

4. Econometric Strategy

As discussed in the introduction, the main objective of my present work is analyse the transmission mechanisms behind the responses to uncertainty shocks in the Brazilian economy. However, before describe the main results of this paper, the following sections presents some aspects of the solution method, data, calibration and estimation.

4.1. Solution Method

We use a third order approximation to solve the policy functions around the deterministic steady state (Born and Pfeifer, 2014). I follow this approach because a first-order approximation of the model would induce a loss of all the dynamics created by volatility since the policy rules follow a certainty equivalence principle. This means that any change in the volatilities of the innovations does not have effect in the policy functions, i.e. agents do not respond to volatility changes up to first order. Thus, policy rules will depend exclusively of $\varepsilon_{a,t}$, $\varepsilon_{i,t}$ and $\varepsilon_{s,t}$. Moreover, a second order approximation only captures the effects of the iterations of these shocks, that is, $\varepsilon_{a,t} \times u_t^{\sigma_a}$, $\varepsilon_{i,t} \times u_t^{\sigma_i}$ and $\varepsilon_{s,t} \times u_t^{\sigma_s}$. Only in the third (or higher) order approximation the shocks $u_t^{\sigma_a}$, $u_t^{\sigma_i}$ and $u_t^{\sigma_s}$ play a role for non-zero policy functions.

4.2. Data

The measure of the total productivity factor the of Brazilian economy is the TFP series of Fenestra et al. (2015), available on the website of the Penn World Table[4]. I decomposed the real interest rate, r_t, as the risk-free international real interest rate plus a specific spread from Brazil. I used the rate of return on US Treasury securities as a measure of the international nominal risk-free interest rate. The international risk-free real rate was constructed by subtracting expected inflation from the US Treasury bond rate as was done in Fernández-Villaverde et al. (2011). Finally, I calculated the expected inflation as the average US inflation in the current month and the previous eleven months. Both the US Treasury rate and the inflation series are obtained from the Federal Reserve Bank of St. Louis[5]. For the data on Brazilian spreads, I used the Global Spread Emerging Markets Bond Index (EMBI) for Brazil published by J.P. Morgan. Neumeyer and Perri (2005) explain in detail the advantages of EMBI data compared to existing alternatives. All series cover the period 1995.Q4 : 2018.Q1. I present the time series in the Appendix.

4 Available at https://fred.stlouisfed.org/series/RTFPNABRA632NRUG.
5 Available at https://fred.stlouisfed.org/series/TB3MS)

4.3. Calibration

The model has a total of 22 parameters. I follow two strategies: from this total, 10 (the parameters that are not of main interest in this study) were calibrated and 12 were estimated. The calibration of the parameters follow Fernández-Villaverde et al. (2011). The parameter that determines the elasticity of labor supply (ω) was set at 2; the rate of capital depreciation, $\delta = 0.014$; the share of capital in production, $\alpha = 0.33$; the elasticity of inverse intertemporal substitution, $\sigma = 2$ (agents risk averse); the intertemporal discount factor, $\beta = (1+r)^{-1}$; the parameter of the cost of capital adjustment, $\phi = 5$; the parameter that controls the average value of the debt, $D = 4$; the cost of maintaining debt, $\Phi_D = 0.001$. The values set for ω, α, σ and δ were collected from Mendoza (1997), Schmitt-Grohé and Uribe (2003), Aguiar and Gopinath (2007) e Neumeyer and Perri (2005). The other parameters are based on the original calibration of Fernández-Villaverde et al. (2011). Below is a table with the calibration summary:

Table 1: Calibrated Parameters.

Parameter	Description	Value
σ	Inverse Elasticity of Intertemporal Substitution	2
η	Inverse Substitution Elasticity of Labor Supply	1000
δ	Capital Depreciation Rate	0.014
α	Share of Capital in relation to GDP	0.33
$\bar{r}$	Steady State Interest Rate	log(0.02)
β	Intertemporal Discount Factor	$(1+r)^{-1}$
Φ_D	External Debt Substitution Elasticity	0.001
$\bar{D}$	External Debt of Steady State	4
ϕ	Capital Adjustment Cost	5

Source: Fernández-Villaverde et al. (2011).

4.4. Bayesian Estimation

We use Bayesian methods to estimate the model parameters that drives the volatility process. The Bayesian techniques consist of using the available information about the economy in the form of prior of the parameter distributions and then using the observed data to update these distributions, thus arriving at the posterior distributions of the estimated parameters (An and Schorfheide, 2007).

The algorithm used in the estimation of the moments of the posterior distribution was the Metropolis Hasting, with 2 Markov Chains with 1.000.000 draws and 20% of the generated parameter vectors were disregarded before using the simulations. The acceptance rate was approximately 24%. The general idea of using the algorithm is to simulate the posterior distribution from a sequence of samples generated from a distribution that is initially unknown. The algorithm uses the fact that under usual conditions the model parameters will be asymptotically normal. Thus, it is possible to perform an efficient exploration of the posterior distribution in the fashion neighborhood, which was previously found using Bayesian estimation methods.

Since there is a problem related to the presence of two innovations (level and volatility) that interact in a non-linear way, I use the particle filter (derivation in Appendix). Robert and Casella (2013) argues that this filter is a Sequential Monte Carlo algorithm that allows for the evaluation of likelihood given some parameter values through resampling simulation methods.

4.5. Priors

The next step is to specify priors distributions for the parameters to be estimated. Each prior is a probability density function of a parameter, constituting a formal way of specifying probabilities for each value that the parameter can assume, usually based on past studies. The values used for the estimation (mean and standard deviation) are based on Fernández-Villaverde et al. (2011). Except for the parameters that determine the standard deviation of processes (8), (12) and (13), the same priors are defined for all other parameters. Table 2 reports the priors for the parameters of the processes (5), (8), (10), (11), (12) and (13):

Table 2: Prior Distributions.

Parameter	Distribution	Mean	Standard Deviation
ρ_a	Beta	0.9	0.02
σ_a	Normal	0.6	0.04
ρ_{σ_a}	Beta	0.9	0.1
η_a	Beta	0.5	0.3
ρ_i	Beta	0.9	0.02
σ_i	Normal	0.6	0.04
ρ_{σ_i}	Beta	0.9	0.1
η_i	Beta	0.5	0.3
ρ_s	Beta	0.9	0.02
σ_s	Normal	0.6	0.04
ρ_{σ_s}	Beta	0.9	0.1
η_s	Beta	0.5	0.3

Source: Fernández-Villaverde et al. (2011).

4.6. Posteriors

Once I obtain the likelihood function and specify the priors distributions, it is possible to obtain the posterior distributions. The posteriors represent the probabilities attributed to different parameter values after observation of the data. Basically, they constitute an update of the probabilities given by the prior, based on the additional information provided by the variables present in the sample. To formally express how the latter relates to the priors, I apply the Bayes' Theorem to the two random events θ e y^*, which yield:

$$p(\theta \mid y^*) = \frac{p(\theta, y^*)}{p(y^*)} \tag{26}$$

$$p(y^* \mid \theta) = \frac{p(\theta, y^*)}{p(\theta)} \Leftrightarrow p(\theta, y^*) = p(y^* \mid \theta)p(\theta) \tag{27}$$

in which $p(\theta \mid y^*)$ is the density of the conditional parameters to the data, $p(\theta, y^*)$ is the joint density of the parameters and data, $p(y^* \mid \theta)$ is the density of the conditional data to the parameters (likelihood), $p(\theta)$ is the unconditional density of the parameters (prior) and $p(y^*)$ is the marginal density of the data. Substituting (30) into (29), I have:

$$p(\theta \mid y^*) = \frac{p(y^* \mid \theta)p(\theta)}{p(y^*)} \tag{28}$$

Table 3 reports the means for the parameters that determine the processes of the exogenous shocks. It is interesting to note that the data are very informative in relation to the parameters that determine the behavior of the exogenous variables, since the estimated values are very different from the values stipulated by the prior distributions.

Table 3: Posterior Distributions.

Parameter	Prior			Posterior	
	Distribution	Mean	Standard Deviation	Mean	68% HPD* Interval
ρ_a	Beta	0.9	0.02	0.94	[0.91 ; 0.94]
σ_a	Normal	0.6	0.4	0.72	[0.55 ; 0.99]
ρ_{σ_a}	Beta	0.9	0.1	0.95	[0.93 ; 0.97]
η_a	Beta	0.5	0.3	0.25	[0.21 ; 0.29]
ρ_i	Beta	0.9	0.02	0.95	[0.93 ; 0.96]
σ_i	Normal	0.6	0.4	0.87	[0.69 ; 1.19]
ρ_{σ_i}	Beta	0.9	0.1	0.96	[0.94 ; 0.98]
η_i	Beta	0.5	0.3	0.38	[0.35 ; 0.41]
ρ_s	Beta	0.9	0.02	0.95	[0.93 ; 0.97]
σ_s	Normal	0.6	0.4	0.85	[0.56 ; 1.24]
ρ_{σ_s}	Beta	0.9	0.1	0.94	[0.91 ; 0.97]
η_s	Beta	0.5	0.3	0.43	[0.40 ; 0.46]

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm. *HPD: Highest Posterior Density.

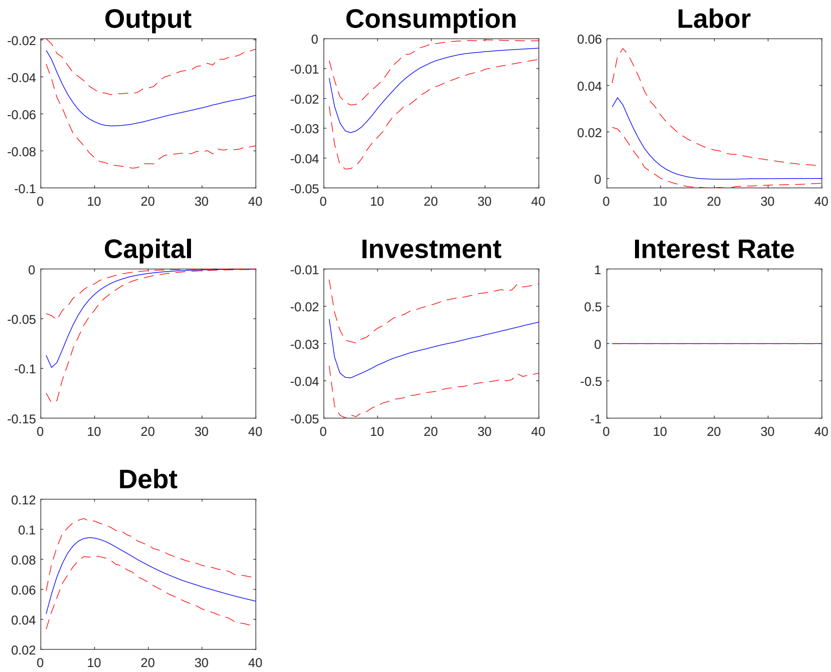
5. Results

We further analyse the model dynamics by using the IRFs of the variables in response to temporary volatility shocks of one standard deviation. As revealed from the Figures 1 to 3, the variables return back to their corresponding steady state values gradually, ensuring stability of the model. Finally, I decompose the variance of aggregate variables among different shocks.

5.1. IRFs to a Volatility Shock on TFP

After a volatility shock in TFP, the precautionary savings motive induces a fall in consumption and, as future levels of technology become more riskier and capital returns are more uncertain, investment falls. The drop in consumption and investment explain a decrease in output. On the other hand, households save more by using external debt, that is, households increase their external assets. A potential driver of the effects found in Figures 3 and 4 are the countercyclical fluctuations of precautionary savings. According to Basu and Bundick (2017), this occurs because of agents' risk aversion in times of recession: individuals tend to be more cautious and raise their savings levels, thus amplifying the effect of a shock of uncertainty.

Figure 1: Impulse Response Function for a Volatility Shock in TFP.



Note: For each plot the red dashed and blue solid lines represent the responses at the posterior mean and 10th, 90th percentiles of the posterior distribution.

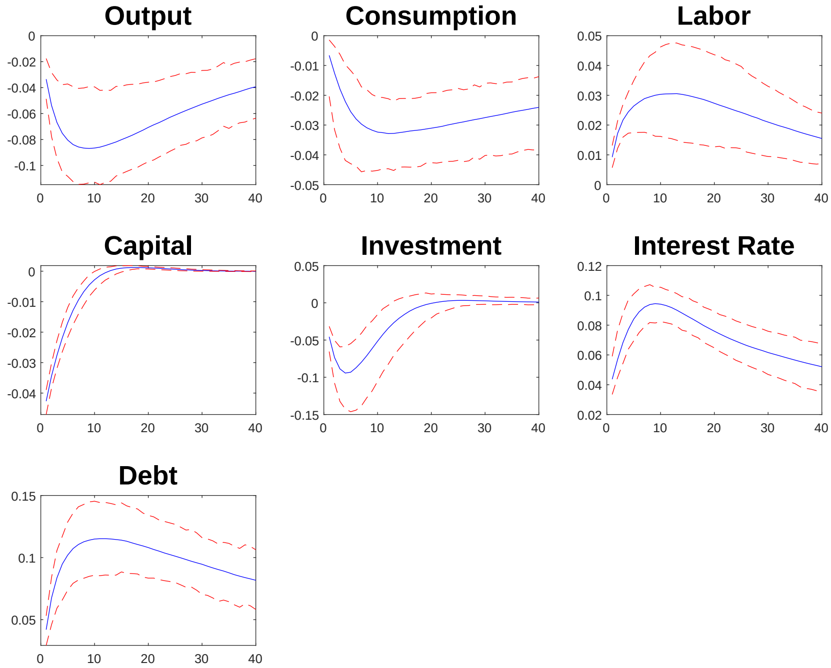
5.2. IRFs to a Volatility Shock on Interest Rates

To better understand the dynamics of volatility shocks on interest rates, it is necessary to model the first-order conditions. The first-order condition with respect to D_{t+1} is used to uncover the precautionary mechanism behind the effects of country spread volatility shocks. In particular, this condition can be rewritten as

$$\frac{1}{1+r_t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} = \Phi_D(D_{t+1} - D) \tag{29}$$

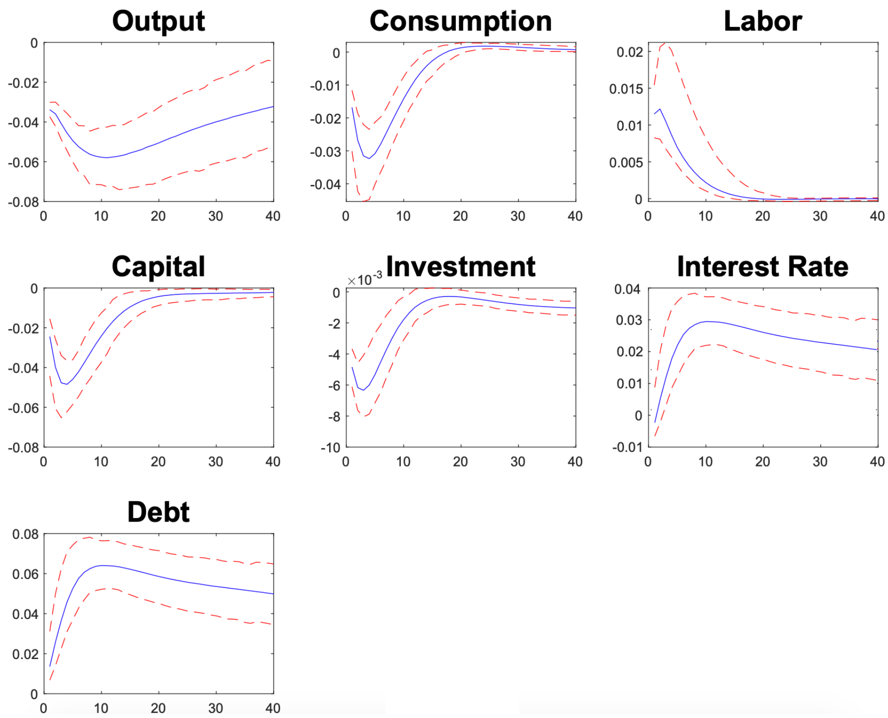
Fernández-Villaverde et al. (2011) show that volatility shocks raise $E_t \frac{\lambda_{t+1}}{\lambda_t}$. Higher real interest rate volatility will increase the future volatility of consumption. With more uncertainty regarding future consumption, a convex marginal utility will then imply graphically that $E_t \lambda_{t+1}$ rises. Furthermore, consumption drops upon impact, increasing marginal utility today and λ_t. In addition, the increase in real interest rate volatility makes the cost of projects more uncertain, resulting in a drop in investment.

Figure 2: Impulse response function for a volatility shock in spreads.



Note: For each plot the red dashed and blue solid lines represent the responses at the posterior mean and 10th, 90th percentiles of the posterior distribution.

Figure 3: Impulse response function for a volatility shock in international real interest rate.



Note: For each plot the red dashed and blue solid lines represent the responses at the posterior mean and 10th, 90th percentiles of the posterior distribution.

5.3. Variance Decomposition

In this section I present the variance decomposition to evaluate the importance of volatility shocks for aggregate fluctuations. Given the non linearity of the model and the resulting interaction of shocks as a consequence of the third order Taylor approximation I can not divide the total variance between the shocks. To solve this problem I simulate the model only with a subset of the shocks (Fernández-Villaverde et al., 2011). In other words, I define the achievements of one or two of the shocks to zero and measure the volatility of the economy with the remaining shocks. Table 4 shows the variance decomposition for the product, investment, consumption and labor between different shocks. Each column of Table 4 refers to a particular simulation: (1) the general specification with all three shocks (productivity, country spreads and volatility); (2) a shock only to productivity; (3) a productivity and interest rate shock (with volatility set at its unconditional value); (4) a shock only in the interest rate; (5) interest rate and volatility shocks; and (6) shocks only for volatility.

Table 4: Variance Decomposition.

	All shocks (1)	TFP only (2)	W/o volatility (3)	Rate level (4)	W/o TFP (5)	Volatility only (6)
σ_Y	7.60	2.81	4.57	3.63	7.00	21.52
σ_C	5.22	5.01	5.07	0.58	1.01	10.29
σ_I	19.01	5.43	11.97	10.63	18.11	15.74
σ_L	15.51	5.84	9.78	7.80	14.19	12.46

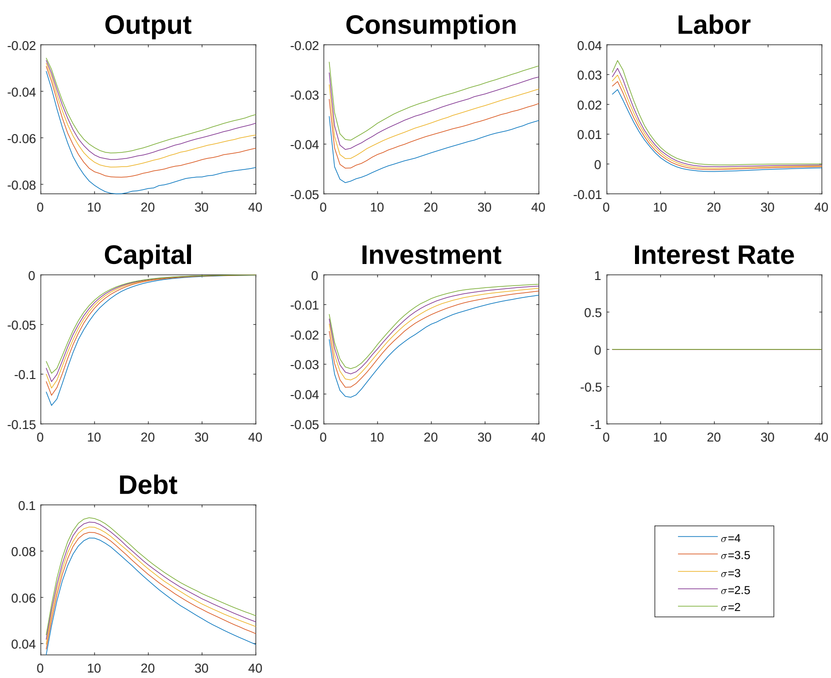
This table reports the variance decomposition for the different structural shocks in the model of Fernández-Villaverde et al. (2011) with stochastic volatility. First column: all shocks; second column: TFP shocks only; third column: without volatility shocks to spread and T-bill rate; fourth column: only level shocks to the spread and the T-bill rate; fifth column: without TFP shocks; sixth column: only shocks to the volatility of spreads and the T-bill rate.

As this present work focuses on the analysis of uncertainty, the column 6 shows that volatility alone makes a relatively important contribution to the fluctuations of output (the standard deviation is 21.52), consumption (the standard deviation is 10.29), investment (standard deviation of 15.74) and labor (standard deviation of 12.46). Based on these results, volatility shocks are important to explain the business cycles of the Brazilian economy.

5.4. Robustness Check

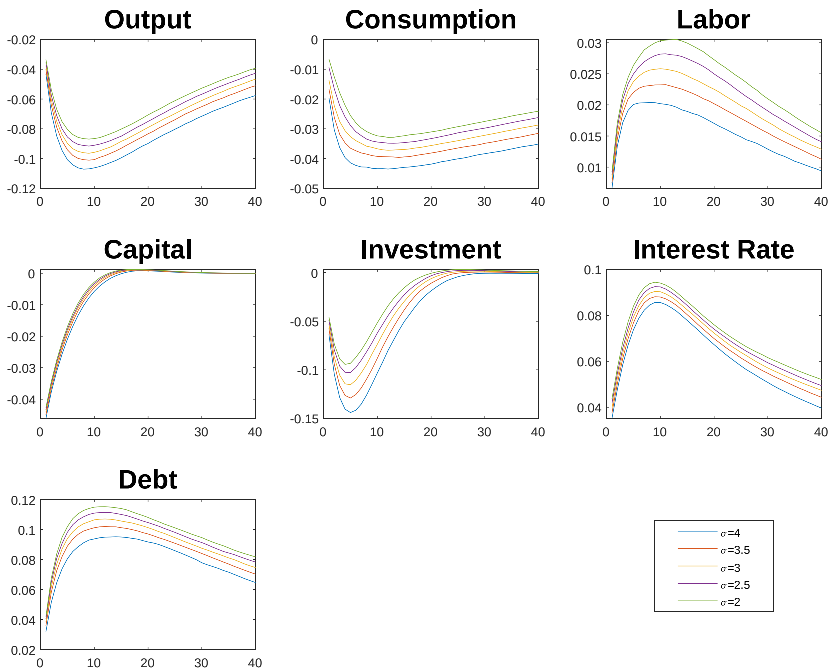
In this subsection I perform a robustness analysis to verify the sensitivity of the results obtained by simulating the model with changes in the household risk aversion coefficient. The degree of risk aversion determines the impact of second moment shocks on DSGE models with stochastic volatility (Bretscher et al., 2017). Thus, I expect that by increasing the value of the degree of risk aversion the effects of volatility shocks will be amplified. I simulate the model for several values of σ ($\sigma = 2$, 2.5, 3, 3.5 and 4). Below I plot the median of the different IRFs considering different values for risk aversion coefficient for 40 trimesters in response to a shock of 1 standard deviation on the TFP and interest rates. In general, the results did not show significant changes in relation to those presented previously, but they show that the results to the uncertainty shocks are stronger when I consider agents more risk averse.

Figure 4: Impulse Responses to a volatility shock on TFP for different values of the risk aversion coefficient.



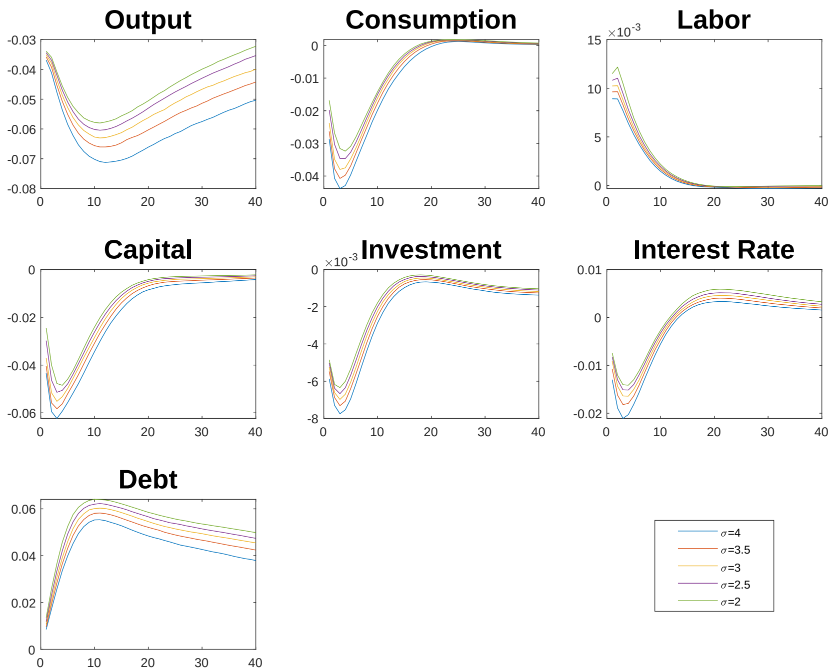
Note: For each plot the irfs are plotted for different values of the risk aversion coefficient ($\sigma = 2$, 2.5, 3, 3.5 and 4).

Figure 5: Impulse Responses to a volatility shock in spreads for different values of the risk aversion coefficient.



Note: For each plot the irfs are plotted for different values of the risk aversion coefficient (σ = 2, 2.5, 3, 3.5 and 4).

Figure 6: Impulse Responses to a volatility shock in international real interest rate for different values of the risk aversion coefficient.



Note: For each plot the irfs are plotted for different values of the risk aversion coefficient (σ = 2, 2.5, 3, 3.5 and 4).

6. Concluding Remarks

In this present work I quantified how recent uncertainty has affected economic activity in Brazil. In particular, after estimating the parameters of the processes that define stochastic volatility using Bayesian techniques, I used a general equilibrium model in such a way that the transmission channels of the shocks could be identified. I solve the model using a third order approximation for the policy functions and the other parameters were calibrated to coincide with several moments of the Brazilian macroeconomic aggregates.

Examining the channels through which uncertainty shocks may affect the Brazilian economy, the impulse-response functions suggest that volatility shocks usually lead to a drop in consumption, investment and production, and an increase in the supply of labor and debt. In addition, uncertainty shocks are relevant to explain the dynamics of economic activity in Brazil, in view of the results obtained. These results are in line with those found by Bloom (2009), Bachmann et al. (2013) and Alexopoulos and Cohen (2015).

Quantify the changes in productivity and real interest rate volatility and their interaction with business cycle fluctuations further enhances understanding of the international macroeconomics. Although the period of analysis of this study ends in January 2018, several events that may occur in the Brazilian economy and politics, as well as in the international scenario, suggest an extension of the uncertainty situation in the country. In particular, the present analysis of the impact of volatility shocks should assist policy makers in formulating more effective macroeconomic interventions.

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Appendix

Particle Filter

Following Born and Pfeifer (2014), let x_t be a AR(1) process, such as

$$x_t = \rho x_{t-1} + e^{\sigma_t} \upsilon_t, \upsilon_t \sim N(0,1) \tag{30}$$

in which the unobserved state σ_t follows a stochastic volatility process

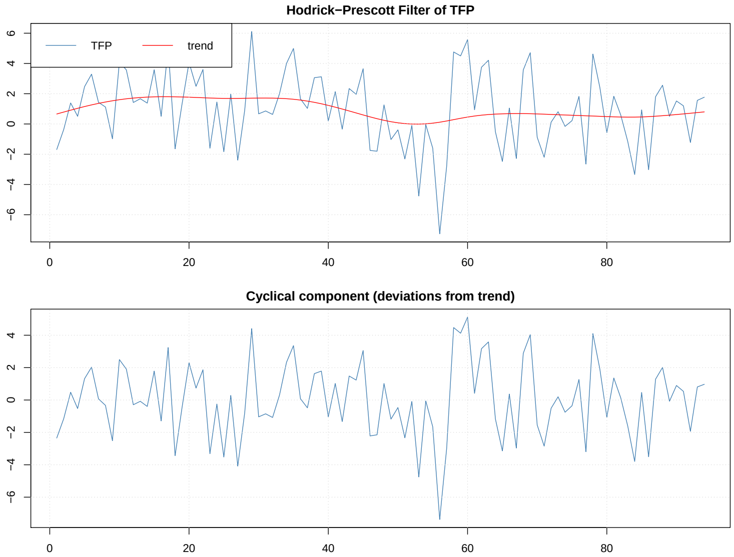
$$\sigma_t = (1-\rho^\sigma)\bar{\sigma} + \rho^\sigma \sigma_{t-1} + \eta \varepsilon_t, \varepsilon \sim N(0,1) \tag{31}$$

in which $\bar{\sigma}$ is the unconditional mean of σ_t. Hence, a filter is required to obtain the so-called filtering density $p(\sigma_t \mid x^t;\Theta)$. Due to the nonlinearity embedded in the stochastic volatility setup of the shocks, I cannot simply employ the Kalman filter as in the case of linearity and normally distributed shocks. Instead, I employ the particle filter, a special application of the more general class of Sequential Monte Carlo methods, to evaluate the likelihood (Fernández-Villaverde et al., 2011). Given the structure in (30) and (31) and some initial value x_0, the factorized likelihood of observing x^T can be written as

$$\begin{aligned} p(x^T;\Theta) &= \prod_{t=1}^{T} p(x_t \mid x_{t-1};\Theta) \\ &= \int p(x_1 \mid x_0,\sigma_0;\Theta) d\sigma_0 \prod_{t=2}^{T} \int p(x_t \mid x_{t-1},\sigma_t;\Theta) p(\sigma_t \mid x_{t-1};\Theta) d\sigma_t \\ &= \int \frac{1}{e^{\sigma_0}\sqrt{2\pi}} exp\left[\frac{-1}{2}\left(\frac{x_1-\rho x_0}{e^{\sigma_0}}\right)^2\right] d\sigma_0 \\ &\times \prod_{t=2}^{T} \int \frac{1}{e^{\sigma_t}\sqrt{2\pi}} exp\left[\frac{-1}{2}\left(\frac{x_t-\rho x_{t-1}}{e^{\sigma_t}}\right)^2\right] p(\sigma_t \mid x^{t-1};\Theta) d\sigma_t \end{aligned} \tag{32}$$

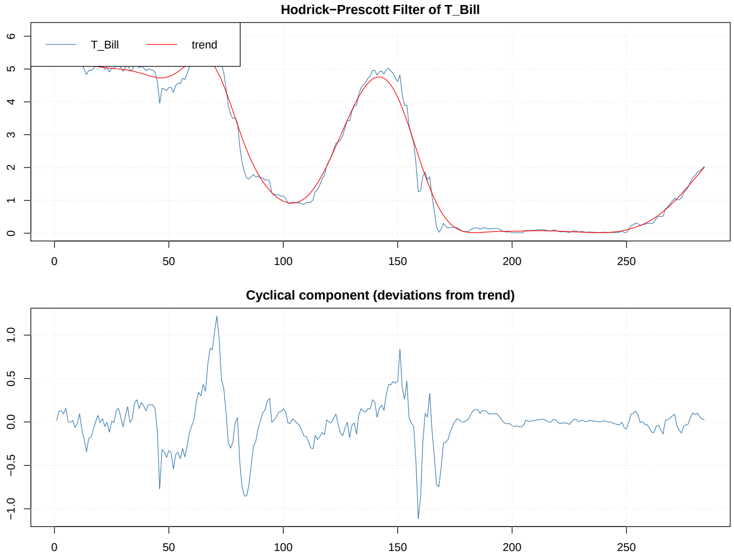
in which x_t is a $(t \times 1)$ vector that stacks the observations on x up to time t, Θ stacks the parameters, and the last equality follows from the assumption of normally distributed shocks. Although I do not have an analytical expression for $p(\sigma_t \mid x_{t-1};\Theta), t = 1,...,T$, and can therefore not compute it directly, I can employ the particle filter to estimate the likelihood by iteratively drawing from $p(\sigma_t \mid x_{t-1};\Theta)$ (Fernández-Villaverde and Rubio-Ramírez, 2007).

Figure 7: Data used to estimate the parameters of the processes that define stochastic volatility - TFP.



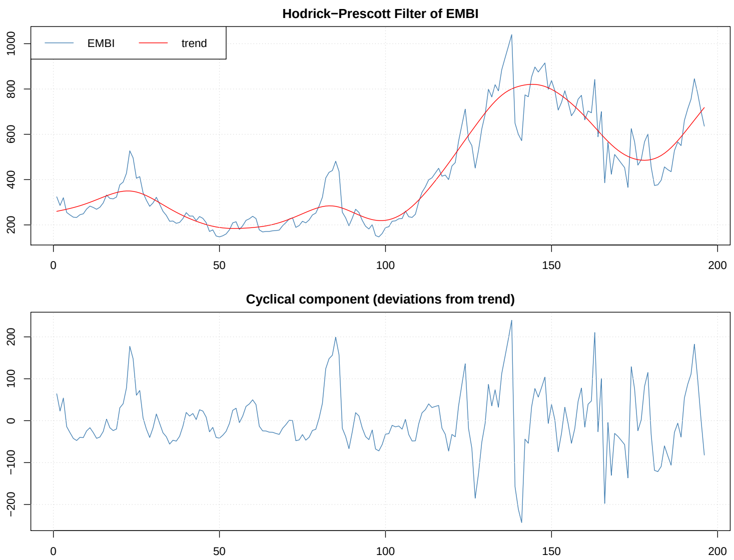
Source: Federal Reserve Bank of St. Louis and J. P. Morgan.

Figure 8: Data used to estimate the parameters of the processes that define stochastic volatility - T-Bill.



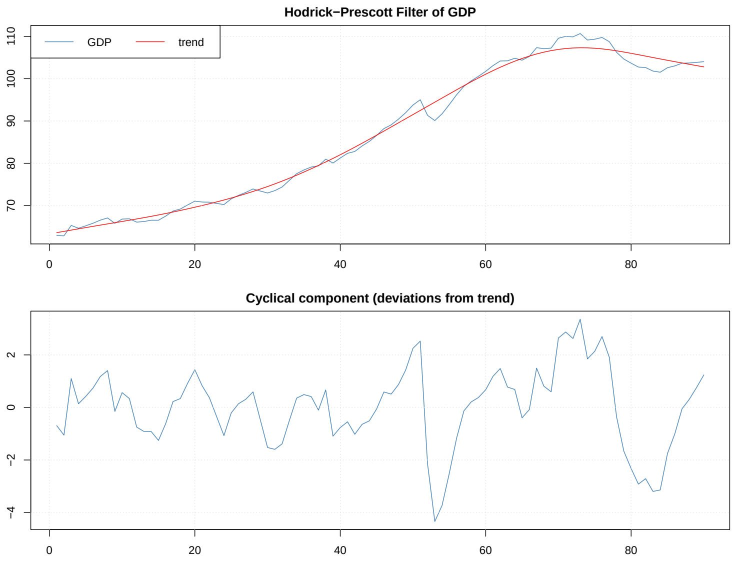
Source: Federal Reserve Bank of St. Louis and J. P. Morgan.

Figure 9: Data used to estimate the parameters of the processes that define stochastic volatility - EMBI.



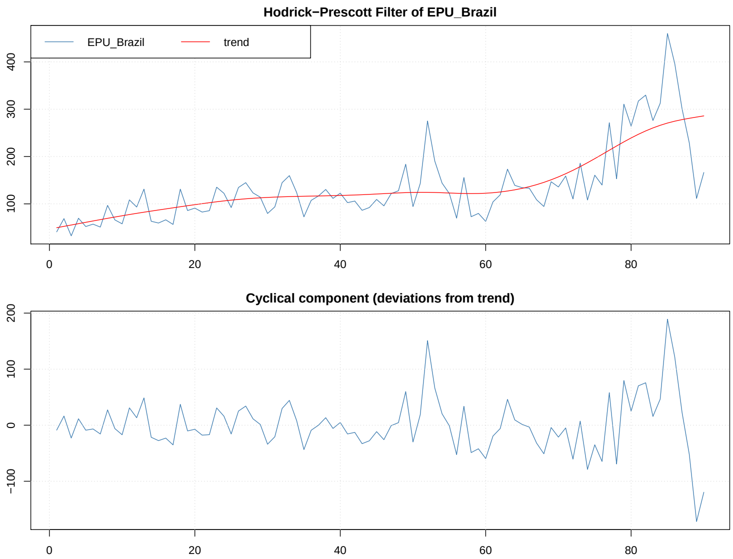
Source: Federal Reserve Bank of St. Louis and J. P. Morgan.

Figure 10: Real GDP - Brazil (Cyclical Component and Trend).



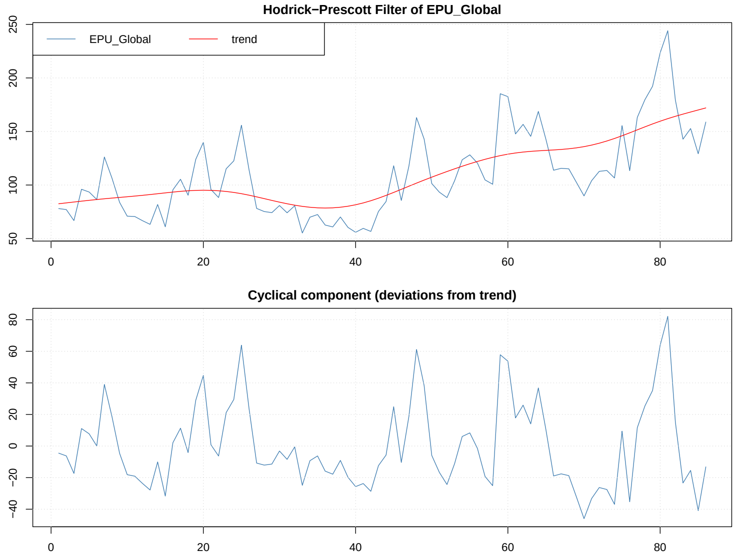
Source: Ipeadata.

Figure 11: Economic Policy Uncertainty - Brazil (Cyclical Component and Trend).



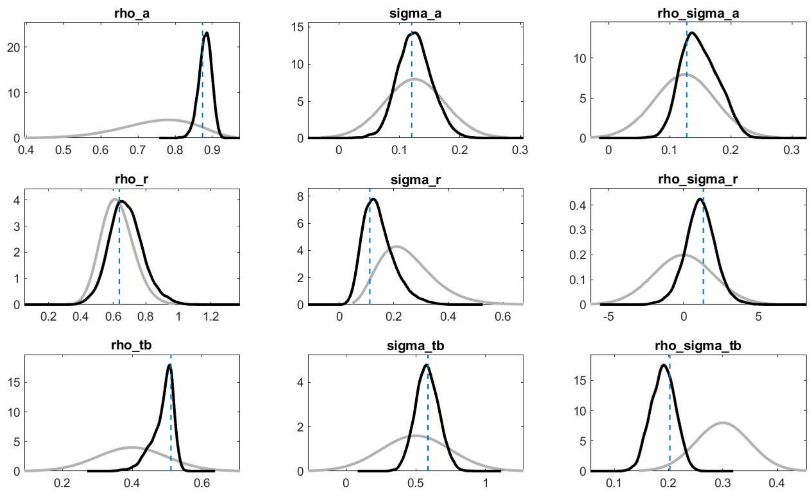
Source: Economic Policy Uncertainty.

Figure 12: Economic Policy Uncertainty - Global (Cyclical Component and Trend).



Source: Economic Policy Uncertainty.

Figure 13: Posteriors of the Estimated Parameters.



Source: Estimated results.