Universidade Federal de Pernambuco
Centro de Ciências Sociais Aplicadas
Departamento de Economia
Programa de Pós-Graduação em Economia

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Coordination Failures and Endogenous Productivity: A General Equilibrium Approach

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Dissertação submetida ao programa de Pós-graduação em Economia da Universidade Federal de Pernambuco, como requisito parcial para obtenção do grau de Mestre em Economia.

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DANIEL MENDONÇA ARAÚJO

COORDINATION FAILURES AND ENDOGENOUS PRODUCTIVITY: A GENERAL EQUILIBRIUM APPROACH

Dissertação ou Tese apresentada ao Programa de Pós-Graduação em Economia da Universidade Federal de Pernambuco, como requisito parcial para a obtenção do título de Mestre em Economia.

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Resumo

Diversas pesquisas empíricas demonstraram que a má alocação de recursos entre firmas heterogêneas pode ter consequências danosas para produtividade dos países. O presente trabalho estuda estas distorções quando estas afetam não apenas a produtividade presente, mas também a dinâmica de produtividade futura, em um ambiente onde as decisões dos agentes podem gerar falhas de coordenação. Esta analise será realizada a partir de um modelo padrão de firmas heterogêneas, com produtividade parcialmente endógena. Neste modelo, as firmas possuem a capacidade de afetar sua produtividade futura por meio de experimentos arriscados, que tomam a forma de choques de produtividade, e as falhas de coordenação são geradas a partir de uma complementaridade de demanda.
Palavras-chave: Falhas de Coordenação. Produtividade Endógena. Má alocação de recursos. inovação via experimentação.

Abstract

Several papers have shown that the misallocation of resources among heterogeneous firms can have damaging consequences for countries' total factor productivity. The present work studies the impact of these distortions when they affect not only the present productivity, but also the dynamics of future productivity, in an environment where the decisions of the agents can generate coordination failures. This analysis will be carried out from a standard model of heterogeneous firms, with partially endogenous productivity. In this model, firms have the ability to affect their future productivity by means of risky experiments, which take the form of productivity shocks, and coordination failures are generated from a complementarity of demand.
Keywords: Coordination Failures. Endogenous Productivity. Misallocation. innovation by experimentation.

Contents

1 INTRODUCTION

A set of recent studies reshaped the economic growth literature by pointing to evidence that microeconomic distortions in the allocation of resources among heterogeneous firms are one of the great causes of cross-country differences in TFP and then in living standards. The advance of this approach was strengthened by contributions such as Hsieh and Klenow (2009), and most recently Da-Rocha, Tavares and Restuccia (2017) and Restuccia and Rogerson (2017) have highlighted the dynamic role of misallocation as a path to further research.

The dynamic effect of the misallocation of resources might be justified by different mechanisms. The start point has been to consider that, in the presence of endogenous productivity, misallocation of resources today may affect investment decisions in R&D and then the growth path.

Along this line of thought, the present work develops a general equilibrium model with partially endogenous productivity in an environment conducive to coordination failures of the agents. The model also has an endogenous entry and a partially endogenous exit. The mechanism from which firms are able to influence their future productivity approaches the literature of "innovation by experimentation", from the model developed by Gabler and Poschke (2013).

The coordination problem in the model is developed as in Schaal and Taschereau-Dumouchel (2015), presenting a mechanism of complementary of demand, where the choice of a firm to maintain its high production in a given period, generates an additional income that increases the demand for the products of other firms in this period, thus adding incentives to production. In the present model the firm will maintain its high productivity in a given period if it does not invest in innovation, due to the costs in terms of current production to innovate. This complementarity of demand allows the possibility of miscoordination and different equilibria, whose conditions will be better developed in the following sections.

Throughout work exercises will be developed, verifying how the misallocation factors such as taxing of the firms with high productivity affect the equilibrium conditions of the model. This work stands out in the literature as it analyzes the dynamic impact of misallocation of resources, in an environment that allows the occurrence of coordination failures.

1.1 Related Literature

The work is mainly related to three main branches in the literature: the misallocation of resources, the work of innovation through experimentation and finally the study of problems of coordination.

Within the first branch, this work is related to seminal papers as Melitz (2003), Restuccia and Rogerson(2008), and Hsieh and Klenow (2009) who first developed analysis on the static damage of the misallocation, and more closely relates to the path highlighted by recent studies like Restuccia and Rogerson (2017) that points the relevance of dynamic factors of the misallocation for the explanation of the development.

The work of Hsieh and Klenow (2009) stands out for its quantitative analysis of how harmful the misallocation of resources can be. Using a general equilibrium model with heterogeneous firms calibrated for China, India and US data, the authors realize that if China and India were able to reach the US level of misallocation it would be possible to achieve gains in total factor productivity (TFP) of these countries in the order of 30% to 50% for China and 40% to 60% for India. These results are striking due to the magnitude of its findings, thus indicating how promising this line of research can be.

Following this line, an important question in the recent literature is "in what way the reaction of the firms to misallocation can affect their long-term productivity". Restuccia and Rogerson (2017) argue that the future of research related to misallocation is the analysis of productivity dynamics. The authors note that recent literature has focused heavily on the static impacts of misallocation, but as studies on productivity dynamics progress an analysis of the dynamic impacts of misallocation is needed. Some authors have been struggling to identify the channels through which agents are able to influence their own productivity dynamics.

In this line of dynamic misallocation, the present work approaches some works such as Atkeson (2010), Caggese (2014), Buera, Kaboski and Shin (2015), Ayerst (2016), Bonfiglioli, Crino, Gancia (2016), Guner, Parkhomenko and Ventura (2016), Lopes-Martin (2016), and Da-Rocha, Tavares and Restuccia (2017), who develop different and alternative models for endogenous productivity.The present work stands out from these due to the environment which includes in the discussion of misallocation the important aspect of coordination failures.

The present work uses the mechanism of innovation by experimentation. This use is justified by several works such as McGuckin et al. (1996), Broda and Weinstein (2010), Lentz and Mortensen (2008), and Bernard et al. (2010). Gabler and Poschke (2013) summarize some relevant facts, such as: Each year about 25% of the goods put up for sale is new or will be discontinued the following year; At least 40% of the goods are sold for only one year, and plants only adopt about half to one third of the technologies they test. Thus, the mechanism is an important channel in the study of productivity endogeneity. The present work is mainly related to the experimentation mechanism as presented by Gabler and Poschke (2013).

In the literature on coordination problems, the present work is mainly related to works analyzing the dynamic aspects of coordination failures, such as Morris and Shin (1999); Chamley (1999); Frankel and Pauzner (2000); Angeletos et al. (2007). Each with different approaches, with regard to the uniqueness or multiplicity of equilibrium.

In general, models with the presence of coordination failures present multiplicity of equilibria, but several techniques can be used to treat this problem. Among these techniques, two stand out due to their great use. The first one refers to the breakdown of the common knowledge, the second concerns the presence of timing frictions in the actions of the agents that modifies the strategic interaction. Some works, such as Angeletos et al. (2007) stand out for their detailed analysis of the multiplicity conditions of different models.

Morris and Shin (1999) develop a dynamic model where the foundamental that represents the strength of the status quo follows a random walk and is common knowledge. The authors demonstrate that their games can be treated as unrelated static games, each with a unique equilibrium. Frankel and Pauzner (2000) analyze a dynamic coordination game and achieve the uniqueness of equilibrium through aggregate shocks and with a idiosyncratic inertia.

Angeletos et al. (2007) realizes that considering a dynamic model if the agents receive private information only in the first period, by iterated elimination of dominated strategies it is possible to obtain a single equilibrium, however, considering a framework where agents perform a learning process each period with new information, the authors demonstrate the presence of multiple equilibrium's even in the presence of private information.

Two interesting surveys stand out in the field of coordination failures: Angeletos and Lian (2016) and Guimarães, Machado and Pereira (2017). The survey presented by Angeletos and Lian (2016) stands out for the great scope of analysis, extending in recent work on global games, beauty contests, and their applications. The authors also highlight the important efforts made in the literature in order to solve weaknesses of workhorse macroeconomic models to relaxations of common knowledge, and advances in the macroeconomic operation of concepts such as "coordination failure" and "animal spirits".

Guimarães, Machado and Pereira (2017) develop their survey from a model of dynamic coordination whose equilibrium uniqueness is obtained from the use of timing frictions. From this point, the authors advance the literature analyzing a set of extensions of their basic model as endogenous hazard rates and ex-ante heterogeneous agents, pointing also important macroeconomic insights that can be obtained from this set of models.

Two studies of coordination are highlighted due to the similarity with the present work: Schaal and Taschereau-Dumouchel (2015) and Machado (2017), in these papers the authors advance the studies of failures of coordination to a framework of general equilibrium, by a demand complementarity mechanism. Starting from a model with multiple equilibria, equilibrium is unified through the use of imperfect information, in Schaal and Taschereau-Dumouchel (2015) the equilibrium holds two stationary states, called by the authors of coordinating traps, while Machado (2017) have one stochastic steady state.

The present work stands out for extending the studies of dynamic misallocation (via innovation through experimentation) to a framework capable of explaining coordination failures by demand complementarity like Schaal and Taschereau-Dumouchel (2015) and Machado (2017), a point that can provide different insights for the search for an understanding of economic growth.

1.2 Work Organization

The paper will be developed in three sections, from this introduction.In section 2 we will present the development of the model, with the analysis of its possible equilibria, section 3 will highlight the role of the social planner in the efficiency of the model, and section 4 will develop comparative static exercises, verifying the impact of the misallocation on the established equilibria in the model.

2 MODEL

2.1 Environment

The model economy is set as a general equilibrium with partially endogenous productivity and consists of a representative household, a final good sector and a intermediate good sector. The intermediate good consist of a continuum of varieties solely used for the production of the final good in line with Dixit-Stiglitz model of monopolistic competition.

The final good is produced by a representative firm, perfectly competitive, that combines a continuum of differentiated intermediate goods, indexed by $j \in J_t$, with J_t being able to be divided in two groups J_t^h and J_t^l, according to the decision of innovate or not innovate taken by the intermediate firms, which are still heterogeneous in productivity. We also have that J_t is endogenously determined.

The intermediate firms can influence future productivity from an experiment. At each period, firms can carry out a costly and risky experiment through a shock on future productivity becomes possible. The model also allows the presence of an additional exogenous shock to productivity that the firms do not have control. The experiment is modeled as an idiosyncratic productivity shock (u) for firm productivity drawn from a distribution H (x). For the realization of the experiment firms pay two costs, the first is a traditional fixed cost f and the second is a "disruption cost" ($\theta \in (0,1)$), also fixed. The model allows firms not to be forced to stay with a draw from an experiment that did not work (negative draw), and can get rid of it at cost 0, thus receiving as a result of the experiment a productivity increase of $u = Max(x, 0)$, where x is the value of the draw if positive. So the expected value of experimentation is positive.

Entry and exit are considered endogenous, so agents decide to enter the market if the expected gain is greater than an entry cost, and exit if their productivity provides a negative expected gain. There is also an exogenous exit probability at each period, in the line of Luttmer (2012), this guarantees that the productivity distribution is stationary. Time is discrete and goes forever.

2.2 Households and Preferences

Households utility is given by:

$$E\Sigma_{t=0}^{\infty}\beta^t U(C_t) \tag{1}$$

where $0 < \beta < 1$ is the intertemporal discount factor, and $C_t \geq 0$.

The utility function adopted is:

$$E\Sigma_{t=0}^{\infty}\beta^t U(C_t) = \begin{cases} E\Sigma_{t=0}^{\infty}\beta^t \frac{C_t^{1-\gamma}}{1-\gamma}, & \text{for } \gamma > 0, \gamma \neq 1 \\ E\Sigma_{t=0}^{\infty}\beta^t log(C_t), & \text{for } \gamma \to 1 \end{cases} \tag{2}$$

where $\gamma > 1$.

The household budget constrain is:

$$C_t + a_{t+1} \leq W_t L_t + a_t(1 + r_t) \tag{3}$$

Where a_t are the assets held at the period t, W_t is the wage rate and R_t is the rental rate of capital, with depreciation $0 < \delta < 1$. We assume that $L_t = 1$.

The Euler Condition is given by:

$$C_t^{-\gamma} = \beta(1 + r_{t+1})C_{t+1}^{-\gamma} \tag{4}$$

2.3 Final Good Producers

The final good is produced by a representative firm, perfectly competitive, that combines a continuum of differentiated intermediate goods, indexed by $j \in J_t$, with J_t being able to be divided in two groups J_t^h and J_t^l, according to the decision to innovate (h) or not innovate (l) taken by the intermediary firms,with firms presenting heterogeneous productivities even within the same group, and using CES Production function.We also have that J_t is endogenously determined, and $m_t \in [0,1]$ is the measure of active firms that innovate in a given period t.

$$Y_t = \left(\int_{j \in J_t} Y_{jt}^{\frac{\sigma-1}{\sigma}} \, dj \right)^{\frac{\sigma}{\sigma-1}} \tag{5}$$

Where Y_t is the total output of the final good, Y_{jt} is the input of intermediate good j, and σ is the elasticity of substitution between varieties, with $\sigma > 1$,his is necessary for the presence of complementarity of demand. The price of the final good is normalized to 1.

Profit maximization, taking the prices as given, yields the usual demand curve and the price of the final good. **Proof in appendix A1.**

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\sigma} Y_t \tag{6}$$

$$1 = P_t = \left(\int_{j \in J_t} P_{jt}^{1-\sigma} \, dj \right)^{\frac{1}{1-\sigma}} \tag{7}$$

2.4 Intermediate Good Producers

Intermediate good j is produced by a monopolist. With production function:

$$Y_{jt} = A e^{z_{jt}} \theta L_{jt}^{1-\alpha} \tag{8}$$

where $0 < \alpha < 1$, and $A > 0$, L_{jt} represents the labor used by the intermediary firm j in period t.

The dynamic of the productivity z_t is given by:

$z_{jt} = z_{jt-1} + u_{jt} + \varepsilon_{jt}$

Where $u = Max(x, 0)$ is the result of the experiment, which is modeled as a draw from the cdf H (u), and assumes the draw value x if the result of the draw is positive and 0 if is negative, due to the assumption that the firm can get rid of it without cost of an unsuccessful experiment.

To perform the experiment it is necessary to pay two fix costs, the disruption cost $\theta \in (0,1)$, and $f > 0$. $\varepsilon_t \sim iidN(0, \zeta_z^{-1})$ represents an unanticipated productivity shock.

We can define: $A_{jt}^h(z_{jt}) = Ae^{z_{jt}}\theta$ and $A_{jt}^l(z_{jt}) = Ae^{z_{jt}}$, from the innovation decision taken by a given firm j in period t, which implies a profit Π_{jt}^h for the firm that decides to innovate, and Π_{jt}^l for the firm that decides not innovate.

2.5 The Problem of the Intermediate Firm

Each intermediate firm maximizes the following Bellman equation:

$$V(z,\mu) = Max\{V(z,\mu)^h, V(z,\mu)^l\} \tag{9}$$

$V(z,\mu)$ represent the decision that the firm must take to innovate or not at each period, where V^h is the value if the firm decides to innovate and V^l a value if the firm decides not to innovate, z represents the productivity of firm j and μ the distribution of firms' productivity in period t.

The value of innovate can be defined as:

$$V(z,\mu)^h = \Pi^h - f - \kappa_f + \frac{1-\chi}{1+r} EMax[V(z+u+\varepsilon), 0] \tag{10}$$

Where χ is the an fixed exogenous probability of exit, κ_f is a fixed operating cost. Firms can, at each period, decide between, continue producing and quit permanently, making that decision after having revealed the values of u and ε. The max operator refers to the optimal decision of exit or continuation.

The value of not innovate is given by:

$$V(z,\mu)^l = \Pi^l - \kappa_f + \frac{1-\chi}{1+r} EMax[V(z+\varepsilon), 0] \tag{11}$$

In the continuation decision, the firm choose a threshold z_{ex} below which the firm exits. The optimal threshold of exit satisfies:

$$V(z_{ex},\mu) = Max\{V(z_{ex},\mu)^h, V(z_{ex},\mu)^l\} = 0 \tag{12}$$

The profits for a firm with $i \in (h,l)$, are given by: **Proof in appendix A1**

$$\Pi^i_{jt} = \frac{1}{\sigma} \left(\frac{A^i_{jt}(z_{jt})}{\bar{A}_t(\mu_t)} \right)^{\sigma-1} P_t Y_t \tag{13}$$

Where μ_t represents the distribution of firms' productivity in a given period, this term will be better presented in the subsection of Productivity Distribution.

2.6 Entry

There is free entry of firms, for a cost κ_e. New entrants draw their initial productivity from a distribution $\eta(z)$, and "e" is the entry rate.Entry is optimal if its value is higher than its cost.

2.7 Productivity Distribution

The law of motion of the productivity distribution (μ), represents the aggregate distribution of the economy and is given by:

$$\mu' = Q\mu + e\eta \tag{14}$$

Where Q represents the productivity transition operator, which summarizes the effect of all the transitions that occurred in the productivity of the firms that remained active in the period transition, thus representing the changes in productivity distribution of the previous period. "e" is the entry rate of the economy and η is the distribution from where firms that enter the market draw their initial productivity. In line with Luttmer (2012) and Gabler and Poschke (2013), a stationary distribution of μ exists as long as the exit rate χ is large enough.

2.8 Definition of Competitive Equilibrium

Competitive equilibrium consists of allocations $\{C_t, L_t\}$, the price vector $\{W_t, r_t\}$, the measure of innovating firms $m_t(\mu_t) \in [0,1]$, probability χ, $\varepsilon_t \sim iidN(0, \zeta_z^{-1})$, and distributions $\{\mu_t\}$, such that:

1)The allocations $\{C_t, L_t\}$ solve the households' problem given prices.

2)The allocation $\{L_t\}$ solve the firms' problem given prices.

3) The market clears

$$L_t = 1 = \left(\int_{j \in J_t^h} L_{jt}^h \, dj + \int_{j \in J_t^l} L_{jt}^l \, dj \right) \tag{15}$$

2.9 Equilibrium

In this section we will establish the functional forms of the final goods production function Y_t, the individual production function of the intermediate firms Y_{jt}^i and the profit of the intermediate firms Π_{jt}^i, due to the optimal choices of capital and labor, where $i \in \{h, l\}$.

Proposition1 : For a measure m_t of innovating firms, the equilibrium level of the final good is given by:

$$Y_t = \bar{A}_t(\mu_t) L_t^{1-\alpha}, \tag{16}$$

where:

$$\bar{A}_t(\mu_t) = \left(\int_{j \in J_t^h} A_{jt}^{h\sigma-1}(z_{jt}) \, dj + \int_{j \in J_t^l} A_{jt}^{l\sigma-1}(z_{jt}) \, dj \right)^{\frac{1}{\sigma-1}}, \tag{17}$$

And the production and profit levels of the intermediate firms of a given type $i \in \{h, l\}$ are given by:

$$Y^i_{jt} = \left(\frac{A^i_{jt}(z_{jt})}{\bar{A}_t(\mu_t)} \right)^{\sigma} Y_t \tag{18}$$

and

$$\Pi^i_{jt} = \frac{1}{\sigma} \left(\frac{A^i_{jt}(z_{jt})}{\bar{A}_t(\mu_t)} \right)^{\sigma-1} P_t Y_t \tag{19}$$

Proof in the Appendix A1.

This proposition 1 is quite similar to that presented in Schaal and Taschereau-Dumouchel (2015) and in Machado (2017), differing mainly due to the fact that firms differentiate themselves in terms of productivity even within the same type $i \in \{h, l\}$, due to the mechanism of innovation through experimentation.

An interesting point to highlight is the dependence of the individual production of intermediate firms on aggregate demand, which occurs due to the existence of complementarity of demand in the monopolistic competition environment. Thus, an environment of low aggregate demand generates disincentives to current production, which may in this model lead to the decision of firms to invest in innovation due to the cost in terms of current production.

As in Schaal and Taschereau-Dumouchel (2015) the production of the final goods firm can be represented by a Cobb-Douglas production function with TFP given by $\bar{A}_t(\mu_t)$ which represents a combination of the productivity levels of the intermediate firms, that is, of the current productivity distribution μ_t. In this model the aggregate output decreases with the increase of the number of innovative firms m_t due to innovation costs.

2.10 The impact of changes on $m(\mu)$

In this section, a comparative static analysis will be presented, verifying how a change in the measure $m_t(\mu_t)$ of firms that have decided to innovate, in a given period t, affects the payoff of innovating and the decision to innovate of a firm j.

It is important to point out that since the model considers a measure of intermediate firms, it will be considered that when an agent makes his decision to innovate he disregards the impact of his decision on the distribution of productivity in the following period.

Table 1 – Glossary

Index	Description
α	Parameter of the Cobb-Douglas production function of the intermediate firm
β	Intertemporal discount factor
σ	Parameter of the CES function of the Firm producing final goods
γ	Utility function parameter
χ	Fixed exogenous probability of exit
θ	Disruption cost $\theta \in (0,1)$
a_t	Assets held at the period t
f	Traditional fixed cost
z_{jt}	Productivity of firm j at time t
z_{ex}	Threshold of productivity below which the firm exits
u	The result for the experiment of firm j at time t
ε	Exogenous productivity shock of firm j at time t
μ_t	Productivity distribution
e	Entry rate of the economy
η	Distribution from where firms that enter the market draw their initial productivity
κ_e	Fixed entry cost
κ_f	Fixed operating cost
m_t	Measure of active firms that innovate in a given period t
τ	Aggregate Distortion
$\tau(z)$	Productivity-dependent distortion

As discussed in the previous section, the decision to innovate of a given firm j is given by:

$$V(z,\mu) = Max\{V(z,\mu)^h, V(z,\mu)^l\} \tag{20}$$

And the innovation payoff given by:

$$\Delta V(z,\mu) = V(z,\mu)^h - V(z,\mu)^l \tag{21}$$

By replacing the forms of profits, this payoff can be rewritten as:

$$\Delta V(z,\mu) = \frac{1}{\sigma}\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}(\mu)^{\sigma-1}} P_t Y_t(\mu)\right] + \frac{1-\chi}{1+r}\Delta V_c - f \tag{22}$$

And finally, for $L_t = 1$, $P_t = 1$, functional forms of A_j^h,A_j^l and the functional form of Y_t:

$$\Delta V(z,\mu) = \frac{1}{\sigma}\left[((Ae^{z_j})^{\sigma-1}(\theta^{\sigma-1} - 1))\bar{A}(\mu)^{2-\sigma}\right] + \frac{1-\chi}{1+r}\Delta V_c(\mu) - f \tag{23}$$

Where:

$$\Delta V_c = EMax[V(z+u+\varepsilon),0] - EMax[V(z+\varepsilon),0] \tag{24}$$

Realize that, as $\theta \in (0,1)$ and $\sigma > 1$, the term $(\theta^{\sigma-1} - 1)$ is negative, so that the first component of the above equation can be interpreted as the present cost of innovation, the second component of the equation can be interpreted as the expected value of the incremental production of u units.

Due to the cost in terms of current productivity of innovating, we have that the term $\bar{A}$ is decreasing in m_t, with this the effect on cost depends on the term σ as forward will be discussed. It will also be necessary to find out the effect of the measure $m_t(\mu)$ in the expected gain. At first observe the profit function of a given firm j:

$$\Pi^i_{jt} = \frac{1}{\sigma}\left(\frac{A^i_{jt}(z_{jt})}{\bar{A}_t(\mu_t)}\right)^{\sigma-1} Y_t(\mu) \tag{25}$$

Note that a change in m can have two effects on profit, an effect with respect to market loss represented in the ratio $\frac{A^i_{jt}(z_{jt})}{\bar{A}_t(\mu_t)}$,and a second effect with respect to the demand externality present in the model, which occurs via the presence of Y_t in the profit function.

Substituting the functional form of Y_t.

$$\Pi^i_{jt} = \frac{1}{\sigma}\left(A^i_{jt}(z_{jt})\right)^{\sigma-1} \bar{A}(\mu)^{2-\sigma} \tag{26}$$

So the effect of an increase in $m_t$in the profit of a given firm depends on the value of σ, already considering $\sigma > 1$. Three scenarios are possible, for $\sigma \in (1,2)$ the externality effect is greater than the substitution effect, for $\sigma = 2$ effects cancel out, and for $\sigma > 2$ the substitution effect is greater than the externality effect.

For $\sigma \in (1,2)$ the increase in m_t reduces the present cost of innovation, $\sigma = 2$ the externality and substitution effects cancel out and the cost is independent of m_t, and for $\sigma > 2$ the cost is increasing in m_t.

Turning now to the term ΔV_c:

$\Delta V_c = EMax[V(z+u+\varepsilon),0] - EMax[V(z+\varepsilon),0]$

The increase in m_t implies an increase in the distribution of productivity in the following periods regardless of the agents' future choices, this is due to the positive productivity gain for firms that decide to innovate, which affects the distribution. It's therefore necessary to say what the expectations behavior of a firm j under the future payoff with respect to the distribution of productivity of the agents.

For $\sigma \in (1, 2)$ the externality effect is greater than the competition effect, so that the future gain will be increasing in the distribution and thus in m_t.

For $\sigma = 2$ the future gains will not depend on the distribution of productivity due to the cancellation of the effects.

And finally to $\sigma > 2$ the surplus of productivity "u" generated in the future will have a decreasing value in m_t, and this is due to the firm preferring to have a superior production in an environment in which the other firms present low productivity, being able to maintain a larger market share.

Hsieh and Klenow (2014) consider $\sigma = 3$,this way we will analyze this scenario in the following discussion.Now we will analyze how the increase in m affects the innovation decision.

- The firm will choose to innovate if:

  $\Delta V \geq 0 \longrightarrow \Pi^h - \Pi^l - f + \frac{1-\chi}{1+r}\Delta V_c \geq 0$

- The firm will choose not to innovate if:

  $\Delta V \leq 0 \longrightarrow \Pi^h - \Pi^l - f + \frac{1-\chi}{1+r}\Delta V_c \leq 0$

- The firm will be indifferent if:

  $\Delta V = 0$

  From i, we say that an firm j will decide to innovate, in a given period t if:
  $\Delta V \geq 0 \longrightarrow \Pi^h - \Pi^l - f + \frac{1-\chi}{1+r}\Delta V_c \geq 0$

$$\frac{1}{\sigma}\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}}P_t Y_t\right] + \frac{1-\chi}{1+r}\Delta V_c - f \geq 0 \tag{27}$$

Under $P_t = 1$, $L_t = 1$ and $Y_t = \bar{A}L^{\alpha}$.

$$\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}(\mu_t)^{\sigma-1}}\right]\bar{A} \geq \sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right) \tag{28}$$

Substituting the functional forms of A_{jt}^h and A_{jt}^l :

$$(Ae^{z_j})^{\sigma-1}(\theta^{\sigma-1} - 1)\bar{A}(\mu_t)^{2-\sigma} \geq \sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right) \tag{29}$$

Then a firm j will decide to innovate if:

$$(Ae^{z_j})^{\sigma-1}\bar{A}(\mu_t)^{2-\sigma} \geq \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1} - 1)} \tag{30}$$

As there is a cost in terms of current production in the decision to innovate, f a larger number of firms have decided to innovate the term $\bar{A}_t$ will be lower than in a scenario with a lower value of $m_t(\mu_t)$. What for $\sigma > 2$ implies that the left side of the inequality increases.

Now, looking at the term on the right side, we can see that considering $\frac{1-\chi}{1+r}\Delta V_c > f$ the numerator signal will be negative, and as $\theta \in (0,1)$ and $\sigma > 1$ the denominator shows a negative sign, so that the right side of the inequality shows a positive sign.

Let us now consider the impact of an increase in $m_t(\mu_t)$ under the term ΔV_c. This term ΔV_c represents the expectation of the present value of a surplus of "u" units of production, where "u" represents a positive productivity draw if the firm decides to innovate. As $m_t(\mu_t$ represents the measure of active firms innovating in t, the high m implies a higher distribution in the following periods. With this, the problem can be identified as in which scenario a surplus production presents greater value, in an environment where most firms have higher productivity or when other firms have low productivity.

As under $\sigma > 2$ the gain of a higher productivity is greater when the other firms present low productivity, the effect of an increase of m_t on the term ΔV_c is negative, so that:

$$\uparrow [(Ae^{z_j})^{\sigma-1}\bar{A}(\mu_t)^{2-\sigma}] \geq \frac{\sigma\left(f - \frac{1-\chi}{1+r} \downarrow \Delta V_c\right)}{(\theta^{\sigma-1} - 1)} \tag{31}$$

such that:

$$(Ae^{z_j})^{\sigma-1} \uparrow \bar{A}(\mu_t)^{2-\sigma} \geq\downarrow B_H \tag{32}$$

It is possible to notice that an elevation of $m_t(\mu_t)$ makes the firms more likely to innovate. The intuition of this result concerns to the fact that when in the period t many firms decided to innovate this decision pressured the rest of firms to also innovate under the risk of losing market space.

2.11 Analysis of the Equilibrium

Each intermediate firm maximizes the following Bellman equation:

$$V(z,\mu) = Max\{V(z,\mu)^h, V(z,\mu)^l\} \tag{33}$$

As discussed in the firm's problem section, $V(z)$ represent the decision that the firm must take to innovate or not at each period, where V^h is the value if the firm decides to innovate and V^l a value if the firm decides not innovate.

The value of innovate can be defined as:

$$V(z,\mu)^h = \Pi^h - f - \kappa_f + \frac{1-\chi}{1+r} EMax[V(z+u+\varepsilon),0] \tag{34}$$

The value of not innovate is given by:

$$V(z,\mu)^l = \Pi^l - \kappa_f + \frac{1-\chi}{1+r} EMax[V(z+\varepsilon),0] \tag{35}$$

Proposition 2: Consider the following condition on parameters:

$$\theta^{2-\sigma} > 1 \tag{36}$$

Under this condition,in a given period t, there exist thresholds $B_L > B_H$, such that:

i)if $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) > B_L$, $\forall\ j \in J_t$, the equilibrium is unique and all firms choose to innovate, $m_t = 1$;

ii)if $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) < B_H$, $\forall\ j \in J_t$, the equilibrium is unique and all firms choose to not innovate, $m_t = 0$;

iii)if $B_l \geq A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \geq B_H$ for some $j \in J_t$, there is an equilibrium in mixed strategies, $m_t \in (0,1)$, which coexists with the two equilibria in pure strategies.

Where $e^{\bar{z}}(\mu)$ is given by:

$$e^{\bar{z}}(\mu) = \left[\int_{j\in J_t} (e^{z_j})^{\sigma-1} dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{37}$$

With:

$$\frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}} \equiv B_H \tag{38}$$

$$\frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)} \equiv B_L \tag{39}$$

Proof in the Appendix A2

Similarly to the result obtained by Schaal and Taschereau-Dumouchel (2015), in the present model the equilibria in pure strategies coexist with the equilibrium in mixed strategies under a condition on the parameters. The result obtained differs due to the importance of the productivity distribution of the firms and the difference between the continuation gains ΔV_c to determine the thresholds, to the determination of the equilibrium.

A balance in which all firms choose to innovate will occur if $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) > B_L$, $\forall\ j \in J_t$ $\forall j \in J_t$, which is more likely to occur in an environment where there is a high productivity distribution. A balance where all firms decide not to innovate will occur if $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) < B_H$, $\forall j \in J_t$, more likely to occur in an environment of low productivity distribution.

The equilibrium in mixed strategies can be realized if for some $j \in J_t$, $B_L > A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) > B_H$, so that $m_t \in (0,1)$, which coexists with the two equilibria in pure strategies.

An interesting aspect that can be perceived is the fact that the decision of not innovate by all firms generates a balance of greater aggregate production, this is due to the cost in terms of current production generated by the innovation, balanced with increments in the future productivity.

3 DISTORTIONS AND PRODUCTIVITY

Recent studies such as Hsieh and Klenow (2009) and Restuccia and Rogerson (2017) emphasize the importance of distortions in the understanding of differences in the level of productivity of the countries, and from this in the analysis of the growth of the countries.

In the present work, due to the development of an endogenous productivity model, distortions affect not only the allocation of resources of the countries, but also the path of productivity evolution.

In the following subsections the impacts of two types of distortions will be developed, first aggregate distortions in the line of Restuccia and Rogerson (2008), and then productivity-dependent distortion in the line of Gabler and Poschke (2013). In this section it will be assumed that any net tax revenue is returned lump-sum to households.

3.1 Aggregate distortions

In the presence of agreggate distortions in line with Restuccia and Rogerson (2008), the values of the choices of innovating and not innovating are now given by:

$$V(z,\mu)^h = (1-\tau)\Pi^h - f - \kappa_f + \frac{1-\chi}{1+r}EMax[V(z+u+\varepsilon),0] \tag{40}$$

$$V(z,\mu)^l = (1-\tau)\Pi^l - \kappa_f + \frac{1-\chi}{1+r}EMax[V(z+\varepsilon),0] \tag{41}$$

Where $\tau \in (0,1)$. From these new values we can establish a new proposition about the behavior of firms.

Proposition 3: Consider the following condition on parameters:

$$\theta^{2-\sigma} > 1 \tag{42}$$

In the presence of an aggregate distortion $\tau \in (0,1)$, we have that:

i) if $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) > B_L* \equiv \frac{B_H}{(1-\tau)}$, $\forall\, j \in J_t$, the equilibrium is unique and all firms choose to innovate, $m_t = 1$;

ii) if $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) < B_H* \equiv \frac{B_L}{(1-\tau)}$, $\forall j \in J_t$, the equilibrium is unique and all firms choose to not innovate, $m_t = 0$;

iii) if $B_L* \geq A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \geq B_H*$ for some $j \in J_t$, there is an equilibrium in mixed strategies, $m_t \in (0,1)$, which coexists with the two equilibria in pure strategies.

Where $e^{\bar{z}}(\mu)$ is given by:

$$e^{\bar{z}}(\mu) = \left[\int_{j\in J_t} (e^{z_j})^{\sigma-1} dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{43}$$

With:

$$\frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}(1-\tau)} \equiv B_H* \tag{44}$$

$$\frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)(1-\tau)} \equiv B_L* \tag{45}$$

Proof in the Appendix A4

It is possible to see that the presence of aggregate distortions modifies the choice of agents, hindering the emergence of the equilibrium in pure strategies where all firms innovate, and facilitating the occurrence of equilibrium in pure strategies where firms do not innovate.

An aspect to be emphasized is that aggregate distortion affects present and future returns in the same way, also presenting a negative effect on capital accumulation due to interference in firm profitability.

3.2 Productivity-dependent distortions

In this subsection we attribute distortions only to firms with productivity $z > z*$. Two types of distortions will be considered, the first concerns a distortion $\tau \in (0,1)$, and then we will analyze the impact of an increasing distortion in productivity, $\tau(z) = \tau_0 + \tau_1(z)$.

3.2.1 Constant productivity-dependent distortion

In this subsection we will consider the first type of productivity-dependent distortion. In this scenario, for $\tau \in (0,1)$, the firms with $z > z*$ show thresholds B_H* e B_L*, while firms with $z < z*$ show thresholds B_H e B_L.

The presence of distortion modifies the choices made by intermediate firms with productivity above threshold z*, hindering the balance in pure strategies where everyone innovates, making it more likely to balance in pure strategies where nobody innovates and also makes difficult the accumulation of capital of the firms subject to the tax. The distortion affects present and future values in a different way, this is due to the presence of the productivity threshold that if the firm exceeds it will be taxed in a different way.

3.2.2 Variable distortion in z

Now consider a distortion $\tau(z) = \tau_0 + \tau_1(z)$ with τ_1 increasing in z in line with Gabler and Poschke (2013).

Proposition 4: In the presence of a productivity-dependent distortion $\tau(z) = \tau_0 + \tau_1(z)$ with τ_1 increasing in z, we have:

i)If for all firms with $z > z*$: $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) > B'_L \equiv \frac{B_L}{(1-\tau(z))}$, while for all the other firms with $z < z*$: $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu)K^{\alpha} > B_L$, the equilibrium is unique and all firms choose to innovate, $m_t = 1$;

ii) If for all firms with $z > z*$: $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) < B'_H \equiv \frac{B_H}{(1-\tau(z))}$, while for all the other firms with $z < z*$: $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu)K^{\alpha} < B_H$, the equilibrium is unique and all firms choose to not innovate, $m_t = 0$;

iii) if $B'_L \geq A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \geq B'_H$ for some firm with z>z* or if $B_L \geq A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu)K^{\alpha} \geq B_H$ for some firm with z<z*, there is an equilibrium in mixed strategies, $m_t \in (0,1)$, which coexists with the two equilibria in pure strategies.

Where $e^{\bar{z}}(\mu)$ is given by:

$$e^{\bar{z}}(\mu) = \left[\int_{j \in J_t} (e^{z_j})^{\sigma-1} dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{46}$$

With:

$$\frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}(1-\tau(z))} \equiv B'_H \tag{47}$$

$$\frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)(1-\tau(z))} \equiv B'_L \tag{48}$$

Proof in the Appendix A4

This type of taxation, although it seems unusual, due to the productivity of the firms be an unobservable feature, they can capture the essence of size-dependent distortions, as Gabler and Poschke (2013) point out.

In this environment the distortion is increasing in z, which produces still more damaging impacts to the dynamics of innovation of the firms, in comparison with the previous subsection, and as well as the previous subsection, distortion affects in different ways current and future profits. This point is important because taxation will have different impacts on the relationship between the present cost of experimentation and the future benefits of experimentation.

4 CONCLUSION

The present work proposed a general equilibrium model with partially endogenous productivity in an environment conducive to coordination failures of the agents.The model also has an endogenous entry and a partially endogenous exit. The mechanism from which firms are able to influence their future productivity approaches the literature of "innovation by experimentation", from the model developed by Gabler and Poschke (2013).

The coordination problem in the model is developed as in Schaal and Taschereau -Dumouchel (2015), the present work extends their analysis to an environment where the intermediate firms are able to influence their future productivity from the realization of experiments,and with this, becoming able to make inferences about the role of productivity-dependent distortions and aggregate distortions in the multiplicity of equilibria and in turn in the productivity dynamics of the economy.

This work can be extended by different fronts in later research. The first concerns the analysis of the multiplicity of equilibria with a view to a possible uniqueness and subsequent calibration of the model to the moments of reality, which would allow important quantitative analyzes of the role of distortions in economic growth. In this model firms can get rid of unsuccessful experiments at zero cost, it may be interesting to relax this hypothesis. Another promising line corresponds to the extension of the model to a framework where firms are able to choose different variances for the productivity shock generated by the experiment, from the payment of a variable cost of disruption, which may provide interesting insights, due to their greater proximity to the actual environment of experimentation.

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6 Appendix

A - Proofs

A1- Equilibrium

Proposition1 : For a measure m_t of innovating firms, the equilibrium level of the final good is given by:

$$Y_t = \bar{A}_t(\mu_t) L_t^{1-\alpha}, \tag{49}$$

where:

$$\bar{A}_t(\mu_t) = \left(\int_{j \in J_t^h} A_{jt}^{h\sigma-1}(z_{jt})\, dj + \int_{j \in J_t^l} A_{jt}^{l\sigma-1}(z_{jt})\, dj \right)^{\frac{1}{\sigma-1}}, \tag{50}$$

And the production and profit levels of the intermediate firms of a given type $i \in \{h, l\}$ are given by:

$$Y_{jt}^i = \left(\frac{A_{jt}^i(z_{jt})}{\bar{A}_t(\mu_t)} \right)^{\sigma} Y_t \tag{51}$$

Proof 1:

a)The Household solves:

$$E\Sigma_{t=0}^{\infty}\beta^t U(C_t) = \begin{cases} E\Sigma_{t=0}^{\infty}\beta^t \frac{C_t^{1-\gamma}}{1-\gamma}, & \text{for } \gamma > 0, \gamma \neq 0 \\ E\Sigma_{t=0}^{\infty}\beta^t log(C_t), & \text{for } \gamma \to 1 \end{cases} \tag{52}$$

st.

i)$C_t + a_{t+1} \leq W_t L_t + a_t(1+r)$

ii)$C_t \geq 0, L_t = 1$

iii)Given a_0

From household maximization, the Euler Condition is given by:

$$C_t^{-\gamma} = \beta(1 + r_{t+1})C_{t+1}^{-\gamma} \tag{53}$$

b)Problem of the firm producing the final good:

The firm producing the final good solves:

$$Max \left[P_t \left(\int_{j \in J_t} Y_{jt}^{\frac{\sigma-1}{\sigma}}\, dj \right)^{\frac{\sigma}{\sigma-1}} - \int_{j \in J_t} P_{jt} Y_{jt}\, dj \right] \tag{54}$$

Let $\frac{\sigma}{\sigma-1} = \epsilon$

We can rewrite the equation so that:

$$Max\{P_t \left(\int_{j \in J_t} Y_{jt}^{\frac{1}{\epsilon}}\, dj \right)^{\epsilon} - \int_{j \in J_t} P_{jt} Y_{jt}\, dj\} \tag{55}$$

FOC:

$\frac{\partial \Pi}{\partial Y_{jt}} = \epsilon P_t \left(\int_{j \in J_t} Y_{jt}^{\frac{1}{\epsilon}}\, dj \right)^{\epsilon-1} \frac{1}{\epsilon} Y_{jt}^{\frac{1}{\epsilon}-1} - P_{jt} = 0$

From the production function:

$Y_t^{\frac{1}{\epsilon}} = \left(\int_{j \in J_t} Y_{jt}^{\frac{1}{\epsilon}}\, dj \right)$

Combining this equation with the FOC, it is possible to obtain:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{\frac{\epsilon}{1-\epsilon}} Y_t \tag{56}$$

Remember that $\frac{\sigma}{\sigma-1} = \epsilon$, such that:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\sigma} Y_t \tag{57}$$

The price of the final good is given by:

$$P_t = \left(\int_{j\in J_t} P_{jt}^{1-\sigma}\, dj\right)^{\frac{1}{1-\sigma}} \tag{58}$$

c) Firms producing intermediate goods:
Let $A^h_{jt}(z_{jt}) = \theta A e^{z_{jt}}$ and $A^l_{jt}(z_{jt}) = A e^{z_{jt}}$ where $\theta \in (0,1)$. So that:

$$Y^i_{jt} = A^i_{jt}(z_{jt}) L^{i1-\alpha}_{jt} \tag{59}$$

Where $i \in \{h,l\}$. From the market clearing conditions we have:

$$L_t = \left(\int_{j\in J^h_t} L^h_{jt}\, dj + \int_{j\in J^l_t} L^l_{jt}\, dj\right) = 1 \tag{60}$$

Where m_t is the measure of innovating firms.
Partial Equilibrium
Given R_t, W_t and the demand Y_t from final good sector.

$$Max P^i_{jt} Y^i_{jt} - W_t L^i_{jt} \tag{61}$$

Replacing the demand of the firm of final goods for intermediate goods, the production function of intermediate goods and getting the first order conditions in terms of capital and labor, can be achieved:

$$W_t = (1-\alpha)\frac{\sigma-1}{\sigma}\frac{P_t Y_t^{\frac{1}{\sigma}} Y_{jt}^{i1-\frac{1}{\sigma}}}{L^i_{jt}} \tag{62}$$

Combining both equations, we obtain:

$$\frac{\sigma-1}{\sigma} Y_t^{\frac{1}{\sigma}} Y_{jt}^{i-\frac{1}{\sigma}} = \frac{1}{A^i_{jt}(z_{jt})}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \tag{63}$$

Remember that:

$$\frac{P^i_{jt}}{P_t} = \left(\frac{Y^i_{jt}}{Y_t}\right)^{-\frac{1}{\sigma}} \tag{64}$$

Assuming $P_t = 1$

$$P^i_{jt} = \frac{\sigma}{\sigma-1}\frac{1}{A^i_{jt}(z_{jt})}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \tag{65}$$

Substituting in the P_t equation

$$P_t = \left(\int_{j\in J^h_t} P^h_{jt}\,dj + \int_{j\in J^l_t} P^l_{jt}\,dj\right) \tag{66}$$

$$P_t = \frac{\sigma}{\sigma-1}\frac{1}{\bar{A}(\mu_t)}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \tag{67}$$

Where $\bar{A}_t(\mu_t)$ is:

$$\bar{A}_t(\mu_t) = \left(\int_{j\in J^h_t} A^{h\sigma-1}_{jt}(z_{jt})\,dj + \int_{j\in J^l_t} A^{l\sigma-1}_{jt}(z_{jt})\,dj\right)^{\frac{1}{\sigma-1}}, \tag{68}$$

Substituting P^i_{jt} and P_t in the demand for intermediate goods:

$$Y^i_{jt} = \left(\frac{P^i_{jt}}{P_t}\right)^{-\sigma} Y_t = \left(\frac{A^i_{jt}(z_{jt})}{\bar{A}_t(\mu_t)}\right)^{\sigma} Y_t \tag{69}$$

So the production of the final good is given by:

$$Y_t = \left(\int_{j\in J^h_t} Y^{h\frac{\sigma-1}{\sigma}}_{jt}\,dj + \int_{j\in J^l_t} Y^{l\frac{\sigma-1}{\sigma}}_{jt}\,dj\right)^{\frac{\sigma}{\sigma-1}} = \bar{A}_t(\mu_t)L^{1-\alpha}_t. \tag{70}$$

From the first order conditions, we can rewrite:

$$L^i_{jt} = (1-\alpha)\frac{\sigma-1}{\sigma}\left(\frac{A^i_{jt}(z_{jt})}{\bar{A}_t(\mu_t)}\right)^{\sigma-1}\frac{P_tY_t}{W_t} \tag{71}$$

The market clearing condition can be rewritten as:

$$L_t = 1 = \left(\int_{j\in J^h_t} L^h_{jt}\,dj + \int_{j\in J^l_t} L^l_{jt}\,dj\right) = (1-\alpha)\frac{\sigma-1}{\sigma}\frac{P_tY_t}{W_t} \tag{72}$$

And Finally:

$$\Pi^i_{jt} = P^i_{jt}Y^i_{jt} - W_tL^i_{jt} = \frac{1}{\sigma}P^i_{jt}Y^i_{jt} \tag{73}$$

$$\Pi^i_{jt} = \frac{1}{\sigma}\left(\frac{A^i_{jt}(z_{jt})}{\bar{A}_t(\mu_t)}\right)^{\sigma-1} P_tY_t \tag{74}$$

Q.E.D

A2- Analysis of the equilibrium

Proposition 2: Consider the following condition on parameters:

$$\theta^{2-\sigma} > 1 \tag{75}$$

Under this condition,in a given period t, there exist thresholds $B_L > B_H$, such that:

i)if $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) > B_L$, $\forall\ j \in J_t$, the equilibrium is unique and all firms choose to innovate, $m_t = 1$;

ii)if $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) < B_H$, $\forall\ j \in J_t$, the equilibrium is unique and all firms choose to not innovate, $m_t = 0$;

iii)if $B_l \geq A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \geq B_H$ for some $j \in J_t$, there is an equilibrium in mixed strategies, $m_t \in (0,1)$, which coexists with the two equilibria in pure strategies.

Where $e^{\bar{z}}(\mu)$ is given by:

$$e^{\bar{z}}(\mu) = \left[\int_{j\in J_t} (e^{z_j})^{\sigma-1} dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{76}$$

Idea of proof 2:

Remember that:

$$V(z,\mu) = Max\{V(z,\mu)^h, V(z,\mu)^l\} \tag{77}$$

Let:

$$\Delta V(z,\mu) = V(z,\mu)^h - V(z,\mu)^l \tag{78}$$

- The firm will choose to innovate if:

  $\Delta V \geq 0 \longrightarrow \Pi^h - \Pi^l - f + \frac{1-\chi}{1+r}\Delta V_c \geq 0$

- The firm will choose not to innovate if:

  $\Delta V \leq 0 \longrightarrow \Pi^h - \Pi^l - f + \frac{1-\chi}{1+r}\Delta V_c \leq 0$

- The firm will be indifferent if:

  $\Delta V = 0$

Where:

$$\Delta V_c = EMax[V(z+u+\varepsilon), 0] - EMax[V(z+\varepsilon), 0] \tag{79}$$

Proof 2:

i)We say that there is an equilibrium in pure strategy with all firms innovating if $\forall j \in J_t$, in a given period t:

$\Delta V \geq 0 \longrightarrow \Pi^h - \Pi^l - f + \frac{1-\chi}{1+r}\Delta V_c \geq 0$

$$\frac{1}{\sigma}\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}} P_t Y_t\right] + \frac{1-\chi}{1+r}\Delta V_c - f \geq 0 \tag{80}$$

Under $P_t = 1$, $L_t = 1$ and $Y_t = \bar{A}L^{\alpha}$.

$$\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}}\right]\bar{A} \geq \sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right) \tag{81}$$

Substituting the functional forms of A_{jt}^h, A_{jt}^l and $\bar{A}$:

$$\frac{(Ae^{z_j})^{\sigma-1}(\theta^{\sigma-1}-1)}{\int_{j\in J_t}(Ae^{z_j}\theta)^{\sigma-1}dj}\left(\int_{j\in J_t}(Ae^{z_j}\theta)^{\sigma-1}dj\right)^{\frac{1}{\sigma-1}} \geq \sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right) \tag{82}$$

$$A(e^{z_j})^{\sigma-1}\left(\int_{j\in J_t}(e^{z_j})^{\sigma-1}dj\right)^{\frac{2-\sigma}{\sigma-1}} \geq \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}} \tag{83}$$

Let :

$$e^{\bar{z}}(\mu) = \left[\int_{j\in J_t}(e^{z_j})^{\sigma-1}dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{84}$$

and:

$$\frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}} \equiv B_H \tag{85}$$

Then:

$$A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) > B_H \tag{86}$$

ii) We say that there is an equilibrium in pure strategy with all firms choosing to not innovate if $\forall j \in J_t$, in a given period t:

$\Delta V \leq 0 \longrightarrow \Pi^h - \Pi^l - f + \frac{1-\chi}{1+r}\Delta V_c \leq 0$

$$\frac{1}{\sigma}\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}} P_t Y_t\right] + \frac{1-\chi}{1+r}\Delta V_c - f \leq 0 \tag{87}$$

Under $P_t = 1$, $L_t = 1$ and $Y_t = \bar{A}L^{\alpha}$.

$$\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}}\right]\bar{A} \leq \sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right) \tag{88}$$

Substituting the functional forms of A^h_{jt}, A^l_{jt} and $\bar{A}$:

$$\frac{(Ae^{z_j})^{\sigma-1}(\theta^{\sigma-1}-1)}{\int_{j\in J_t}(Ae^{z_j})^{\sigma-1}dj}\left(\int_{j\in J_t}(Ae^{z_j})^{\sigma-1}dj\right)^{\frac{1}{\sigma-1}} \leq \sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right) \tag{89}$$

$$A(e^{z_j})^{\sigma-1}\left(\int_{j\in J_t}(e^{z_j})^{\sigma-1}\right)^{\frac{2-\sigma}{\sigma-1}} \leq \frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)} \tag{90}$$

Let :

$$e^{\bar{z}}(\mu) = \left[\int_{j\in J_t}(e^{z_j})^{\sigma-1}dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{91}$$

and:

$$\frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)} \equiv B_L \tag{92}$$

Then:

$$A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \leq B_L \tag{93}$$

iii) It is now necessary to verify whether equilibria in pure strategies and equilibrium in mixed strategies coexist, which would occur if $B_L > B_H$.

$$\frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)} > \frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}} \tag{94}$$

So this would only happen if:

$$\theta^{2-\sigma} > 1 \tag{95}$$

If $\theta^{2-\sigma} \leq 1$ the balance is always unique.

Let us now present the balance in mixed strategies. Firms are indifferent in their innovation decision if:

$$\frac{1}{\sigma}\left[\frac{A_j^{h\sigma-1}-A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}}P_tY_t\right]+\frac{1-\chi}{1+r}\Delta V_c - f = 0 \tag{96}$$

The equation can be rewritten as:

$$A(e^{z_j})^{\sigma-1}\left(\int_{j\in J_t^h}(e^{z_j}\theta)^{\sigma-1}+\int_{j\in J_t^l}(e^{z_j})^{\sigma-1}\right)^{\frac{2-\sigma}{\sigma-1}} = \frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)} \tag{97}$$

Note that if all firms innovate we have the balance i and if no firm innovate we have the balance ii.

Q.E.D

A3- Distortions and Productivity

Proposition 3: Consider the following condition on parameters:

$$\theta^{2-\sigma} > 1 \tag{98}$$

In the presence of an aggregate distortion $\tau \in (0,1)$, we have that:

i) if $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \geq B_L* \equiv \frac{B_H}{(1-\tau)}$, $\forall\, j \in J_t$, the equilibrium is unique and all firms choose to innovate, $m_t = 1$;

ii) if $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \leq B_H* \equiv \frac{B_L}{(1-\tau)}$, $\forall j \in J_t$, the equilibrium is unique and all firms choose to not innovate, $m_t = 0$;

iii) if $B_L* > A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) > B_H*$ for some $j \in J_t$, there is an equilibrium in mixed strategies, $m_t \in (0,1)$,which coexists with the two equilibria in pure strategies.

Where $e^{\bar{z}}(\mu)$ is given by:

$$e^{\bar{z}}(\mu) = \left[\int_{j\in J_t} (e^{z_j})^{\sigma-1} dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{99}$$

Proof 3

In the presence of an aggregate distortion $\tau \in (0,1)$ it is possible to obtain the thresholds from the same idea used in proof 2, thus we have:

i)We say that there is an equilibrium in pure strategy with all firms innovating if $\forall j \in J_t$, in a given period t:

$\Delta V \geq 0 \longrightarrow (1-\tau)[\Pi^h - \Pi^l] - f + \frac{1-\chi}{1+r}\Delta V_c \geq 0$

$$(1-\tau)\frac{1}{\sigma}\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}} P_t Y_t\right] + \frac{1-\chi}{1+r}\Delta V_c - f \geq 0 \tag{100}$$

Under $P_t = 1$, $L_t = 1$ and $Y_t = \bar{A}L^{\alpha}$.

$$\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}}\right]\bar{A} \geq \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(1-\tau)} \tag{101}$$

Substituting the functional forms of A_{jt}^h, A_{jt}^l and $\bar{A}$:

$$\frac{(Ae^{z_j})^{\sigma-1}(\theta^{\sigma-1}-1)}{\int_{j\in J_t}(Ae^{z_j}\theta)^{\sigma-1}}\left(\int_{j\in J_t}(Ae^{z_j}\theta)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \geq \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(1-\tau)} \tag{102}$$

$$A(e^{z_j})^{\sigma-1}\left(\int_{j\in J_t}(e^{z_j})^{\sigma-1}\right)^{\frac{2-\sigma}{\sigma-1}} \geq \frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}(1-\tau)} \tag{103}$$

Let :

$$e^{\bar{z}}(\mu)=\left[\int_{j\in J_t}(e^{z_j})^{\sigma-1}dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{104}$$

and:

$$\frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}(1-\tau)} \equiv B_H* \tag{105}$$

Then:

$$A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \geq B_H* \tag{106}$$

ii) We say that there is an equilibrium in pure strategy with all firms choosing to not innovate if $\forall j \in J_t$, in a given period t:

$\Delta V \leq 0 \longrightarrow (1-\tau)[\Pi^h - \Pi^l] - f + \frac{1-\chi}{1+r}\Delta V_c \leq 0$

$$(1-\tau)\frac{1}{\sigma}\left[\frac{A_j^{h\sigma-1}-A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}}P_tY_t\right]+\frac{1-\chi}{1+r}\Delta V_c - f \leq 0 \tag{107}$$

Under $P_t = 1$, $L_t = 1$ and $Y_t = \bar{A}L^{\alpha}$.

$$\left[\frac{A_j^{h\sigma-1}-A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}}\right]\bar{A} \leq \frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(1-\tau)} \tag{108}$$

Substituting the functional forms of A^h_{jt}, A^l_{jt} and $\bar{A}$:

$$\frac{(Ae^{z_j})^{\sigma-1}(\theta^{\sigma-1}-1)}{\int_{j\in J_t}(Ae^{z_j})^{\sigma-1}dj}\left(\int_{j\in J_t}(Ae^{z_j})^{\sigma-1}dj\right)^{\frac{1}{\sigma-1}} \leq \frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(1-\tau)} \tag{109}$$

$$A(e^{z_j})^{\sigma-1}\left(\int_{j\in J_t}(e^{z_j})^{\sigma-1}\right)^{\frac{2-\sigma}{\sigma-1}} \leq \frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)(1-\tau)} \tag{110}$$

Let :

$$e^{\bar{z}}(\mu)=\left[\int_{j\in J_t}(e^{z_j})^{\sigma-1}dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{111}$$

and:

$$\frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)(1-\tau)} \equiv B_L* \tag{112}$$

Then:

$$A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \leq B_L* \tag{113}$$

iii) It is now necessary to verify whether equilibria in pure strategies and equilibrium in mixed strategies coexist, which would occur if $B_L > B_H$.

$$\frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)(1-\tau)} > \frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}(1-\tau)} \tag{114}$$

So this would only happen if:

$$\theta^{2-\sigma} > 1 \tag{115}$$

If $\theta^{2-\sigma} \leq 1$ the balance is always unique.

Let us now present the balance in mixed strategies. Firms are indifferent in their innovation decision if:

$$(1-\tau)\frac{1}{\sigma}\left[\frac{A_j^{h\sigma-1}-A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}}P_tY_t\right]+\frac{1-\chi}{1+r}\Delta V_c - f = 0 \tag{116}$$

The equation can be rewritten as:

$$A(e^{z_j})^{\sigma-1}\left(\int_{j\in J_t^h}(e^{z_j}\theta)^{\sigma-1}+\int_{j\in J_t^l}(e^{z_j})^{\sigma-1}\right)^{\frac{2-\sigma}{\sigma-1}} = \frac{\sigma\left(f-\frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)(1-\tau)} \tag{117}$$

Q.E.D

Proposition 4: In the presence of a productivity-dependent distortion $\tau(z) = \tau_0 + \tau_1(z)$ with τ_1 increasing in z, we have:

i)If for all firms with $z > z*$: $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) > B'_L \equiv \frac{B_L}{(1-\tau(z))}$, while for all the other firms with $z < z*$: $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu)K^{\alpha} > B_L$, the equilibrium is unique and all firms choose to innovate, $m_t = 1$;

ii) If for all firms with $z > z*$: $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) < B'_H \equiv \frac{B_H}{(1-\tau(z))}$, while for all the other firms with $z < z*$: $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) < B_H$, the equilibrium is unique and all firms choose to not innovate, $m_t = 0$;

iii) if $B'_L \geq A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \geq B'_H$ for some firm with z>z* or if $B_L \geq A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu)K^{\alpha} \geq B_H$ for some firm with z<z*, there is an equilibrium in mixed strategies, $m_t \in (0,1)$, which coexists with the two equilibria in pure strategies.

Where $e^{\bar{z}}(\mu)$ is given by:

$$e^{\bar{z}}(\mu) = \left[\int_{j\in J_t} (e^{z_j})^{\sigma-1} dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{118}$$

Proof 4:

While firms with z <z * behave in the same way as the firms presented in Proof 2, firms with z> z * behave differently as we will show below.

i)We say that there is an equilibrium in pure strategy with all firms innovating in the presence of a productivity-dependent distortion $\tau(z) = \tau_0 + \tau_1(z)$ with τ_1 increasing in z, if for all the firms with $z < z*$: $A(e^{z_j})^{\sigma-1} e^{\bar{z}}(\mu) \geq B_H$, as proved in proposition 2, and for all firms with $z > z*$: $A(e^{z_j})^{\sigma-1} e^{\bar{z}}(\mu) \geq B'_H \equiv \frac{B_H}{(1-\tau(z))}$.

To obtain this second result, we start from the following equation.

$\Delta V \geq 0 \longrightarrow (1-\tau(z))[\Pi^h - \Pi^l] - f + \frac{1-\chi}{1+r}\Delta V_c \geq 0$

$$(1-\tau(z))\frac{1}{\sigma}\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}} P_t Y_t\right] + \frac{1-\chi}{1+r}\Delta V_c - f \geq 0 \tag{119}$$

Under $P_t = 1$, $L_t = 1$ and $Y_t = \bar{A}L^{\alpha}$.

$$\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}}\right]\bar{A} \geq \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(1-\tau(z))} \tag{120}$$

Substituting the functional forms of A^h_{jt}, A^l_{jt} and $\bar{A}$:

$$\frac{(Ae^{z_j})^{\sigma-1}(\theta^{\sigma-1}-1)}{\int_{j\in J_t}(Ae^{z_j}\theta)^{\sigma-1}}\left(\int_{j\in J_t}(Ae^{z_j}\theta)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \geq \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(1-\tau(z))} \tag{121}$$

$$A(e^{z_j})^{\sigma-1}\left(\int_{j\in J_t}(e^{z_j})^{\sigma-1}\right)^{\frac{2-\sigma}{\sigma-1}} \geq \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}(1-\tau(z))} \tag{122}$$

Let :

$$e^{\bar{z}}(\mu) = \left[\int_{j\in J_t} (e^{z_j})^{\sigma-1} dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{123}$$

and:

$$\frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}(1-\tau(z))} \equiv B'_H \tag{124}$$

Then:

$$A(e^{z_j})^{\sigma-1} e^{\bar{z}}(\mu) \geq B'_H \tag{125}$$

ii) We say that there is an equilibrium in pure strategy with all firms choosing not innovating in the presence of a productivity-dependent distortion $\tau(z) = \tau_0 + \tau_1(z)$ with τ_1 increasing in z, if for all the firms with $z < z*$: $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \leq B_L$, as proved in proposition 2, and for all firms with $z > z*$: $A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \leq B'_L \equiv \frac{B_H}{(1-\tau(z))}$.

To obtain this second result, we start from the following equation.

$\Delta V \leq 0 \longrightarrow (1-\tau(z))[\Pi^h - \Pi^l] - f + \frac{1-\chi}{1+r}\Delta V_c \leq 0$

$$(1-\tau(z))\frac{1}{\sigma}\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}}P_t Y_t\right] + \frac{1-\chi}{1+r}\Delta V_c - f \leq 0 \tag{126}$$

Under $P_t = 1$, $L_t = 1$ and $Y_t = \bar{A}L^\alpha$.

$$\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}}\right]\bar{A} \leq \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(1-\tau(z))} \tag{127}$$

Substituting the functional forms of A^h_{jt}, A^l_{jt} and $\bar{A}$:

$$\frac{(Ae^{z_j})^{\sigma-1}(\theta^{\sigma-1}-1)}{\int_{j\in J_t}(Ae^{z_j})^{\sigma-1}dj}\left(\int_{j\in J_t}(Ae^{z_j})^{\sigma-1}dj\right)^{\frac{1}{\sigma-1}} \leq \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(1-\tau(z))} \tag{128}$$

$$A(e^{z_j})^{\sigma-1}\left(\int_{j\in J_t}(e^{z_j})^{\sigma-1}\right)^{\frac{2-\sigma}{\sigma-1}} \leq \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)(1-\tau(z))} \tag{129}$$

Let :

$$e^{\bar{z}}(\mu) = \left[\int_{j\in J_t}(e^{z_j})^{\sigma-1}dj\right]^{\frac{2-\sigma}{\sigma-1}} \tag{130}$$

and:

$$\frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)(1-\tau(z))} \equiv B'_L \tag{131}$$

Then:

$$A(e^{z_j})^{\sigma-1}e^{\bar{z}}(\mu) \leq B'_L \tag{132}$$

iii) It is now necessary to verify whether equilibria in pure strategies and equilibrium in mixed strategies coexist, which would occur if $B'_L > B'_H$.

$$\frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)(1-\tau(z))} > \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)\theta^{2-\sigma}(1-\tau(z))} \tag{133}$$

So this would only happen if:

$$\theta^{2-\sigma} > 1 \tag{134}$$

If $\theta^{2-\sigma} \leq 1$ the balance is always unique.

Let us now present the balance in mixed strategies. Firms are indifferent in their innovation decision if:

$$(1-\tau(z))\frac{1}{\sigma}\left[\frac{A_j^{h\sigma-1} - A_j^{l\sigma-1}}{\bar{A}^{\sigma-1}} P_t Y_t\right] + \frac{1-\chi}{1+r}\Delta V_c - f = 0 \tag{135}$$

The equation can be rewritten as:

$$A(e^{z_j})^{\sigma-1}\left(\int_{j\in J_t^h}(e^{z_j}\theta)^{\sigma-1} + \int_{j\in J_t^l}(e^{z_j})^{\sigma-1}\right)^{\frac{2-\sigma}{\sigma-1}} = \frac{\sigma\left(f - \frac{1-\chi}{1+r}\Delta V_c\right)}{(\theta^{\sigma-1}-1)(1-\tau(z))} \tag{136}$$

Q.E.D