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GUILHERME BARROS CORRÊA DE AMORIM

# RANDOM-SUBSET VOTING

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Supervisor: Prof. Dr. Ana Paula Cabral Seixas Costa.

Co-supervisor: Prof. Dr. Danielle Costa Morais.

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GUILHERME BARROS CORRÊA DE AMORIM

**RANDOM-SUBSET VOTING**

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**BANCA EXAMINADORA**

Prof[a]. Dr[a]. Ana Paula Cabral Seixas Costa (Orientadora)
Universidade Federal de Pernambuco

Prof. Leandro Chaves Rêgo, PhD (Examinador interno)
Universidade Federal do Ceará

Prof[a]. Dr[a]. Maísa Mendonça Silva (Examinadora interna)
Universidade Federal de Pernambuco

Profa. Pascale Zaraté, Docteur (Examinadora externa)
Université Toulouse 1 Capitole

Profa. Isabelle Linden, Docteur (Examinadora externa)
Université de Namur

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## ABSTRACT

Most of the voting procedures in the literature assume that voters have preferences that are complete, transitive and subject to the independence of irrelevant alternatives property. A basic premise concerning the rationality of the individuals is that any voter is able to evaluate any pair of alternatives in a set and select his/her preferred one, or indicate indifference between them. Nevertheless, some researchers have highlighted that voters, as humans, have limited capacity to deal with and consequently compare big sets of alternatives. In this study, we propose the Random-Subset Voting (RSV), a voting procedure that allows the voters to evaluate less alternatives. Instead of analyzing the entire choice set, each voter evaluates a random subset of a pre-determined size. The proposed model was tested under Borda with three different approaches: mathematical modelling, Monte Carlo simulations and experiments. It was also tested under plurality, approval and Condorcet with Monte Carlo simulations. The results for all methods suggest that, for big and homogeneous populations, the proposed model leads to equivalent results to those found in the traditional methods, having the advantage of allowing the voters to make decisions by analyzing less options. We advocate that RSV can be a tool to be implemented in our societies and in our organizations, having important social and economic implications.

Keywords: Voting methods. Probabilistic voting. Bounded rationality. Decision theory.

## RESUMO

A maioria dos procedimentos de votação apresentados na literatura assumem que os eleitores têm preferências completas, transitivas e sujeitas à independência das alternativas irrelevantes. Uma premissa básica relacionada à racionalidade dos indivíduos é que qualquer eleitor é capaz de avaliar qualquer par de alternativas em um conjunto de opções e indicar se prefere alguma delas ou se é indiferente. No entanto, alguns pesquisadores destacam que os eleitores, sendo seres humanos, têm capacidade limitada para lidar com e, consequentemente, comparar grandes conjuntos de alternativas. Neste trabalho, propomos o modelo de Votação baseado em Subconjuntos Aleatórios de alternativas (VSA), um processo de votação que permite que os eleitores avaliem menos alternativas. Em vez de analisar todo o conjunto de opções, cada votante avalia um subconjunto aleatório de tamanho pré-determinado. O modelo proposto foi testado com o método de Borda através de três diferentes abordagens: modelagem matemática, simulações Monte Carlo e experimentos. O VSA foi avaliado com os métodos pluralista, de aprovação e de Condorcet com simulações Monte Carlo. Os resultados para todos os métodos sugerem que, para populações grandes e homogêneas, o modelo proposto leva a decisões coletivas equivalentes aos dos respectivos modelos tradicionais, tendo a vantagem de permitir que o eleitor tome decisões avaliando menos alternativas. Nós acreditamos que o VSA pode ser uma ferramenta a ser implementada em nossas sociedades e em nossas organizações, gerando importantes implicações sociais e econômicas.



Palavras-chave: Métodos de votação. Votação probabilística. Racionalidade limitada. Teoria da decisão.

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SUMMARY

# 1 INTRODUCTION

By the end of the eighteenth century, with the French Revolution and the American War of Independence, the modern world started to progressively migrate from the old monarchies to the newborn democratic states. The ideals of liberty, equality and fraternity, along with the nascent possibility of a direct participation of the population in the social decisions, flourished, shaping the development of the western world.

At that time, together with this great shift in history, some scholars started to envision decision models and voting schemes that would work as the basic engines of the new paradigms that were being established. The French mathematicians Jean-Charles de Borda and Jean Antoine Nicolas Caritat, the Marquis de Condorcet, proposed relevant theoretical background on voting theory and played important roles in the beginning of our recent democratic endeavor (BLACK, 1958).

More than 150 years later, in the mid twentieth century, the studies of Kenneth Arrow fostered a new cycle in the development of the Social Decision Theory, putting him as one of the all-time major contributors to the field. In 1950, his paper *A Difficulty in the Concept of Social Welfare*, see Arrow (1950), presented what he called the doctrines of citizens' sovereignty and rationality, and proposed the well-known Possibility Theorem stating that, under some conditions, every social welfare function would be either imposed or dictatorial.

The framework on which Arrow has built his theory is based on deterministic and rational assumptions, which means that there is no space for subjective or probabilistic reasoning. For him, and for hundreds of his followers, modelling rational voters' preferences that are complete, transitive and subject to independence of irrelevant alternatives (IIA) was good enough to cope with the complexity faced at the time.

Under this rational mindset, most models assume the rather intuitive reasoning that the more alternatives, the better. In other words, the bigger the choice-sets, the more likely the decision maker will choose the alternative that best fits his/her preferences, maximizing the utility. Chernev, Bockenholt and Goodman (2015) presents a comprehensive list of studies in which a large variety of options leads to a greater satisfaction for the decision makers.

In the meantime, whilst the rational models gained ground, the literature of Social Decision Theory faced a gradual shift from pure rational to fuzzy or probabilistic models of individual behavior. Allais (1953) and Simon (1955) mark important milestones in this trend and state the foundations of what was later referred to as “bounded rationality”.

For Allais (1953) and Simon (1955), the models that set the “economic man” as the ground for the developments in decision theory needed a drastic review in order to support the new findings of the psychologists and the behavioral economists. These researchers have suggested that, in real situations, the observed behavior of people substantially differed from the behavior preconized by the traditional rational models.

Tversky and Kahneman (1981) corroborate this reasoning and present many real decision scenarios in which people systematically violate the rational premises. They link these violations to the psychological facets that guide the perception of the decision and to the process of evaluation of alternatives.

Some scholars have focused their attention on exposing that people sometimes fail to present transitive or complete preferences (ANAND, 1993; CAMERER, 1998; DANAN; ZIEGELMEYER, 2006; FISHBURN, 1991; NURMI, 2014a). Other researchers question the validity of the independence of irrelevant alternatives property (CHENG; LONG, 2007; FRY; HARRIS, 1996; MCFADDEN; TRAIN; TYE, 1977; ZHANG; HOFFMAN, 1993).

Another premise that is also questioned by many studies is the one that argues that the more alternatives, the more satisfied the decision maker. Iyengar & Lepper (2000), Schwartz, et al. (2002) and Schwartz (2003) argue that people are not able to evaluate a large amount of data in order to make many decisions. They present substantial evidences that giving more alternatives to the individuals might hinder their ability to make good decisions.

It is verified that individuals can face an overwhelming effect when dealing with too many options in decisions (SCHEIBEHENNE, 2008; SCHEIBEHENNE; GREIFENEDER; TODD, 2010) . Chernev, Bockenholt and Goodman (2012) present a mindful review of the literature on choice overload and propose a model in which several variables are related to the perception of choice overload. These variables include the number of alternatives, the complexity of the choice set and the preference uncertainty.

Into the political arena, very few researchers have focused on the choice overload issue. Nurmi (1983) presents his considerations concerning the difficulty of the task of casting a vote. He compares voting systems in regard to the implementation criterion (a measure of complexity and difficulty of casting votes) and states that some methods require more cognitive effort than others.

Botti and Iyengar (2006) analyze how the policy makers could provide "people with the ability to make choices that individually customize public goods and services. Yet in doing so, they must be mindful of people's cognitive limitations, knowledge about their own preference, and negative emotional responses that may complicate choices and thwart individual welfare."

This reasoning has led us to question whether the models we adopt in order to make social decisions take into consideration these non-rational aspects of human behavior. More specifically, are there voting schemes designed to mitigate the overwhelming effect faced by the individuals when too many alternatives are in the choice-set?

Botti and Iyengar (2006) approaches this question by arguing that reducing the number of alternatives at different steps in the decision process could be an insightful way to ease the cognition burden faced by individuals when dealing with the information overload resulting from the excessive number of options.

Besedes et al. (2015), in the paper *Reducing Choice Overload without Reducing Choices*, corroborates Botti and Iyengar (2006) and tackles the overload issue by proposing changes in the choice architecture in a way that large decisions are split into series of smaller ones. In this approach, the set of alternatives is divided into smaller random subsets that are sequentially presented to the voter in order to progressively gather his/her preferences.

This approach, however, despite the benefit of easing the voting task, still requires a considerable cognitive effort as the decision makers keep on evaluating all alternatives of the choice-set.

This reasoning then leads to our main research questions:

a) How can we effectively reduce the cognitive overload faced by voters when dealing with too many options?
b) Do the voters really need to express their preferences over the entire set of alternatives?
c) Is it possible to reach the same social outcomes with less cognitive effort from the voters?
d) Which impact the reduction of the number of alternatives evaluated by each voter might have in the outcome of the election?
e) How could such a shrinkage of the choice-set be employed in real voting scenarios?

## 1.1 GOALS

In order to answer to these research questions, we have proposed the Random-Subset Voting (RSV), a voting procedure in which each voter do not analyze the entire set of alternatives, but a random-subset of it. In RSV, we tackle the cognitive overload faced by the voters by randomly reducing the number of alternatives that each voter has to analyze when making decisions.

The procedure, as will be described in details in the third chapter, is in fact proposed as a framework, or a meta-procedure, in which any traditional voting method can be tied to.

The main contributions of this study are:

a) The presentation of a bibliographic review concerning the deterministic and probabilistic voting procedures, as well as a review of the literature that studies the rationality of individuals in the context of decision theory;
b) Proposing and proving a theorem that shows that, under some conditions, Random-Subset Borda (RSV implemented with Borda) produces results equivalent to the traditional Borda itself;
c) Designing and conducting simulations of RSV with Borda, plurality, approval and Condorcet voting schemes, and investigating how the method behaves under different voting scenarios;
d) Designing and conducting experimental applications that provide empirical evidences for RSV.

## 1.2 METHODOLOGY

In order to accomplish these goals, this thesis takes the quantitative approach of research with theoretical and empirical methods of investigation.

Before enumerating and describing the three methods applied, we present some remarks on the process used to select the appropriate literature to support this study.

The main search engines used to look for the papers, books and other references that guided the development of this research were the Google Scholar[1] and the Portal de Periódicos da CAPES[2].

Our initial searches focused on terms like “voting procedures” and “probabilistic voting schemes”. Then, we have investigated some correlated terms: “stochastic voting”, “voting and lotteries” and “probabilistic social choice”.

These exploratory searches have led to the first references that formed the primary bibliography studied. From this bibliography, we have started the process of studying the references of each paper listed on this primary bibliography.

Along with this look at the past of each reference, we have also, with the help of the Google Scholar engine, looked for the papers that came after and which explicitly cite the documents in our bibliography. It allowed us to identify the papers that came after the references we initially had.

This process of looking for references and citations has repeated during the entire development of this study and other terms were added to the search: “rationality”, “bounded rationality”, “simulation of voting”, “large electorates” and so on.

With this approach, we have built a big net or graph of bibliographic references that supported the development of this research.

After these brief comments on the process of mapping the important references for this thesis, we present the three methods employed to evaluate the Random-Subset Voting procedure: the mathematical analysis, the simulations and the experiments.

The first method to evaluate RSV and to examine its properties was the mathematical analysis. In this theoretical approach, we have investigated whether the voters really need to express their preferences over the entire choice-set and we have studied the possibility of reaching the same social outcomes with less cognitive overload.

In this analysis, we have followed a classical deductive reasoning to prove a theorem that shows that, under some conditions, Borda and the Random-Subset Borda lead to identical results.

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[1] http://scholar.google.com

[2] http://www.periodicos.capes.gov.br/

Although being useful in the analytical sense, this deductive method did not provide enough insights to lead to more practical understanding on RSV. In order to tackle this limitation of the mathematical analysis, and thus to evaluate the effective impact of reducing the number of alternatives, we have then implemented a series of simulations.

According to Axelrod (2005), the simulation, as a scientific methodology in Social Sciences, can support the validation and the improvement of models upon which the simulation is based. Corroborating Axelrod, Walliman (2011) states that the simulation technique consists of creating a parameterized representation of the system, using mathematical or physical models. It is frequently applied in what-if scenarios.

As our second methodological approach, we have then implemented a software and conducted simulations to model elections for Borda, plurality, approval and Condorcet and their respective Random-Subset versions.

The simulations allowed us to play with the main parameters of the model and to stablish some basic perception on how RSV behaves with varying populations and choice-sets. The simulations, however, as the mathematical analysis, have the limitation of being under the theoretical domain. We needed then some empirical evidences of its application in real voting scenarios.

We have therefore modelled two different experiments in which real participants used RSV to make social choices. These experiments were conducted among members of the academic community in the Universidade Federal de Pernambuco and the Instituto Federal de Pernambuco, in Brazil.

More details on the three approaches implemented in this study will be described in the following chapters. In the next section, we describe the general structure of the thesis.

## 1.3 STRUCTURE OF THE THESIS

This study is organized as follows.

In chapter one, we present the introduction, with the motivation of the proposed procedure, the goals of this work and the methodology applied.

The second chapter is dedicated to examine the literature that supports this study. The relevant work is divided into five main sections. The first section focuses on deterministic voting schemes; the second, on probabilistic ones. Sections three and four revise the literature concerning rationality and bounded rationality. The last topic brings a review on models that consider large electorates.

Chapter three presents the Random-Subset Voting procedure followed by its main properties and the mathematical analysis of the method.

In chapters four and five, we present the two simulations and the two experiments implemented in order to evaluate how RSV works under specific scenarios. We also present some analysis and discussions concerning the proposed procedure.

The RSV proposal, the mathematical modeling, the simulation 1 and the experiment 1 have already been published in the International Conference on Group Decision and Negotiation (GDN) conference, in 2016 (AMORIM; COSTA; MORAIS, 2016), and in the Journal of Artificial Societies and Social Simulations (JASSS), in 2018 (AMORIM et al., 2018). The simulation 2 and the experiment 2, presented in chapters four and five, will be soon formatted as papers and sent for review.

In chapter six, we draw the conclusions of the work and present the limitations and the directions for future researches.

After the conclusions, we present the last two sections of this document: the references and the appendix.

# 2 THEORETICAL BACKGROUND AND LITERATURE REVIEW

In this chapter, we present the literature review along with the theoretical background that supports this study.

The chapter is in great part dedicated to presenting the most important deterministic and probabilistic voting schemes in the literature. These models are introduced in the first and the second sections, respectively.

We then discuss the findings of scholars on the rational aspects of decision making. In the third section, we analyze the main premises in regard to the rationality of the individuals. In the following, the non-rational aspects are studied. In the fifth section we deepen our understanding on elections in which the number of voters is large. Finally, in the last section we guide some concluding remarks of the chapter.

## 2.1 DETERMINISTIC VOTING SCHEMES

In general terms, a voting scheme is a procedure of eliciting preferences of a set of citizens in regard to a set of alternatives and, after combining these preferences, reach a final collective choice.

In this section, we present an overview of the main deterministic voting schemes found in the literature. Each subsection describes a method and its main features. We start by plurality, followed by Borda, Condorcet and approval. We then describe Nanson's, Copeland's, Hare's and Coombs' methods. In the last subsection, a list of other deterministic methods is presented.

### 2.1.1 Plurality

Plurality is one of the simplest and most used voting systems in the world. In plurality, which is sometimes referred to as the "first-past-the-post" method, the voters are only required to choose one alternative from the list of candidates. The winner is the candidate with more votes.

For instance, suppose that *15* voters have to choose among three alternatives (*a*, *b* and *c*). Suppose that the preferences of the voters can be represented by a full ranking of the alternatives. Table 1 shows the preferences of the voters.

Table 1 – Preferences of 15 voters

| # voters | Ranking |
|---|---|
| *2* | $a \succ b \succ c$ |
| *2* | $a \succ c \succ b$ |
| *3* | $b \succ c \succ a$ |
| *5* | $b \succ a \succ c$ |
| *2* | $c \succ a \succ b$ |
| *1* | $c \succ b \succ a$ |

Source: The author (2020).

From the preferences of Table 1, the count of votes for candidates *a*, *b* and *c* are, respectively 4, 8 and 3. Alternative *b* is then the collective choice.

In spite of its importance and its wide application, a very critical flaw is found in the method: the possibility of leading to victory a candidate disapproved by the majority of the voters. In order words, the plurality method does not satisfy the *Condorcet winner* criterion. It also does not satisfy the *Condorcet loser* criterion, i.e. a candidate that loses to any other candidate in the pairwise comparisons may win the election.

Plurality could also encourage voters to vote for candidates different from their preferred ones when their favorite option does not have chances of winning the election (BRAMS; FISHBURN, 2002).

A very well-known modification on plurality is the plurality-runoff. In this method, the voters, at first, follow the same plurality rules, i.e. they choose one alternative from the choice set. If the winner candidate has more than *50%* of the votes, the result is reached. When this is not the case, the two most preferred alternatives go to a second voting round. The voters will then choose one alternative between these top two.

In spite of not resolving the Condorcet winner issue of plurality, it definitely avoids the Condorcet loser to be elected (NURMI, 1983).

### 2.1.2 Borda

The Borda voting scheme (initially named *élection par ordre de mérite*, or election by order of merit) was proposed in 1781 by Jean-Charles de Borda, a French mathematician and political scientist (BORDA, 1781).

Borda proposes a point system in which the voter declares his/her preferences over the alternatives through a complete ranking. The higher the element in the ranking, the more points it gets. Table 2 presents the relation proposed by Borda between position in the ranking and the points of the alternatives on the individual rankings.

Table 2 – Borda points

| **Position** | **Points** |
|---|---|
| *1* | *a+(m-1)b* |
| *2* | *a+(m-2)b* |
| *3* | *a+(m-3)b* |
| … | … |
| *m-2* | *a+2b* |
| *m-1* | *a+b* |
| *m* | *a* |

Source: Adapted from Borda (1781).

Even though Borda proposed the method with generic *a* and *b*, he exemplified this method with *a=b=1*. The final count of each alternative would be the sum of the points in each individual ranking. A definite axiomatization of the Borda voting rule is presented in Young (1974).

Applying Borda to the same voting scenario presented in Table 1, we would get 34 points for *b*, *30* for *a* and *26* for *c*. Table 3 details the computation of the points.

Table 3 – Computing the Borda score

| **# voters** | **Ranking** | ***a*** | ***b*** | ***c*** |
|---|---|---|---|---|
| *2* | $a \succ b \succ c$ | *6* | *4* | *2* |
| *2* | $a \succ c \succ b$ | *6* | *2* | *4* |
| *3* | $b \succ c \succ a$ | *3* | *9* | *6* |
| *5* | $b \succ a \succ c$ | *10* | *15* | *5* |
| *2* | $c \succ a \succ b$ | *4* | *2* | *6* |
| *1* | $c \succ b \succ a$ | *1* | *2* | *3* |
| **Points** | | *30* | *34* | *26* |

Source: The author (2020).

When proposing his solution, Borda raised some concerns regarding the plurality method. His argument was based on a core example in which 21 voters have to choose among alternatives $A$, $B$ and $C$.

For 13 voters, $B \succ A$ and $C \succ A$. For 8 voters, $A \succ B$ and $A \succ C$. In this case, according to Borda, it is reasonable that the alternative $A$ must not be the social choice of these 21 decision makers. However, if, among the 13 voters, 7 set $B \succ C$ and 6 set $C \succ B$, a plurality count would lead to the ranking in Table 4.

Table 4 – Borda example

| **Alternative** | **Votes** |
|---|---|
| A | 8 votes |
| B | 7 votes |
| C | 6 votes |

Source: (BORDA, 1781).

In other words, Borda argued that in scenarios with more than two alternatives, the traditional "first-past-the-post" model could easily lead to a winner that is disapproved by the majority of the voters.

The main argument against the Borda method, was that it could lead to an alternative that would not be the Condorcet winner. In addition, it was later proven that the Borda count, in some contexts, can be subject of strategic manipulation. These arguments, however, did not inhibited the adoption of Borda's scheme into the elections of the *Académie Royale des Sciences*, in Paris, for about twenty years. Other recent real applications of Borda are found in (FRAENKEL; GROFMAN, 2014; REILLY, 2002).

### 2.1.3 Condorcet

Jean Antoine Nicolas Caritat, the Marquis de Condorcet, was a French mathematician and social philosopher who, in the end of the 18$^{th}$ century, gave important contributions to the social choice field. His most famous work is the *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix* (Essay on the Application of Analysis to the Probability of Majority Decisions) (CONDORCET, 1785).

In Condorcet (1785), he presented his main argument concerning the preferences of the majority of the voters as a central aspect in the social decision. As stated by Young (1988), Condorcet argued that:

"Enlightened voters honestly attempt to judge what decision will best serve society. They may occasionally judge wrongly. But assuming that they are more often right than wrong, the majority opinion will very likely be 'correct'" (YOUNG, 1988).

This reasoning had an important influence of the probability theory, newly developed at the time.

In the method proposed by Condorcet, the voters presented their opinions about the alternatives by a series of pairwise comparisons, indicating which alternative was preferred. This approach, considering that each individual is able to evaluate all the alternatives in a consistent manner, i.e. avoiding intransitive preferences, would lead to a ranking for each voter. Condorcet then draw a matrix in which the cells recorded the number of rankings where one alternative was better ranked than the other. Considering the preferences of the *15* voters of Table 1, the Condorcet method leads to the matrix in Table 5.

Table 5 – Condorcet matrix (15 voters and 3 alternatives)

| **Alternatives** | ***a*** | ***b*** | ***c*** |
|---|---|---|---|
| ***a*** | - | *6* | *9* |
| ***b*** | *9* | - | *10* |
| ***c*** | *6* | *5* | - |

Source: The author (2020).

From Table 5, one could easily notice that $b \succ a$ for 9 voters, $b \succ c$ for 10 voters and $a \succ c$ for 9 voters, which gives $b \succ a \succ c$.

The Condorcet method, however, could lead to situations in which the final collective ranking presents cycles and, therefore, generate inconclusive results. Young (1988) presents a voting matrix with three alternatives and thirteen voters.

Table 6 – Condorcet matrix (13 voters and 3 alternatives)

| **Alternatives** | ***a*** | ***b*** | ***c*** |
|---|---|---|---|
| ***a*** | - | 8 | 6 |
| ***b*** | 5 | - | 11 |
| ***c*** | 7 | 2 | - |

Source: (YOUNG, 1988).

From Table 6, one could notice that $a \succ b$ for 8 voters, $b \succ c$ for 11 voters and $c \succ a$ for 7 voters, which is contradictory, since $a \succ b$, $b \succ c$ and $c \succ a$.

In order to deal with cyclic collective preferences, Condorcet proposed that, among the propositions in the cycle, the one with the smallest plurality should be removed. In this example, we should remove $c \succ a$. This leads to $a \succ b \succ c$.

From the Condorcet proposal, the social decision researchers derived the notion of Condorcet methods, which designate voting procedures that select the Condorcet winner, the alternative that would win by majority on pairwise comparisons. Furthermore, a method is said to meet the Condorcet loser criterion, when it does not select the alternative that loses to any other on pairwise comparisons.

The Marquis de Condorcet, who defended the pairwise comparison approach, intensively criticized the Borda's proposal. Both belonged to the *Académie Royale des Sciences*, in Paris, in the late XVIII century and, mainly because of their distinct backgrounds, they were rivals in the academy. While Condorcet was a great theoretician, Borda had more practical inclinations with focus on ship design, fluid mechanics, variation calculus, metric system and many other fields (MASCART, 1919).

The Condorcet ideas were extensively studied in the literature, and the advantages and drawbacks of his method led to the developments of many contributions (ADAMS, 1997; GEHRLEIN, 2006; GEHRLEIN; FISHBURN, 1976).

### 2.1.4 Approval

The approval voting scheme was mainly studied by Brams and Fishburn (1978) in the United States in the late 70s. In this method, the voters vote for as many candidates as they approve. In simply terms, the voter simply receives the list of candidates and decides who he/she approves and who he/she do not approve. The winner is the candidate that receives more votes.

In order to exemplify the approval method, we present in Table 7, an adaptation of the example of Table 1.

Table 7 – Preferences of 15 voters under approval voting

| # voters | Ranking | # of approves | *a* | *b* | *c* |
|---|---|---|---|---|---|
| 2 | $a \succ b \succ c$ | *2* | *2* | *2* | *0* |
| 2 | $a \succ c \succ b$ | *1* | *2* | *0* | *0* |
| 3 | $b \succ c \succ a$ | *1* | *0* | *3* | *0* |
| 5 | $b \succ a \succ c$ | *1* | *0* | *5* | *0* |
| 2 | $c \succ a \succ b$ | *1* | *0* | *0* | *2* |
| 1 | $c \succ b \succ a$ | *2* | *0* | *1* | *1* |
| **Points** | | | *4* | *11* | *3* |

Source: The author (2020).

In Table 7, we included the new column indicating how many alternatives were approved by the voters for each possible ranking. In the case of $a \succ b \succ c$, for example, only $a$ and $b$ were approved by the *2* voters. This means that $a$ and $b$ got two points each.

The final counts for alternatives $a$, $b$ and $c$ and were, respectively, *4*, *11* and *3*, which leads to the final collective ranking $b \succ a \succ c$.

Although being enthusiastically defended by Brams, who argues that "approval allows voters to vote for as many candidates as they find acceptable" and in spite of its real applications in many professional organizations[3], the method has been largely criticized. First, as highlighted by Niemi (1984) and Saari, Newenhizen and Van (1988), it can be subject to strategic manipulation. It is also known that approval does not necessarily chooses the Condorcet winner and could lead to the Condorcet loser (GEHRLEIN; LEPELLEY, 1998).

### 2.1.5 Nanson

The Nanson's method was presented in 1907 by Edward J. Nanson, an Australian Professor of Mathematics in the University of Melbourne. The method consists of a multi-stage procedure that starts with an ordinary Borda run. After calculating the scores, the alternative with the smallest score is eliminated. The Borda score are then calculated considering the subset, and, again, the alternative with the smallest score is excluded. This process repeats until no more elimination is possible (MCLEAN, 1996).

[3] Institute of Management Sciences, IEEE, Mathematical Association of America and American Statistical Association (WEBER, 1995).

This ingenious Nanson's proposal has one main fundamental improvement: it does choose the Condorcet winner, whenever it exists. It is also particularly resistant to manipulation (FAVARDIN; LEPELLEY, 2006).

One of the main limitations of Nanson's proposal is presented by Nurmi (1983), who argues that the method fails to attend to the monotonicity property[4].

It has been used in real elections in the University of Melbourne (between 1926 and 1982), in the University of Adelaide (since 1968) and in the state of Michigan (in the 1920s) (NARODYTSKA; WALSH, 2011).

### 2.1.6 Copeland

The Copeland method was proposed in 1951 by A. H. Copeland in his seminar *A reasonable social welfare function* that took place at the University of Michigan. It is a Condorcet winner method, i.e. if there is a Condorcet winner, the Copeland method will select it (COPELAND, 1951).

In this method, each alternative gets a score that indicates the number of times the alternative wins and loses on pairwise comparisons. Taking Table 5 as example, we compute the number of times each alternative wins and loses (see Table 8).

Table 8 – Copeland result of Table 5

| **Alternatives** | ***Count*** | ***Score*** |
|---|---|---|
| ***a vs. b*** | 6 vs. 9 | *b* scores 1 point |
| ***a vs. c*** | 9 vs. 6 | *a* scores 1 point |
| ***b vs. c*** | 10 vs. 5 | *b* scores 1 point |

Source: The author (2020).

In this example, *b* scored 2 points, *a* scored 1 point and *c* scored 0. Alternative *b* is then the Copeland's choice.

Compared to the Condorcet method itself, Copeland's has an important improvement: it gives not only the Condorcet winner, when it exists, but also a ranking of the

[4] The criterion of monotonicity states that if alternative *a* wins when a given method is used and then some voters change their preferences in a way that *a* is ranked higher than before and all other preferences remain the same, then *a* should remain the winner.

alternatives in a way that, whenever the Condorcet winner is removed from the list, the next one in the ranking is the new Condorcet winner.

A known drawback of Copeland's method is the possibility of leading to ties. Also, it is questioned for putting excessive importance in the number of victories, instead of in the magnitude of them (MERLIN; SAARI, 1997; SAARI; MERLIN, 1996).

### 2.1.7 Hare (Single Transferable Vote)

The Hare procedure of Single Transferable Vote (STV) was designed by Thomas Hare, in England, and by Carl George Andrae, in Denmark (HARE, 1859). The method consists of a series of eliminations of the alternatives with fewer votes, transferring these votes to the alternatives with more votes. The goal is to elect a subset of the set of alternatives.

The method was used in real scenarios for the first time in Tasmania, in 1896. In the beginning of the 20th century it was adopted in some city councils in the US (BENNETT, 2011). Today, it is used in Ireland, Australia, Malta, South Africa and in several academic institutions in the US (BRAMS; FISHBURN, 2002; DORON, 1979; TIDEMAN, 1995).

The STV procedure is typically divided into the following steps (DORON, 1979):

1. The voters are required to set their preferences over the alternatives; in this case, a complete ranking for each voter.
2. Calculate the quota, the threshold for electing a candidate.

$$q = \frac{n}{(k+1)} + 1, \tag{2.1}$$

   where
   $n$ = number of voters;
   $k$ = number of seats.

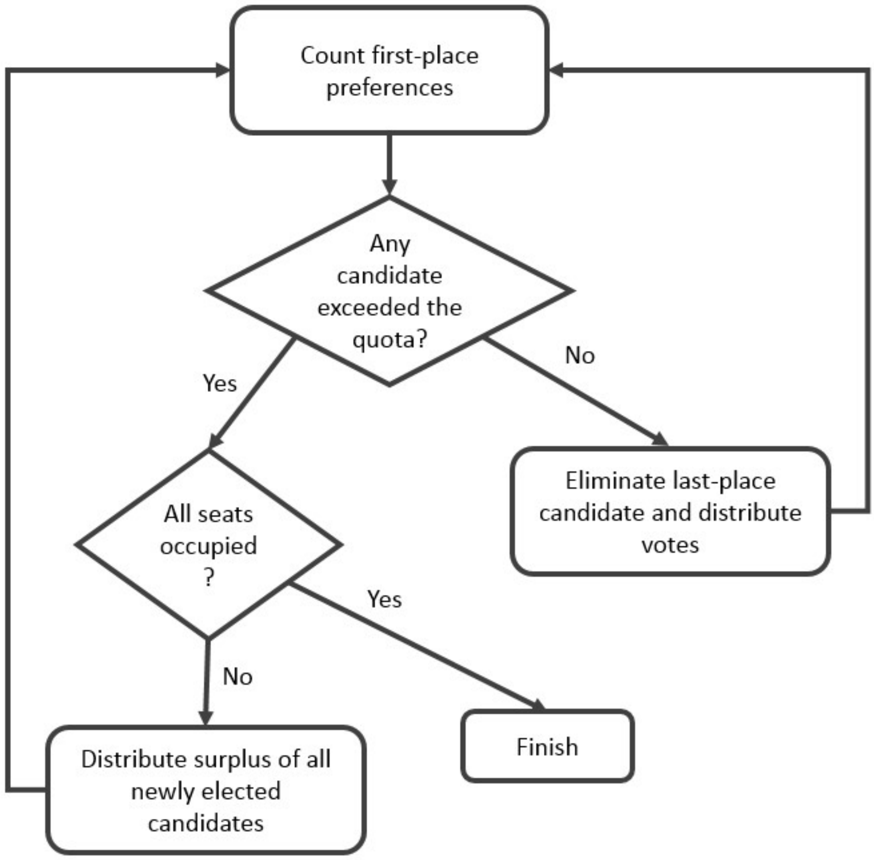
3. If the candidate(s) with more first places passed the quota, then they have their seats guaranteed and their exceeding votes are transferred to the remaining alternatives according to the preferences of the voters. Whenever the

alternative(s) in the first place do not reach the quota, then the lowest performing alternative is eliminated and its votes are distributed in the same way.

4. The third step repeats until all the seats were allocated.

This process is illustrated by Figure 1.

Figure 1 – Hare's method steps




Source: Adapted from (DORON; KRONICK, 1977).

Some scholars have noticed inconsistencies in STV (DORON, 1979; DORON; KRONICK, 1977). They have shown that the method presents some perverse aspects that lead to situations in which a candidate would win with 10,000 votes, but would lose with an additional 5,000 votes. Also, Nurmi (1983) proves that the Hare procedure do not necessarily pick the Condorcet winner. Nonetheless, if there is a Condorcet loser, the procedure does not select it.

### 2.1.8 Coombs

Coombs (1964) proposed a modification in Hare's procedure in which, according to Levin and Nalebuff (1995), "instead of deleting the candidate with the fewest first-place votes, we eliminate the candidate with the most last-place votes.". The election ends when no more seats is available. According to Nurmi (1983), the Coombs's procedure can be regarded as "the search for those alternatives that in the view of the majority are 'least tolerable.'"

In contrast to Hare's, Coombs procedure always requires running all predetermined steps in order to find out the elected members. Similarly, to Hare's, Coombs's voting scheme, although excluding the Condorcet loser, do not necessarily choose the Condorcet winner.

### 2.1.9 Other methods

Along with these methods, the literature in deterministic voting schemes presents a much broader range of procedures. The Table 9 presents a list of other deterministic procedures.

Table 9 – Other deterministic procedures

| **Method** | ***References*** |
|---|---|
| Black's | (BLACK, 1958) |
| Dodgson's | (DODGSON, 1876) |
| Schwartz's | (SCHWARTZ, 1972) |
| SNTV | (BAKER; SCHEINER, 2007;COX, 1991) |
| Range Voting | (SMITH, 2000) |
| Bucklin's | (HOAG; HALLETT, 1926) |
| Kemeny's | (KEMENY, 1959) |
| Schulze's | (SCHULZE, 2003) |

Source: The author (2020).

An overview of the main deterministic voting procedures can be found in Nurmi (1983).

In the next section, we present a review of the probabilistic voting schemes.

## 2.2 PROBABILISTIC VOTING SCHEMES

The literature of probabilistic or stochastic voting models presents an important number of contributions that, in spite of its tiny practical application in recent real-world election scenarios, is building a solid background for future developments and potential adoption.

We start this section with a brief overview of the decision by lot concept. We then examine some of the most important probabilistic voting schemes – Intriligator's, Zeckhauser's and Fishburn's proposals. In the last subsection, we present further references on probabilistic voting models.

### 2.2.1 Decision by lot (Random selection)

By decision by lot, or random selection, we simply mean that the social choices are made at random. A lottery is used to pick the candidates, not considering the preferences of the citizens in regard to the alternatives.

In his book *Election by lot at Athens*, James Wycliffe Headlam (HEADLAM, 1891) brings some insights about the ideas that drove the Athenian democracy in the VI and V centuries BC. He states that election by lottery was on the basis of the ancient Athenian democracy, playing an important role in the selection of political officials.

Many recent scholars have shown that it was not only in Greece that this idea had some fertile soil to flourish. The list of cities and countries that employed some sort of social decisions by lot include Venice, Bologna, Parma, Florence and other Italian cities; Barcelona, in Spain; some cities in Switzerland; some boroughs in England (CARSON; MARTIN, 1999; STONE, 2011).

Although not extensively discussed, the random selection tool has received some attention in the last decades. In a path back to the original concepts of democracy, Barbara Goodwin is an important milestone. In her book *Justice by Lottery* (GOODWIN, 1992), she argues that decision by lot can be a good substitute for reason in some contexts.

A few years before Goodwin, Jon Elster published the *Solomonic Judgements* (ELSTER, 1989). In this publication, he argues that the use of random selection in social choices is the result of the recognition of the limits in human reasoning. He draws a direct critique of the rational paradigm we are all immersed in and, by citing Kant, he says:

"As in Kant´s critique, the first task of reason is to recognize its own limitations and draw the boundaries within which it can operate. The irrational belief in the omnipotence of reason is what I call hyperrationality". (ELSTER, 1989)

Elster argues that uncertainty, indifference, indeterminacy and incommensurability are on the basis of the use of lotteries and he inquires the nonadoption of lotteries in contexts where they would be reasonable or justifiable. Elster, after meditating over several rationality aspects and conflicts, presents a comprehensive, though inductive, analysis of the implications of decision by lot.

Carson and Martin (1999) complement Elster's work by depicting a very broad view of random selection, presenting a historical perspective and some case studies around the globe. They encourage the adoption of random selection and characterize means of promoting it in the society.

Peter Stone is the most active researcher in random selection in the literature in the recent years (STONE, 2007; 2009a,b; 2010; 2011; 2014). His contributions focus on structuring arguments that justifies the application of lotteries in social decisions. Also, he proposes that "When reasons cannot determine the option to select on their own, the agent must resort to some form of non-reasoned decision-making". In this context, randomizing comes as a tool to deal with this decision-making process.

The implementation of choices through randomization is also the focus of Dowlen's studies. In his developments, he suggests that lotteries can be very relevant tools against tyranny and factionalism. He affirms that "the immunity from willful manipulation is an integral feature of every genuine lottery" (DOWLEN, 2009).

A good overview on randomness and legitimacy in selecting democratic representatives can be found in (PARKER, 2011).

In spite of these developments, very few applications of random selection can be found: citizens and policy juries in US, planning cells in Germany and departmental committees in Australia (CARSON; MARTIN, 1999). In fact, the implementation of such systems is still regarded by many with the same skepticism of Headlam (1891) more than a century ago. He states that it is hard to comprehend how such a system prevailed in a civilized community. He quotes:

> There is no institution of ancient history which is so difficult of comprehension as that of electing officials by the lot. We have ourselves no experience; of the working of such a system, any proposal to introduce it now would appear so ludicrous that it requires some effort for us to believe that it ever did prevail in a civilized community. There can be few people who, when they first hear that it existed at Athens and in other Greek states, do not receive the information with incredulity. (HEADLAM, 1891).

This random selection ideal, however, along with the review of some premises regarding the rationality of the individuals, encouraged the development of some hybrid models of decision. Models that do not consider full randomization, but rather a mix of random selection and voting. In the next subsections, we present the main probabilistic voting schemes found in the literature.

### 2.2.2 Intriligator

One of the most prominent models of probabilistic voting was proposed by Intriligator (1973). In his model, he proposes a reorientation on the way one regards the preferences of the voters. Instead of taking the traditional approaches, i.e. ranking or choosing alternatives, he proposes that each voter will present his/her preferences through individual probability vectors indicating the relative frequency of choosing each candidate.

He considers a society with $n$ individuals (indexed $i = 1, 2, \dots, n; n \geq 2$) and $m$ alternatives (indexed $j = 1, 2, \dots, m; m \geq 2$). The individual probability vectors, named $q_i$, are summarized as follows:

$$q_i = (q_{i1}, q_{i2}, \dots, q_{im});\ q_{ij} \geq 0\ for\ all\ i, j;$$

By taking $q_{i1}$as probability vectors, Intriligator derives that:

$$\sum_{j=1}^{m} q_{ij} = 1; for\ all\ i. \tag{2.2}$$

His method consists of calculating, from the individual probability vectors, the collective probability vector. Intriligator defines $p$ as the vector in which each position $p_j$

represents the social probability of choosing alternative $j$. The rule of calculating the social probability vector, named average rule, is defined by Equation (*2.3*).

$$p_j = \frac{1}{n}\sum_{i=1}^{n} q_{ij}\,;\ for\ all\ j. \qquad (2.3)$$

The final step of Intriligator's proposal is to define the social choice through a random mechanism that takes into account the vector $p$.

Some important extensions to this work can be found in Barbera (1979) and Intriligator (1982). Intriligator (1982) adjusted the concept of average rule to the weighted average rule. And Barbera (1979) extended the Intriligator's model to include two wide classes of decision schemes: supporting size and point voting, probabilistic adaptations of majority and positional voting.

### 2.2.3 Zeckhauser

Zeckhauser (1969) considered an expansion of the set of alternatives including not only the basic alternatives, but also lotteries between them. A clear example is the one in which 101 members of a club have to decide over 3 options (*A*, *B* or *C*). Fifty members prefer *A* to *B* to *C*. Another fifty members prefer *C* to *B* to *A*. One member prefers *B* to (*A* and *C*), i.e. he/she is indifferent between *A* and *C*. A traditional voting procedure might reasonably lead to *B* (the mean) as the group choice.

However, a coalition of the 100 members who prefer *A* or *C* would opt for an even-chance lottery between *A* and *C* than picking *B* for sure. In this case, especially when the game is repeated, opting for an expanded view of the alternatives set might be appropriate. In such cases, the coalition, would regard the lotteries between alternatives as viable alternatives.

In a complementary probabilistic approach, Zeckhauser (1973) proposes the random dictator model. In this approach, the voters write the name of their preferred candidates on a ballot. The ballots are then collected and placed in a referendum drum. After properly shuffling the drum, the candidate is selected at random.

### 2.2.4 Fishburn

Fishburn (1972) complements the developments of Zeckhauser and the notes made by Shepsle (1970) by investigating the existence of admissible and undominated lotteries and the connection between simple majorities and lotteries.

He also explores some of the Intriligator's assumptions in the probabilistic model of social choice. Fishburn argues that potential confusion might arise from the notion of the intensity of the individual preference. He shows that other notions of intensity of preference in the literature lead to some contradictions in Intriligator's reasoning (FISHBURN, 1975).

Fishburn also proposes the Maximal Lottery Methods, a family of selection rules that addresses the problem of cyclic majorities and/or tied votes through a random mechanism. This proposal is based on pairwise comparisons (FISHBURN, 1984).

### 2.2.5 Further references on probabilistic voting and general analysis

The developments on probabilistic voting models include other important contributions. Gibbard (1977) studies the manipulation of voting schemes that combines voting with chance. Burden (1997) presents a good overview of deterministic and probabilistic voting schemes.

Nurmi, Kacprzyk and Fedrizzi (1996) proposes a different approach to deal with some inconsistencies regarding aggregation of opinions. He discusses the use of rough sets theory and its link with decision theory (PAWLAK, 1982). Finally, he associates fuzzy preference with rough sets and randomness, presenting preliminary remarks about how they can be mixed in order to tackle the problems related to spatial voting games.

Coughlin and Nitzan (1981) use game theory to model the strategies of the candidates. They work on finding the local electoral equilibria in case of probabilistic voting methods.

A consolidated picture of the probabilistic voting schemes is presented in (BURDEN, 1997; COUGHLIN, 1992; NURMI, 2002).

By thoroughly analyzing all these deterministic and probabilistic methods as a whole, one could observe a clear duality. In one side, we have the decision by deterministic schemes; in the other, the decision by lot. Between these extremes, a full spectrum of methods can be observed.

Dowlen (2009) brings some insights about this duality and the applicability of the lot tool depending on the context. In accordance with Stone (2009a), he argues that new theoretical problems can arise from the necessity to identify contexts in which indeterminacy requires lottery and the situations in which it is not required.

In general terms, these methods consist of solutions for bringing lottery into the social decision systems we implement in our communities. Each scholar finds some reasonable argument to defend the use of sortition, or the combination of sortition and voting, when choosing alternatives.

## 2.3 RATIONALITY

In addition to the review of the most relevant deterministic and probabilistic voting schemes in the literature, we must present a quick overview of the theories concerning the rationality of the decision makers. This section focuses on this issue and analyze it in regard to completeness and transitiveness of the preferences of the voters and to the independence of irrelevant alternatives property.

The notions of rationality applied in Social Choice Theory derives from the neoclassical maximization theory in Economics. Paul A. Samuelson, one of the greatest economists in the XX century, developed this maximization theory by stating that the rational economic agent acts in terms of maximizing his utility, or self-interest (SAMUELSON, 1948). Goodwin et al. (2015) argues that this maximization “statement has often been interpreted to mean that pursuit of self-interest is the only thing that is done by rational economic actors – and that anything else is irrational.”

In the context of social decisions, Arrow (1950) structured the foundations of the rational Social Choice Theory by proposing a set of *natural* conditions concerning the tastes of individuals and by evaluating how these preferences would merge to form the collective decision. Downs (1957) presents a straightforward view of Arrow’s fundamental premises in regard to rationality:

> A rational man is one who behaves as follows:
> 1. He can always make a decision when confronted with a range of alternatives.

> 2. He ranks all the alternatives facing him in order of his preference in such a way that each is either preferred to, indifferent to, or inferior to each other.
> 3. His preference ranking is transitive.
> 4. He always chooses from among the possible alternatives which ranks highest in his preference ordering.
> 5. He always makes the same decision each time he is confronted with the same alternatives (DOWNS, 1957).

In summary, in order to be considered rational, the preferences of the voters must by complete, transitive and subject to independence of irrelevant alternatives. In addition, the voter must not act strategically, i.e. do not choose his/her preferred alternative in order influence the collective result.

### 2.3.1 Complete preferences

For Arrow and Downs, any rational voter is capable of ranking all the alternatives in the choice set ($A$) by indicating preference ($\succ$) or indifference ($\sim$). To simplify the model, Arrow proposes the preference or indifference relation ($\succcurlyeq$):

$$a \succcurlyeq b \iff a \succ b \text{ } or \text{ } a \sim b \tag{2.4}$$

By considering that the preference or indifference relation ($\succcurlyeq$) is complete, Arrow states that:

$$\forall\, x, y \in A, \text{ either } x \succcurlyeq y \text{ or } y \succcurlyeq x. \tag{2.5}$$

### 2.3.2 Transitive preferences

The preferences of a decision-maker are said to be transitive when:

$$\forall\, x, y, z \in A, \text{ if } x \succcurlyeq y \text{ and } y \succcurlyeq z, \text{ then } x \succcurlyeq z. \tag{2.6}$$

### 2.3.3 Independence of Irrelevant Alternatives

The preference of a voter $v$ is said to be subject to the Independence of Irrelevant Alternatives (IIA) property if, for any $x, y \in A$, adding or removing alternatives from $A$ does not change the preference of $v$ over $x$ and $y$.

If $x \succcurlyeq y$ when considering the set $A$, then $x \succcurlyeq y$ when considering $A'$, whenever $A \subset A'$ or $A' \subset A$.

More on IIA can be found in (BJORN; VUONG, 1985; CATO, 2013; RAY, 1973).

## 2.4 BOUNDED RATIONALITY

The neoclassical maximization theory has been severely criticized by many economists and psychologists who questioned the traditional rational models in Economics. The contributions of Allais (1953), Kahneman and Tversky (1979), Simon (1955) and many others gave rise to more relaxed and flexible premises regarding the assumptions on the behavior of the agents. This movement naturally yielded to more complex and sophisticated economic models.

At the same time, this trend in the Economics field directly influenced the Social Choice Theory, leading to a considerable attention to models that cope with the non-rationality of the citizens.

Most of the contributions in this area started with empirical evidences, captured by the researchers, that were not contemplated by the rational framework. Ariely (2008), for example, presents a series of experiments in which individuals are required to present their preferences over travel destinations, subscriptions, images, etc. He shows that adding some *decoy* alternative into the alternatives set unconsciously guide the decision maker into a predicted option. Ariely also discuss the impact of emotions on the decisions people make. In a skillfully designed experiment, he illustrates that some men, when subject to some emotions like sexual arousal, anger, frustration, etc., tend to react in a way they would not normally do; and this reaction has impact over their choices.

Elster (1989) and Goodwin (1992) present relevant links between the premise of non-rationality (or partial rationality) of the human beings and the importance of proposing models that not only consider the non-rational behavior, but takes benefits from it.

In the next sub-sections, we present other contributions to this topic, focusing on four different aspects: incomplete preferences, intransitive preferences, non-IIA and choice overload.

### 2.4.1 Incomplete preferences

Even though it might seem reasonable that individuals are able to compare any two alternatives in a choice set and state their preferences or indifferences, many scholars have been arguing that the completeness assumption might not be verified in some experimental situations. Danan and Ziegelmeyer (2006) present experiments in which the participants could, at a very small cost, postpone a decision. He shows that a significant percentage of participants postponed some decisions. According to them, this fact suggests a link between indecisiveness and incomplete preferences. In addition, they propose a model that provides an individual measure of preference completeness.

From a more theoretical standpoint, Nau (2006) presents an axiomatization of incomplete preferences, using probability and utility theories. Ok, Ortoleva and Riella (2012) extends the Nau's approach by analyzing the completeness property in regard to two different forms of indecisiveness: indecisiveness in beliefs and indecisiveness in tastes.

### 2.4.2 Intransitive preferences

The transitivity of individual preferences also plays an important role in the perception of the rationality. By analyzing the definition of transitivity and its consistent sense of logic, it seems odd to envision a situation in which this property would not be verified. Anand (1993) states that:

> There is some sense in which intransitive choices appear to be a profound form of error. Transitive preferences are just a matter of logic, and it is natural to suppose that their negation must be logically inconsistent. Any observed violations would be judged as irrational in much the same way that we take behavior based on incorrect syllogistic inference so to be. (ANAND, 1993)

Tversky (1969), nevertheless, gave a very compelling example of a context in which intransitivity could not be regarded as an odd behavior.

> Consider, for example, a person who is about to purchase a compact car of a given make. His initial tendency is to buy the simplest model for $2089. Nevertheless,

> when the salesman presents the optional accessories, he first decides to add power steering, which brings the price to $2167, feeling that the price difference is relatively negligible. Then, following the same reasoning, he is willing to add $47 for a good car radio, and then an additional $64 for power brakes. By repeating this process several times, our consumer ends up with a $2593 car, equipped with all the available accessories. At this point, however, he may prefer the simplest car over the fancy one, realizing that he is not willing to spend $504 for all the added features, although each one of them alone seemed worth purchasing (TVERSKY, 1969).

Tversky also presented two controlled experiments in which consistent intransitivity could be obtained. The first, investigated the behavior of subjects with respect to transitivity in the context of gambling. The second experiment focused on the perception of the participants in the selection of college applicants.

Bar-Hillel and Margalit (1988) brings the discussion of the issue into a more subjective plane. They defend that the preferences could never be intransitive, though the choices could. Choices are externalizations of inner preferences.

Two additional papers focus on intransitivity and incomplete preferences at the same token. Nurmi (2014b) defends that presenting incomplete and/or intransitive preferences is not necessarily inconsistent with the maximization principle of the traditional rational choice theory. Mandler (2005) presents contexts where the revealed preferences differ from the psychological ones, and argues that there are situations in which the preferences are transitive, but not complete; and situations in which they are complete, but not transitive.

In spite of these contributions and many conducted experiments, there are still some skepticism about the real intransitivity of preferences and this topic will is still an open field of research (FISHBURN, 1991; REGENWETTER; DANA; DAVIS-STOBER, 2011).

### 2.4.3 Non-IIA preferences

As well as completeness and transitivity, Independence of Irrelevant Alternatives (IIA) has been studied in the context of rationality. Most of the experiments in this topic focus on demonstrating that the inclusion of an alternative can change the preference of two other alternatives.

Luce and Suppes (1965) presents the pony-bicycle example and discuss the implications of including a second bike in the choice set. In their experiment, some children were presented to a choice situation in which they had to choose between a pony and a blue bicycle. The average results indicate that they were indifferent between these options. However, when one includes a red bike in the choice set, the kids opted for the blue bike.

Tversky (1972) corroborates Luce and Suppes (1965) with the Beethoven/Debussy and the Paris/Rome examples. Other examples can be found in (ARIELY, 2008; DEBREU, 1960; GRETHER; PLOTT, 1979; MCFADDEN; TRAIN; TYE, 1977; TVERSKY; SLOVIC; KAHNEMAN, 1990).

Scholars have also employed other techniques in order to evaluate IIA. Fry and Harris (1996) and Cheng and Long (2007) used Monte Carlo techniques to test the IIA property in Logit models. Samuelson (1985) presents a model of likelihood of IIA in the context of probabilistic choice models.

### 2.4.4 Choice Overload

In spite of presenting choice overload under this bounded rationality section, this issue is, in fact, not directly considered as a topic in the traditional bounded rationality literature. It is often analyzed in the context of consumer behavior by measuring the inclination of buying certain products when more or less alternatives are in the choice set.

Frequently, individuals are not able to fully process the information required to make an optimal choice (BOTTI; IYENGAR, 2006). Also, with the increase on the number of alternatives, the amount of information to be processed increases, hampering the task of evaluating the options (SCHWARTZ, 2004).

Iyengar and Lepper (2000) present three experimental studies in which the predisposition to purchase chocolate or gourmet jams are measured in terms of the number of alternatives (six vs. twenty-four). They conclude that there are statistical evidences that offering more options may undermine satisfaction and motivation.

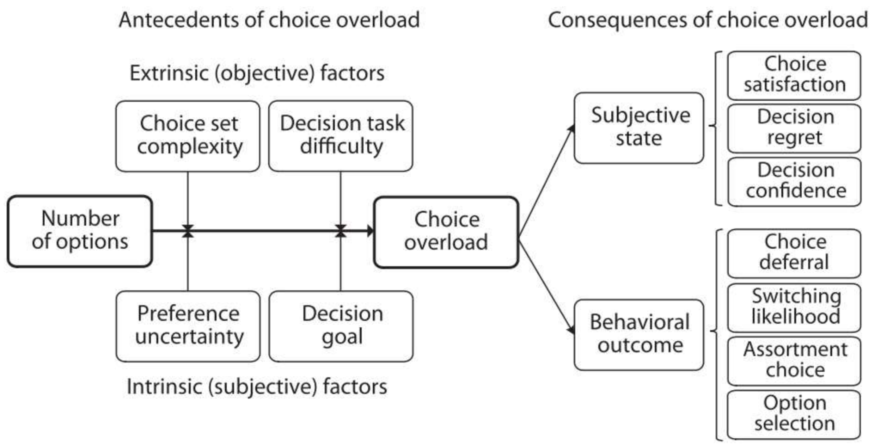
Schwartz (2002) present a relevant analysis of the negative effects of the proliferation of options. By analyzing his findings of Schwartz (2000), they state:

> He suggested that as options are added within a domain of choice, three problems materialize. First, there is the problem of gaining adequate information about the options to make a choice. Second, there is the problem that as options expand people's standards for what is an acceptable outcome rise. And third, there is the problem that as options expand, people may come to believe that any unacceptable result is their fault, because with so many options, they should be able to find a satisfactory one (SCHWARTZ et al., 2002).

Reed (2011) and Reed, Kaplan and Brewer (2012), in a series of experiments, evaluate how the subjective satisfaction of the participants variated as the number of options increased. The results of both papers show that a hyperbolic discounting function is a good approach to model the satisfaction levels. The findings of Kaplan and Reed (2013) corroborate these results.

Chernev, Bockenholt and Goodman (2012) carry out a thorough review and a meta-analysis on the choice overload literature, summarizing his findings in the conceptual model of Figure 2.

Figure 2 – Conceptual model of the impact of assortment size on choice overload




Source: (CHERNEV; BOCKENHOLT; GOODMAN, 2015).

This model links extrinsic and intrinsic factors, as well as the number of options, to the perception of the choice overload. The extrinsic factors include the complexity of the choice set and the difficulty in the decision task. The intrinsic factors consider preference uncertainty and the decision goal.

Furthermore, the model traces the consequences of choice overload from two different perspectives: the subjective state of the individual (satisfaction, regret and confidence), and the behavioral outcome (choice deferral, switching likelihood, assortment choice and option selection).

Another meta-analytic review of the choice overload literature can be found in (SCHEIBEHENNE; GREIFENEDER; TODD, 2010).

In the next section, we present a brief overview of the literature that studies elections with large electorates.

## 2.5 LARGE ELECTORATES

The traditional models of social choice usually present arguments and structure their reasoning by examples and counter-examples with very few voters choosing from very small sets of alternatives.

However, as stated by Gehrlein and Fishburn (1978), large electorates are not uncommon and the use the probabilistic and statistical methods could help in the comparison of voting procedures. Under the impartial culture assumptions, they compare Condorcet, Borda and simple majority when the number of voters tends to infinity. Gerhlein (1992) takes a similar approach to evaluate the Condorcet efficiency criterion of the Borda voting rule.

The contributions of Myerson and Weber (1993) comes from the proposal of a theory of voting equilibria for plurality, approval and Borda voting rules, focusing on large scale elections.

Campbell (1999) brings some insights concerning the "strategic calculus" of the voters that could not envision a gain in the act of voting. When the electorate is large, the probability of being pivotal in the election is insignificant; thus, the cost of the task might not justify the act of voting. He also studies how decisive minorities affect the outcome of the elections and attempt to provide a theoretical foundation for this issue.

Kurrild-Klitgaard (2001) empirically examines the Condorcet paradox of voting in a large electorate. He claims that too much attention is dedicated to the cyclical collective preferences issue. Nevertheless, very few empirical researches have provided evidences of in real-world large electorate scenarios.

More recently, (Laslier (2009), Mandler (2013) and Mckelvey and Patty (2006) have presented important contributions on this topic.

## 2.6 FINAL CONSIDERATIONS

In this chapter, we have presented a general overview of the literature in regard to deterministic and probabilistic voting models, rationality and bounded rationality when making decisions. We have also introduced a quick review on large electorates and how it may affect the voting process.

From a wider perspective, we have presented how the scholars, especially those from Behavioral Economics and Psychology, have started in the last decades to review some of the fundamental premises concerning the rationality of the voters. It has been shown that, in opposition to the main assumptions of the traditional utility theory in Economics, people could systematically make non-rational decisions.

Moreover, we have characterized how the choice overload, and the consequent overwhelming effect faced by the voters, is an aspect of the rationality of the individuals that needs to be taken into account when adopting a voting scheme.

The voting models introduced so far (both deterministic and probabilistic) lacks, however, of such a view on how this overwhelming effect, and the consequent incompleteness, intransitiveness and non-IIA behavior, may impact the social choices.

This gap, along with the premise of large electorates, has then motivated the development of the Random-Subset Voting approach. The proposed method, which was mainly inspired by the Intriligator's probabilistic voting model and by the Besedes et al. (2015) approach to mitigate the effects of choice overload, will be presented in details in the next chapter.

# 3 RANDOM-SUBSET VOTING (RSV)

In our literature review, we have studied deterministic and probabilistic voting schemes and investigated how the premises of rationality and non-rationality of the citizens may affect our models of individual behavior.

From this analysis, we have verified that very few studies propose solutions to mitigate the overwhelming effect faced by the voters that have to decide upon big choice-sets. Besedes et al. (2015) and Botti & Iyengar (2006) present the idea of reducing the cognitive cost of deciding by breaking the evaluation of the alternatives by the voters into smaller steps.

These solutions, however, although presenting an initial approach to deal with this issue, still requires to the voters to evaluate all alternatives in the choice-set.

This fundamental gap motivated the development of the Random-Subset Voting procedure, which is introduced in this chapter.

The chapter is divided into four sections. In the first, we present RSV. The main properties of the proposed method are described in the second section, followed by the mathematical analysis of the procedure, introduced in the third. In the last section we present the final considerations of the chapter.

## 3.1 RANDOM-SUBSET VOTING

The main insight that inspired RSV was the proposal of Intriligator (1973). His probabilistic model of social choices defends that, after the elicitation of the preferences of the voters, the aggregation would not directly lead to the final ranking, but to a weighted probabilistic ranking that would be the basis of the random mechanism that elect the choice(s). So, the candidate with more votes would be the one with more chances of being elected in the random step of the procedure. The candidate with less votes would be the one with less chances of being chosen.

This idea has guided us the following questions. Why not to bring the probabilistic step to a stage before the elicitation of preferences? Do the voters really need to express their preferences over the entire set of alternatives?

Assume that $V = \{v_1, v_2, v_3, \ldots, v_n\}$ is a set of $n$ voters who are required to indicate their preferences over $m$ alternatives on a set $A = \{a_1, a_2, a_3, \ldots, a_m\}$.

Random-Subset Voting states that instead of leaving the voters to indicate their preferences over $A$, each of them will vote from $A_i$, a random subset of $A$ of size $r$. This means that, for each voter $v_i$, a mechanism will generate the subset $A_i$ by randomly picking $r$ alternatives from $A$. The voters will express their preferences over each random subset of alternatives.

In summary, the method is divided into three steps:

1. A fair random mechanism is used to generate $n$ random subsets of $r$ alternatives ($r < m$) and assign each of them to a voter $v_i$ [5];
2. Each voter $v_i$ indicates his/her preferences over $A_i$;
3. The individual preferences are aggregated and the result of the election is reached.

By fair, we mean that the alternatives will have the same probability of being chosen as elements of the random subsets, i.e. each random selection of $r$ alternatives is an independent random sampling process with replacement.

The main goal of using RSV, instead of the traditional methods, is that it randomly reduces the number of alternatives of the voters, allowing them to make more accurate decisions. Our assumption is that, when analyzing less alternatives, the voters are less subject to the risk of facing the cognitive overload caused by the increase in the amount of information to deal with.

As already mentioned, four methods were tested under RSV in this study: Borda, plurality, approval and Condorcet. In most part of the study, the Borda method was the focus. Plurality, approval and Condorcet were only in the scope of the second simulation, which will be presented in the next chapter.

[5] The idea is that the random-subsets of alternatives will be delivered anonymously to the voters a few days (or weeks) before the election.

In order to facilitate the notation, we have called Random-Subset Borda (or RSB) the application of Borda under the RSV scheme. The same reasoning applies to Random-Subset Plurality (RSP), Random-Subset Approval (RSA) and Random-Subset Condorcet (RSC).

We extend this abbreviation including the number of random alternatives in the scenario: RSMr, in which M is the voting method and *r* is the number of random alternatives. For example, RSB3 indicates that we are dealing with Random-Subset Borda with 3 random alternatives; RSP5 refers to Random-Subset Plurality with 5 random alternatives; and so on.

## 3.2 RSV PROPERTIES

In order to have a better perspective on the method, it is important to respond to a fundamental question: what justifies the formulation of this new procedure? This rather simple question leads us to envisage what motivated its development and which properties and attributes the proposed method has that justifies its existence in itself.

The main motivation behind RSV comes from the challenge brought by Nurmi (2014a) of addressing the construction of decision models based on less demanding assumptions in regard to the rationality of the individuals. Also, we have pursued the reasoning of Botti and Iyengar (2006) that states that:

"By limiting people's choice sets at different stages throughout the choice process, policy makers may ease the cognitive burden that comes with information overload" (IYENGAR, 2006).

RSV then fills a gap in the Social Choice Theory literature proposing a way to allow the voters to express their preferences with less risk of facing the cognitive overload when dealing with too many options. Moreover, the method presents some interesting properties to be taken into account when one decides to put it in action.

The first and more direct one has to do with *manipulability*. As stated by Gibbard (1977), the manipulation of a voting process by an elector (strategic voting) occurs when he/she misrepresents his/her preferences in order to get the desired results. In RSV, the manipulation of the outcomes by the voters, in this sense, is significantly diminished by the random reduction of alternatives in the subsets. The voters will not be able to freely play with the options in order to act strategically. Also, since the main application of RSV is in the contexts of large electorates, the risk of having any voter in a pivotal position in the poll is close to zero. He/she will thus be guided by the goal of expressing his/her real preferences among the options.

RSV also might play an important preventive role in *vote buying* practices. Since the voters will not vote from the entire choice set and the random selection of alternatives to each voter is done anonymously, those who would intend to influence the results of the elections through vote buying would have a much harder work. The higher cost and risk of the illegal practice could prevent it.

In the perspective of Nurmi's *implementation criteria* (NURMI, 1983), RSV considerably reduces the *difficulty* of the task of casting the votes. If, for example, the Borda method is used by the citizens to evaluate a set of 30 alternatives, the task will not be easy. However, the use of Random-Subset Borda with 3 alternatives will make the task much easier.

In the case of multi-stage procedures, Nurmi takes the number of ballots as a measure of the *complexity* of a voting scheme. In this sense, RSV also reduces the complexity of the task.

## 3.3 MATHEMATICAL MODELING

In this section, we tackle the research question that asks whether it is possible to reach the same outcomes with less cognitive effort from the voters. We state and prove a theorem showing that, under some conditions, the final rankings of two elections, one using the Random-Subset Borda with $r$ random alternatives and one using Borda itself will be the same, with probability *1*, for a large enough $n$.

Based on Gehrlein and Fishburn (1978), we assume large electorates and focus on $n \to \infty$.

As stated in the last section, let $A$ be the set of $m$ alternatives and $V$ be the set of $n$ voters. Define $\mathcal{A}$ as the set of all possible complete orderings of $A$. Define $\mathcal{A}(j,k) \subseteq \mathcal{A}$ as the subset of $\mathcal{A}$ in which the alternative $a_j$ is in the *k-th* position in the ordering.

Define *p(x)* as the fraction of voters whose preferences are described by the ordering $x \in \mathcal{A}$. So, considering the Borda method, in which the alternative in the *k-th* position gets *(m-k)* points, the average score of alternative $a_j$ is:

$$\overline{X_j} = \sum_{k=1}^{m-1}(m-k)\sum_{x\,\in\mathcal{A}(j,k)} p(x). \qquad (3.1)$$

Now, consider the method Random-Subset Borda with *r* random alternatives ($2 \leq r < m$). The alternative in the *q-th* position in the randomly selected subset gets $(r - q)$ points.

Let $X_{i,j}^r$ be the random variable that indicates the score of the alternative $a_j$ for the voter $v_i$. Note that, if $a_j \notin A_i$, then $X_{i,j}^r = 0$.

Define $\overline{X_{n,J}^r}$ as the random variable that indicates the average score of alternative $a_j$ in the RSB election.

$$\overline{X_{n,j}^r} = \frac{1}{n}\sum_{i=1}^{n} X_{i,j}^r. \qquad (3.2)$$

Lemma 1 proves that, if all voters satisfy independence of irrelevant alternatives (IIA), the expected average score of any alternative in RSB is equal to the average score in traditional Borda, except for a positive scale factor.

**Lemma 1**: Suppose that every voter in *V* satisfies IIA and that $2 \leq r < m$. Thus, we have:

$$\mathbb{E}\left(\overline{X_{n,J}^r}\right) = \frac{r(r-1)}{m(m-1)}\overline{X_J}.$$

*Proof of Lemma 1*:

Considering that alternative *j* can be in anyone of the *m-th* positions in the set *A* and that it can be in any of the *r-th* positions in the selected subset, it follows that:

$$\mathbb{E}\left(\overline{X_{n,J}^r}\right) = \sum_{k=1}^{m-1}\left(\sum_{q=1}^{r-1}(r-q)\frac{\binom{k-1}{q-1}\binom{m-k}{r-q}}{\binom{m}{r}}\sum_{x\,\in\mathcal{A}(j,k)} p(x)\right) =$$

$$= \frac{1}{\binom{m}{r}} \sum_{k=1}^{m-1} \left( \sum_{q=1}^{r-1} (r-q) \binom{k-1}{q-1} \binom{m-k}{r-q} \sum_{x \in \mathcal{A}(j,k)} p(x) \right).$$

Thus, using the binomial coefficient definition and inverting the two innermost summations, we get:

$$\mathbb{E}\left(\overline{X_{n,J}^r}\right) = \frac{1}{\binom{m}{r}} \sum_{k=1}^{m-1} \left( \left( \sum_{x \in \mathcal{A}(j,k)} p(x) \right) \sum_{q=1}^{r-1} (m-k) \binom{k-1}{q-1} \binom{m-k-1}{r-q-1} \right) =$$

$$= \frac{1}{\binom{m}{r}} \sum_{k=1}^{m-1} \left( (m-k) \left( \sum_{x \in \mathcal{A}(j,k)} p(x) \right) \sum_{q=1}^{r-1} \binom{k-1}{q-1} \binom{m-k-1}{r-q-1} \right).$$

Finally, using results about the hypergeometric distribution and Equation 3.1, we get:

$$\mathbb{E}\left(\overline{X_{n,J}^r}\right) = \frac{\binom{m-2}{r-2}}{\binom{m}{r}} \sum_{k-1}^{m-1} \left( (m-k) \sum_{x \in \mathcal{A}(j,k)} p(x) \right) =$$

$$= \frac{r(r-1)}{m(m-1)} \overline{X_J} \quad \blacksquare$$

Since the random variables $X_{i,j}^r$, with $i=1,2,...,n$, are independent and limited, the first strong law of Kolmogorov implies that:

$$\overline{X_{n,j}^r} \xrightarrow{n\to\infty} \frac{r(r-1)}{m(m-1)} \overline{X_j}. \tag{3.3}$$

Finally, we prove the theorem that states that, for a big enough electorate, with probability *1*, the result of elections using Borda and Random-Subset Borda, with $2 \leq r < m$, are the same.

**Theorem 1**: Suppose that every voter in *V* has preferences over *A* satisfying IIA and completeness and that $2 \leq r < m$. Thus, we have that:

$$\overline{X_{j_1}} > \overline{X_{j_2}} \Leftrightarrow P\left(\lim_{n\to\infty} \overline{X^r_{n,j_1}} > \lim_{n\to\infty} \overline{X^r_{n,j_2}}\right) = 1, \forall a_{j_1}, a_{j_2} \in A.$$

*Proof of Theorem 1*:

From *(3.3)*, we can affirm that

$$P\left(\lim_{n\to\infty} \overline{X^r_{n,J_1}} = \frac{r(r-1)}{m(m-1)}\overline{X_{J_1}}\right) = 1$$

and

$$P\left(\lim_{n\to\infty} \overline{X^r_{n,J_2}} = \frac{r(r-1)}{m(m-1)}\overline{X_{J_2}}\right) = 1.$$

Thus,

$$P\left(\lim_{n\to\infty} \left(\overline{X^r_{n,J_1}} - \overline{X^r_{n,J_2}}\right) = \frac{r(r-1)}{m(m-1)}\left(\overline{X_{J_1}} - \overline{X_{J_2}}\right)\right) = 1. \quad \blacksquare$$

This theorem, in general terms, states that Borda and RSB converge when the population is large enough and when the voters are considered rational in regard to the IIA property.

In the next two chapters, we present the simulations and experiments conducted to test the RSV method.

## 3.4 FINAL CONSIDERATIONS

In this chapter, we have introduced the Random-Subset Voting procedure, a method that randomly reduces the number of alternatives the voters have to evaluate in a social choice. The main goal of the method is proposing a structured solution for the choice overload issue faced by the voters in many voting scenarios. The proposed solution can also have important effects on manipulation and vote buying practices.

In the mathematical modeling, we have proved a theorem that shows that Borda and Random-Subset Borda, when the population is large enough, lead to the same outcomes.

This analytical approach, while being quite useful, do not provide the necessary support to introducing the method in real elections. The theorem studies the behavior of the method when *n* tends to infinity. But what would happen in finite and realistic scenarios?

Another important questions that needs to be investigated are: how would RSV behave with other voting schemes? Does the convergence verified in Borda would take place? In which circumstances?

In the next chapter, we present the simulations implemented in order to give us more practical insights on RSV. Moreover, we study the behavior of the proposed schema not only with Borda, but with other important voting methods.

# 4 SIMULATIONS

The results of the mathematical modeling inspired the development of a Monte Carlo simulation software in which we could deepen our analysis on RSV. In this chapter, we detail the simulation techniques used and the results of the simulations.

In the social choice field, we can find many Monte Carlo simulation approaches. Ludwin (1976) simulates a three-candidate single-seat election and compares voting methods. Bordley (1983) develops a general method for evaluating election schemes and uses it to compare six well-known voting schemes. Merrill (1984) simulates elections in a randomly generated society and analyzes the Condorcet efficiency criteria for seven different methods.

Bissey, Carini and Ortona (2004) presents a program to simulate preferences of voters and compare the representativeness and the governability of eleven voting systems. Nurmi (1992) shows some theoretical assumptions concerning the models of simulations of voting procedures. Other important studies in the field are (ADAMS, 1997; CHAMBERLIN, J., 1986; CHAMBERLIN, J. R.; COHEN, 1978; GEHRLEIN; LEPELLEY, 2000; JOHNSON, 1999; SCHUMACHER; VIS, 2012).

With the simulations, we mainly intended to study the impact of the shrinkage of the choice-set in the outcomes of the elections. More specifically, we evaluated choice-sets of different sizes in scenarios with varying numbers of voters and alternatives.

The chapter is divided into two main sections that present the simulations that have been implemented in order to evaluate RSV. In the first, we focus on Random-Subset Borda. In the second, we investigate how RSV works under three additional voting methods: plurality, approval and Condorcet. At the end we present the final considerations of the chapter.

## 4.1 SIMULATION 1

In simulation 1, we have simulated the Random-Subset Borda (RSB) method, comparing the outcomes with the results of the traditional Borda method itself.

The simulation proposed assumes that voters have preferences that are complete, transitive and subject to IIA. The reason for this approach is primarily to assure that, when comparing voting systems, we have the same underlying premises supporting them. In other terms, the main point we want to evaluate is how RSB performs in comparison to Borda when the same premises regarding rationality are taken into account.

The simulation software was developed in Python and is publicly available at the CoMSES Net Computational Model Library[6].

In order to facilitate the presentation, we propose the following definitions:

a) *Configuration*: tuple ($n$; $m$) that indicates respectively the number of voters and the number of alternatives of a given simulation;
b) *Scenario*: randomly generated list of $n$ rankings representing the preferences of the voters over $m$ alternatives. For each configuration, we can generate many distinct scenarios;
c) *Unit Simulation*: for a given scenario, a unit simulation is the execution of Borda and RSBr. For each unit simulation, we set four parameters: number of voters ($n$), number of alternatives ($m$), number of random alternatives ($r$) and the distribution of preferences;
d) *Match index (MI)*: when comparing two rankings, the match index counts the number of alternatives in the same positions in both rankings. A full match is verified when all alternatives of both rankings are in the same positions. In this case, the match index is equal to the number of alternatives ($MI = m$). On the other hand, when $MI$ counts zero, we have minimal $MI$ and none of the alternatives match. This concept is as an adaptation of the Kemeny's distance (KEMENY, 1959).

We divide this simulation section into six subsections. In the first, we present how the random preferences of the voters were generated. In the second, we describe the unit simulation. In subsections three and four, we present the results of the simulation for several runs and scenarios. In the fifth subsection, we identify thresholds of convergence between Borda and RSB. Finally, we present some analysis of the results.

[6] www.openabm.org/model/4758/

### 4.1.1 Random preferences generation

In the context of modeling and simulating voting procedures, the random generation of preferences is by far the most important issue. Gehrlein and Fishburn (1981) describe three methods of generating profiles: impartial culture (IC), impartial anonymous culture (IAC) and maximal culture (MC). Apart from the details of each one, all three procedures are neutral regarding the alternatives and characterize "close" elections. Gehrlein (2004) and Gerlein (2006) evaluate how specific variations on IAC affects the probability of observing Condorcet's paradox. Chamberlin and Featherston (1986) present what Tideman and Plasmann (2012) denominated unique unequal probabilities (UUP). In this approach, as stated by Tideman and Plasmann (2012), "each candidate occupies a specifiable ranking niche (first, second, etc.), and that for each possible ranking of the candidates described by these niches, there is a constant probability that this ranking will be used by a voter.". An overview of the most common methods for profile generation is presented in Tideman and Plasmann (2012).

Our approach uses the UUP model of Chamberlin and Featherston (1986) but takes a different path in order to simplify its implementation. Instead of setting to each possible ranking a predetermined probability, we associate in our model each alternative to a Gaussian distribution ($N_i$) with specific mean and variation. This distribution is the basis of the random number generator used to set the preferences of each voter.

The algorithm of random preferences generation works as follows:

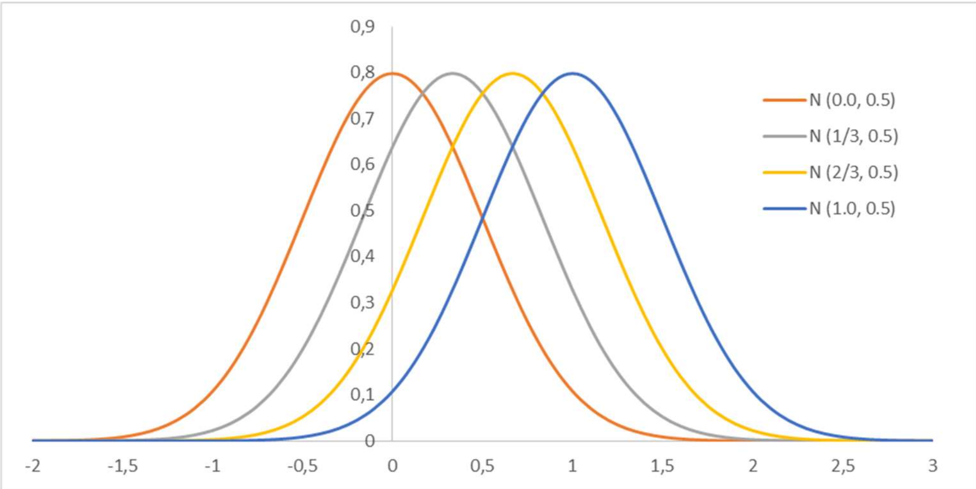
```
For each alternative ai, define Ni(μi, σi);
For each voter vj:
      For each alternative ai:
            Generate xi, random number generated from Ni(μi, σi);
      Ai <- Order the alternatives according to xi;
```

At the end of this process, the algorithm will produce a scenario, i.e. $n$ random rankings of $m$ alternatives, one ranking for each voter.

For $N_i$, we propose a fixed value for $\sigma(0.5)$ and means ranging linearly from *1* to *0*. In spite of the apparent arbitrariness of the values of the standard deviation and the range of means, they were intended to guarantee that all alternatives have a chance (a very small chance for the least preferred ones) of being chosen as the first candidate. These values for sigma and for the range of means are the result of a set of non-structured exploratory simulations run in order to empirically investigate which parameters would fit the experiments.

As an illustration of a configuration with *4* alternatives, the distributions for $a_1$, $a_2$, $a_3$ and $a_4$ are, respectively, $N_1(1.0,\ 0.5)$, $N_2(0.67,\ 0.5)$, $N_3(0.33,\ 0.5)$, $N_4(0.0,\ 0.5)$. Figure 3 depicts the normal curves for each alternative.

Figure 3 – Gaussian distributions for 4 alternatives




Source: The author (2020).

Each distribution in Figure 3 guides the random number generation process of each alternative.

Also, observe that this approach, in spite of leaving room for a great level of variation in the probability rankings and being good enough to cope with the complexity required for the behavior of the voters in our simulations, models homogenous voters.

This procedure will generate a set of 24 possible rankings of 4 alternatives.

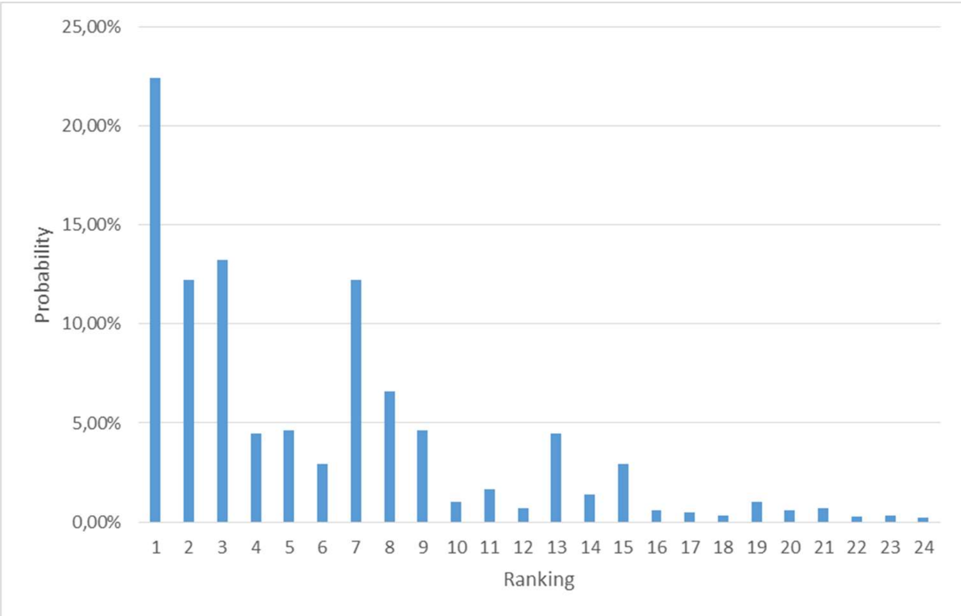
Table 10 and Figure 4 present the probabilities of each ranking for the distributions in Figure 3 when using the algorithm of random preference generation proposed.

Table 10 – Rankings and Probabilities

| # | ranking | prob. (%) | # | ranking | prob. (%) |
|---|---|---|---|---|---|
| 1 | $a_1, a_2, a_3, a_4$ | *22.42* | *13* | $a_3, a_1, a_2, a_4$ | *4.44* |
| 2 | $a_1, a_2, a_4, a_3$ | *12.25* | *14* | $a_3, a_1, a_4, a_2$ | *1.41* |
| 3 | $a_1, a_3, a_2, a_4$ | *13.24* | *15* | $a_3, a_2, a_1, a_4$ | *2.91* |
| 4 | $a_1, a_3, a_4, a_2$ | *4.44* | *16* | $a_3, a_2, a_4, a_1$ | *0.60* |
| 5 | $a_1, a_4, a_2, a_3$ | *4.64* | *17* | $a_3, a_4, a_1, a_2$ | *0.46* |
| 6 | $a_1, a_4, a_3, a_2$ | *2.92* | *18* | $a_3, a_4, a_2, a_1$ | *0.31* |
| 7 | $a_2, a_1, a_3, a_4$ | *12.25* | *19* | $a_4, a_1, a_2, a_3$ | *1.00* |
| 8 | $a_2, a_1, a_4, a_3$ | *6.59* | *20* | $a_4, a_1, a_3, a_2$ | *0.60* |
| 9 | $a_2, a_3, a_1, a_4$ | *4.64* | *21* | $a_4, a_2, a_1, a_3$ | *0.70* |
| 10 | $a_2, a_3, a_4, a_1$ | *1.00* | *22* | $a_4, a_2, a_3, a_1$ | *0.28* |
| 11 | $a_2, a_4, a_1, a_3$ | *1.67* | *23* | $a_4, a_3, a_1, a_2$ | *0.32* |
| 12 | $a_2, a_4, a_3, a_1$ | *0.70* | *24* | $a_4, a_3, a_2, a_1$ | *0.21* |

Source: The author (2020).

Figure 4 – Rankings and Probabilities




Source: The author (2020).

Note that our random preferences generation proposal, as showed in the example of Table 10 and Figure 4, implements the unique unequal probabilities of Chamberlin and Featherston (1986), i.e. associates a probability to every possible ranking.

### 4.1.2 Unit simulation

A unit simulation, as the name suggests, represents the most basic software piece in our simulation system. It is the basis of all simulations that will be presented in the next subsections.

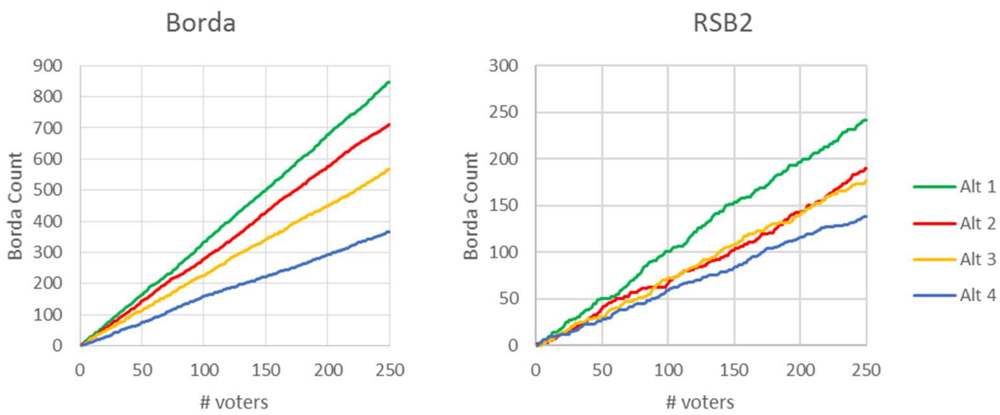
Simply put, for a generated scenario, the unit simulation randomly selects the subsets, computes the Borda count for the traditional Borda and the Random-Subset Borda, and finally calculates the match index. It works as follows:

```
Create scenario (n; m);
For each voter vi, with i ranging from 1 to n:
      Generate Ai (random subset of alternatives of size r);
Compute the Borda count for the traditional Borda;
Compute the Borda count for RSBr;
Calculate the match index (MI).
```

The generation of each random subset $A_i$ have to guarantee that the alternatives will have the same chance of being assigned to any random subset.

Figure 5 shows the results of a unit simulation for a (*250*; *4*) configuration with *2* random alternatives. The distributions of Figure 3 were used for generating the random preferences. For convenience, we have computed the evolution of the Borda count as the voters cast their votes. In practice, it is the equivalent of publishing the partial results of the election after each vote cast.

Figure 5 – Unit simulation of Borda and RSB2




Source: The author (2020).

The lines in the graph present the evolution of the Borda count for all four alternatives in both methods. Note that, as the number of voters increase, the rankings of Borda and RSB2 tend to converge. In this simulation, the match index reached 4 (full match) after 221 votes.

In the next subsection, we propose the multi-r simulation.

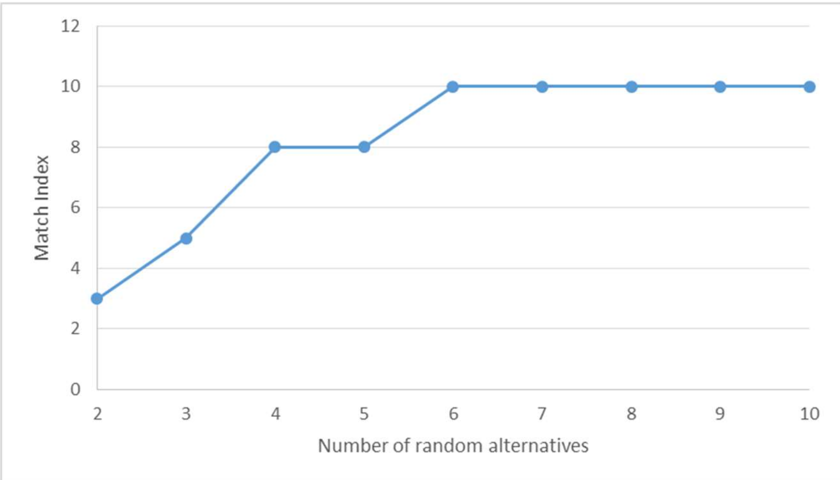
### 4.1.3 Multi-r simulation

For a given scenario, we define a multi-r simulation as the execution of all possible unit simulations for a single scenario. In other words, for each value of *r* (ranging from *2* to *m*), a unit simulation is executed and the match index is calculated.

The main goal of the multi-r simulation is to study how changes in the size of the random subsets impacts the match index.

In Figure 6, we present the results of the multi-r simulation of a (*1,000*; *10*) scenario. The graph indicates how *MI* evolves as the number of random alternatives increases.

Figure 6 – Multi-r simulation for (1,000; 10)




Source: The author (2020).

Note that, as the number of random alternatives increase, the match index tends to reach *m*. It is a rather intuitive result, since the upper bound of the number of random alternatives is in this case *10*. In this simulation, the value of *MI* for *r=2* was *3*. It means that the comparison of the final rankings of Borda and RSB2 presented only *3* matches. For values of *r* ranging from *3* to *5*, *MI* was, respectively, *5*, *8* and *8*. For the remaining values of *r* (from *6* to *10*), *MI* was equal to *10* (full match).

This means that, for this specific generated scenario with *1,000* voters and *10* alternatives, using RSB with 6 or more random alternatives have led to same outcome of Borda itself.

In the next subsection, we show how we have executed the multi-r simulations for *15* pre-determined configurations.

### 4.1.4 Multi-Configuration simulation

In the multi-r simulation of the last subsection, we have presented how Borda and RSB behaved for a unique configuration (*1,000* voters and *10* alternatives). In this subsection, we introduce the multi-configuration simulation.

In basic terms, the multi-configuration simulation is the run of several multi-r simulations for different configurations. While the multi-r simulation focuses on studying the impact of different values of *r*, the multi-configuration simulation incorporates the analysis of the effect of changing the number of voters and the number of alternatives.

We have implemented our simulation software to run the *15* configurations presented in Table 11.

Table 11 – Configurations of simulation 1

| ***m*** | **n** | | | | |
|---|---|---|---|---|---|
| 10 | 500 | 1,000 | 2,000 | 5,000 | 10,000 |
| 20 | 2,000 | 5,000 | 10,000 | 20,000 | 40,000 |
| 50 | 5,000 | 10,000 | 40,000 | 70,000 | 100,000 |

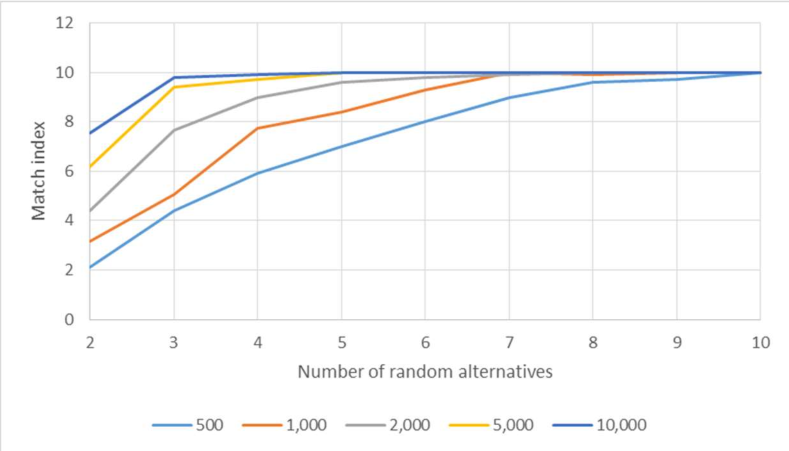
Source: The author (2020).

For each configuration, we have generated *20* random scenarios and executed a multi-r simulation for each one. We have then calculated the average values of *MI* ($\overline{MI}$) for each configuration.

Figure 7 presents the results of the simulations for *m=10*. Each line in the graph represents, for each configuration, the variation of the average match index as the number of random alternatives increase. Note that, as expected, *r=10* implied $\overline{MI}$*=10* for all configurations; it is the traditional Borda method.

For small values of *r*, *3* for example, the mean of *MI* for the configurations (*500; 10*), (*1,000; 10*), (*2,000; 10*), (*5,000; 10*) and (*10,000; 10*) were, respectively, *4.4, 5.05, 7.65, 9.4* and *9.8*. For *r*=*8*, these values reached *9.6, 9.9, 10, 10* and *10*.

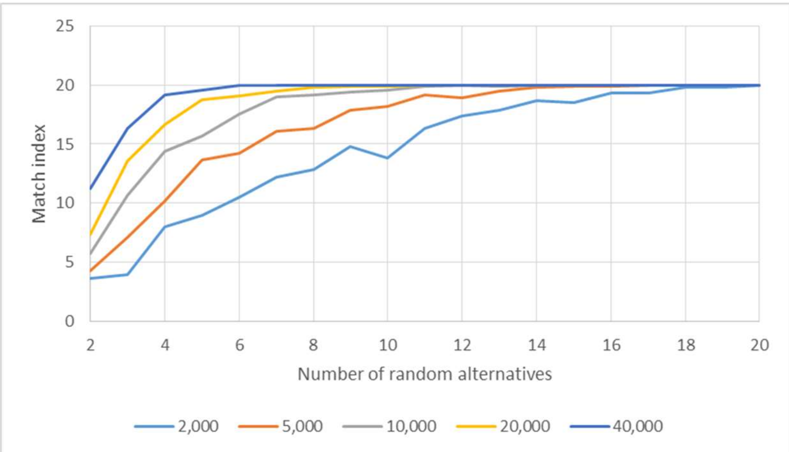
Figure 7 – Multi-configuration simulation for m=10




Source: The author (2020).

In Figure 8, the lines indicate the evolution of the average match index for the configurations in which *m*=*20*. One can easily observe that for *n*=*40,000*, the match index reaches its peak at *r*=*6* staying unchanged for the remaining values of *r*. For *n* equals to *20,000*, *10,000*, *5,000* and *2,000* the methods converged at *r* equals to *11*, *12*, *17* and *20*.

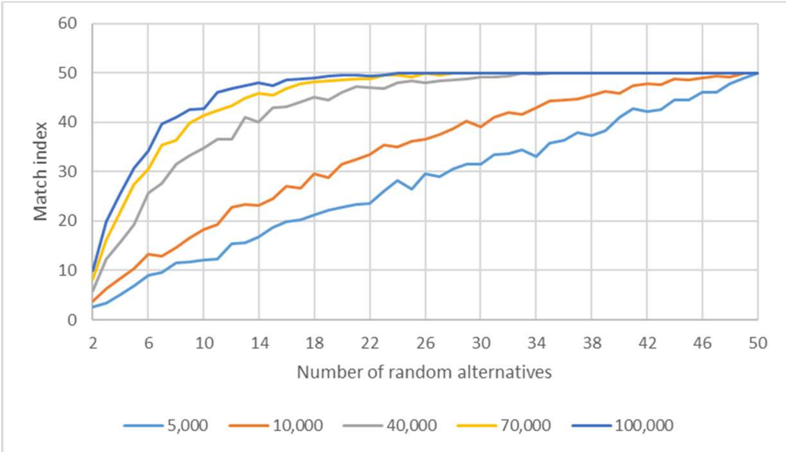
Figure 8 – Multi-configuration simulation for m=20




Source: The author (2020).

A similar analysis of Figure 9 shows that, for *m=50*, the methods converge at *r* equals to *50*, *50*, *38*, *30* and *25* for *m* equals to *5,000*, *10,000*, *40,000*, *70,000* and *100,000*, respectively.

Figure 9 – Multi-configuration simulation for m=50




Source: The author (2020).

### 4.1.5 Convergence simulation

The multi-configuration simulation detailed in the last subsection supported us in understanding how *MI* evolves when we vary the number of alternatives, the number of voters and the number of random alternatives.

In this subsection, we focus on a more practical question: for a given number of alternatives, how many voters and random alternatives are necessary to have a full match of Borda and RSBr?

In order to have a first glance of the answer, we have implemented a simulation in which, for a given number of alternatives, we gradually increase the number of voters and check whether the full match is reached. When it is reached, we decrement *r* and repeat the same process. This cycle repeats until *r*=2. The algorithm that follows presents the simulation structure:

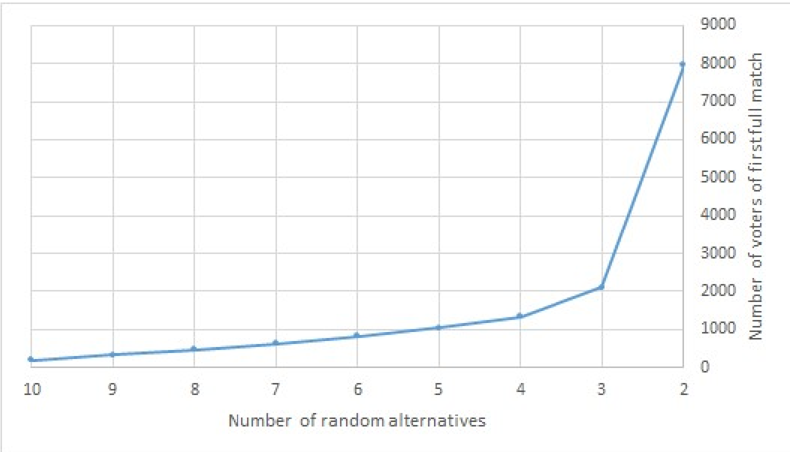
```
Define r=m
Define n=100
Define list = []
While r>=2:
      Create scenario (n; m)
      Single-run for the scenario with r random alternatives
      Compute MI
      If MI = m:
            r = r-1
            Save tuple (r; n) in list
      Increment n
```

In summary, this simulation identifies the population in which the first full match is verified for each value of *r*.

For each value of *m* (*10*, *20* and *50*) we have run *20* times the simulation. At the end of the process, we have a list of tuples that indicates the number of voters *m* in which a full match was verified with *r* random alternatives.

Figure 10 presents the results of simulations for *m=10*. For each value of *r*, the graph depicts the mean of the population (among the 20 runs) in which a full match of Borda and RSBr was first verified.

Figure 10 – Convergence for m=10



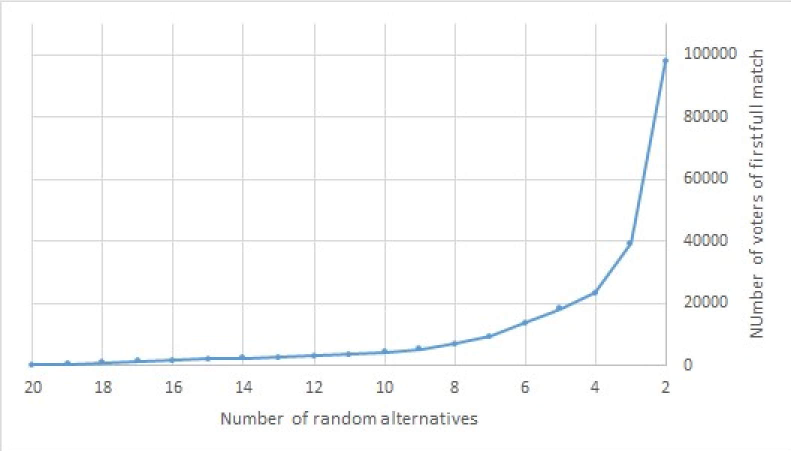

Source: The author (2020).

The behavior of the curve indicates that, for values of *r* ranging from *10* to *3*, the corresponding population rose steadily from *200* to *2,000*. For *r=2*, the number of voters increased exponentially and reached *8,000*.

A similar exponential growth is verified in Figure 11 (*m=20*). For *r* ranging from *20* to *5*, one can identify a gradual increase in the population. Afterwards, the slope of the curve changes and the subsequent values of *n* grow rapidly.

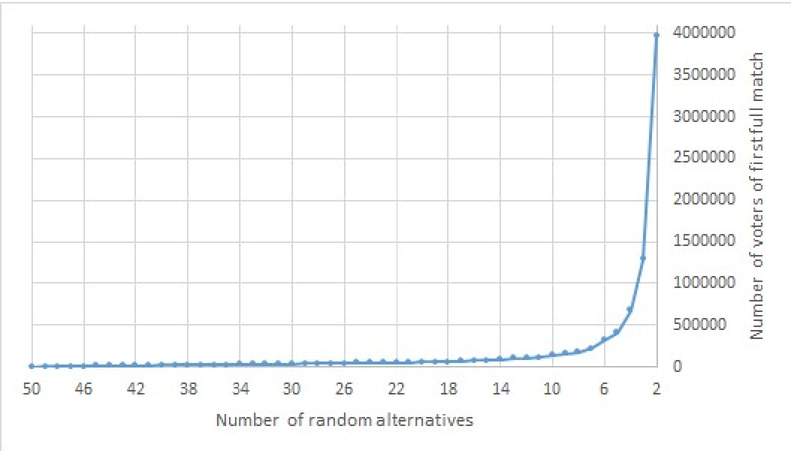
For the simulations with *m=50* (Figure 12), the exponential behavior of the curve is even more tangible. After *r=10*, when the number of voters was around *140,000*, the curve begins a clear change in its behavior, leading to approximately *4,000,000* voters when *r=2*.

Figure 11 – Convergence for m=20




Source: The author (2020).

Figure 12 – Convergence for m=50




Source: The author (2020).

### 4.1.6 Analyses

In this section, we have introduced the simulation of the Borda method under the RSV context. We have described how our Monte Carlo simulation software was implemented and showed the results of the progressively complex simulations developed.

We have started by the unit simulation, which is the atomic part of every simulation implemented. We have then proposed the multi-r, the multi-configuration and the convergence simulations.

The results of the simulations corroborated the mathematical analysis showing that, regarding the assumptions of transitivity, completeness, IIA and large electorate, Random-Subset Borda matched Borda.

The multi-configuration simulations showed how the match index evolved when increasing the number of random alternatives. In the case $m=10$ and *10,000* voters, for example, three random alternatives were enough to lead to a *MI* very close to *10*, which suggests that this size for the population was good enough to get close to the full match. When *40,000* voters and *20* alternatives were tested, the minimum number of random alternatives to reach the full match was 6. For $m=50$, the number of voters to guarantee the match has significantly increased. In order to have a full match around $r=20$, the number of voters had to be close to *100,000*.

In all the multi-configuration simulations, we could verify that the more voters or the more random alternatives, the earlier the match.

In the convergence simulations, our goal was providing a more practical instrument to guide the decision of setting the number of random alternatives in each election. For $m=10$, for example, the results suggest that three random alternatives were enough for a population of *2,000* voters. In the case of $m=20$ and $m=50$, for the same value of r, the population was *40,000* and *1,350,000* voters, respectively.

The simulations suggested, in general terms, some important relationships between the number of alternatives ($m$), the number of voters ($n$), the number of random alternatives ($r$) and the match index (*MI*):

a) On keeping $m$ and $n$ fixed, increasing $r$ led to an increase in *MI*. In spite of the variations identified in the results this trend is verified in all simulations;
b) On keeping $m$ and $r$ fixed, increasing $n$ led to an increase in *MI*;

c) For a given $m$, in order to reach a full match ($MI = m$), one can either increase $n$ or $r$;

For fixed values of $m$ and $r$, the simulations also suggest the existence of convergence thresholds for $n$. In other words, one can expect that after a certain number of voters the probability of having $MI = m$ tends to 1.

These simulation presents, however, some limitations that need to be addressed.

First, we have focused only on Borda. In fact, our main intent was to examine, using simulation techniques, how the mathematical analysis of RSV would behave. In spite of reaching quite motivating results, it would be crucial to our model to be tested under other voting schemes.

Second, we have identified that the random preferences generation model, inspired by Chamberlin and Featherston (1986) and Tideman and Plasmann (2012), could be attuned to lead to more efficient simulations.

In fact, since we have the statistical control of the results of the simulations when the number of voters is big enough, we simply need to compare the results of the random-subset version of the method with the expected final ranking derived from the random preferences generation process. We do not need to run both methods, just the RSV.

These two limitations, along with the necessity of stressing the proposed RSV method with more tests, motivated the development of the simulation 2, which is presented in the following section.

## 4.2 SIMULATION 2

In this section, we introduce the second Monte Carlo simulation implemented in order to test the Random-Subset Voting method.

In this new simulation, we deepen our analysis in the RSV scheme by presenting new simulations of the method and by focusing on the main limitation of the first one: only the Borda method was tested. In addition to the Borda method, we have implemented RSV with plurality, approval and Condorcet (Copeland), comparing and evaluating the convergence of each method.

The choice among these three new methods to be tested under RSV is based on the following reasoning. First, plurality and approval are methods that, while being very present in the literature, have inputs from the voters that are different from Borda. While Borda requires a complete ranking of the alternatives from the voters, plurality requires only the most preferred one and approval requires the alternatives that the voters consider approved by them. So, testing RSV under these three new methods enriches the analysis and allows more insights on how the use of methods with different inputs of the voter might impact the results. Second, by using Copeland's method, we intend to do the opposite. We wanted to test how a method with exactly the same input as Borda would behave with RSV.

In this second simulation, we have also made some improvements in the simulation structure since there is no need to run both the regular method and the RSV. The premises of the model, based on the unique unequal probabilities, state that each candidate is preset in a ranking niche (first, second, and so on). And, as the population gets bigger, each alternative will progressively fit its expected position in the ranking. Therefore, as we are dealing with large populations, we could identify the final theoretical ranking of the traditional method without the need of running the simulation.

We divide this simulation section into three subsections. In the first, we present the setup of the simulation, highlighting the differences from the first simulation. In the second subsection, we present the results, followed by the analysis in subsection three.

### 4.2.1 Simulation setup

The definitions of configuration and scenario of the simulation 1 are kept untouched. However, a few modifications on the definitions of unit simulation and match index need to be described.

The definitions of the new unit simulation and the match index, in this second experiment, are:

a) *Unit simulation*: for a given scenario, the unit simulation computes the final collective ranking of the alternatives, considering a specific method and the number *r* of random alternatives.
b) *Match index* (*MI*): after a unit simulation, the match index counts the number of alternatives that is in the expected position of each alternative in the final ranking.

The generation of random preferences of the population (the scenario creation algorithm), follows the same structure of the first simulation.

The new unit simulation is defined as follows:

```
Create scenario (n; m);
For each voter vi, with i ranging from 1 to n:
      Generate Ai (random subset of alternatives of size r);
Aggregate the results and compute final collective ranking;
Calculate the match index (MI).
```

We have proposed a fixed value for $\sigma$ (0.5) and means ranging linearly from *1* to *0*. This procedure leads to *n* lists of *m* alternatives indicating the preferences of each voter.

In order to evaluate the random-subset voting applied to Borda, plurality, approval and Condorcet, we have proposed a few modifications in the design of the first simulation. We have chosen to structure the unit simulations in a way that more emphasis is given to the number of voters, rather than the number of random alternatives. In other words, instead of analyzing how the variations on *r* affects the match index, we variate the population for the same number of random alternatives.

All simulations considered the same basic premises of the first experiment: independence of irrelevant alternatives, transitivity and completeness.

In this study, we first simulate the traditional methods and then we go for the Random-Subset versions of each scheme. In both situations, the configurations with *10* and *20* alternatives were simulated.

Table 12 presents the simulations for the Random-Subset versions of the methods. For *m=10*, we have tested *2*, *3*, *5* and *7* random alternatives; the population ranged from *200* to *35,000* with gaps of *100*. For *m=20*, the values of r were *3*, *5*, *7* and *11*, and the number of voters ranged from *500* to *150,000*, with gaps of *500*. For each configuration, *10* independent runs were simulated and the average match index was calculated.

Table 12 – Simulations of the second experiment

| ***m*** | **Number of random alternatives** | | | |
|---|---|---|---|---|
| *10* | *2* | *3* | *5* | *7* |
| *20* | *3* | *5* | *7* | *11* |

Source: The author (2020).

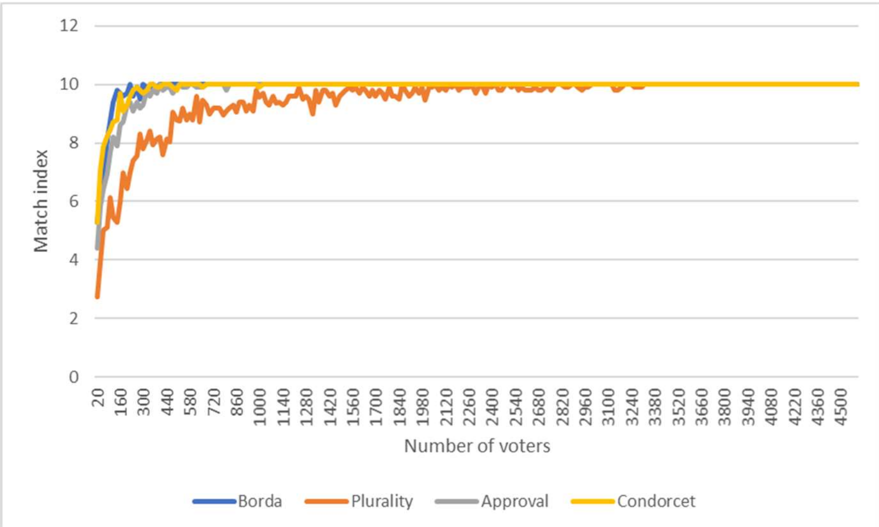
In the next section we present the results of the simulations.

### 4.2.2 Results

In this section, we present the results of the simulations. First, we exhibit the results of the traditional Borda, plurality, approval and Condorcet. Then we present the results of the Random-Subset versions of each method.

Figure 13 and Figure 14 shows the evolution of the match index for the four traditional methods. In both simulations, the curves move towards convergence, suggesting that the more voters participate in the election, the closer the match index to the number of alternatives. The results show that Borda and Condorcet converge first, followed by approval and, then, plurality.

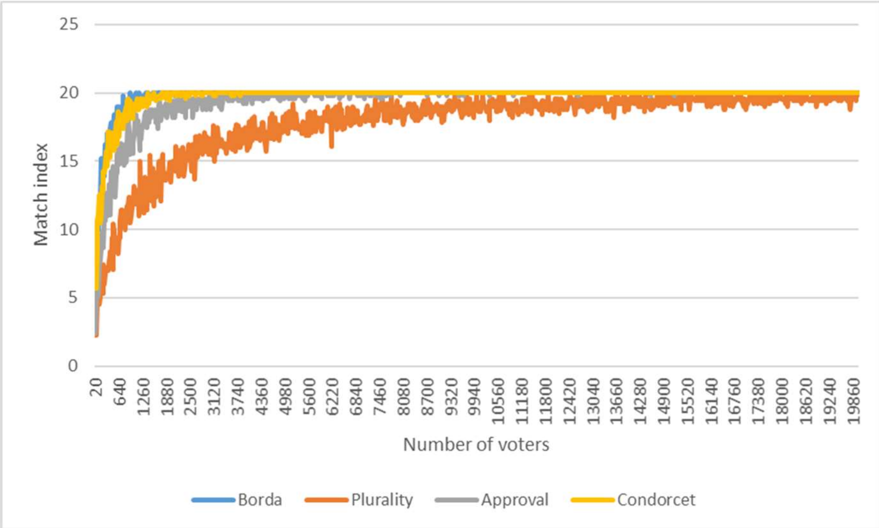
Figure 13 – Simulations for m=10 – traditional methods




Source: The author (2020).

Observing the graphs, one can verify that, for *m=10*, all methods converged before *3,500* voters. For *m=20*, only plurality did not converge before *20,000* voters. However, there is a clear trend towards convergence in case of more voters simulated.

Figure 14 – Simulations for m=20 – traditional methods

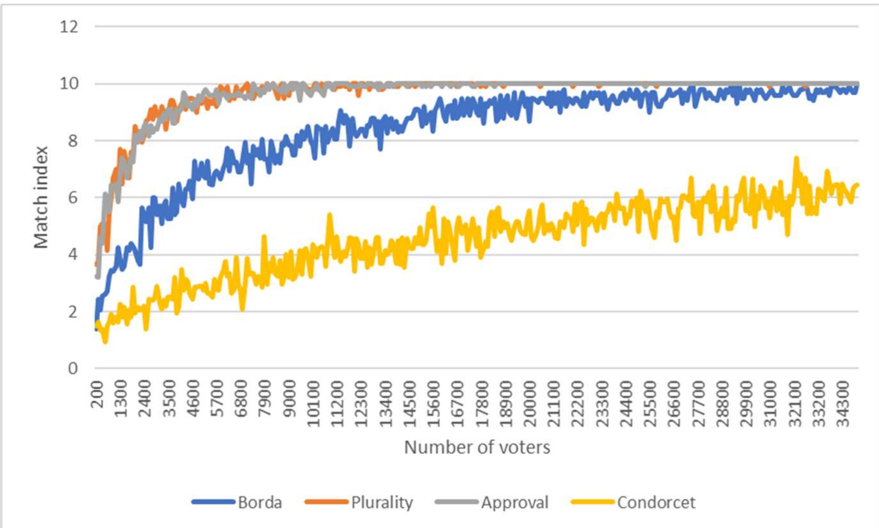



Source: The author (2020).

For the Random-Subset versions of the methods, the graphs below exhibit the match index for the simulations of Table 12. All simulations included the four voting methods tested under RSV with fixed number of random alternatives and variable number of voters.

Figure 15 shows the results of the simulations for *10* alternatives and random subsets of size *2*.

Figure 15 – Simulations for RSV m=10, r=2




Source: The author (2020).

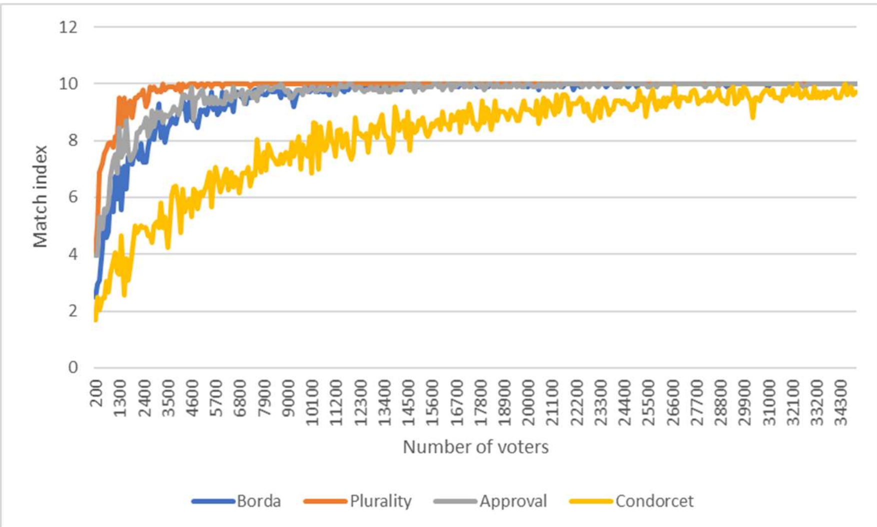
The results of Figure 15, show, for Borda, plurality, approval and Condorcet, the evolution of the match index as the number of voters increase. For all of them the match index increased with the number of voters. The speed of the rise, however, varied considerably among the methods. While plurality and approval very quickly converged, which means that the match index reached *10*, Borda and Condorcet took a bit longer to reach the full match. In fact, in spite of the clear trend towards convergence, none of these two methods converged until *35,000* voters.

It is important to notice that in this simulation, the curves of plurality and approval presented the same shapes. The reason is that only one choice is available between the 2 alternatives of the random subset.

The results of the simulations for *m=10* and *r=3* is shown in Figure 16. Again, we can regard the direct relation between the number of voters and the match index. For *r=3*, however, as expected from the first simulation, the methods converged more quickly than *r=2*. Note that, for plurality, for example, with *r=2*, the convergence happened around *15,000* voters, with very few oscillations after this point. For *r=3*, *6,000* voters were enough to converge.

In the case of Copeland, for *r=2*, the match index after *35,000* voters was around *6*. For *r=3*, it was very close to *10*.

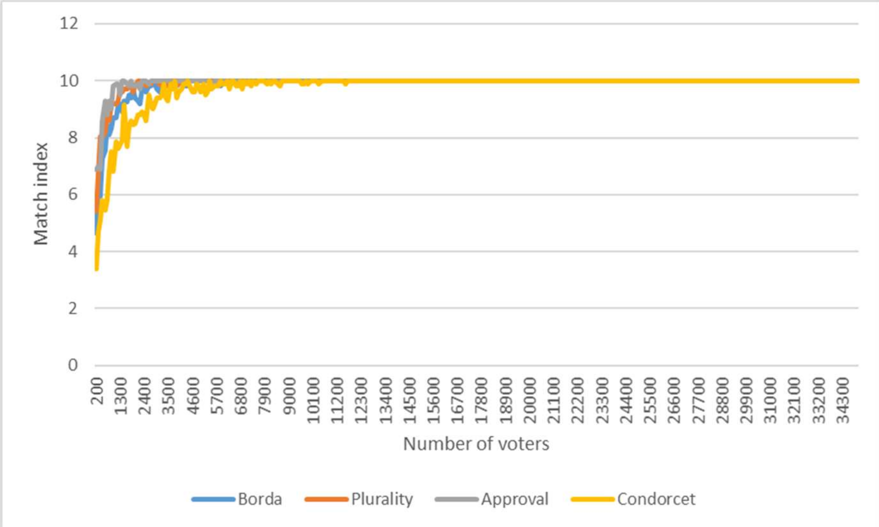
Figure 16 – Simulations for RSV m=10, r=3




Source: The author (2020).

In Figure 17 and Figure 18, the results of *m=10* for *5* and *7* random alternatives is shown.

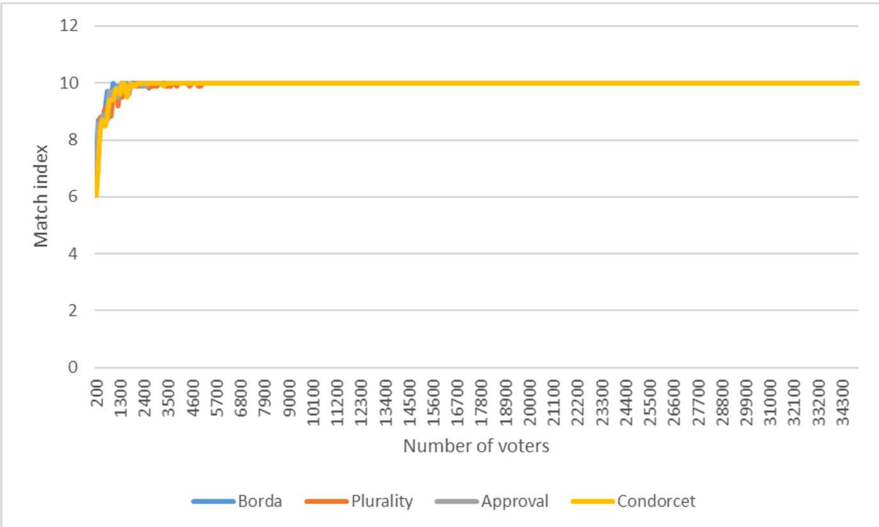
Figure 17 – Simulations for RSV m=10, r=5




Source: The author (2020).

In these simulations, all methods converged. For *r=5*, *12,000* voters were enough to make all methods reach the full match. In the case of *7* random alternatives, the full match for all four methods happened before *5,000* voters.

Figure 18 – Simulations for RSV m=10, r=7

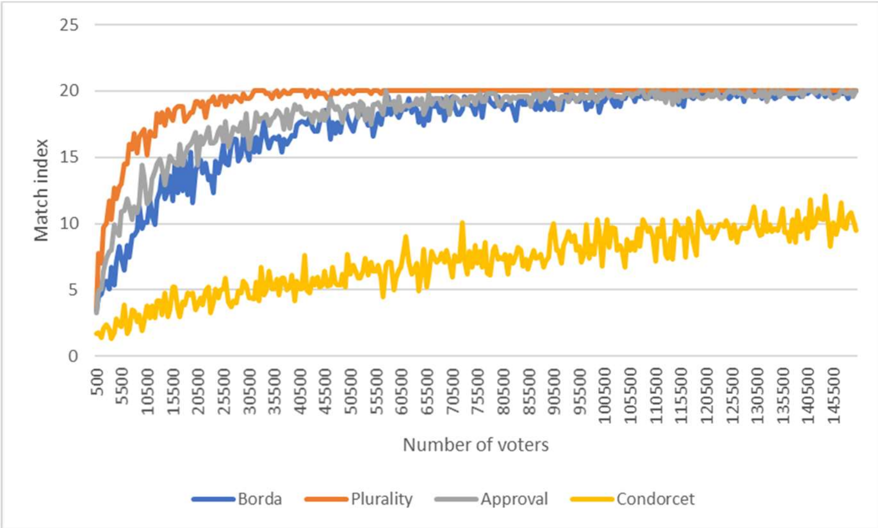



Source: The author (2020).

The next four figures present the results for simulations for *20* alternatives.

In Figure 19, *r=3*, the plurality method converged around *60,000* voters. Approval and Borda presented similar evolution and reached match index above *19* for *150,000* voters. For Copeland, the mean match index at the end of the simulation was *9.5*.

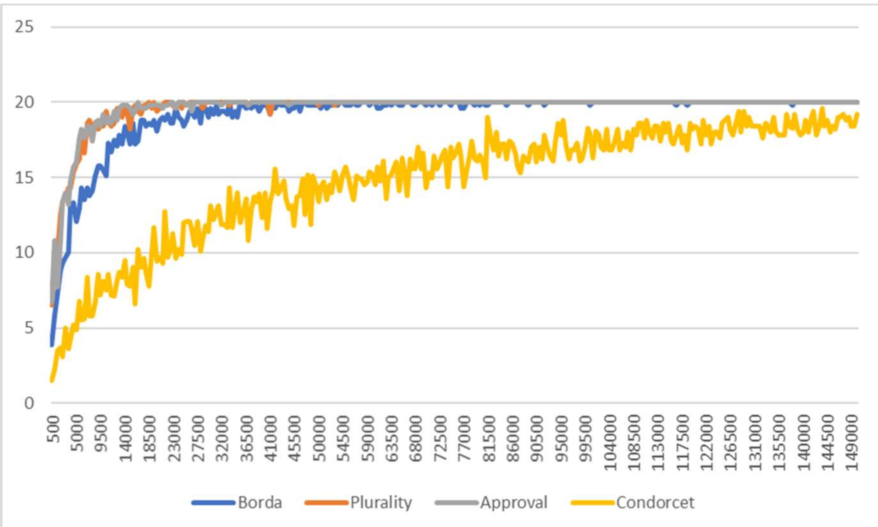
Figure 19 – Simulations for RSV m=20, r=3




Source: The author (2020).

For *5* random alternatives, Figure 20, plurality and approval converged around *50,000* voters and Borda close to *130,000*. Condorcet did not converge after *150,000*.

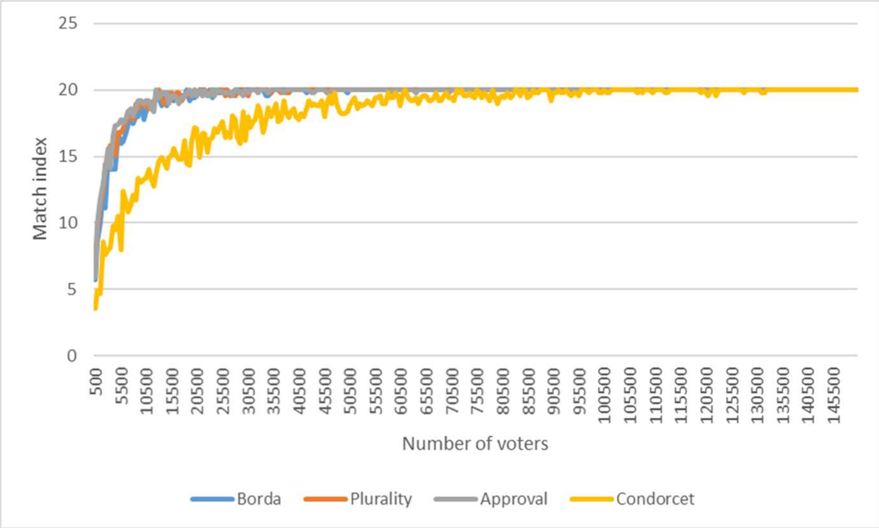
Figure 20 – Simulations for RSV m=20, r=5




Source: The author (2020).

For *r=7*, Figure 21, all four methods converged. Again, Condorcet was the last one with *n* around *130,000*. Borda, plurality and approval had similar behavior, having the full match before *50,000* voters.

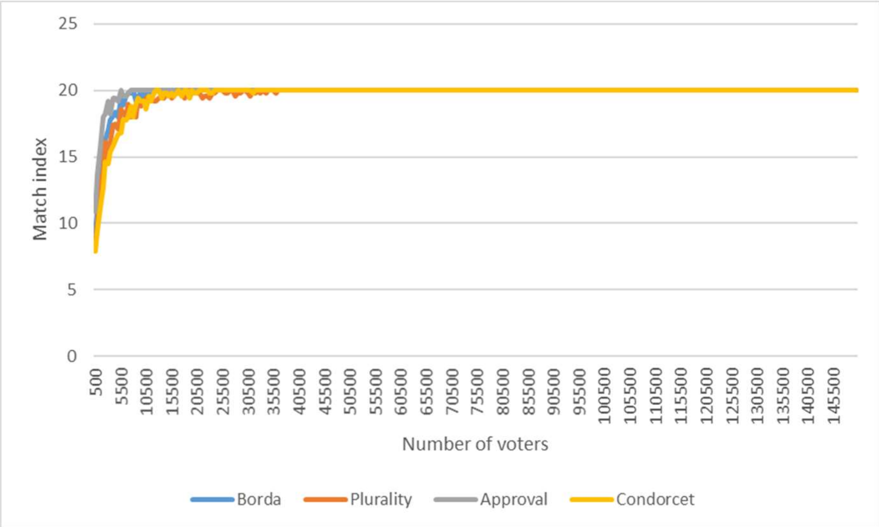
Figure 21 – Simulations for RSV m=20, r=7




Source: The author (2020).

The last graph, Figure 22, shows the results of the simulations with *m=20* and *r=11*. In this scenario, all methods converged very quickly with approval and Borda reaching the full match a bit earlier.

Figure 22 – Simulations for RSV m=20, r=11




Source: The author (2020).

In the next section, we discuss these results.

### 4.2.3 Analyses

The results indicate that, for Borda, as already expected from the results of the first experiment, and for plurality, approval and Condorcet (Copeland), the simulations converged to the expected outcome. In all simulations, in spite of the different methods, and the different inputs of the voters, we can observe that, as the number of voters increases, the match index reaches the number of alternatives, which means that all alternatives matched their expected positions in the final collective rankings.

The simulations also corroborate the conclusions of the first simulation in regard to the number of random alternatives and the number of voters. When more alternatives are in the random subset, more quickly the match index increases. Similarly, the more voters, the higher the match index.

It also validated the specific thresholds and convergence points, when only the Borda method was under investigation. This new study with *10* and *20* alternatives for Borda led to results equivalent to those found in simulation 1.

It is important to notice that, as the value of *r* increased, the convergence of the methods gradually evolved to the results of the traditional methods.

When analyzing, and comparing, the four voting schemes, the simulations of the traditional methods showed that Borda and Condorcet converged first, followed by approval and plurality. When studying the results of the RSV simulations, different behaviors regarding the convergence speed of each method was verified. The plurality method, although being the one in which less information from the voters is required about their preferences, had the best performance with RSV in scenarios where the random subsets were small. In these scenarios, Copeland presented the worst performance. When the number of random alternatives is big, approval and Borda performed better.

A first analysis on this intriguing result leads us to the following reasoning. One might expect that Borda and Condorcet, as in the simulations of the traditional methods, while requesting more information about the voters' preferences, would lead more quickly to the final collective expected ranking. And plurality, with less information about the preferences would result in slower convergence speeds. The results of the simulations with random subsets, however, contradict the reasoning of the traditional methods that the more information about the preferences, the quicker the convergence.

One initial hypothesis is that when Borda and Condorcet are used with Random-Subset Voting, many alternatives that are doomed to be in the end of the list comes frequently in the random subsets and gets points. For Borda, for example, when *10* alternatives are in the choice set, and $r=3$, the alternatives gets *3*, *2* and *1* points, and the remaining ones, that did not appear in the random subset, do not score. So, although having more information of the preferences of the voters over the three alternatives in the random subsets, RSB will make *7* alternatives consistently gets 0 points in the votes. And these seven alternatives will, in many cases, be preferred to those in the random subsets. Thus, when poorer alternatives score and mid or high options do not, the speed of the convergence is affected. This behavior, however, do not inhibit the convergence to proceed, though in slower speed.

For plurality, the convergence in the traditional method is the slower one, since the method request less information from the voters about their preferences. For RSP, however, when three random alternatives are presented to the voters, only one alternative gets *1* point and all the others get zero. So, it is less probable that some poor alternatives score. Those poor alternatives in the random subsets will get the same score as those out of the random subset: zero. As the number of random alternatives increases, however, more alternatives are selected for the random subsets and this effect is minimized. The same reasoning applies to approval.

This rationale, while explaining in some degree the apparent contradiction found in the simulations, might be investigated in more depth in future studies. Also, we let for future researches the analysis on the differences found in the convergence of Borda and Condorcet in the simulations. Since both methods required the full ranking of the voters, it was expected that their convergence speeds would be similar. The results, however, shows this is not the case. More investigation needs to be undergone in this issue.

From a more practical perspective, the results of the simulation of these four methods serves as a good starting point for the application of RSV in real polls. If, for example, *10* alternatives need to be evaluated by *5,000* citizens, the use of Random-Subset Plurality (RSP) with random-subsets of size *3* would lead to the same outcome of the use of the traditional plurality.

If, instead of using RSP, the policy makers decide to use RSB, keeping the same *10* alternatives and *3* random alternatives, there would be necessary a population of around *20,000* voters to guarantee the convergence.

In addition, we can observe, in the results, oscillations in the values of the match index as the number of voters increases. These variations are due to the intrinsic probabilistic structure of the simulations and reflect natural fluctuations that usually happens in real elections. Of course, the more runs executed for the configurations, the less variations would be observed. In this study, we have opted for *10* runs. This means that each point in the lines represent the mean of *10* runs of the same configuration. In spite of these variations, the trends are quite clear, indicating that the number of runs was good enough to evaluate tendencies of the simulations proposed.

## 4.3 FINAL CONSIDERATIONS

In this chapter, we have presented the structure and the results of two Monte Carlo simulations developed to test the RSV proposal.

In the first simulation, we have investigated how the Random-Subset Voting performed with the Borda method. The results of the simulation corroborated the theorem presented on chapter three and provided many relevant insights for RSV. The main outcome of simulation 1 was the relationships identified between the three main parameters set for the simulation: the number of voters ($n$), the number of alternatives ($m$) and the number of random alternatives ($r$).

The second simulation allowed us to evaluate how RSV performed with three more voting methods: plurality, approval and Condorcet (Copeland). The results indicated what we have intuitively expected: all methods converged. Also, the relationships between $n$, $m$ and $r$ were as in the first simulation.

Some results however were quite interesting. In particular, we have verified that plurality had the best performance with RSV for small values of $r$. Copeland presented the slowest convergence curve among the methods. For big random subsets, approval and Borda presented the best performance.

We have proposed some rationale in regard to these intriguing results, but future studies need to be conducted in order to investigate in more details how RSV behaves with other methods.

Another issue that might be examined in more depth in subsequent studies is the random generation of preferences used in both simulations. We have based our design in the UUP model proposed by Chamberlin and Featherston (1986), using the random variable $N_i(\mu_i, \sigma_i)$ to generate the random preferences. The parameters set for $\mu$ (*1* to *0*) and $\sigma$ (*0.5*) were defined after some exploratory tests of the model before the simulations. These tests consisted of a series of *ad hoc* simulations in which we have experimented many variations for the parameters. Future implementations can evaluate in a more systematic way how variations on these parameters affect the results of the simulations. Also, one could implement simulations that considers heterogeneous populations.

At the end, the simulations proved to be a useful technique to evaluate RSV.

However, as a software, the simulation presents a fundamental limitation: it cannot capture some aspects of the methods that would emerge only from its application in real social choice environments.

In the next chapter, we present two web experiments in which real voters used RSV to express their preferences and make social choices.

# 5 WEB EXPERIMENTS

In the last two chapters, we have introduced the Random-Subset Voting method, presented its main properties and proved that Random-Subset Borda and Borda lead to the same outcome when the population is large enough. Furthermore, we have described how we have implemented Monte Carlo simulation softwares to test RSV under different voting schemes and scenarios.

Even though some practical aspects of RSV could be investigated from the mathematical analysis and the simulations, other issues could only be studied through the examination of the behavior of the method in a real voting setting. In other words, we need empirical evidences of the RSV operation.

In this chapter, in order to evaluate how Random-Subset Voting performs in real scenarios, we implement two python-based web experiments in which respondents were required to participate in a poll. In both experiments, Random-Subset Borda was used.

The main research questions we intend to answer with the experiments are:

a) Which impact the reduction of the number of alternatives evaluated by each voter might have in the outcome of the election?
b) How could such this shrinkage of the choice-set be employed in real voting scenarios?

This chapter is divided into three sections. In the first, in which we present the experiment 1, we focus on evaluating how RSV would work in a web poll in which participants determined the juices they prefer the most. In the experiment, a fictitious fruit processing industry wanted to know the preferences of the participants in regard to juices.

In the second experiment, which is described in the section two, a more realistic case has been designed. The participants were students that voted for real projects that were about to be implemented in their school by the administration.

In the last section, we present the final considerations of the experiments.

## 5.1 EXPERIMENT 1

In this first experiment, the participants were invited to join a fictitious survey of an industry of fruit concentrate for juices. They had to indicate their preferences over different juices (made from concentrate) of typical fruits in Brazil. The main goal of the experiment was to empirically evaluate the consequences of the random reduction of the number of alternatives proposed by RSV.

This experiment was divided into three interconnected sub-experiments, in which the participants, using Borda and Random-Subset Borda, ranked different lists of juices. The general structure of the experiment and each sub-experiment, along with the list of the 15 fruits, will be described in details in the first subsection – experiment setup.

By proposing juices, we intended to let the voters express preferences that come mostly from their perception of taste. Even though other items like real candidates or projects could have been the object of this experiment, it was expected that the analysis of these alternatives would have required more cognitive effort from the respondents, decreasing their predisposition to follow all steps of the experiment.

When selecting the type of juice we would set for the experiment, we have opted for juices from concentrates. By choosing this kind of juice, instead of those made directly from the fruits, we intended to present to the participants alternatives less susceptible to variations, since they tend to be more homogeneous. Using natural fruits might have led to uncontrolled parameters like ripeness or preparation steps of the juices introducing variations in the tastes for the same fruit.

The participants were mainly students and employees from *Universidade Federal de Pernambuco* and *Instituto Federal de Pernambuco*, in Brazil. The invitation to participate was also sent by e-mail and through social networks.

We have organized this section as follows. In the first subsection we present the setup of the experiment, describing its general structure and the sub-experiments. In the second, we show the results. Finally, we analyze the results in the third subsection.

### 5.1.1 Experiment setup

In this subsection, we present the details of how we have set up the experiment.

We start by introducing in Table 13 the list of the *15* fruits used.

Table 13 – Fruits used in the experiment

| # | **Fruit** |
|---|---|
| 1 | Strawberry (*fragaria vesca*) |
| 2 | Passion fruit (*passiflora edulis*) |
| 3 | Mangaba (*hancornia speciosa*) |
| 4 | Acerola (*malpighia emarginata*) |
| 5 | Cashew fruit (*anacardium occidentale*) |
| 6 | Guava (*psidium guajava*) |
| 7 | Graviola (*annona muricata*) |
| 8 | Pitanga (*eugenia uniflora*) |
| 9 | Sugar-apple (*annona squamosa*) |
| 10 | Pineapple (*ananas comosus*) |
| 11 | Cajá (*spondias mombin*) |
| 12 | Grapes (*vitis sp.*) |
| 13 | Tamarind (*tamarindus indica*) |
| 14 | Mandarin orange (*citrus reticulata*) |
| 15 | Mango (*mangifera sp.*) |

Source: The author (2020).

All these fruits and concentrates are very common in Pernambuco, the Brazilian state in which we have conducted the experiment, and most of the people in the region have tried all of them.

The experiment is divided into three sub-experiments and one independent ranking step that are embedded into the ten steps presented in Figure 23.

Figure 23 – Steps of experiment 1


Source: The author (2020).

Steps 1, 2 and 10 (in blue) were used, respectively, to introduce the experiment to the participants, to fill up a quick form with their personal information and to present the final message. The remaining 7 steps (3 to 9) consisted of sequential lists of juices for ranking.

Sub-experiment 1 (in yellow) consists of steps 3 and 6, in which the voters rank 2 and 4 alternatives respectively. In steps 4 and 7 (sub-experiment 2, in green) the voters rank 3 and 5 juices. In grey, steps 5 and 8 constitute the third sub-experiment. In step 5, the voters rank 3 alternatives; in step 8, they rank 7.

Finally, in step 9, the participants were required to rank all 15 alternatives.

Table 14 details the sub-experiments, the steps and the subsets of fruits used.

Table 14 – Sub-experiments and fruits

| Set | Sub-experiment | Fruit |
|---|---|---|
| Set 1 | Sub-experiment 1 (yellow)<br>Steps: 2 random alternatives<br>Step 6: 4 alternatives | Strawberry<br>Passion fruit<br>Mangaba<br>Acerola |
| Set 2 | Sub-experiment 2 (green)<br>Steps 4: 3 random alternatives<br>Step 7: 5 alternatives | Cashew fruit<br>Guava<br>Graviola<br>Pitanga<br>Sugar-apple |
| Set 3 | Sub-experiment 3 (grey)<br>Steps 5: 3 random alternatives<br>Step 8: 7 alternatives | Pineapple<br>Cajá<br>Grape<br>Tamarind<br>Mandarin orange<br>Mango<br>Acerola |
| Set 4 | Independent ranking step (red)<br>Step 9: 15 alternatives | All fruits |

Source: The author (2020).

The fruits of set 1 were used in the sub-experiment 1 - steps 3 and 6. In the step 3, the participants ranked two random alternatives from the set 1; in the step 6, they ranked the entire set. The same idea was applied to set 2 (sub-experiment 2; steps 4 and 7), and to set 3 (sub-experiment 3; steps 5 and 8).

In order to avoid any bias in the ranking tasks, the alternatives were exhibited in random orders for each participant in each step.

Step 9 was not part of any of the sub-experiments and consisted of an independent ranking step used to provide some insights in regard to the time took by the participants to cast their votes and to the consistency of their preferences.

The size of the sets and of the random subsets for each sub-experiment (*2* and *4*, for sub-experiment 1; *3* and *5*, for sub-experiment 2; and *3* and *7*, for sub-experiment 3) has been chosen in a way that some combinations of $m$ and $r$ could be tested. Although no specific method was set in order to define these numbers, two reasoning have intuitively driven the process.

First, from the experience with the simulations, we knew that the relations between $m$, $n$ and $r$ play an important role in the expected convergence of Borda and RSB and it would have been important to experiment different values for these parameters. Also, since we could not anticipate how many voters would join the experiment, only $m$ and $r$ were effectively under our control.

The choice of relatively small values for $m$ (*4*, *5* and *7*) was mainly based on the upper bound of the simulations for $m=10$. In these simulations, the minimum number of voters for reaching the convergence was *2,000*, which would represent a very long, and perhaps unattainable, experiment to conduct with real voters in our context. Smaller values for $m$ would require fewer voters.

Second, it was important to set values for $m$ and $r$ that had different proportions between them. In sub-experiment 1, $r=2$ and $m=4$ lead to random subsets that are composed by *50%* of the alternatives in the original choice-set. For sub-experiment 2 ($r=3$, $m=5$) the proportion was *60%*. For the third sub-experiment ($r=3$, $m=7$) the proportion was about *42%*.

From a functional perspective, the ranking task was accomplished by a drag and drop tool in which the participants were asked to change the order of the juices in a list. The most preferred juices were positioned on the top of the list, the least preferred ones, at the bottom.

Figure 24 illustrates the step 4 of the experiment. In the screen, the participant could, using the mouse or his/her own fingers, change the positions of the juices according to his/her preferences. In this illustration, acerola was the most preferred juice and passion fruit (*maracujá*) was the least preferred. Samples of all the screens used in the experiment are presented in the Appendix A.

Figure 24 – Step 4 of the experiment; ranking 4 alternatives (set 1)


Source: The author (2020).

The results of the experiment are presented in the next section.

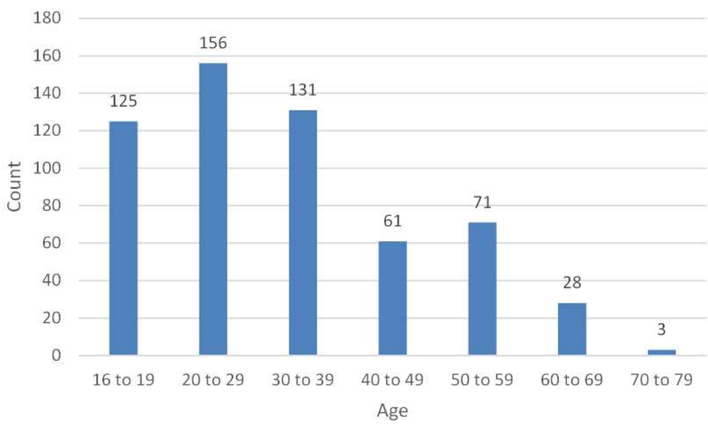
### 5.1.2 Results

This subsection, in which we describe the results of the experiment 1, is divided into three subsections, one for each sub-experiment.

Five hundred and seventy-five (*575*) respondents from *16* to *72* years old participated in the experiment.

Figure 25 shows the distribution of the ages of the participants.

Figure 25 – Distribution of ages among the participants




Source: The author (2020).

The majority of the participants, *156*, around *27%*, aged between *20* and *29*. More than *71%* were under *40*. Only three participants aged over *70*. Concerning their sexes, *241* (*41.9%*) were male and *334* (*58.1%*) were female.

Among the *575* participants, *439* (or *76.3%*) completed the experiment. The remaining *23.4%* did not follow all steps. Our analysis in this study only considered the *439* complete valid answers.

In the subsections that follow, we present the results of each sub-experiment, the analysis of the time spent in each step and an IIA analysis.

#### *5.1.2.1 Sub-experiment 1*

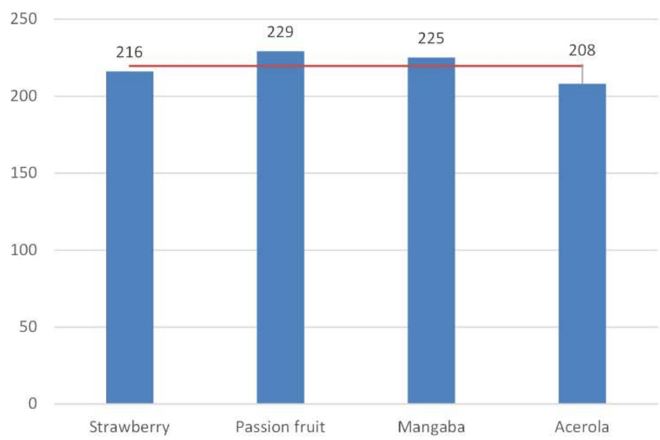
The sub-experiment 1 consisted of two steps. In the first, the participants had to rank two random alternatives from the set 1; in the second, they had to rank the entire set 1.

Before presenting the results of the experiment, we dedicate some attention to the analysis of the number of times each alternative were selected for the *439* random subsets.

In

Figure 26, the bars indicate the number of times each juice appeared in the random subsets; the red line highlights the expected number of appearances for the alternatives: *219.5*.

Figure 26 – Distribution of random alternatives – set 1




Source: The author (2020).

A quick hypothesis testing for the proportion suggests, as expected, that the random selection of fruits for the subsets followed a uniform distribution. The null hypothesis is that the proportion is equal to *0.25*; the alternative is that the proportion is not equal to *0.25*. We have implemented a two-tailed test. Table 15 presents the p-values.

Table 15 – P-values for experiment 1 – sub-experiment 1

| ***Juice*** | **Occurrences** | **Proportion** | **P-value** |
|---|---|---|---|
| Strawberry | *216* | *0.246* | *0.84* |
| Passion fruit | *229* | *0.261* | *0.60* |
| Mangaba | *225* | *0.254* | *0.76* |
| Acerola | *208* | *0.237* | *0.53* |

Source: The author (2020).

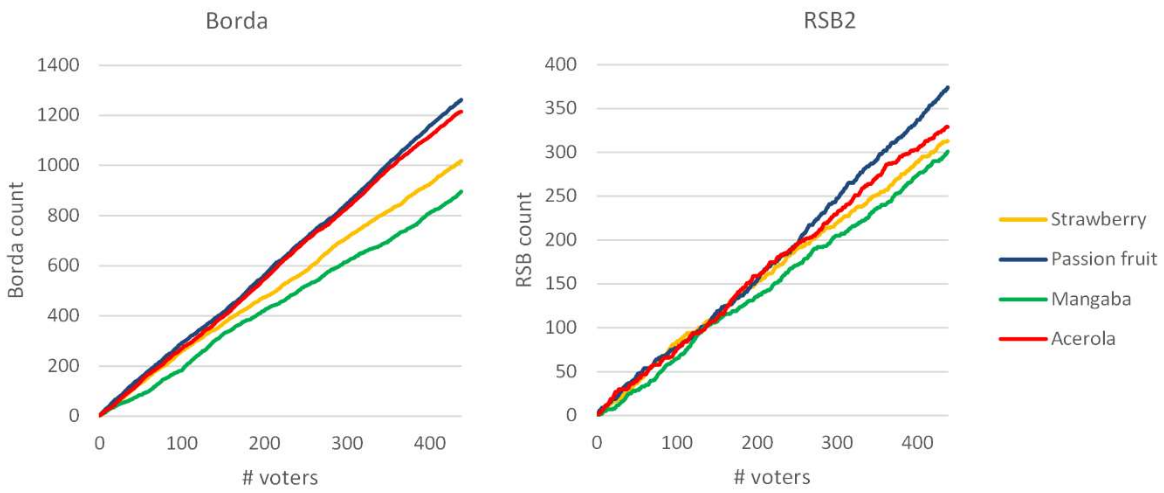
Since the p-values were consistently higher than the significance level of *0.05*, we could not reject the null hypothesis that the proportion was equal to *0.25*.

After this analysis of distribution of random alternatives, we present the results of the sub-experiment 1 (see Figure 27).

In this graph, we can identify the evolution of the Borda count for Borda and RSB with *r=2* for each alternative of set 1. These counts show that the final social rankings of the methods converged in this case. Table 16 and Table 17 present the final counts for both methods.

A deeper analysis of the RSB results show that after *243* votes the rankings converged.

Figure 27 – Borda vs. RSB2 – set 1




Source: The author (2020).

Table 16 – Ranking for set 1 – Borda count

| Pos. | Alternative | Borda count |
|---|---|---|
| 1 | Passion fruit | *1262* |
| 2 | Acerola | *1215* |
| 3 | Strawberry | *1018* |
| 4 | Mangaba | *895* |

Source: The author (2020).

Table 17 – Ranking for set 1 – RSB count

| Pos. | Alternative | RSB count |
|---|---|---|
| 1 | Passion fruit | *374* |
| 2 | Acerola | *329* |
| 3 | Strawberry | *313* |
| 4 | Mangaba | *301* |

Source: The author (2020).

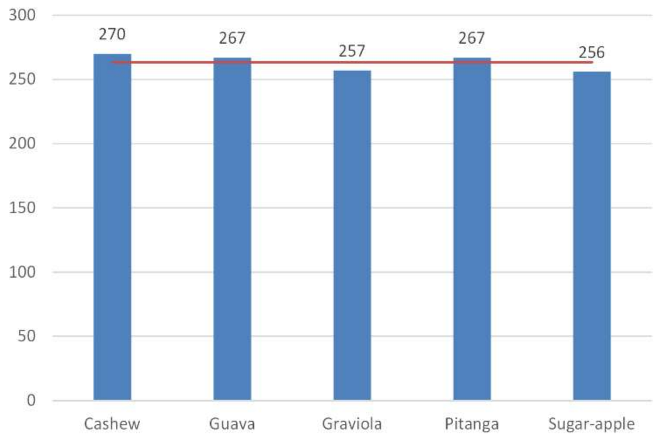
In the next subsection, we present the results of sub-experiment 2.

#### 5.1.2.2 *Sub-experiment 2*

The sub-experiment 2 followed the same reasoning of the sub-experiment 1, but focused on the five alternatives of set 2.

Figure 28 and Table 18 present the distribution of random alternatives and the p-values of the hypothesis testing for the proportion of occurrences of each juice in the random subsets.

Figure 28 – Distribution of random alternatives – set 2




Source: The author (2020).

Table 18 – P-values for experiment 1 – sub-experiment 2

| *Juice* | Occurrences | Proportion | P-value |
|---|---|---|---|
| Cashew | *270* | *0.205* | *0.793* |
| Guava | *267* | *0.203* | *0.886* |
| Graviola | *257* | *0.195* | *0.799* |
| Pitanga | *267* | *0.203* | *0.886* |
| Sugar-apple | *256* | *0.194* | *0.796* |

Source: The author (2020).

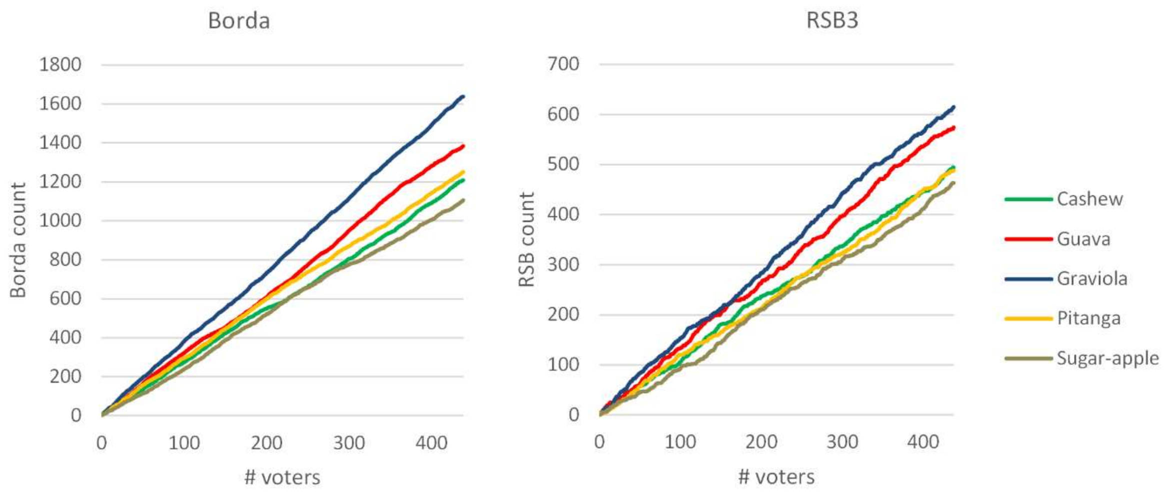
As well as in sub-experiment 1, the collected data suggest that the sampling process for generating the random subsets was unbiased.

The results of the sub-experiment in

Figure 29 shows the evolution of the counts for Borda and RSB with *r=3*. It indicates that the greater the number of voters, the closer the convergence of the results.

In this case, in contrast to the results of the sub-experiment 1, we did not reach a full match between the final rankings. However, the trends in the figure suggest that a few more participants would have led to the convergence of the rankings. Table 19 and Table 20 summarize the results of the second experiment.

Figure 29 – Borda vs. RSB3 – set 2




Source: The author (2020).

Table 19 – Ranking for set 2 – Borda count

| Pos. | Alternative | Borda count |
|---|---|---|
| 1 | Graviola | *1638* |
| 2 | Guava | *1384* |
| 3 | Pitanga | *1250* |
| 4 | Cashew | *1208* |
| 5 | Sugar-apple | *1105* |

Source: The author (2020).

Table 20 – Ranking for set 2 – RSB count

| Pos. | Alternative | RSB count |
|---|---|---|
| 1 | Graviola | *615* |
| 2 | Guava | *574* |
| 3 | Cashew | *494* |
| 4 | Pitanga | *488* |
| 5 | Sugar-apple | *463* |

Source: The author (2020).

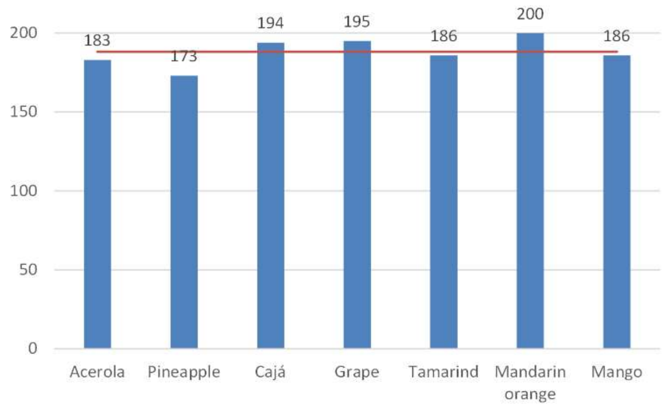
The results of sub-experiment 3 is presented in the following subsection.

#### *5.1.2.3 Sub-experiment 3*

The sub-experiment 3 followed the same reasoning of the sub-experiments 1 and 2, but focused in the seven juices of set 3.

Figure 30 and Table 21 present the distribution of random alternatives and the p-values of the hypothesis testing for the proportion of occurrences of each juice in the random subsets of sub-experiment 3.

Figure 30 – Distribution of random alternatives – set 3




Source: The author (2020).

Table 21 – P-values for experiment 1 – sub-experiment 3

| ***Juice*** | **Occurrences** | **Proportion** | **P-value** |
|---|---|---|---|
| Acerola | *183* | *0.139* | *0.815* |
| Pineapple | *173* | *0.131* | *0.492* |
| Cajá | *194* | *0.147* | *0.790* |
| Grape | *195* | *0.148* | *0.755* |
| Tamarind | *186* | *0.141* | *0.922* |
| Mandarin | *200* | *0.152* | *0.590* |
| Mango | *186* | *0.141* | *0.922* |

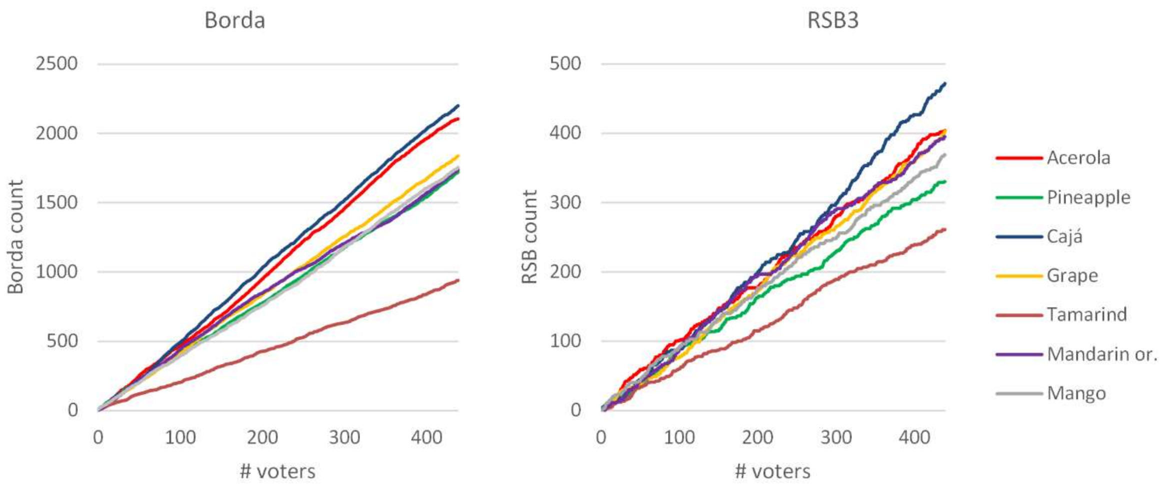
Source: The author (2020).

Even though the number of pineapples in the random subsets was relatively small (*173*), the data of set 3 also indicates that the random sampling method was unbiased.

The results of the sub-experiment are presented in Figure 31, in Table 22 and Table 23.

In spite of the small difference in the final rankings, the overall convergence of the results does not appear so clear as in the sub-experiments 1 and 2. In fact, we have identified only one change in the position of the alternatives in the rankings. While mango was ahead mandarin in Borda, the opposite was found in RSB3. The evolution of the counts and the final result, however, suggest that adding more participants would have made RSB with *r=3* closer to Borda in this scenario.

Figure 31 – Borda vs. RSB3 – set 3




Source: The author (2020).

Table 22 – Ranking for set 3 – Borda count

| Pos. | Alternative | Borda count |
|---|---|---|
| 1 | Cajá | *2199* |
| 2 | Acerola | *2105* |
| 3 | Grape | *1837* |
| 4 | Mango | *1754* |
| 5 | Mandarin Or. | *1736* |
| 6 | Pineapple | *1720* |
| 7 | Tamarind | *941* |

Table 23 – Ranking for set 3 – RSB count

| Pos. | Alternative | RSB count |
|---|---|---|
| 1 | Cajá | *472* |
| 2 | Acerola | *404* |
| 3 | Grape | *403* |
| 4 | Mandarin Or. | *395* |
| 5 | Mango | *369* |
| 6 | Pineapple | *330* |
| 7 | Tamarind | *261* |

Source: The author (2020).

In the next section, we analyze the results of the experiment.

### 5.1.3 Analyses

In this subsection, we analyze the results of experiment 1. It is divided into three main subsections. In the first, we present the analyzes of the convergence of RSB and Borda in the three sub-experiments conducted. In the second subsection, we introduce some considerations in regard to the time took by the participants to cast their votes. Finally, in the third subsection, we study the consistency of the votes.

#### *5.1.3.1 Convergence analysis*

In this experiment, the respondents were invited to indicate their preferences of juices in several lists that were presented to them in a very specific order. In total, *575* people participated and the main goal was evaluating how the Borda method worked under the Random-Subset framework.

The results suggest that Borda and RSB converge when the number of voters is big enough, corroborating the findings of the mathematical analysis and the simulations.

In the first sub-experiment, in which *2* random alternatives were selected from the set of *4*, the convergence was fully verified; both Borda and RSB have led to the same outcome. In sub-experiments 2 and 3, the full match was not verified. However, the trends in the graphs indicate that a bigger set of voters would have improved the results.

Nonetheless, a more thorough analysis of this data has brought an important question: what impact the variations of the number of occurrences of the alternative in the random subsets have had in the final results? In this context, it is important to emphasize that one of the main premises of RSV is the fairness of the distribution of alternatives in the random subsets.

Even though the statistical analysis of the distributions of alternatives suggests that no bias was present and that the variations on the occurrences of each alternative were due to the natural oscillations of the random selection, some alternatives appeared much more frequently than others. In sub-experiment 1, for example, while passion-fruit appeared *229* times in the subsets, acerola was only in *208* subsets. For experiments 2 and 3, similar situations were verified.

This issue has motivated the proposition of the weighted count for the RSB. It is calculated by simply dividing that absolute count of RSB for each juice by its the number of occurrences.

In Table 24, Table 25 and Table 26 we present the weighted counts for all three sub-experiments.

Table 24 – Weighted count for sub-experiment 1

| **Position** | **Juice** | **Borda count** | **Occurrences** | **RSB count** | **RSB weighted count** |
|---|---|---|---|---|---|
| 1 | Passion fruit | *1262* | *229* | *374* | *1.633* |
| 2 | Acerola | *1215* | *208* | *329* | *1.582* |
| 3 | Strawberry | *1018* | *216* | *313* | *1.449* |
| 4 | Mangaba | *895* | *225* | *301* | *1.338* |

Source: The author (2020).

Table 25 – Weighted count for sub-experiment 2

| **Position** | **Juice** | **Borda count** | **Occurrences** | **RSB count** | **RSB weighted count** |
|---|---|---|---|---|---|
| 1 | Graviola | *1638* | *257* | *615* | *2.393* |
| 2 | Guava | *1384* | *267* | *574* | *2.150* |
| 3 | Pitanga | *1250* | *267* | *488* | *1.828* |
| 4 | Cashew | *1208* | *270* | *494* | *1.830* |
| 5 | Sugar-apple | *1105* | *256* | *463* | *1.809* |

Source: The author (2020).

Table 26 – Weighted count for sub-experiment 3

| **Position** | **Juice** | **Borda count** | **Occurrences** | **RSB count** | **RSB weighted count** |
|---|---|---|---|---|---|
| 1 | Cajá | *2199* | *194* | *472* | *2.433* |
| 2 | Acerola | *2105* | *183* | *404* | *2.208* |
| 3 | Grape | *1837* | *195* | *403* | *2.067* |
| 4 | Mango | *1754* | *186* | *369* | *1.984* |
| 5 | Mandarin orange | *1736* | *200* | *395* | *1.975* |
| 6 | Pineapple | *1720* | *173* | *330* | *1.908* |
| 7 | Tamarind | *941* | *186* | *261* | *1.403* |

Source: The author (2020).

In sub-experiments 1 and 2, the use of the weighted count did not lead to any change in the final rankings of the RSB count. For sub-experiment 3, however, the number of occurrences of mango and mandarin orange in the random subsets differed considerably, leading to weighted counts that put these two juices in different positions in the ranking compared to the ranking of the RSB count.

While in the RSB ranking mandarin orange was in 4th and mango in 5th, in the RSB weighted ranking they inverted their positions. In this case, the RSB weighted ranking matched the Borda ranking. In the table, we have highlighted in yellow the RSB count and the RSB weighted count for mango and mandarin.

In addition to the convergence analysis, which was the focus of the experiment, we have conducted some analysis concerning the time taken by the participants while casting their votes and the consistency of the votes. In the following subsections we present both analyses.

#### *5.1.3.2 Time analysis*

In order to have a first image of the effort required by the participants in the steps, we have, along with the experiments, computed the duration of each ranking task. Table 27 summarizes the average duration for each set size.

Table 27 – Average duration of ordering elements

| **Set size** | **2** | **3** | **4** | **5** | **7** | **15** |
|---|---|---|---|---|---|---|
| Avg. duration (seconds) | *50.92* | *14.31* | *17.45* | *20.21* | *34.42* | *80.49* |

Source: The author (2020).

Except from the set with two elements, we could verify that the durations are consistent with what one could intuitively expect: the more alternatives, the more time the participants would take to rank them. Note that rankings for *3*, *4* and *5* juices took on average *14.31*, *17.45* and *20.21* seconds, respectively. For seven alternatives, the average duration increased to *34.42* seconds. In the last step, ranking *15* elements, the participants took on average *80.49* seconds to accomplish the task.

The relatively high average duration of ranking two elements was due to the fact that this step was the first ordering task of the participants. Since it was the first time they used the drag and drop component, a learning time is included in the *50.92* seconds they took on average to rank the first set[7].

---

[7] In case of reproducing this experiment, we recommend including a dummy initial step before step 3 to let the voters learn to use the tool. In our experiment, using the step 3 as a learning step did not had impact in the choice that the participants have made. However, it had impact in the time of the step three.

This time analysis on this experiment served as a basic overview of how long can take the vote casting task. It provides a first impression of the complexity of the task itself. More experiments on this issue should be implemented in the future.

In the next subsection, we introduce the consistency analysis of the experiment 1.

#### *5.1.3.3 Consistency analysis*

The consistency analysis was designed to evaluate to what extent the voters were consistent in their decisions. The consistency, in this context, was measured by the relative position of the alternatives between parts 1 and 2 of the sub-experiments.

For each respondent, we have compared the order of the elements in the subsets to the order of the same elements in the full sets.

For example, a voter in part 1 of sub-experiment 1 was required to rank alternatives strawberry and passion fruit. He/she ranked [passion fruit, strawberry]. In part 2, he/she was required to rank the whole set 1 (strawberry, passion fruit, mangaba and acerola) and the result was [acerola, strawberry, mangaba and passion fruit]. Note that when more alternatives were presented to the voter, the relative positions of alternatives passion fruit and strawberry changed. This means that the voter was not consistent.

This consistency analysis measured to what extent the participants acted rationally in regard to the independence of irrelevant alternatives property. In order words, we intended to evaluate whether adding or removing alternatives from the choice set would lead to different relative positions of the alternatives in the rankings (RADNER; MARSCHAK, 1954) and (SEN, 1971).

Among all subsets and sets in the ranking steps, nine possible combinations of subset-set were attained. The combinations and the results of the IIA analysis are presented in Table 28.

Table 28 – IIA analysis of the experiment data

| # | Subset - Set | Steps | IIA | Non-IIA | % IIA | % Non-IIA |
|---|---|---|---|---|---|---|
| 1 | RS Set 1 - Set 1 | 3 - 6 | *355* | *84* | *80.9%* | *19.1%* |
| 2 | RS Set 2 - Set 2 | 4 - 7 | *287* | *152* | *65.4%* | *34.6%* |
| 3 | RS Set 3 - Set 3 | 5 - 8 | *301* | *138* | *68.6%* | *31.4%* |
| 4 | RS Set 1 - Set 4 | 3 - 9 | *351* | *88* | *80.0%* | *20.0%* |
| 5 | RS Set 2 - Set 4 | 4 - 9 | *254* | *185* | *57.9%* | *42.1%* |
| 6 | RS Set 3 - Set 4 | 5 - 9 | *268* | *171* | *61.0%* | *39.0%* |
| 7 | Set 1 - Set 4 | 6 - 9 | *199* | *240* | *45.3%* | *54.7%* |
| 8 | Set 2 - Set 4 | 7 - 9 | *118* | *321* | *26.9%* | *73.1%* |
| 9 | Set 3 - Set 4 | 8 - 9 | *59* | *380* | *13.4%* | *86.6%* |

Source: The author (2020).

Each line in the table presents the quantity and the percentage of respondents who declared consistent (IIA) and inconsistent (non-IIA) preferences.

In the case of the sub-experiment 1 (RS Set 1 - Set 1), which were linked to the steps 3 and 6, from the *439* respondents, *355* (or *80.9 %*) declared preferences in step 3 that were consistent with their preferences in step 6; *84* participants (*19.1%*) declared inconsistent preferences in this situation.

For the sub-experiments 2 and 3, second and third lines in Table 28, *65.4%* and *68.6%*, respectively, declared consistent preferences.

The remaining six subset-set combinations compared the sets and subsets of experiments 1, 2 and 3 to the set 4 (with *15* alternatives). The lines 4 to 9 in the Table 28 present the consistency rates in these cases. It could be verified that the percentage of consistent preferences drop from *80.0%* (when comparing the random subset of set 1 to set 4) to *13.4%* (when comparing set 3 to set 4). In this last case, only *59* of the *439* participants declared preferences in the set 3 (with *7* juices) that were consistent the preferences in set 4 (with *15* juices).

This brief consistency analysis corroborates the studies on IIA suggesting that people often fail to act rationally. More studies on how consistently the voters act under RSV can be undergone in the future.

In the next section, we present experiment 2.

## 5.2 EXPERIMENT 2

In the second experiment, we have implemented a voting scenario in which students had the opportunity to participate in the decisions of their campus.

The research questions we tackled with this experiment were the same we have developed on experiment 1. We focus on studying whether the voters have to express their preferences over the entire choice-set and on investigating the impact of reducing the number of alternatives in the results of an election.

However, instead of using juices, we have chosen real projects as the alternatives. This approach allowed us to have a better feeling on how RSV might behave in more realistic voting scenarios.

Furthermore, we have improved the methodology employed. In the first experiment, each voter participated in both Borda and RSB. For experiment 2, the voters were randomly separated into the control and the test groups.

The experiment was conducted among students of the Instituto Federal de Pernambuco (IFPE), Ipojuca campus, in Pernambuco, Brazil. In total, *539* participated.

Before going into de the details of the experiment, we need to bring some important remarks concerning the alternatives chosen for the experiment. The alternatives, in this experiment, were real projects of the campus.

Altogether, twelve projects were selected to be ranked by the students. The selection of these projects was made after a thorough discussion with the campus administration, being in consonance with the real demands of the community. Also, the implementation of this experiment is part of the new democratic models that are being set in the campus with the goal of achieving more community participation in the decisions of the administration. As in the first experiment, Borda and Random-Subset Borda (RSB) were used. Again, the experiment was divided into three independent sub-experiments.

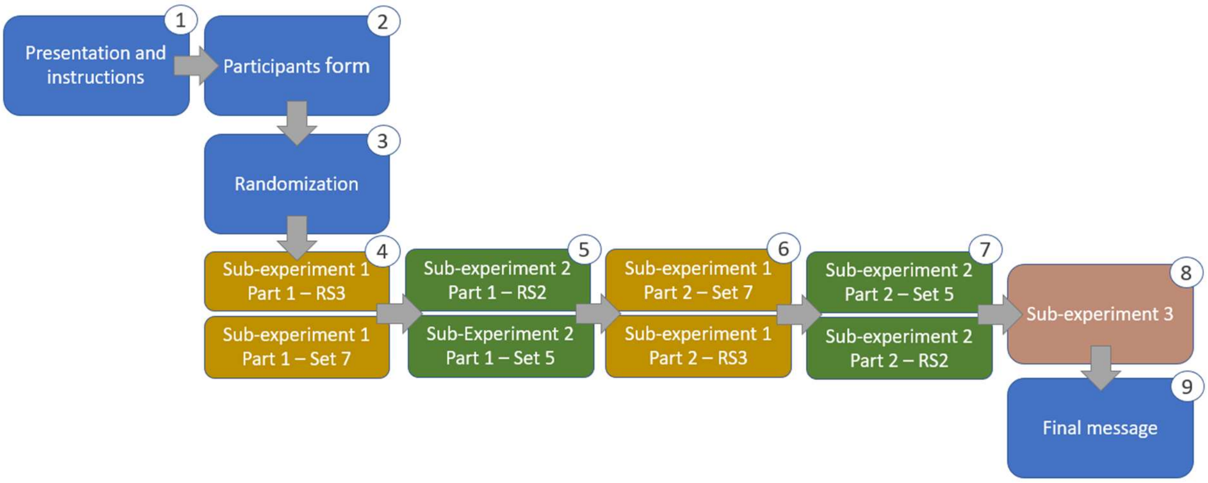
This section is organized as follows. In the first subsection we detail the experiment setup. Then, in the second, we describe the results. The analyses of the experiment are presented in the third subsection.

### 5.2.1 Experiment setup

In this subsection, we describe how we have structured the experiment 2.

As in the first one, this experiment is composed by three interconnected sub-experiments (named 1, 2 and 3). The nine steps of the experiment and the list of alternatives that were in each sub-experiment are exhibited in Figure 32 and Table 29.

Figure 32 – Steps of experiment 2




Source: The author (2020).

Table 29 – Alternatives of experiment 2

| # | **Alternative** | **Sub-experiments** |
|---|---|---|
| 1 | Implement a project, articulated with the Ipojuca municipality and the police office, in order to ensure the security of the campus (bus stop and surroundings) | 1, 3 |
| 2 | Refurbish the campus toilettes | 1, 3 |
| 3 | Upgrade internet availability and speed in the campus | 1, 3 |
| 4 | Plant trees and remodel the common areas of the campus | 1, 3 |
| 5 | Install a canteen and/or foodtruck in the campus | 1, 3 |
| 6 | Fix and replace the students' desks | 1, 3 |
| 7 | Refurbish and enlarge the covered parking area | 1, 3 |
| 8 | Promote partnerships with the municipality of Ipojuca and of the surroundings in order implement better transport of the students to the campus | 2, 3 |
| 9 | Build a sports court | 2, 3 |
| 10 | Celebrate and deepen partnerships with companies to promote internships' programs | 2, 3 |
| 11 | Refurbish the campus concierge | 2, 3 |
| 12 | Refurbish the classrooms | 2, 3 |

Source: The author (2020).

As shown in Table 29, alternatives 1 to 7 were reserved for sub-experiment 1 and alternatives 8 to 12, for sub-experiment 2. In sub-experiment 3, all twelve alternatives were in the choice set.

In the steps 1 and 2, the instructions were presented and the participants filled a basic form with the following descriptive data: sex, age, course, hometown, semester and course time.

In the third step, randomization, the participants were randomly split into the control and the test groups for sub-experiments 1 and 2.

In sub-experiment 1, which consisted of steps 4 and 6, seven projects were evaluated by the students. In step 4, the participants in the test group were required to rank random subsets of three alternatives (RSB3). Those in the control group, ranked the full set of 7. In step 6, which can be regarded as a complementary consistency vote for experiment 1, those in the test group ranked the set of 7 and those in the in the control group ranked the random subset of *3*.

Sub-experiment 2 (steps 5 and 7) had the same structure of the first one, except that a set of five alternatives was selected and the size of the random subset was *2*. In step 5, the participants in the test group ranked *2* random alternatives. In step 7, those in the test group ranked the full set of *5*. The participants in the control group ranked *5* alternatives in part 1 and *2* random alternatives in the part 2. The random selection of participants for the groups in sub-experiments 1 and 2 were independent events.

It is important to highlight that steps 4 and 5 are the main ones in sub-experiments 1 and 2. In these steps, the results of the test and the control groups were verified. Steps 6 and 7 were used to test the consistency of the votes.

As in experiment 1, no specific method was used to determine the sizes of the choice-sets. We have based our decision on the empirical evidences of the simulations and on the different proportions on *r* and *m*.

In the third sub-experiment, step 8, the participants were asked to rank all twelve alternatives. This experiment did not lead to any specific convergence or consistency analysis. Since sub-experiments 1 and 2 used disjoint sets of alternatives, it was not possible to get a full view of the preference of the students over the *12* alternatives proposed by the administration of the campus. We have then implemented this last experiment with all *12* alternatives in order to give to the campus administration a full view of the results. In addition, we have used this last step in the time analysis, which will be presented in the end of this section.

Another aspect of the experiment setup that needs to described is the sampling process used, as it directly impacts the representativeness of the sample.

At the time we conducted the experiment, about *700* registered students were regularly taking courses in the campus. Since our goal was to effectively run an election in the campus, one premise we decided to set was to allow every student to join.

Therefore, we have visited all classes in the campus, presented the alternatives to the students and invited them to participate. The voting was voluntary.

The experiment was implemented by using a web application designed in Python and was available to the students through their smartphones. Appendix B presents the screens of the experiment.

In the next subsection, we present the results of the experiment.

### 5.2.2 Results

In order to better describe the results, we have divided the subsection into five topics. In the first, we present the basic descriptive statistics of the experiment, emphasizing the main attributes of the sample. In topic two, we present the criteria used for a vote to be considered valid, followed by the distribution of valid votes. Finally, in the third, fourth and fifth topics we present, respectively, the results of sub-experiments 1, 2 and 3.

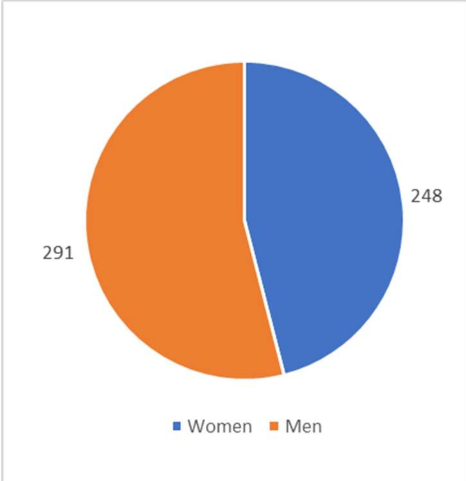
#### *5.2.2.1 Descriptive statistics*

Our database registered *539* participants after the two weeks in which the experiment was conducted. Considering the population of about *700* students that were regularly taking classes at the moment the election took place, we have reached around *77%* of participation.

For each student, we have gathered the following data: sex, age, hometown, course and course time.

The distribution of the sex of the students is presented in Figure 33. From the *539* students that participated, *291* (*53.99%*) were men and *248* (*46.01%*) were women.

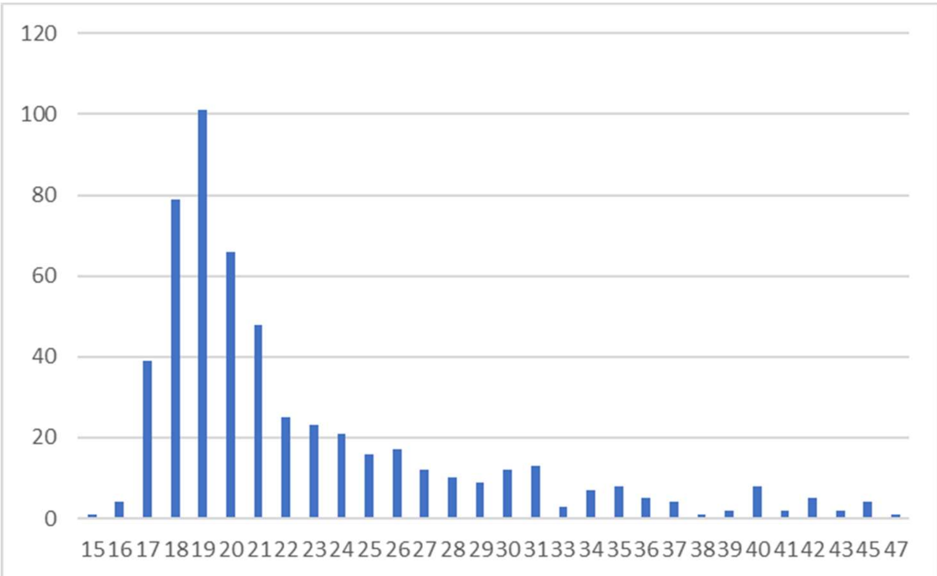
Figure 33 – Sex of the participants




Source: The author (2020).

The average age of the sample was *22.87* years old and the distribution of ages is presented in Figure 34.

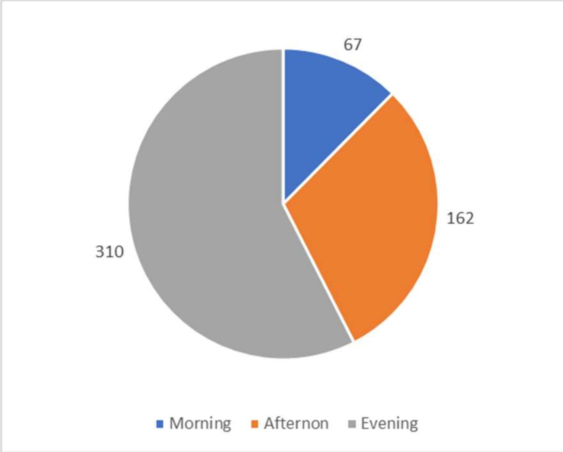
Figure 34 – The histogram of the ages of the participants




Source: The author (2020).

Our data has also shown that most of the participants (*310*, or *57.51%*) study in the evening and only a few in the morning (*67*, or *12.43%*). The students in the afternoon counted *162* (*30.06%*). The distribution of the course time is exhibited in Figure 35.

Figure 35 – Distribution of students in the morning, afternoon and evening




Source: The author (2020).

In Table 30, we present the hometowns of the student.

Table 30 – Cities of the participants

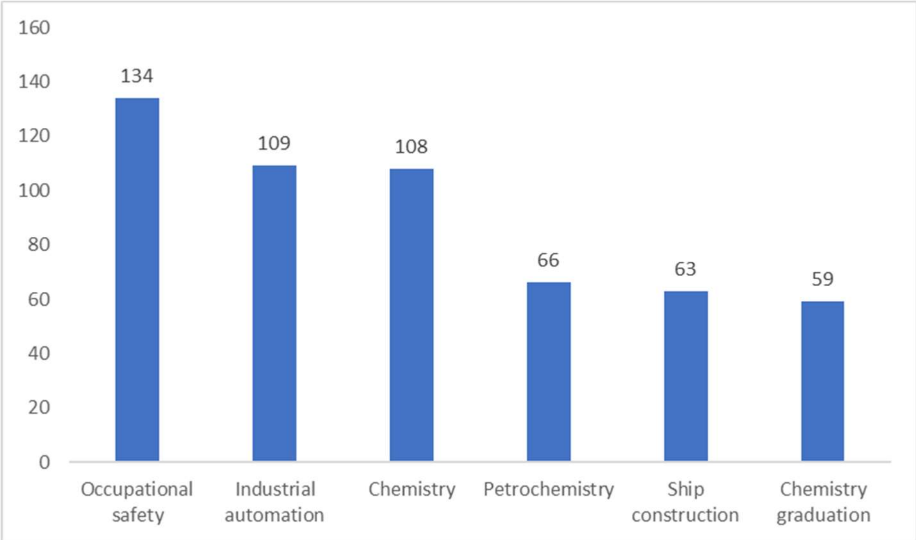
| City | # | % |
|---|---|---|
| Cabo de Santo Agostinho | *248* | *46.21%* |
| Ipojuca | *207* | *38.40%* |
| Sirinhaém | *43* | *7.98%* |
| Jaboatão dos Guararapes | *13* | *2.41%* |
| Rio Formoso | *13* | *2.41%* |
| Recife | *6* | *1.11%* |
| Escada | *5* | *0.93%* |
| Olinda | *2* | *0.37%* |
| Other | *2* | *0.37%* |

Source: The author (2020).

The campus is located in Ipojuca, a city 40 kilometers far from Recife, the capital of the state of Pernambuco. The students, however, are not only from Ipojuca. Most part of them (*46.21%*) are, in fact, from Cabo de Santo Agostinho, a city 10 km far from Ipojuca. In second place, comes Ipojuca itself with *207* students, or *38.40%*. The other cities together represent only *15.58%*, or *84* students.

The courses of the students are presented in Figure 36. From the *539* students of the campus, *134* study Occupational Safety, *109* study Industrial Automation and *108*, Chemistry. Petrochemistry, Ship Construction and the Chemistry graduation counted, respectively, *66*, *63* and *59*.

Figure 36 – Courses of the participants



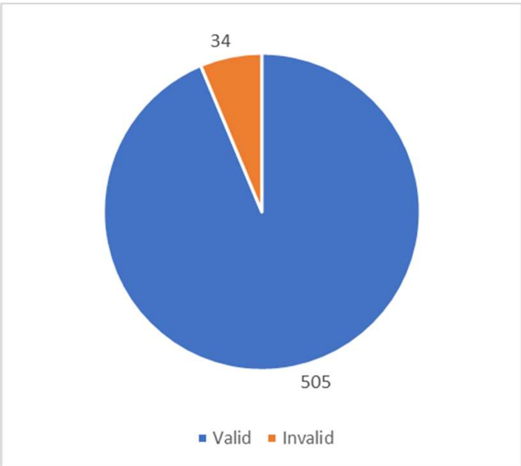

Source: The author (2020).

### *5.2.2.2 Validation of votes*

A vote was considered to be valid if all steps of the experiments were completed. A participant that started the voting process and stopped it at any step but the final message (step 9) did not have his/her vote counted in the poll.

Figure 37 depicts the number of valid and invalid votes. From the *539* cast votes, *505* (*93.69%*) were considered valid and *34* (*6.31%*) were invalid.

Figure 37 – Distribution of valid and invalid votes




Source: The author (2020).

In the following subsections we present the results of sub-experiments 1, 2 and 3.

### *5.2.2.3 Sub-experiment 1*

Sub-experiment 1 consisted of steps 4 and 6. In step 4, *248* participants were randomly selected to the test group, which ranked three random alternatives from the set of seven of sub-experiment 1. *257* students were randomly selected to the control group that ranked the entire set of seven options. In step 6, the students in the control group ranked three, and those in the test group ranked seven alternatives.

In this subsection, we focus on presenting the convergence results of step 4. The results of step 6, along with the study of the randomization processes, will be presented in the analyses section.

In Table 31 and Table 32, we exhibit the final rankings and the Borda count for the control and the test groups of experiment 1.

Table 31 – Result of sub-experiment 1 – control group

| **Pos.** | **Alternative** | **Borda Count** |
|---|---|---|
| 1 | Alt. 1 | *1540* |
| 2 | Alt. 5 | *1371* |
| 3 | Alt. 3 | *1032* |
| 4 | Alt. 4 | *1030* |
| 5 | Alt. 2 | *822* |
| 6 | Alt. 6 | *763* |
| 7 | Alt. 7 | *638* |

Table 32 – Result of sub-experiment 1 – test group

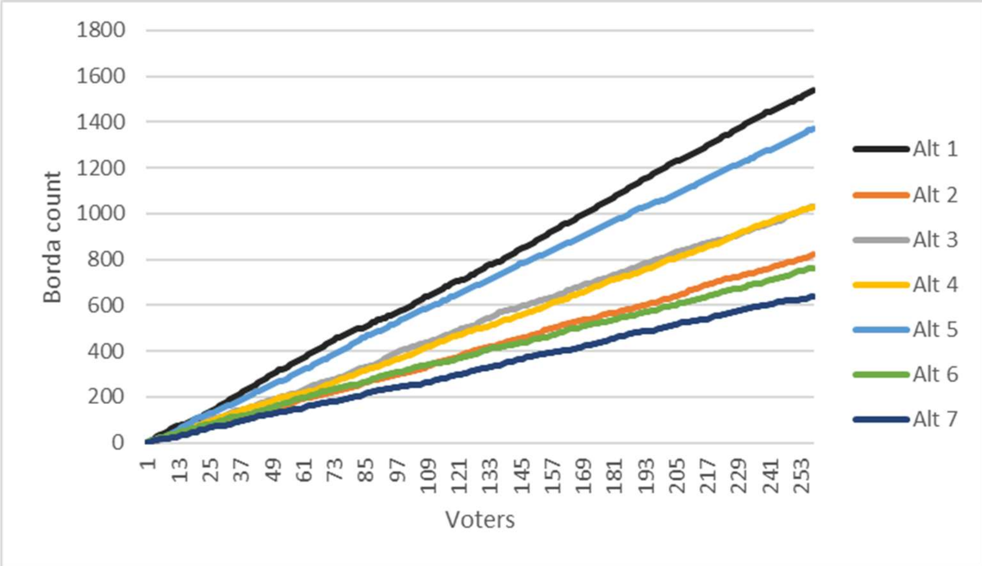
| **Pos.** | **Alternative** | **RSB Count** |
|---|---|---|
| 1 | Alt. 1 | *286* |
| 2 | Alt. 5 | *266* |
| 3 | Alt. 3 | *242* |
| 4 | Alt. 4 | *201* |
| 5 | Alt. 2 | *172* |
| 6 | Alt. 6 | *168* |
| 7 | Alt. 7 | *153* |

Source: The author (2020).

Note that both the test and the control groups have led to the same outcome: alternative 1 in the first place, followed by 5, 3, 4, 2, 6 and 7.

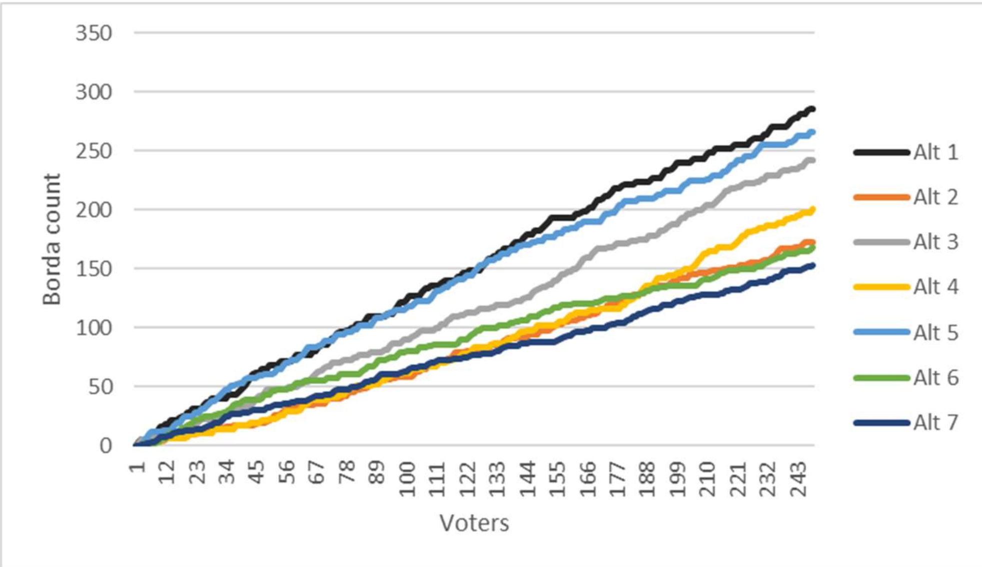
In order to have a better perspective of the results, we present, in Figure 38 and Figure 39, the evolution of the Borda count for each group of the sub-experiment 1.

Figure 38 – Evolution of the Borda count for the control group of sub-experiment 1




Source: The author (2020).

Figure 39 – Evolution of the Borda count for the test group of sub-experiment 1




Source: The author (2020).

The lines in the graphs show, for example, how alternative 1 has consistently led the election in both test and control groups. On the other hand, alternative 7 was progressively driven to the last position.

In the next subsection, we present the results of sub-experiment 2.

#### *5.2.2.4 Sub-experiment 2*

Sub-experiment 2 followed that same structure of sub-experiment 1, except that the sizes of the choice-set and the random subset were, respectively, five and two.

It consisted of steps 5 and 7. In the first step, *262* participants were randomly selected to the test group, which ranked two random alternatives from the set of five. *243* students were randomly picked to the control group that ranked the entire set of five options. In step 7, the students in the control group ranked two, and those in the test group ranked five alternatives.

In this subsection, we focus on presenting the convergence results of step 5. The results of step 7 and the study of the randomization procedures implemented will be described in the analyses section.

Table 33 and Table 34 present the Borda count of sub-experiment 2 for the control and the test groups.

Figure 40 and Figure 41 show the evolution of the Borda count in both groups.

Table 33 – Result of sub-experiment 2 – control group

| Pos. | Alternative | Count |
|---|---|---|
| 1 | Alt. 10 | *1093* |
| 2 | Alt. 8 | *849* |
| 3 | Alt. 9 | *603* |
| 4 | Alt. 11 | *566* |
| 5 | Alt. 12 | *534* |

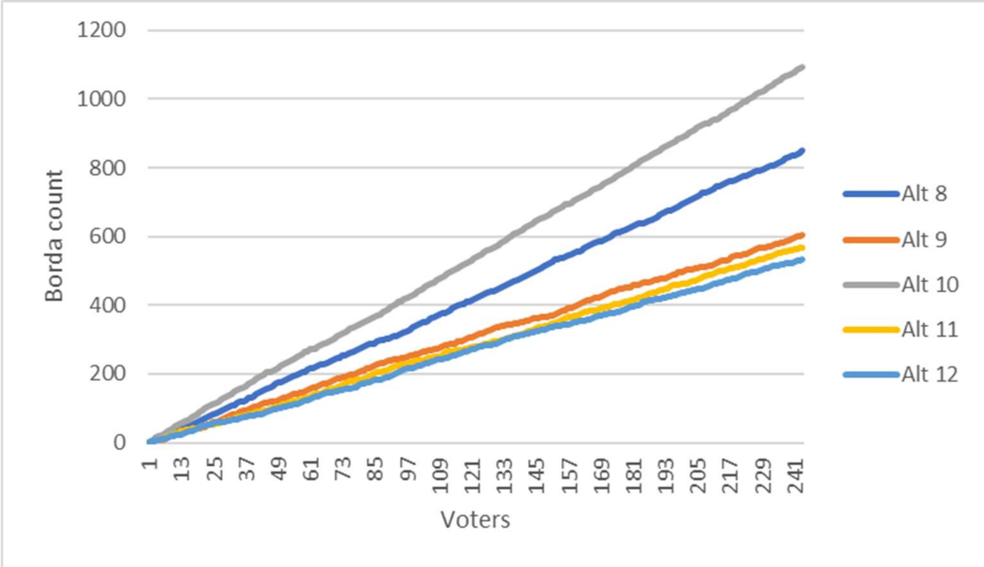
Source: The author (2020).

Table 34 – Result of sub-experiment 2 – test group

| Pos. | Alternative | Count |
|---|---|---|
| 1 | Alt. 10 | *217* |
| 2 | Alt. 8 | *158* |
| 3 | Alt. 9 | *139* |
| 4 | Alt. 12 | *138* |
| 5 | Alt. 11 | *134* |

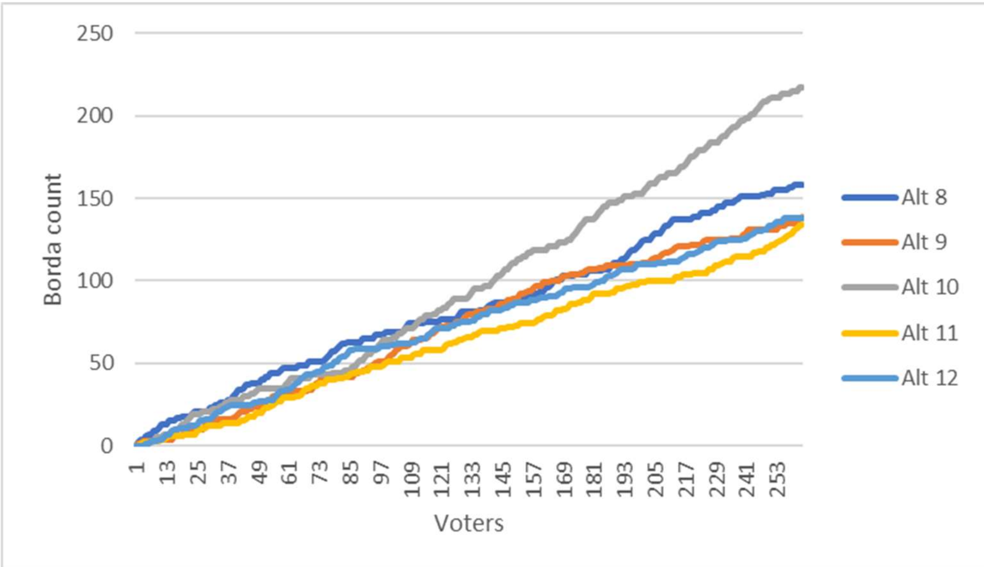
Source: The author (2020).

Figure 40 – Evolution of the Borda count for the control group of sub-experiment 2




Source: The author (2020).

Figure 41 – Evolution of the Borda count for the test group of sub-experiment 2




Source: The author (2020).

In this experiment, the final rankings of the test and the control groups were not identical. Although projects 10, 8 and 9 kept their positions in both rankings, alternatives 11 and 12 did not. In the analysis section, we discuss these results.

In the next subsection, we present the results of sub-experiment 3.

### *5.2.2.5 Sub-experiment 3*

The third part of the experiment, in which the participants ranked all *12* alternatives, was not considered in the convergence analysis itself. It was designed to gather the perception of the students about the whole set of projects in order to guide the decisions of the campus administration.

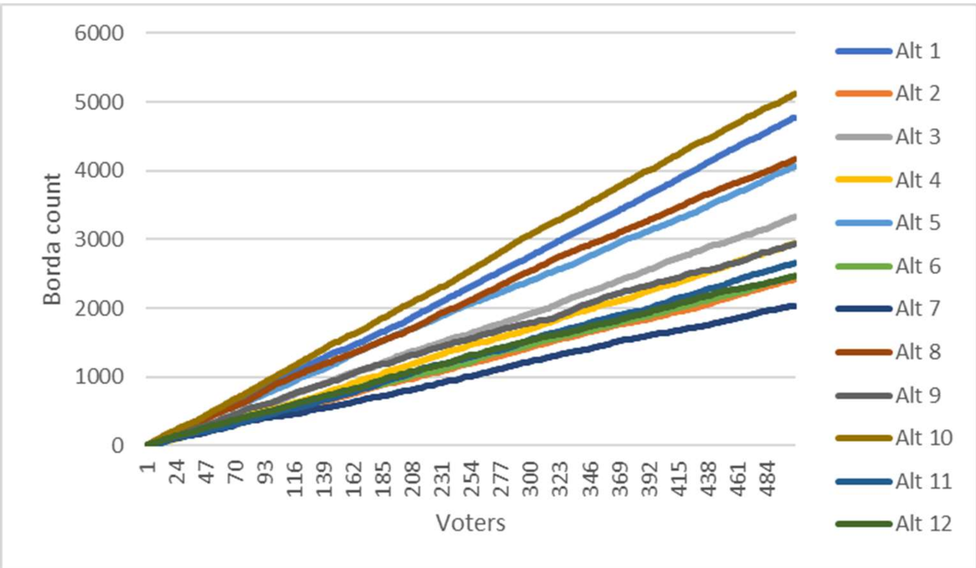
The result of the sub-experiment 3 is presented in Table 35 and in Figure 42.

Table 35 – Result of sub-experiment 3

| **Pos.** | **Alternative** | **Count** |
|---|---|---|
| 1 | Alt. 10 | *5118* |
| 2 | Alt. 1 | *4775* |
| 3 | Alt. 8 | *4166* |
| 4 | Alt. 5 | *4069* |
| 5 | Alt. 3 | *3325* |
| 6 | Alt. 4 | *2936* |
| 7 | Alt. 9 | *2936* |
| 8 | Alt. 11 | *2654* |
| 9 | Alt. 12 | *2472* |
| 10 | Alt. 6 | *2464* |
| 11 | Alt. 2 | *2427* |
| 12 | Alt. 7 | *2036* |

Source: The author (2020).

Figure 42 – Evolution of the Borda count of sub-experiment 3




Source: The author (2020).

The final rankings showed projects 10 and 1 on the top. The project with the least importance for the students was the alternative 7. The results of sub-experiment 3 are consistent with those of sub-experiments 1 and 2.

In the next section, we analyze the results of this experiment.

### 5.2.3 Analyses

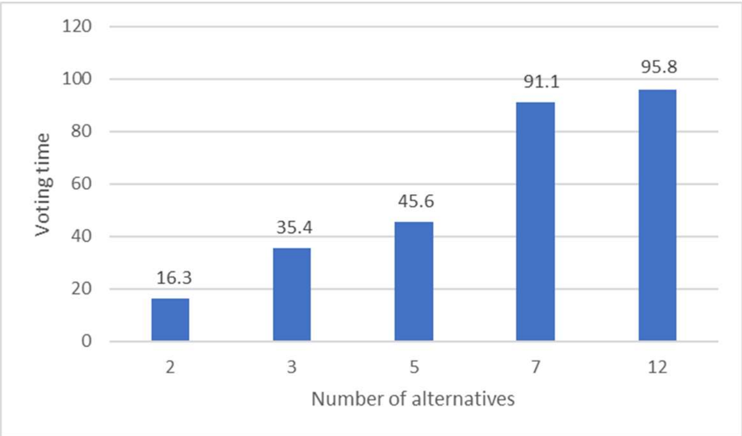
In this section, we analyze the results of experiment two. It is organized as follows. In the first subsection, we present the time analysis of the experiment, followed by the analysis of the randomness, in subsection two. We then describe the consistency and the convergence analyses in the third and in the fourth subsection.

#### *5.2.3.1 Time analysis*

In order to have an initial estimate of the effort of the task proposed to the participants, we have measured how long it took for the voters to cast their votes. The time of every step of sub-experiments 1, 2 and 3 was registered and the figures and tables below present the results.

In Figure 43, the average voting time for each set size is presented. For size 2, the average voting time was *16.3* seconds. For *3*, *5*, *7* and *12* alternatives, the voting time was, respectively, *35.4*, *45.6*, *91.1*, *95.8*.

Figure 43 – Average voting time for experiment 2




Source: The author (2020).

In this experiment, as verified in the first one, the results showed a direct relation between the number of alternatives and the time spent by the participants in the task. It suggests a quite intuitive reasoning: the more alternatives for evaluating, the more time needed for the voters to cast their votes.

The standard deviation for each set size is presented in Table 36.

Table 36 – Standard deviations of the voting times for experiment 2

| **Set size** | **2** | **3** | **5** | **7** | **12** |
|---|---|---|---|---|---|
| Standard deviation | *12.9* | *24.2* | *26.1* | *46.4* | *50.7* |

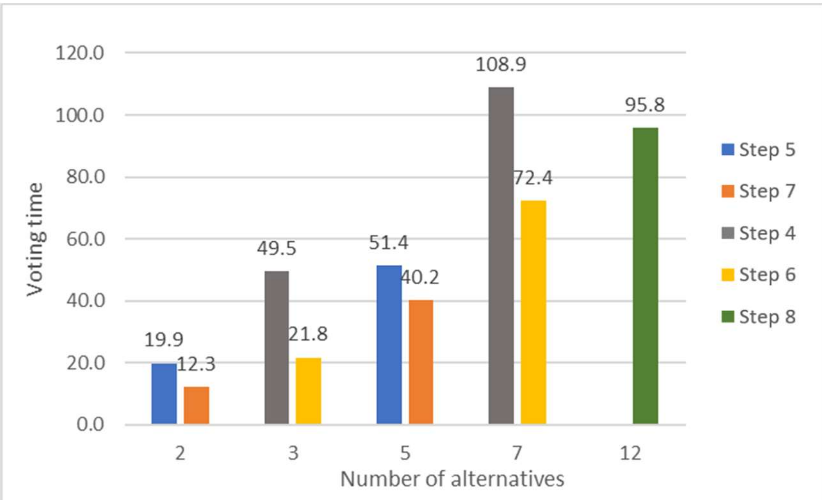
Source: The author (2020).

We have then investigated whether the order in which the subsets were exhibited to the participants had any influence on the time spent by them in the task. In other words: does ranking, for example, the set of *7* in step 4 would lead, on average, to the same time of ranking *7* in step 6?

In order to respond to this question, we have, for each set size, separated the sample into two sets, according the step in which the vote took place.

Figure 44 presents the averages voting times for each set size in different steps.

Figure 44 – Average voting time for each set size in different steps




Source: The author (2020).

For the set size of *2*, the participants ranked the subsets into two different moments. Some of them, those who were selected for the test group, did it in step 5 with *19.9* seconds on average. Those in the control group, step 7, accomplished the task in *12.3* seconds on average. For three alternatives, the average voting times were *49.5* and *21.8* seconds on steps 4 and 6, respectively. When ranking *5* projects, the average voting times were *51.4* and *40.2* seconds on steps 5 and 7, respectively. For *7* alternatives, the average voting times were *108.9* and *72.4* seconds on steps 4 and 6.

From these results, we can see that for the same set size, the participants took less time to accomplish the task in the second part of the experiments. We can also notice that the gap in steps 4 and 6 (*3* and *7* alternatives) is greater than in steps 5 and 7 (*2* and *5* alternatives).

We have tested the hypothesis of different means for each set size. The samples are large and independent. Table 37 present the resulting t-values.

Table 37 – T-values for the time analysis

| **Set size** | **2** | | **3** | | **5** | | **7** | |
|---|---|---|---|---|---|---|---|---|
| Step | 5 | 7 | 4 | 6 | 5 | 7 | 4 | 6 |
| Avg. voting time | 19.9 | 12.3 | 49.5 | 21.8 | 51.4 | 40.2 | 108.9 | 72.4 |
| Std. dev. | 14.6 | 9.2 | 25.0 | 13.0 | 23.8 | 27.3 | 33.3 | 50.1 |
| n | 262 | 243 | 248 | 257 | 262 | 243 | 248 | 257 |
| **t-values** | **7.05** | | **15.54** | | **4.89** | | **9.67** | |

Source: The author (2020).

Considering a significance level of *0.01* and *242* and *247* degrees of freedom, the critical t-value is about *2.59*. From the t-values of Table 37, one can then affirm that there is sufficient evidence to support the claim that the participants took less time to rank same size sets in the second part compared to the first part of the experiments.

We have two hypotheses to explain this behavior. In the first, we suggest that a learning process was present while using the voting system. The students had to rank alternatives with a drag and drop tool that, while being quite simple to use, was new to them and required some quick practice. The other one is related to the fact that, in the second part of each experiment, the participants had already been exposed to some (or all) alternatives of the experiment. Those in the test group voted on random subsets of *2* and *3* in the first part and on the full sets of *5* and *7* in the second part. So, the alternatives which have been analyzed in the first part appeared again in the second, the consistency vote. These facts apparently had an impact and explain the important differences in the voting times. This rationale should be tested in future experiments.

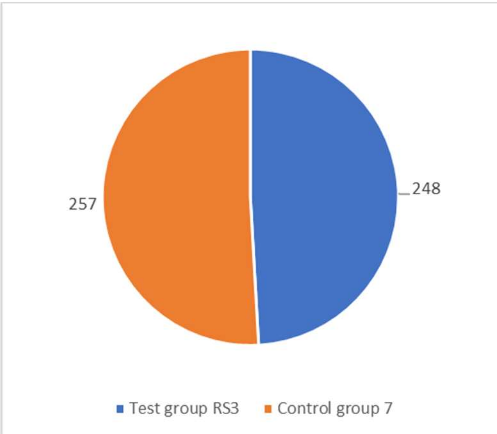
In the following subsection, we present the randomness analysis of the experiment.

*5.2.3.2 Randomness analysis*

The experiment included two important random procedures. The first was the partition of the students into the control and into the test groups for sub-experiments 1 and 2. The second was the selection of the alternatives for the random subsets.

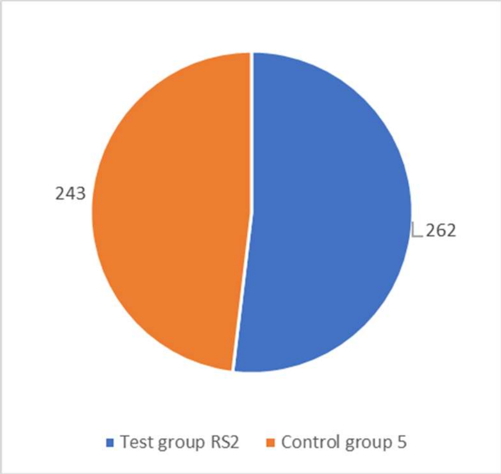
Figure 45 and Figure 46 present the number of participants who have randomly been allocated to the test and the control groups for each experiment. For sub-experiment 1, *248* (*49.11%*) students were in the test group, i.e. voted on random subsets of size *3*, and *257* (*50.89%*) presented their preferences over the full set of *7* alternatives. In sub-experiment 2, *262* (*51.88%*) voted on random subsets of size *2*, and *243* (*48.12%*) voted on the full set of *5*.

Figure 45 – Number of participants in the test and control groups of sub-experiment 1




Source: The author (2020).

Figure 46 – Number of participants in the test and control groups of sub-experiment 2




Source: The author (2020).

A quick statistical analysis of the distribution of the participants into the test and the control groups strongly suggests that the selection of participants for the groups was made at random. In Table 38 and Table 39, the p-values indicate that we cannot reject the null hypothesis that the proportions are equal to *0.5*.

Table 38 – P-values for experiment 2 – sub-experiment 1 – partition of groups

| ***Group*** | **Occurrences** | **Proportion** | **P-value** |
|---|---|---|---|
| Test group | *248* | *0.491* | *0.689* |
| Control group | *257* | *0.509* | *0.689* |

Source: The author (2020).

Table 39 – P-values for experiment 2 – sub-experiment 2 – partition of groups

| ***Group*** | **Occurrences** | **Proportion** | **P-value** |
|---|---|---|---|
| Test group | 262 | 0.519 | 0.398 |
| Control group | 243 | 0.481 | 0.398 |

Source: The author (2020).

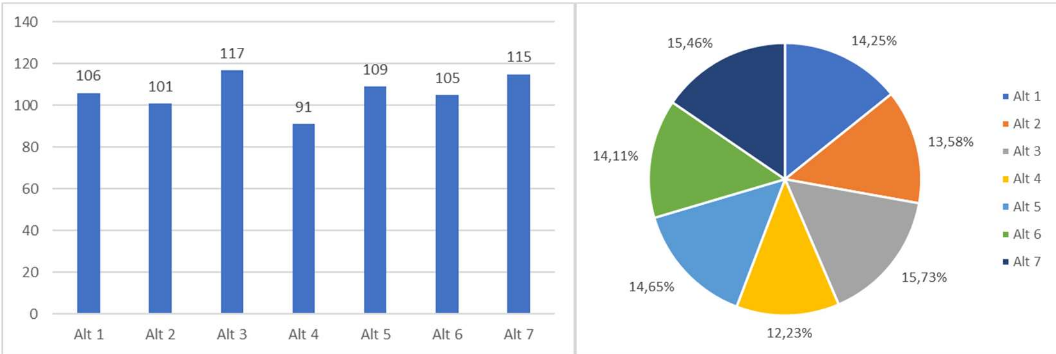
While the first part of the randomness analysis involves the participants and their groups, the second part involves the selection of alternatives for the random subsets.

As preconized by Amorim et al. (2018), the random selection of alternatives for each subset has to be a fair procedure. By fair, it was meant that any alternative would have the same chance of being selected to any random subset. The fairness, in the experiment, was implemented by using the appropriate functions of the random library of Python.

In order to evaluate the fairness of the selection of alternatives for the random subsets, we have computed how many times each alternative appeared in the subsets.

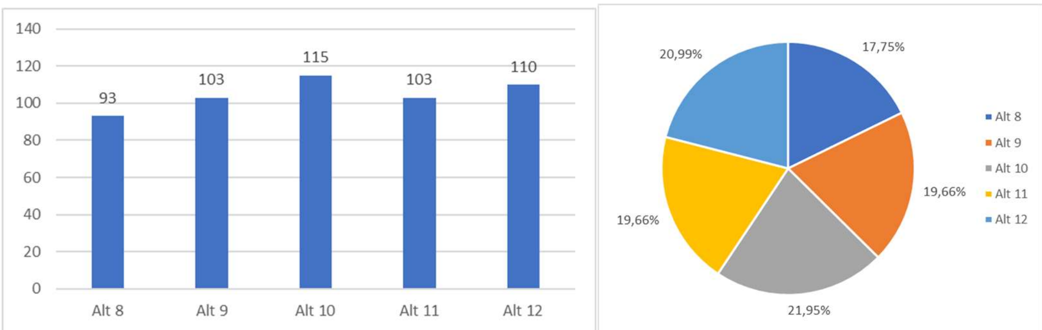
Figure 47 and Figure 48 show how many times each alternative appeared in the random subsets for sub-experiments 1 and 2. The figures also indicate the percentages.

Figure 47 – Count and percentages of appearances of alternatives in the random subsets of sub-experiment 1




Source: The author (2020).

Figure 48 – Count and percentages of appearances of alternatives in the random subsets of sub-experiment 2




Source: The author (2020).

As with the selection of participants for the test groups, we have tested the proportions found in order to verify the fairness of the procedure.

The hypothesis testing has led to the p-values in Table 40 and Table 41. The p-values in both tables suggest that the random selection of alternatives for the subsets was fair.

Table 40 – P-values for experiment 2 – sub-experiment 1 – selection for random subsets

| *Group* | Occurrences | Proportion | P-value |
|---|---|---|---|
| Alt 1 | *106* | *0.142* | *0.986* |
| Alt 2 | *101* | *0.136* | *0.749* |
| Alt 3 | *117* | *0.157* | *0.517* |
| Alt 4 | *91* | *0.122* | *0.355* |
| Alt 5 | *109* | *0.147* | *0.870* |
| Alt 6 | *105* | *0.141* | *0.938* |
| Alt 7 | *115* | *0.155* | *0.598* |

Source: The author (2020).

Table 41 – P-values for experiment 2 – sub-experiment 2 – selection for random subsets

| *Group* | Occurrences | Proportion | P-value |
|---|---|---|---|
| Alt 8 | *93* | *0.177* | *0.362* |
| Alt 9 | *103* | *0.197* | *0.889* |
| Alt 10 | *115* | *0.219* | *0.431* |
| Alt 11 | *103* | *0.197* | *0.889* |
| Alt 12 | *110* | *0.210* | *0.688* |

Source: The author (2020).

In the next subsection, we present the consistency analysis of experiment 2.

#### *5.2.3.3 Consistency analysis*

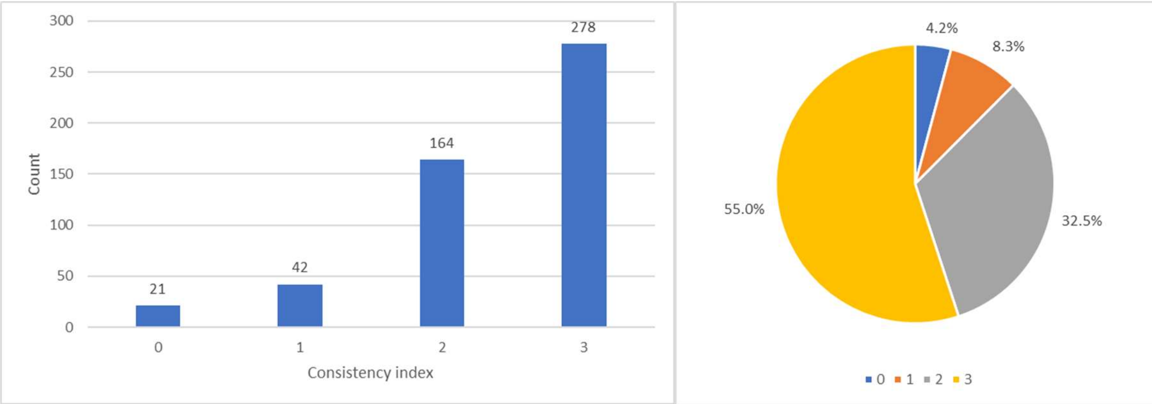
As in experiment 1, the consistency analysis was designed to evaluate to what extent the voters were consistent in their decisions in steps 1 and 2 of each sub-experiment.

In this second experiment, however, we have proposed a more detailed analysis of the consistency of the votes. Instead of evaluating whether the voters followed or not the IIA property, we have measured how consistent the vote was. The variable used to measure the consistency, called consistency index (*CI*), simply counted the number of pairs of alternatives that kept the same relative position in both steps. For sub-experiment 1, with three random alternatives, the consistency index varied from *0* to *3*. Zero meant that none of the alternatives kept their relative positions compared to the others; three meant that all three alternatives kept their relative positions. For sub-experiment 2, with two random alternatives, the values for the consistency index were *0* or *2*.

Figure 49 and Figure 50 show the consistency index for sub-experiments 1 and 2.

For sub-experiment 1, *278* from *505* (or *55.0%*) participants were fully consistent in their decisions; *32.5%* had consistency index equals to two. For *8.3%* of the voters, consistency index counted 1; and for only *4.2%* none of the alternatives kept their relative positions in the consistency step.

Figure 49 – Consistency index for sub-experiment 1




Source: The author (2020).

The consistency index from sub-experiment 1 can be viewed from the perspectives of those in the test and in the control groups.

Those in the test group (*248* participants), ranked three alternatives in part 1 and seven in part 2. Those in the control group (*257* participants), ranked seven alternatives in part 1 and three in part 2.

Table 42 presents the count and the percentage of participants for each *CI* in the test and in the control groups for sub-experiment 1.

Table 42 – Consistency index for the test and control groups in sub-experiment 1

| ***Consistency index*** | **Test group (3-7)** | | **Control group (7-3)** | |
|---|---|---|---|---|
| | **Count** | **%** | **Count** | **%** |
| *0* | *11* | *4.4%* | *10* | *3.9%* |
| *1* | *19* | *7.7%* | *23* | *8.9%* |
| *2* | *91* | *36.7%* | *73* | *28.4%* |
| *3* | *127* | *51.2%* | *151* | *58.8%* |
| TOTAL | *248* | *100%* | *257* | *100%* |

Source: The author (2020).

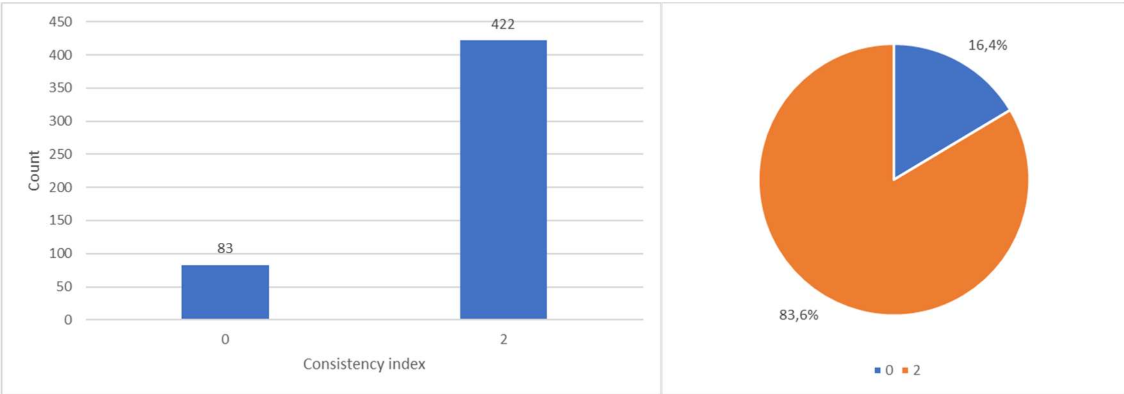
The results in Table 42 shows that *51.2%*, or *127* participants of the test group, were fully consistent. In the control group, the count increased to *151* (*58.8%*). This means that those who ranked *7* alternatives first and then moved to the subset of *3* acted more consistently than those who ranked *3* and then moved to *7*.

For *CI=2*, a decrease in the percentage was verified from the test group (*36.7%*) to the control group (*28.4%*). For *CI* equals to *0*, *1* the count moved from *11*, *19*, in the test group, for *10*, *23*, in the control group.

Note that while an increase was verified for *CI=3*, the opposite happened for *CI=2*.

For sub-experiment 2, Figure 50, *422* (or *83.6%*) of the participants were fully consistent. *16.4%*, or *83* voters, were not consistent.

Figure 50 – Consistency index for sub-experiment 2




Source: The author (2020).

As in sub-experiment 1, we present in Table 43 the consistency index for test and control groups in sub-experiment 2.

In this case (see Table 43), about *84%* of the participants were fully consistent in the test group. For the control group, the percentage dropped to *83.1%*.

Table 43 – Consistency index for the test and control groups in sub-experiment 2

| ***Consistency index*** | **Test group (2-5)** | | **Control group (5-2)** | |
|---|---|---|---|---|
| | **Count** | **%** | **Count** | **%** |
| *0* | *42* | *16.0%* | *41* | *16.9%* |
| *2* | *220* | *84.0%* | *202* | *83.1%* |
| TOTAL | *262* | *100%* | *243* | *100%* |

Source: The author (2020).

In the consistency analysis, we have evaluated to what extent the IIA property was valid. We have seen that, for sub-experiment 1, almost *90%* of the participants had *3* or *2* pairwise matches when comparing part 1 to part 2. For sub-experiment 2, more than *80%* were fully consistent.

These results, while being intuitively acceptable, can invite us to investigate why about *20%* of the students did not act rationally in regard to the IIA property. We propose three possible explanations for this effect.

The first one is the lack of motivation of the participants in the poll. Some students might not have been interested enough to participate in the choice experiment and did not expressed their preferences properly, bypassing some steps or do not dedicating the appropriate time to it. Although it is a reasonable explanation, the alternatives were projects to be implemented in the campus in a participatory democracy context. Also, these projects could have a direct impact in the campus life. We thus believe the students would be motivated enough to engage to vote. The number of participants endorses this argument. Nonetheless, a deeper investigation would be necessary to very such argument.

The second possible interpretation is the fact that the experiment did not consider the possibility of ranking alternatives in the same positions. Tie was not possible. So, if some projects had the same importance to a voter, he/she would not have a way to express his/her opinion properly. This could had led to a relative position reversal for some alternatives.

The third one has been discussed in the literature under the decoy and attraction effects theory. As stated by Trueblood et al. (2013), adding or removing alternatives from the choice set might lead to a change in the preference of the voters. It seems reasonable to presume that when two alternatives are compared in a set of only two or in a set of *20*, their relative positions can change. A voter might have a different perception of the two options when *18* extra alternatives have to be considered.

In this study, it was not possible to develop our analysis in order to isolate the impact of these three issues. In the future, they can be investigated more closely.

Another aspect of the consistency analysis was comparing the behavior of the test and the control groups.

In sub-experiment 1, the first impression was that the control group performed a bit better. *58.5%* of the participants presented $CI=3$. For the test groups it dropped to *51.2%*. However, for $CI=2$, the test group performed better than control group.

For sub-experiment 2, the variation presented in the results of the consistency analysis for the test and the control groups were not significant. More investigation needs to be conducted in order to evaluate the impact of adding or removing alternatives from the choice set in the context of RSV.

In the next subsection, we present the analysis of the convergence between Borda and RSB.

#### *5.2.3.4 Convergence analysis*

The results of the convergence of sub-experiments 1 and 2 corroborate the findings of the mathematical analysis and the simulations, showing that the final rankings of Random-Subset Borda and Borda converged. In fact, a small variation in the result of sub-experiment 2 has been noticed. While in the control group, alternative 11 was preferred to alternative 12, in the test group the latter was preferred to the first.

This change in the relative positions of projects 11 and 12 can be explained by the uneven distribution of alternatives in the random subsets. While project 11 appeared *103* times, project 12 appeared *110* times. It is a small and statistically acceptable difference, but, in this case, it has had an impact in the result.

As in the analysis of the experiment 1, we have computed the weighted count in order to evaluate to what extent the distribution of alternatives in the random subsets indeed impacted the results.

Table 44 and Table 45 below present the weighted count for sub-experiments 1 and 2.

Table 44 – Weighted count for sub-experiment 1 (experiment 2)

| Pos. | Alternative | Borda count | Occurrences | RSB count | RSB weighted count |
|---|---|---|---|---|---|
| 1 | Alt. 1 | 1540 | 106 | 286 | 2.698 |
| 2 | Alt. 5 | 1371 | 109 | 266 | 2.440 |
| 3 | Alt. 3 | 1032 | 117 | 242 | 2.068 |
| 4 | Alt. 4 | 1030 | 91 | 201 | 2.209 |
| 5 | Alt. 2 | 822 | 101 | 172 | 1.703 |
| 6 | Alt. 6 | 763 | 105 | 168 | 1.600 |
| 7 | Alt. 7 | 638 | 115 | 153 | 1.330 |

Source: The author (2020).

Table 45 – Weighted count for sub-experiment 2 (experiment 2)

| Pos. | Alternative | Borda count | Occurrences | RSB count | RSB weighted count |
|---|---|---|---|---|---|
| 1 | Alt. 10 | 1093 | 115 | 217 | 1.887 |
| 2 | Alt. 8 | 849 | 93 | 158 | 1.699 |
| 3 | Alt. 9 | 603 | 103 | 139 | 1.350 |
| 4 | Alt. 11 | 566 | 103 | 134 | 1.301 |
| 5 | Alt. 12 | 534 | 110 | 138 | 1.255 |

Source: The author (2020).

The RSB weighted count had different impacts in the sub-experiments.

In sub-experiment 2, while the use of the absolute RSB count has put alternatives 12 and 11 in fourth and fifth position, respectively, the RSB weighted count has inverted their positions, making them match their positions in the control group. In this case, the weighted count has compensated the differences found in the absolute RSB count.

In sub-experiment 1, however, the use of the weighted count has altered the original result. While the final rankings of both control and test groups matched when using the absolute counts, they did not when we implemented the RSB weighted count, since alternatives 3 and 4 changed their positions.

The point is that alternatives 3 and 4 have been perceived as almost equivalent among the participants. This equivalence can be verified in the evolution of the Borda count of the control group in Figure 38. In this case, any oscillation in the Borda count would lead to changes in the relative positions of the alternatives in the final rankings. More experiments should be conducted in the future in order to investigate similar scenarios.

In the next section, we present the final considerations of the chapter.

## 5.3 FINAL CONSIDERATIONS

In this chapter, we have presented the two experiments designed to empirically test the Random-Subset Voting method and to evaluate how the random reduction of alternatives in the choice-sets would affect the result of the elections.

In the first experiment, the participants presented their preferences over *15* juices. In the second, the students evaluated *12* different projects for their campus. In spite of some minor variations, Borda and RSB converge when the population is big enough, corroborating the findings of the mathematical analysis and of the simulations, presented earlier in this study. The experiments allowed us to defend the argument that the voters do not necessarily need to express their preferences over the entire set of alternatives and still get the same collective outcome.

This central result was complemented by some important analyses concerning time, randomness and consistency.

In regard to the time took by the participants to cast their votes, we have verified a very straightforward phenomena: the more alternatives to evaluate, the more time to accomplish the task. Also, we have identified statistically significant differences in the duration of the ordering task for the same set size when comparing the test and the control groups.

In the consistency analysis, we have found, in both experiments, evidences of the non-rationality of the voters. In both experiments, the participants consistently violated the IIA property. In other words, adding or removing elements from the choice-sets have led to different perceptions of the voters in regard to the alternatives. These violations, however, were not significant enough to promote considerable impacts in the convergences identified.

Concerning the randomness, the analyses have showed that there are sufficient statistical evidences to support that both the partition of the sample into the test and the control groups and the selection of alternatives for the subsets have been made at random.

As expected, however, the absolute values of the alternatives in the random subsets varied, leading to uneven distribution of alternatives. In order to overcome this issue, we have proposed the weighted RSB count. This new count seemed to be useful in mitigating the impact of these variations.

The next chapter presents the conclusions of this thesis.

# 6 CONCLUSIONS AND FUTURE RESEARCH

In this study, we have investigated how voters can face an overwhelming psycho-emotional state when dealing with choice-sets with too many alternatives. We have demonstrated that many scholars, especially those from Psychology and Behavioral Economics, have shown that an excessive number of alternatives might hinder the voter's ability of making good choices.

Very few researchers, however, have proposed solutions for dealing with this issue. Botti and Iyengar (2006) and Besedes et al. (2015) have approached this question by proposing that large decisions can be split into series of smaller ones. This technique, although effectively reducing the cognitive overload in some sense, still requires the voters to present their preferences over all alternatives of the choice-set.

We have then explored how we could systematically reduce the information overload experienced by voters when dealing with too many options and addressed the challenge introduced by Nurmi (2014a) of constructing social choice rules based on less demanding assumptions in regard to the rationality of the voters.

Our main result was the proposal of the Random-Subset Voting (RSV) procedure, a new way of combining probability and voting. With RSV, a voter does not rank the entire set of alternatives, but a random subset of it. This subset is selected by a fair random mechanism that assures that all candidates will have the same chance of being assigned to each voter.

Since RSV explicitly reduces the number of alternatives each voter evaluates, it minimizes the complexity and the difficulty of the task of casting votes. Furthermore, RSV proved to be less subject to manipulation and to vote buying practices. This is due to the fundamental attribute of the method: the random selection of alternatives to the subsets.

The Random-Subset Voting procedure was studied with four different traditional voting schemes: Borda, plurality, approval and Condorcet. The proposed method, in fact, is regarded as a framework, or a meta-solution, to which any voting scheme might be attached.

The method was analyzed under three different approaches: mathematical analysis, simulations and experiments. The application of these three methods to stress the RSV idea has proven to be quite useful.

In all experiments and simulations and in the mathematical modeling, we could find strong evidences that, when the population is large enough, the random-subset version of the method converges to the same final collective result of the method itself. We have demonstrated this result analytically and experimentally with Borda. It was corroborated by the Monte Carlo simulations implemented with Borda, plurality, approval and Condorcet.

a) Mathematical modeling

With the mathematical modeling, we could analytically study the method and its premises, proving a theorem that shows that Borda and Random-Subset Borda, under independence of irrelevant alternatives, completeness and transitivity, and when the population is big enough, lead to the same outcome. This key result motivated the development of the simulations and experiments, which provided more practical insights about RSV.

b) Simulations

The simulations allowed us to play with several parameters in order to evaluate how RSV works with different sizes for the set of voters, the set of alternatives and the set of random alternatives. With the simulations, we could grasp the basic intuition on how RSV would run in real scenarios. In addition, the simulations allowed us to identify thresholds of convergence that could serve as starting points in the definition of the parameters of real elections.

In the simulations, it was also possible to have important insights on how different methods (Borda, plurality, approval and Condorcet) behave under RSV. We have shown that when few random alternatives are in the choice set, plurality and approval performs better than Borda and Condorcet. For higher values of *r*, Borda and Condorcet presented similar results, performing better than plurality and approval. These results provide a groundwork for future studies on how different schemes work under Random-Subset Voting and might help the policy makers to choose which method applies to different scenarios.

One final aspect concerning the simulations that requires our attention is the fact that the source of variability on the generation of random scenarios were in the means of the random variables implemented in the simulations. Each alternative had a different, but fixed, mean in the model. The variance was fixed too. In spite of being quite useful for our intents, this has led to a homogeneous population. In future implementations, one might play with these parameters in order to create more complex scenarios.

c) Experiments

The experiment has given us a taste of a real application of RSV and allowed us to investigate some empirical aspects of the method that could not be studied in the mathematical modelling nor in the simulations. In these experiments, apart from the convergence of the methods, which have also been verified in most scenarios, we have deepened our understanding on the analysis of the time spent by the respondents while voting, of the consistency of the votes and of the randomness issues involving RSV.

In the time analysis, it was verified a quite intuitive reasoning: the more alternatives to evaluate, the more time and effort to the participants to accomplish the task. In the consistency analysis, we have found that the voters have acted consistently in a non-rational manner in respect to the IIA property. The more alternatives in the subset or in the random subset, the stronger the possibility of breaking IIA.

In spite of this lack of consistency in their choices, the convergence of the final rankings was, in general, observed in the experiments.

d) Technological innovations

To run the simulations and the experiments, we have implemented two different softwares that are, along with the RSV itself and all the analysis we have presented so far, important outcomes of this study. These technological innovations are available to the scientific community and the general public for use and for upgrade.

One specific improvement to be implemented in the experiments in the future is to model it as a website in which users could create their own online poll using RSV. They would set the parameters, i.e. the alternatives, the number of random alternatives, the voting method that would run under RSV, and the voters would be invited to participate. It would serve as a framework for stimulating more use of RSV and for collecting more data about its application.

e) Limitations of the method

Some limitations of the proposed method need be highlighted.

The first, and perhaps the most direct one, is that fact the only a few voting schemes have been used under RSV: Borda, plurality, approval and Condorcet. In fact, only the Borda method was subject to mathematical analysis and the experiments. Plurality, approval and Condorcet were present only in the simulations. In future developments of this work, one might consider presenting and proving the general mathematical convergence of RSV for any voting procedure. In addition, the RSV experiments must focus on methods other than Borda. Plurality, approval, Condorcet and other methods might be applied.

Another limitation of this study concerns the fairness criterion and the randomization steps of the experiments. It is important to emphasize that all experiments were conducted considering the fairness criterion proposed by Amorim et al. (2018), which preconized that all alternatives would have the same chance of being allocated to the random subsets. Nevertheless, a quick review of the distribution of alternatives in the subsets showed that some alternatives appeared much more frequently in the subsets than others.

This issue suggests that the proposed fairness criterion, although being fair in the probabilistic sense, could be adjusted in order to be stricter. Future developments on RSV and new experiments could implement a random method that controls the number of times each alternative appears in the random subsets. In this stricter fairness property, instead of resorting only on the random allocation of alternatives to the subsets, one would control the number of appearances of the alternatives in the choice sets in order to make them as close as possible.

In the analysis of the results of the experiments, we have compensated this fairness matter by proposing the RSB weighted count, which considered not only the absolute count of the alternatives, but also the number of occurrences of them in the subsets.

The same idea could have been applied in the random splitting of the participants in the control and in the test groups. One could implement the random selection of participants to the control and the test groups in a way that the sets have the same number of participants. Note, however, that while the random splitting of participants is a methodological issue, the change in the fairness property to make it stricter is a variation of the original RSV method itself.

Future works on RSV can also deeper the investigations concerning the time and the effort of the voters when analyzing too many alternatives. Experiments in neuroscience might be employed in the context of RSV in order to examine the complexity of the voting task under different methods.

f) Why a new method?

Many questions might be raised about RSV and its applicability. The first one, and perhaps the most important, is: what are the advantages of RSV? In other words, what justifies the creation of this new voting scheme?

As mentioned, the main motivation behind the proposal of RSV was the cognitive overload faced by the voters when analyzing too many alternatives. RSV was mainly designed to deal with this issue. The results were quite positive in this sense, by showing that one could randomly reduce the number of alternatives and get the same results. This means that policy makers can use RSV in large population scenarios and will still get the same results.

However, it is expected that, as verified in the experiments, the voters change their preferences when alternatives are added to or removed from the choice set. In fact, we expect that when less alternatives are given to a voter, a more accurate perception of his/her preferences over these alternatives can be gathered than when these alternatives are in bigger sets.

Suppose that citizens in a province have to evaluate (rank) 30 projects. After eliciting their preferences and aggregating the results, the top 5 projects will be implemented. Assume that instead of analyzing 30 projects, each citizen will be asked to evaluate only three randomly chosen projects. If IIA holds and if the number of citizens is big enough, it is expected that voting from the full set with 30 alternatives and from the random subset with 3 would lead to the same final social rankings. However, as verified in the experiments, the rationality of the individuals can be considered to be bounded, in general, and, thus, IIA do not necessarily hold. It would not intrigue us if some inconsistencies in the individual rankings of 30 and of 3 projects are verified. Actually, it is expected that the rankings of the voters with three random alternatives will reflect more precisely their real preferences (over these three alternatives only) than the rankings of 30 alternatives in which these three options are melt into.

It is expected then that, when reducing the number of alternatives, the voters have different perceptions about some alternatives and then change their relative positions. This reversal of preferences behavior has been identified in the experiments and, in spite of the lack of consistency of some voters, the convergence was still reached.

Nonetheless, one could argue that when the inconsistency is high enough, we would face nonconvergent scenarios. In these cases, Borda and RSB would not converge.

So, even though we have proved the convergence and have provided simulations and experiments that show the same final collective rankings for Borda, plurality, approval and Condorcet, it is reasonable to expected that the results of RSV can differ from those of the traditional methods when the inconsistency of the voters' behavior is high enough.

If we assume that each RSV decision leads to qualitatively better evaluations of the alternatives, since the voters are analyzing less options, it is reasonable to conclude that, when the population is big enough, the RSV final collective ranking represents better the preferences of the voters than the traditional methods. More experiments need to be implemented in order to investigate this reasoning.

g) RSV and the current electoral systems

Another important question: is it possible to apply RSV in the context of our current electoral systems?

As a matter of fact, the elections we usually participate, especially those of representatives in councils and chambers, present too many alternatives in the choice set and seems a proper field for applying RSV. In the city of Recife, Brazil, in 2016, for example, 933 people presented their candidature to the city council and each voter, from the electorate of more than one million voters, had to choose one in 933 options. It is likely that none of the voters analyzed all alternatives.

From this example, and from similar ones in Brazil and around the world, one could infer that RSV has potential for immediate application in these decisions. However, we are aware that, in spite of presenting some significant theoretical and experimental results, much work has to be accomplished in RSV in order to bring it to the broad political arena.

One of the main concerns about the method, raised from feedbacks we have received so far, is on the possibility of allocating "poor alternatives" to the voters. Keeping with the example of Recife, if three random candidates are set to a voter and he/she does not have interest at all in any of their propositions, this would lead to a complete lack of interest in participating in the political debate and making choices.

Indeed, when people are the alternatives, this poor alternatives issue might play an important role and, in extreme cases, raise the abstention levels because voters will not be interested in voting in candidates they absolutely disregard.

This poor alternatives issue, however, might not play a big role when considering projects, actions or initiatives. In these situations, the goal of the voters would be to evaluate the alternatives assigned to them and declare their preferences. All the alternatives might be good enough, and the goal would be, for example, to define the subset of projects that will be implemented when a specific budget is available. Our second experiment with real projects goes in this direction.

This analysis between the differences when considering people or projects as alternatives, raise an important issue that could be investigated in more depth. The voting methods usually take general alternatives in their analysis without any discussion or examination on their nature. The voting methods implemented in order to choose people, or representatives, might differ in many aspects from those applied when choosing projects or initiatives in a community, especially if a big online community is in place.

h) Organizations

One possible implementation of RSV is in the context of organizations. Many decisions models and several tools are already in place in the companies in order to support the decision making. In fact, for big institutions, where large collective decisions might take place, Random-Subset Voting can be a tool to be considered in the process. In fact, our second experiment, in spite of being implemented in way that the results could be generalized for the society as a whole, is, at the end of the day, an application of RSV in an organization.

Therefore, RSV can also be regarded as a tool to foster the democratic mindset in public and private institutions.

i) Economic and social impacts

Another application of RSV can be in the e-Participatory Budgeting or Digital Democracies initiatives that are coming into life in the last decades and which is expected to increase very rapidly in the next ones. With the massive growth of the communication technologies and the social media, many researchers and activists start to envision a society in which people will participate more directly into the daily political life. In fact, some argue that we are moving from our indirect democracy back to the origins of the Athenian ideal of direct participation of the citizens in the issues of the communities (CHADWICK, 2009; DAHLBERG, 2011; PATEMAN, 2012; PEIXOTO, 2009; SANTOS, 1998; SINTOMER, 2010; SINTOMER; HERZBERG; RÖCKE, 2008).

In this scenario, the citizens could be systematically requested to analyze projects to be implemented in their cities and prioritize them. This vision will dramatically change the way we see politics and will have important social and economic impacts in the society.

In this context, as in the end of the eighteenth century, we are facing very important transformations in our societies and some of them might lead to new forms of democracy. The vast use of communication tools, the great possibilities of the online world, the rapid increase in the interest of political issues by the population and many other factors have been fostering this great shift.

Just like Borda and Condorcet did a few centuries ago, new voting schemes and decision models need to be in place in order to support this unique transformation process we are all immersed in. We propose Random-Subset Voting as a tool to support the policy makers and the organizations in this great democratic endeavor.

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# APPENDIX A – SCREENS OF THE FIRST EXPERIMENT

In this appendix, we present the print screens of the first experiment.

Figure A1 – Screen 1 of the experiment


Source: The author (2020).

Figure A2 – Screen 2 of the experiment


Source: The author (2020).

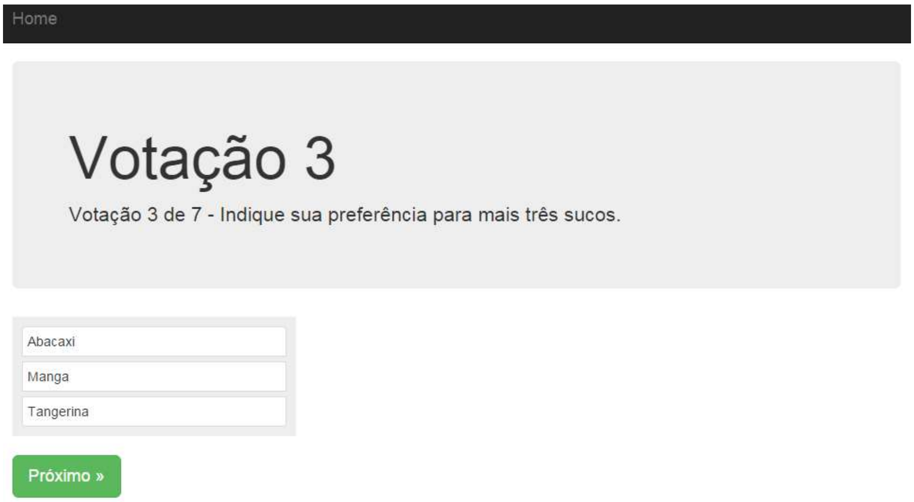
Figure A3 – Screen 3 of the experiment


Source: The author (2020).

Figure A4 – Screen 4 of the experiment

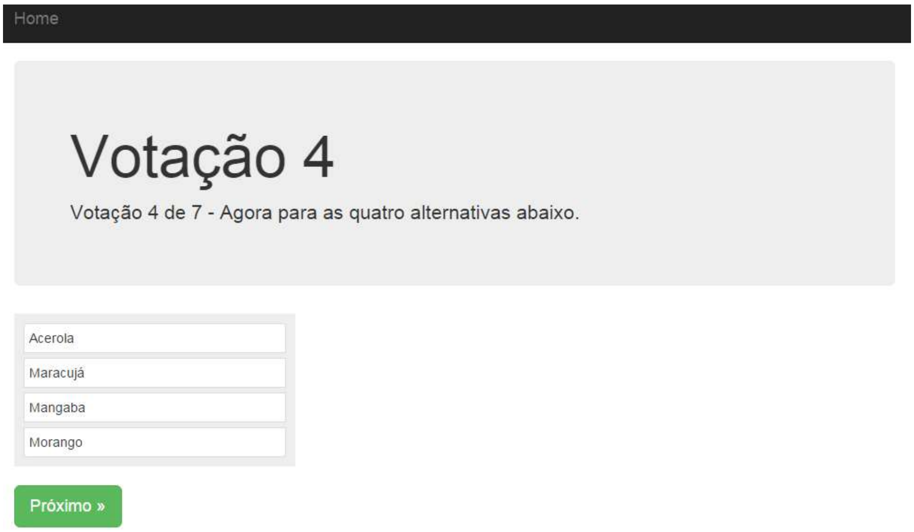

Source: The author (2020).

Figure A5 – Screen 5 of the experiment




Source: The author (2020).

Figure A6 – Screen 6 of the experiment




Source: The author (2020).

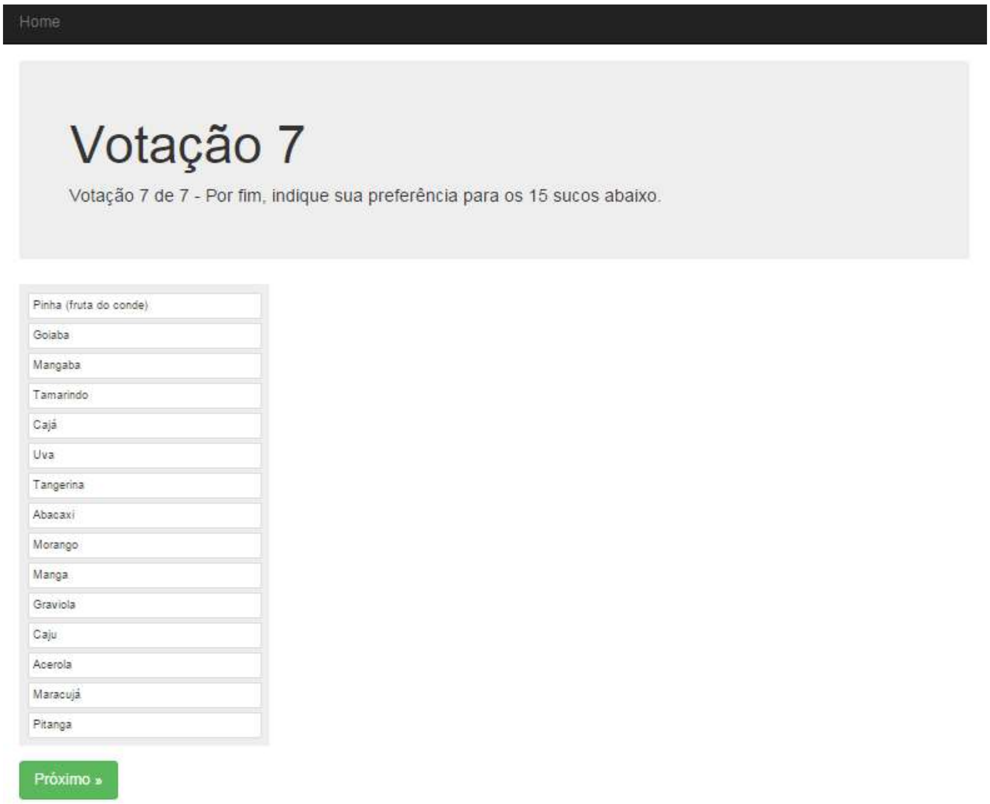
Figure A7 – Screen 7 of the experiment


Source: The author (2020).

Figure A8 – Screen 8 of the experiment


Source: The author (2020).

Figure A9 – Screen 9 of the experiment

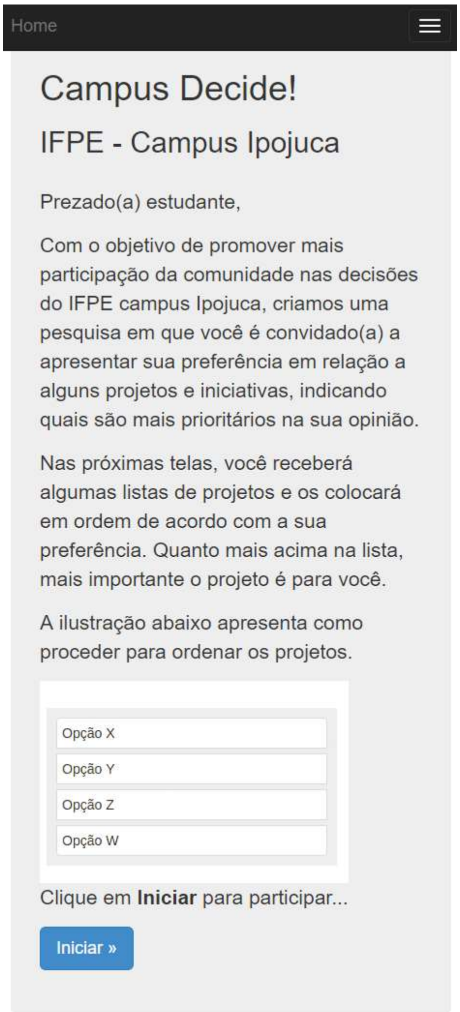



Source: The author (2020).

# APPENDIX B – SCREENS OF THE SECOND EXPERIMENT

In this appendix, we present the print screens of the web application used in the poll of the second experiment. In Figure , we present the screen of step 1.

Figure B1 – Screen of step 1




Source: The author (2020).

A free translation of the text in Figure  gives:

*Dear student,*

*With the goal of stimulating more participation of the community in the decisions of the IFPE campus Ipojuca, we have created a survey in which you are invited to present your preferences in regard to some projects and initiatives, indicating your priority.*

*In the next screens, you will rank lists of projects according to your preference. The more on the top, the more important the project is for you.*

*The illustration below presents how to proceed in order to rank the projects.*

*Click Begin to participate.*

The illustration presented in this first step displays a list of alternatives in an animated GIF that shows how to proceed to rank the alternatives.

In Figure , we present the step 2 of the experiment.

Figure B2 – Screen of step 2


Source: The author (2020).

In the screen of step 2, the basic information about the participant is requested. The text on it says:

*Basic information…*

*Before starting the voting process, we ask you to register some basic information.*

Then, the following fields are displayed: age, sex, hometown, course, semester and course time.

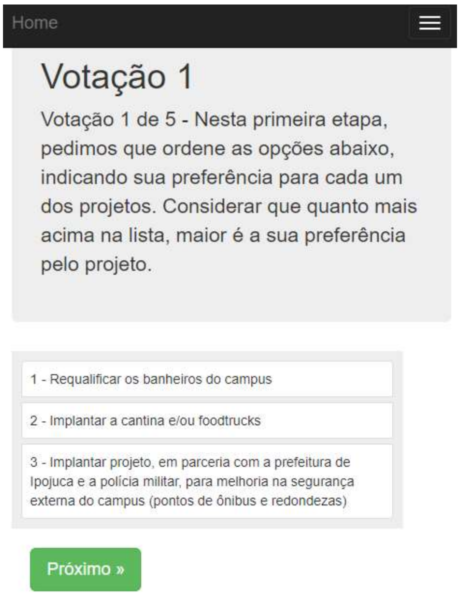
For the screen of step 4 (Figure ), the text in English is:

*Vote 1*

*Vote 1 of 5 – In this stage, we ask you to rank the options below, indicating your preference for each project. Consider that the more on the top of the list, the higher your preference for the project.*

Note that there is no screen for step 3 (randomization) of the experiment. The randomization is an inner step.

Figure B3 – Screen of step 4




Source: The author (2020).

Figure B4 – Screen of step 5

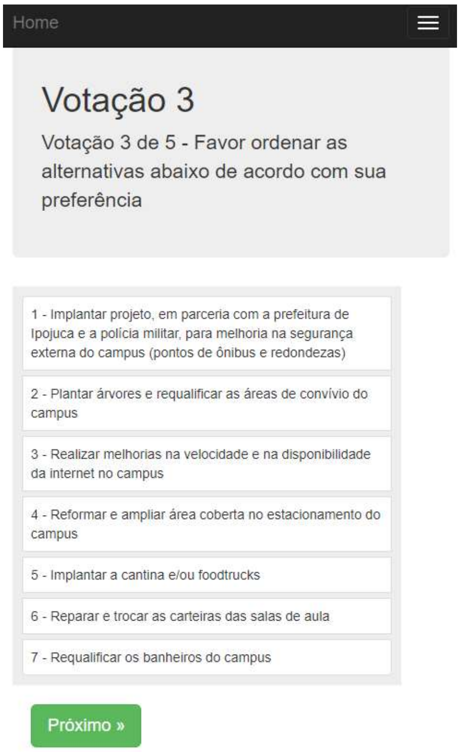

Source: The author (2020).

For the screens of steps 5 (Figure ), 6 (Figure ), 7 (Figure ) and 8 (Figure ), the same text is displayed to the participant, changing only the voting step. A free translation of the text gives:

*Vote X*

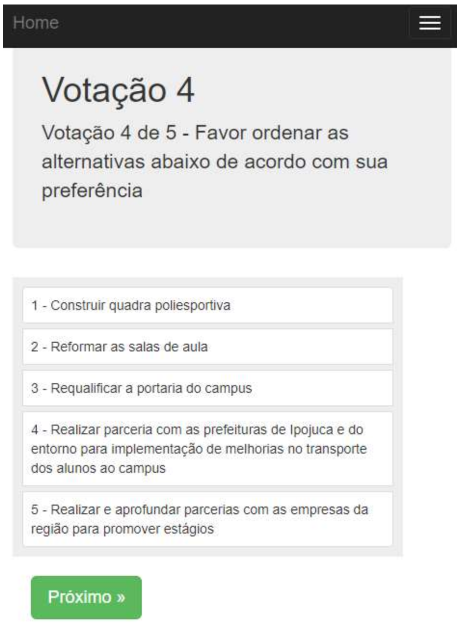
*Vote X of 5 – Please rank the alternatives below according to your preference.*

Figure B5 – Screen of step 6



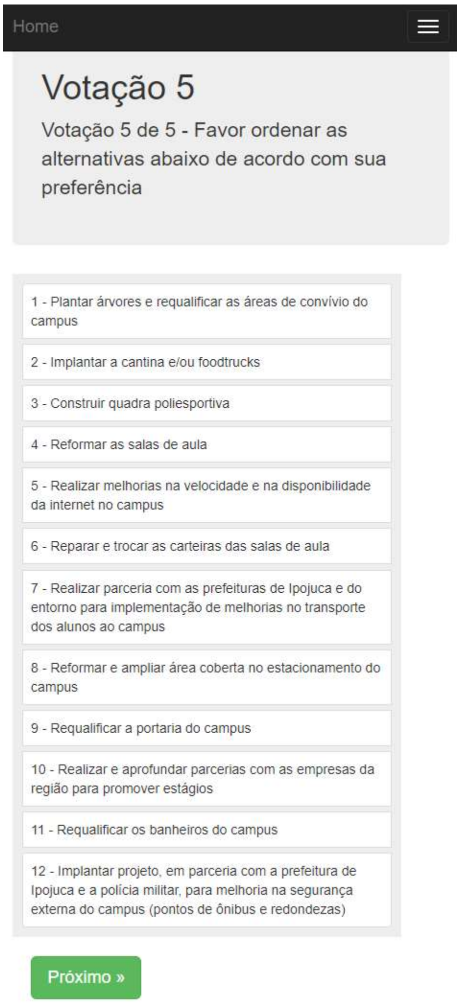

Source: The author (2020).

Figure B6 – Screen of step 7




Source: The author (2020).

Figure B7 – Screen of step 8




Source: The author (2020).

Figure B8 – Screen of step 9


Source: The author (2020).

The final step, Figure B8, displays:

*We thank you for the participation*

*For more information about our research, please contact…*