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JOÃO MATEUS MARQUES DE SANTANA

MODELING MAINTENANCE SERVICE CONTRACTS OF A MULTI-EQUIPMENT AND TWO-CLASS PRIORITY GENERALIZED RENEWAL PROCESS

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ABSTRACT

Due to increasing equipment technology, maintenance activities for complex devices require considerable training, resources, and spare parts for adequate execution. Also, manufacturers often adopt protectionist actions, such as limiting information about maintenance actions, and not providing spare parts to the market. In this context, performing maintenance actions in-house has become costly and inefficient, thus organizations resort to hiring external maintenance services, such as independent service providers or the manufacturer itself. This situation leads to the interaction between different agents; particularly, between service provider and customer (equipment owner). In this work, a model for maintenance service contracts is developed based on a Stackelberg game formulation, considering the interaction between manufacturer, which acts as service provider, and customers, who decide whether to buy a device and which kind of service to hire. Customers are divided into two distinct classes: class 1 is composed by large organizations, which prefer higher equipment availability over cost; class 2 is formed by small organizations, which prefer to pay lower prices for services, even if equipment availability is compromised. Equipment failure-repair behavior follows a generalized renewal process and, since there are multiple devices, each bought by a different customer, a queue system is formed. A discrete event simulation approach is proposed for the solution of the model, overcoming limitations imposed by analytical methods due to model complexity. An application example is presented, along with sensitivity analysis over several of the model's parameters, with the objective of better understanding model behavior.

Keywords: Maintenance service contracts. Stackelberg game. Generalized renewal process. Discrete event simulation.

RESUMO

Devido à evolução da tecnologia, a execução adequada de atividades de manutenção para equipamentos complexos requer treinamento, recursos, e disponibilidade de sobressalentes. Além disso, fabricantes frequentemente adotam ações protecionistas, como a limitação de informações sobre práticas de manutenção e restrição da oferta de partes sobressalentes no mercado. Nesse contexto, realizar manutenção dentro da organização se torna custoso e ineficiente; por isso, é comum a contratação de serviços de manutenção externos, como serviços terceirizados de manutenção ou o próprio fabricante do equipamento. Essa situação leva à interação entre diferentes agentes; particularmente, entre o provedor do serviço e o cliente (dono do equipamento). Neste trabalho, um modelo para contratos de serviços de manutenção é desenvolvido, baseado em uma formulação de jogo de Stackelberg, considerando a interação entre fabricante, que age como um provedor de serviço, e clientes, que decidem pela compra de um equipamento e pelo tipo de serviço contratado. Clientes são divididos entre duas classes distintas: classe 1 é composta por grandes organizações, com preferência por alta disponibilidade de equipamento, mesmo a custos altos; classe 2 é formada por pequenas organizações, que preferem pagar menos pelos serviços, mesmo que isso prejudique a disponibilidade do equipamento. A dinâmica de falhas e reparos dos equipamentos segue um processo de renovação generalizado e, visto que existem múltiplos equipamentos, cada um comprado por um cliente diferente, um sistema de filas com prioridades é formado. Uma abordagem por simulação de eventos discretos é proposta para a solução do modelo, superando limitações impostas por métodos analíticos devido à complexidade do modelo. Um exemplo de aplicação é apresentado, assim como uma análise de sensibilidade sobre vários parâmetros, com o objetivo de permitir melhor entendimento sobre o comportamento do modelo.

Palavras-chave: Contratos de serviços de manutenção. Jogo de Stackelberg. Processo de renovação generalizado. Simulação de eventos discretos.

ILLUSTRATION LIST

TABLE LIST

SYMBOLS

Symbol	Unit	Description
A_k	–	Decision alternative k
A^*	–	Optimal decision alternative
$A^{(c)*}$	–	Optimal decision alternative for a class c customer
c	–	Customer class; $c = 1$ for priority class, $c = 2$ for nonpriority class
C_b	$ 10^3	Device purchase price
C_r	$ 10^3	Mean cost to repair (incurred to service provider)
$C_s^{(c)}$	$ 10^3	Price charged for a repair (incurred to customer of class c)
$C_s^{(c)*}$	$ 10^3	Maximum price a customer of class c pays for a repair
$D_j^{(c)}$	h	Total downtime of device j in class c
$D_{i,j}^{(c)}$	h	Total downtime of device j in class c during coverage period i
m	–	Number of service crews
M	–	Total number of customers (of both classes)
$M^{(c)}$	–	Number of customers in class c
n_{rep}	–	Number of replications for Monte Carlo simulation
$N_j^{(c)}$	–	Total number of failures for customer j in class c
$N_{i,j}^{(c)}$	–	Number of failures for device j in class c during coverage period i
$O_j^{(c)}$	h	Total overtime of device j in class c
$O_{i,j}^{(c)}$	h	Overtime of device j in class c during coverage period i
$P_w^{(c)}$	$ 10^3	Price of extended warranty contract charged to a customer of class c
$P_w^{(c)*}$	$ 10^3	Maximum price a customer of a class c pays for extended warranty
p_0	–	Empty system probability, i.e., probability of no failed units
$p_{a,b,c}$	–	Probability of a failed priority units, b failed nonpriority units, unit of class c being repaired
q	–	GRP-Weibull distribution parameter for repair effectiveness
r	h	Repair duration, excludes time in queue
$R^{(c)}$	$ 10^3 / h	Revenue per operational hour generated by a class c device

Symbol	Unit	Description
s_0	–	Empty system state, i.e., state at which there are no failed units
$s_{a,b,c}$	–	State where there are a failed priority units, b failed nonpriority units, unit of class c being repaired
T	h	Total analysis coverage duration; covers base warranty and extended warranty periods
T_1	h	Duration of base warranty
T_2	h	Duration of extended warranty
$T_j^{(c)}$	h	Total operational time of device j in class c ($D_j^c + T_j^c = T$)
$T_{i,j}^{(c)}$	h	Operational time of device j in class c during coverage period i
$t_a^{(c)}$	h	Time of next failure of an equipment of class c
$t_a^{(c)\prime}$	h	Time of next failure for a device of class c
t_d	h	Time of next repair completion
u	h	Time between occurrence of a failure and beginning of its repair
$U(w)$	–	Utility of wealth w
v_i	h	Equipment virtual age after the i^{th} repair
w_{A_k}	$ 10^3	Wealth resulting from strategy A_k
x_i	h	Time between the $(i-1)^{th}$ repair and the i^{th} failure
y	h	Time between occurrence of a failure and completion of its repair
α	h	GRP-Weibull distribution scale parameter
β	–	GRP-Weibull distribution shape parameter
$\theta_i^{(c)}$	$ 10^3 / h	Penalty per hour of overtime during the i^{th} warranty period for a device of class c
λ	h^{-1}	Average failure rate (for the exponential distribution)
λ^c	h^{-1}	Average failure rate for a device of class c
μ	h^{-1}	Average repair rate of a service crew
$\mu^{(c)}$	h^{-1}	Average repair rate when repairing a device of class c
π	$ 10^3	Manufacturer's profit
π_{A_k}	$ 10^3	Manufacturer's profit from only customers that choose strategy A_k
$\tau^{(c)}$	h	Maximum unpenalized time for the OEM to return a failed device of class c to operational state

CONTENT

1 INTRODUCTION

1.1 Opening Remarks

Organizations often depend on physical assets for executing their primary operations and, consequently, generating revenue. According to Murthy, Karim & Ahmadi [1], equipment and components must operate within required specifications for productivity to be assured. Safety is also affected by maintenance quality (Quinlan, Hampson & Gregson [2]), since malfunctions and failures may cause damage to workers and physical assets. In this context, adequate maintenance management is essential in guaranteeing occupational health and safety, equipment availability, and productivity; these factors directly influence organization's ability to satisfy final customer.

Maintenance management is responsible for analyzing maintenance needs, planning and executing maintenance activities, with the objective of assuring that equipment meets manufacturer specifications, as well as guaranteeing availability of devices. However, organizations commonly use technology-intensive equipment, for which maintenance activities require specialized personnel training, knowledge, and spare parts; also, original equipment manufacturers (OEM) may not provide precise information about maintenance of their devices. Thus, due to modern devices' increasing complexity, organizations have decreased the amount of maintenance activities executed in-house, often hiring external agents, such as the OEM or third-party providers (Pascual, Godoy & Figueroa [3]).

Outsourcing maintenance activities can be beneficial to organizations. De Almeida *et al.* [4] emphasize that in-house maintenance may be more expensive and less reliable than outsourced services, since external agents can offer more skilled and specialized workmanship, while also being obliged to contract-defined performance requirements; this is especially applicable when the organization makes use of multiple technology-intensive devices, from different models and manufacturers. Murthy & Jack [5] also highlight some reasons that lead to organizations hiring external agents for execution of maintenance:

- Reduction of costs, since less resources and personnel are employed into maintenance activities;
- Availability of expert knowledge, which would be too costly to maintain in-house;
- Better cost predictability, since risks are shared with the service provider;

- Availability of advanced technology, such as tools for maintenance, which becomes expensive for in-house maintenance;
- Focus on the organization's key role, since maintenance activities become the service provider's responsibility.

In this work, we approach the problem of maintenance service contracts (MSC) for technology-intensive equipment, extending the models proposed by Zaidan [6], Guedes [7] and Moura *et al.* [8]. These authors proposed models for MSC considering different approaches regarding customers, OEM, and devices. Zaidan [6] proposed a model in which the OEM offers maintenance services for a single customer, and device's failure behavior follows a generalized renewal process. Guedes [7] and Moura *et al.* [8] considered the interactions between OEM and multiple customers, where customers belong to one of two distinct priority classes; regarding equipment failure-repair behavior, Guedes [7] considered a homogeneous Poisson process, while Moura *et al.* [8] employed a nonhomogeneous Poisson process.

We propose an extension to these models by employing a two-class priority queue, such as proposed by Guedes [7] and Moura *et al.* [8], while also modeling devices' failure-repair behavior by considering generalized renewal processes.

The present model considers a market with two customer classes. In fact, class 1 is composed by big organizations, with high potential for revenue generation, having the need for high equipment availability, even at a higher cost, while class 2 is formed by smaller organizations, which have considerable market share, not as high as class 1; these organizations have preference for cheaper maintenance services, even if it means that device availability is compromised.

The OEM offers priority and nonpriority types of services, so that the different customer classes can hire adequate maintenance services for their needs. OEM has limited capability for simultaneous repairs, thus, when waiting in queue for a repair, failed equipment with priority service starts being repaired before failed equipment with nonpriority service. Then, OEM defines adequate service prices for the different available service options, while customers analyze these prices and make their decision on buying (or not) a device, and which type of service to hire. Due to their characteristics, priority services are more expensive, but allow equipment to return faster to operational state, while nonpriority services result in worse equipment availability, but cost less.

1.2 Justification

Maintenance activities can be costly to organizations, although they are extremely important due to their positive effects, such as assurance of adequate operation of equipment. Especially for complex equipment, maintenance activities have become considerably relevant for organizations. This relevance has even given opportunity for businesses to have maintenance activities as their main revenue source, as is the case of independent service providers (Guajardo *et al.* [9]). As mentioned by Cruz & Rincon [10], growing equipment complexity makes difficult for third-party service providers to offer maintenance services, since OEMs tend to adopt protectionist actions, such as not providing enough detailed information or spare parts to the market.

Product warranties provide protection to both OEM and equipment buyer, making post-sale support an important aspect for product sale (Murthy & Djamaludin [11]). In addition to product's base warranty, extended warranties also play an important role in the context of maintenance services, since a significant number of customers tend to purchase extra protection against failures (Lutz & Padmanabhan [12]). Indeed, without warranty, customers are exposed to risks such as of excessively expensive repairs, critical equipment failures, poor repair quality, and others (Damnjanovic & Zhang [13]).

An important aspect of the MSC context is the interaction among different agents. On one hand, service providers and customers have conflicting objectives, since customers want high equipment availability, with fast repairs when devices fail. On the other hand, service providers might want to serve many customers to increase their revenue, forming queues, and thus reducing equipment availability. These conflicting interactions can be modeled by adoption of a game theory approach, where each agent's objectives can be modeled, and an equilibrium can be reached (Greve [14]).

A Stackelberg game (SG) approach can be employed to model the relationship between customers and service provider. The original SG formulation (Gibbons [15]) considers the interaction between two agents, a leader and a follower, where the leader acts first, then the follower observes the leader's decision in order to make its own. In the context of MSC, OEM can be seen as a leader, since it is the only service provider available to the market, while customers are the followers, observing the prices and service options offered by the OEM, then deciding which option to hire. This formulation of SG for MSC was used by Murthy & Yeung

[16], Murthy & Asgharizadeh [17], Ashgarizadeh & Murthy [18], Moura *et al.* [8], among other authors.

As previously mentioned, the present formulation considers that OEM offers maintenance services to several customers, where these customers can be from two different classes. The existence of multiple customers can be modeled with queueing theory, considering the interaction service capability and demand (Gross *et al.* [19]). Since customers may belong to two different classes, each with different preferences, a priority queue formulation can be employed, where priority customers receive better service, accepting to pay more for this benefit. In the field of MSC, Moura *et al.* [8] used a two-class priority queue for modeling a similar situation, considering complex medical equipment.

In a realistic scenario, complex technology-intensive equipment may suffer degradation over time. A generalized renewal process (Yañez, Joglar & Modarres [20]) can be considered to model such behavior, while also allowing for generalization by covering a broad range of scenarios. In addition to modeling increasing, constant, and decreasing failure rates, generalized renewal processes can also cover different levels of repair effectiveness, such as imperfect, perfect, and minimal repairs. Thus, this approach allows for a flexible and realistic model, which can be used in a wide variety of situations.

1.3 Objective

1.3.1 General Objective

This work intends to analyze and model MSC for technology-intensive equipment, considering the strategies of manufacturer (service provider) and multiple customers (device buyers) in two distinct priority classes. We aim at proposing a simulation-based approach for achieving a solution, modeling equipment subject to a generalized renewal process and in a two-class priority queue.

1.3.2 Specific Objectives

In order to achieve the general objective, some specific goals are defined:

- Knowledge about models for MSC, understanding existing methodologies and opportunities for improvement;
- Understanding of game theory and Stackelberg game, and how they can be used in modeling of MSC.

- Familiarization with priority queues, knowledge about analytical methods and simulation-based approaches for modeling priority queues;
- Knowledge about generalized renewal processes;
- Development of a simulation method for a two-class priority queue for equipment subject to a generalized renewal process;
- Solution of the model, using game theory methodology along with simulation results.

1.4 Dissertation Layout

The content present in each of the following Chapters of this text are briefly described below:

- **Chapter 2** presents the theoretical background and literature review for essential elements and subjects involved in the problem proposed in this text and its solution;
- **Chapter 3** gives a detailed description of the problem, explaining features of the proposed model, and introducing the agents' decision problems;
- **Chapter 4** develops the solution for the proposed model;
- **Chapter 5** presents an application example, along with a sensitivity analysis over several of the model's parameters;
- **Chapter 6** concludes remarks.

2 THEORETICAL BACKGROUND AND LITERATURE REVIEW

2.1 Stackelberg Game

Stackelberg game (SG) is a non-cooperative sequential game developed by Heinrich von Stackelberg in a leader-follower interaction. It was originally proposed to evaluate the equilibrium of a duopoly, where competing companies decide the optimal quantity to be produced (Gibbons [15]). Its solution corresponds to a sub-game perfect Nash equilibrium of a two-stage game, with perfect information and players with different profiles (Amir [21]).

According to Greve [14], in a SG, (i) there are two companies in a given market: one is well established and the other is a new entrant that produces homogeneous products; (ii) the players (companies) are rational and intend to maximize their profits; (iii) both companies are aware of all production costs; (iv) all production is consumed by the market; (v) the companies do not cooperate, then they make their decisions without negotiation with the other; vi) the game develops in two stages and the companies have different levels of bargain power. One company is called leader, as it is already established in the market and has more market share, while the other, the follower (new entrant), has a smaller market share; vii) before deciding how much to produce, the leader is aware that the follower observes its action and will react to the decision made in the first stage. Then, the leader anticipates the action of the follower and determines its production level to maximize its profit. It is possible to anticipate the other player's action due to the assumption of perfect information; and, viii) higher production volumes lead to shorter market prices, thus the companies determine a quantity to produce that maximizes their profits considering this relationship between supply and demand.

Although normally employed to model duopoly situations, it is possible to tailor the SG to consider more general negotiations/conflicts, which are characterized by one company having more bargain power than the other. It is exactly what happens in the case here studied; in the context of MSCs for technology-intensive equipment, SG can be used to model the interaction between OEM (leader), who is a monopolist and holds knowledge and technology to execute maintenance, and customers (followers).

Murthy & Yeung [16] were the first authors to introduce the SG as modeling approach in the context of MSCs. Murthy & Asgharizadeh [17] expanded the problem by creating a game between a customer and a manufacturer, assuming perfect repairs. Ashgarizadeh & Murthy [18] and Murthy & Asgharizadeh [22] incorporated multiple customers and service channels. Esmaeili, Shamsi Gamchi & Asgharizadeh [23] considered a three-level service contract between a manufacturer, a customer and an independent third agent. All aforementioned models intend to maximize the clients' expected utility, considering parameters like risk aversion, revenue generated by the system, maintenance costs and times to repair. In such papers, the leader always performs maintenance, provides the maintenance options for the customer and obtains greater payoff, since they charge the prices that maximize their own profits.

The SG formulation presented here is based on the abovementioned works. Generally, in the context of complex equipment, the OEM has a well-trained staff, spare parts and dominates equipment technology. Thus, OEM behaves as a leader, acts first and is the only maintenance service provider. Customers, who decide whether to buy a device, need to guarantee minimum levels of availability for their equipment. However, they do not have expertise in the maintenance of complex equipment. Thus, customers can be considered as followers, once they react to the OEM’s action. The relationship between the SG formulation and the MSC context can be seen in Table 2.1, which was adapted from Moura *et al.* [8] and highlights the following points: players, power structure, actions taken by the players, game dynamics, solution, and equilibrium of the game.

Table 2.1 – Relationship between the SG formulation and the EW context.

	Stackelberg Game	**Extended Warranty Model**
Description	A new entrant must decide if it enters a market in which there is only one company (monopoly). Both companies need to determine their productions to maximize their profits	An organization (customer) must decide if it buys a technology-intensive device for which it will be necessary to outsource maintenance actions provided only by the OEM (maintenance market is monopolistic)
Players	Two companies: leader and follower	Multiple companies: OEM (leader) and multiple customers (followers)
Power structure	Leader is established in market and has more power than the follower	The OEM has the power, due to its knowledge on maintenance technology
Actions	Companies determine how much to produce	OEM determines the prices of each option offered. Customers choose the option that maximizes their utilities
Game development	First, the leader determines how much to produce. Then, the follower determines how much to produce in order to maximize its own profit (considering the leader’s action)	First, the manufacturer determines each service type price based on customer maximum willingness to pay for services. Then, customers choose their action based on the given prices

	Stackelberg Game	Extended Warranty Model
Solution	Backwards induction (Osborne & Rubinstein [24]). First, the follower's optimal production is found. Then the leader's optimal production is solved	Backwards induction. First, customers' optimal action is found. Then, the OEM determines the prices that maximize its expected profit, and consequently influences the customers' decision
Equilibrium	It is reached when the optimal quantity to be produced for the market is determined, considering the sum of the productions of both companies. The payoffs of each company derive from their actions	It is reached when customers choose a maintenance policy, after equipment acquisition. The payoffs of each company derive from their actions

Adapted from Moura et al. [8]

For further and more comprehensive coverage on the Stackelberg formulation, the reader may consult Fudenberg & Tirole [25], Gibbons [15], and Osborne & Rubinstein [24].

2.2 Generalized Renewal Process

Renewal processes (RPs) and non-homogeneous Poisson processes (NHPPs) may be adopted to model perfect and minimal repair situations respectively. According to Lins & Droguett [26], such methods have simplifying assumptions that may be unreal in many practical situations such as the technology-intensive equipment maintenance context. To overcome the limitations of RP and NHPP, Kijima & Sumita [27] developed a probabilistic virtual age based model, known as the generalized renewal process (GRP), that deals with all classes of maintenance actions. According to this model, q (rejuvenation parameter) may generally assume values between 0 and 1:

- $q = 0$ represents a perfect repair (as good as new);
- $q = 1$ corresponds to a minimal repair (as bad as old);
- $0 < q < 1$ indicates imperfect repair (better than old, worse than new).

Cases where $q < 0$ and $q > 1$ are also possible, corresponding to worse than old and better than new conditions, respectively. Generally, GRP may be classified into two types, Kijima types I and II, according to the method used to calculate the virtual age. Further details on Kijima models and other virtual age-based representations can be found in Moura *et al.* [28], Wang & Yang [29], Guo, Ascher & Love [30], Tanwar, Rai & Bolia [31], Ferreira, Firmino & Cristino [32] and Oliveira, Cristino & Firmino [33].

For the approach presented here, Kijima type I is used, so that equipment virtual age follows Equation (2.1), according to which maintenance actions only act on the degradation incurred during x_i, which is the time between $(i-1)^{\text{th}}$ repair and the i^{th} failure; thus, the virtual age v_i proportionally increases over time when $q > 0$.

$$v_i = v_{i-1} + qx_i = q * \sum_{j=1}^{i} x_j \tag{2.1}$$

The cumulative distribution function (CDF) for the time between the $(i-1)^{th}$ repair and the i^{th}failure can be determined from the CDF of the time until a failure conditioned on the virtual age v_{i-1} as seen in Equation (2.2):

$$F(x_i|v_{i-1}) = P(X \leq x_i|X > v_{i-1}) = \frac{F(v_{i-1} + x_i) - F(v_{i-1})}{1 - F(v_{i-1})} \tag{2.2}$$

In this analysis, time until a failure follows a conditioned Weibull distribution because of its flexibility and ability to fit various degradation stages. Then, Equation (2.2) can be rewritten as Equation (2.3), where α is the scale parameter, also called equipment characteristic life, and β is the shape parameter. Note that, for $i = 1$, we have the Weibull distribution itself because, for convenience, $v_0 = 0$. In this text, this distribution is also referred to as GRP-Weibull for simplicity.

$$F(x_i|v_{i-1}) = 1 - exp\left[\left(\frac{v_{i-1}}{\alpha}\right)^{\beta} - \left(\frac{v_{i-1} - x_i}{\alpha}\right)^{\beta}\right] \tag{2.3}$$

When there are reasonably sufficient failure data available, Maximum Likelihood Estimators (MLE) can be used to find GRP parameters α, β, and q. To that end, the procedure described in Yañez, Joglar & Modarres [20] can be followed, allowing for the estimation of GRP parameters for the device of interest.

In the model presented in this text, we consider a generalization of the GRP, due to the existence of multiple devices in two priority classes, and a single service crew. In this sense, if the number of failed units is greater than the service capability, a queue is formed. Therefore, a priority queue formulation is employed to incorporate such behavior, since classic GRP formulation does not consider multiple devices.

Analytical solutions are usual in cases of queuing models based only in exponential distributions. However analytical approaches become limited when considering more complex models, such as GRP-Weibull distributed times until failures, as well as the addition of game formulation and a two-class priority queue into the problem (Moura *et al.* [8]). Therefore, a

DES algorithm is adopted to obtain the GRP-queue system measures; this DES formulation is further explained in Section 4.3.

2.3 Queueing Theory

2.3.1 Queueing Models

Queueing models can be described as birth-and-death processes, where births represent arrivals at the system, and deaths correspond to departures (Hillier & Lieberman [34]; Gross *et al.* [19]). The system itself is responsible for processing arriving clients (inputs); in this sense, departure occurs when processing finishes. The system is capable of serving (processing) a certain number of clients; if the total number of clients in the system surpasses its processing capability, a queue is formed, thus some clients wait until a server finishes processing.

A queueing model can be defined by a set of information: distribution of arrival times; distribution of service durations; number of servers; number of places in the system, i.e., maximum number of clients the system can receive at a time; population size, i.e., total number of customers; and the queue's discipline, a rule for the order at which queued customers begin being served. These parameters can be summarized with Kendall's notation – initially proposed by Kendall [35] and later extended by Lee [36] –, which is in the form A/S/n/K/P/D, where:

- A: Type of arriving process; indicates how times between or until arrivals behave; usually, A = M indicates a Markovian process (exponentially distributed times), A = G indicates a general behavior;
- S: Type of servicing process; indicates how service times behave; follows a similar notation to the arriving process;
- n: number of parallel servers, i.e., number of servers of the system;
- K: system capacity, i.e., maximum number of customers the system can hold at a given time; when K = ∞ or is given in context, it can be omitted;
- P: population size, i.e., the number of customers that might eventually enter the system; when P = ∞ or is given in context, it can be omitted;
- D: queue's discipline, i.e., rule for the order of service for queued customers; e.g.: FCFS (first-come, first-served), SIRO (service in random order); when D = FCFS or is given in context, it can be omitted;

In this work, Markovian is used instead of M, since the presented model has a parameter M. For instance, a queueing system with exponential times between arrivals and service times, one server, infinite capacity, infinite population, and FCFS discipline, can be translated into M/M/1/∞/∞/FCFS or simply M/M/1, or yet respectively Markovian/Markovian/1/∞/∞/FCFS and Markovian/Markovian/1 (following the notation used in this text). A system with general arriving and servicing distributions, two parallel servers, system capacity of five, population of 100, and service in random order, can be denoted as G/G/2/5/100/SIRO.

In the context of reliability and maintenance analysis, queues can be used for modeling equipment failures and repairs. In this sense, a queueing system can represent a set of devices subject to failures, along with one or more repair service crews. The set of devices is modeled by the queueing system's clients, while the service crews are represented by the system's servers; arrivals correspond to device failures, and departures occur when repairs are finished. When the number of failed devices is greater than the number of service crews at a given time, a queue is formed. Table 2.2 shows a summary of the correspondences between a classic queueing model and the maintenance context.

Table 2.2 – Correspondence between queueing model's elements and their equivalents in a reliability and maintenance analysis context

Queueing model's element	Maintenance context
Client	Device
Server	Service crew
Arrival	Failure of a device
Service	Repair
Departure	Completion of a repair
Queue	Failed devices waiting for repair to start

Source: This Research (2017)

2.3.2 Priority Queues

In queueing systems with priority disciplines, customers are divided into classes with different priority levels. Suppose there are 2 classes of customers (class 1 and class 2), each one with a given arrival rate ($\lambda^{(1)}$ and $\lambda^{(2)}$, respectively), and repair rate ($\mu^{(1)}$ and $\mu^{(2)}$, respectively). Users of class 1 have non-preemptive priority over customers of class 2. Thus, after a server finishes a job, the next user to be served belongs to the highest priority class among queued customers. If there is more than one user of the same priority class, they will be served according to a FCFS discipline.

The states of this system can be denoted as (a, b, c), where a and b are the number of class 1 and 2 customers in the system, respectively, and c is the class of the client being served (Gross *et al.* [19]). For illustration, $(3,4,1)$ represents the state at which there are three customers of class 1 and four customers of class 2 in the system, while a class 1 customer is being served. This notation considers a queue system with exactly one server.

In queueing theory, steady state probabilities are the probabilities of observation for each possible state in the long run, i.e., the mean proportion of time the system stays in each of the states. These probabilities can be denoted as $p_{a,b,c}$, where the subscripts are the same as previously defined. Note that p_0 indicates the probability that the system is empty.

To obtain these probabilities, it is necessary to define a set of equations based on the relations between the states of the system. These equations were obtained by equating, for a given set (a, b, c), the rate at which the system enters such state with the rate at which the system leaves the same state. These are the steady-state balance equations (Gross *et al.* [19], Hillier & Lieberman [34]).

For the priority system here described, each balance equation is related to a given state (a, b, c) and is labeled by $s_{a,b,c}$; $M^{(1)}$ and $M^{(2)}$ are the total numbers of customers and μ_1 and μ_2 are the service rates for each priority class, respectively. The resulting system of equations may be solved if one of them is disregarded and substituted by $q_0 + \sum_{a=1}^{M^{(1)}} q_{a,0,1} + \sum_{b=1}^{M^{(2)}} q_{0,b,2} + \sum_{a=1}^{M^{(1)}} \sum_{b=1}^{M^{(2)}} \sum_{c=1}^{2} q_{a,b,c} = 1$. The general set of equations relating the steady state probabilities for this system is given in Appendix 1.

For illustration, an example encompassing four customers, two of each class ($M^{(1)} = 2$, $M^{(2)} = 2$) can be presented. The complete set of steady state equations for such case is provided in Appendix 2. With those probabilities, it is possible to obtain the average arrival rate at the system from each of these classes, as seen in Equations (2.1) and (2.2).

$$E[\lambda^{(1)}] = 2\,\lambda^{(1)}(p_0 + p_{0,1,2} + p_{0,2,2}) + \lambda^{(1)}(p_{1,0,1} + p_{1,1,1} + p_{1,1,2} + p_{1,2,1} + p_{1,2,2}) \tag{2.1}$$

$$E[\lambda^{(2)}] = 2\,\lambda^{(2)}(p_0 + p_{1,0,1} + p_{2,0,1}) + \lambda^{(2)}(p_{0,1,2} + p_{1,1,1} + p_{1,1,2} + p_{2,1,1} + p_{2,1,2}) \tag{2.2}$$

Conversely to the equation system presented in Gross *et al.* [19], which describes a priority queue with infinite population, the one here developed has been tailored for populations of size $M = M^{(1)} + M^{(2)}$. While an infinite population queue model has a constant rate of arrival at the system for each class, the rate of arrival of a queueing system with limited population has an arrival rate that varies depending on the number of clients currently in system. Therefore, the rate of arrival is the sum of the rates of all clients who may still arrive.

Note that the number of possible states and, consequently, the number of balance equations quickly grows as the number of customers in each class increases. For instance, the number of states of this model is $2\,M^{(1)}M^{(2)} + M^{(1)} + M^{(2)} + 1$. A problem with 5 customers of class 1, and 35 customers of class 2 results in 391 states, while a problem with 15 customers of class 1, and 90 customers of class 2 has 2806 states. Due to this situation, the analytical tractability of priority queues becomes burdensome.

Moreover, the description above fits into a two-class priority Markovian/Markovian/1/∞/$M^{(1)} + M^{(2)}$/2-class-FCFS queue, with finite population of size $M^{(1)} + M^{(2)}$, divided in two priority classes. 2-class-FCFS was used to indicate that the queue discipline is composed of two priority classes, where each of them follows FCFS. In the model proposed in this dissertation, a more general queue model is employed by making times until arrivals (failures) for each customer follow a GRP-Weibull distribution. Such queue may be referred to as GRP/Markovian/m/∞/$M^{(1)} + M^{(2)}$/2-class-FCFS. The use of GRP-Weibull-distributed times until failures for each equipment generalizes the case of a Markovian/Markovian/1/∞/$M^{(1)} + M^{(2)}$/2-class-FCFS queue as well as incorporating characteristics such as equipment wear-out and repair efficiency. However, using the GRP-Weibull causes the queue system to be even more complex. Indeed, the balance equations presented above become no longer valid. Thus, once more, the need for a simulation approach is denoted.

3 MODEL DESCRIPTION

3.1 Game Description

The OEM sells a device and offers different types of maintenance services, chosen by buyers at the moment of purchase. Buyers, also called customers, are organizations that intend to generate revenue by using the device. Due to their different profiles, customers can be classified in the two previously mentioned classes; class 1 consist of large organizations with requirement of high availability levels, and class 2 is formed by small organizations which seek to reduce maintenance costs.

A base warranty for the device is provided by the OEM; depending on the hired maintenance service, this base warranty can be extended. Customers decide whether to buy a device and, if they buy it, which service type to hire. Service options offered by the OEM are divided into priority and nonpriority services. Thus, customers decide whether to hire priority service, and whether to extend equipment warranty. According to the preferences of each customer class, class 1 customers always opt for one of the priority services when they purchase a device, while class 2 customers opt for nonpriority services.

The set of alternatives available to the customer follows the model proposed by Moura *et al.* [8], as described as follows. In total, there are five possible customer strategies, with one option not to buy a device, two options for hiring priority services, and two options for hiring nonpriority services. These strategies are given below:

- A_0: Not buying a device. Therefore, customers and OEM have no costs and receive no revenue, since no purchase or contract exists, and the OEM does not manufacture the device;
- A_1: Buying a device with priority service and extended warranty. Device has price C_b, and includes a base priority warranty with duration T_1; extended warranty costs $P_w^{(1)}$ and has duration T_2; $T = T_1 + T_2$ is the total coverage period. During base and extended warranty, every failure is repaired by the OEM without additional cost to the customer; as a priority service, devices covered by this plan begin repair before queued nonpriority devices, but follow FCFS order when in queue with other priority devices. The OEM must return the device to operational state (i.e., finish repairing the unit) within a time $\tau^{(1)}$, otherwise a penalty must be paid to the customer. Let y be the time between a failure and completion of its repair. The

penalty is proportional to the overtime, $y-\tau^{(1)}$, i.e., the time spent after the limit $\tau^{(1)}$. During base warranty, penalty is equal to $\theta_1^{(1)}\left(y-\tau^{(1)}\right)$, and during extended warranty, penalty is equal to $\theta_2^{(1)}\left(y-\tau^{(1)}\right)$, where $\theta_i^{(1)}$ is a priority penalty rate for warranty period i, with $i=1$ corresponding to the base warranty, and $i=2$ being the extended warranty period. Note that only class 1 customers hire this type of service;

- A_2: Buying a device with nonpriority service and extended warranty. Device has price C_b, and includes a base nonpriority warranty with duration T_1; extended warranty costs $P_w^{(2)}$ and has duration T_2. During base and extended warranty, every failure is repaired by the OEM without additional cost to the customer; as a nonpriority service, failed units must wait for queued priority equipment to be repaired before repair starts, following FCFS among nonpriority queued devices. If the OEM does not return the device to operational state within a time $\tau^{(2)}$, a penalty must be paid to the customer; during base warranty, penalty is equal to $\theta_1^{(2)}\left(y-\tau^{(2)}\right)$; and during extended warranty penalty is equal to $\theta_2^{(2)}\left(y-\tau^{(2)}\right)$. Note that only class 2 customers hire this type of service;
- A_3: Buying a device with priority service, and priority on-call service after warranty expiration. Device has price C_b and includes a base priority warranty with duration T_1; after expiration of base warranty, the customer receives on-call service for a duration T_2, paying $C_s^{(1)}$ for each repair. $T=T_1+T_2$ is the total coverage period. During base warranty, every failure is repaired by the OEM without additional cost to the customer; if the OEM does not return the device to operational state within a time $\tau^{(1)}$, a penalty must be paid to the customer, equal to $\theta_1^{(1)}\left(y-\tau^{(1)}\right)$. After base warranty expiration, device is covered by priority on-call service, where the customer must pay a fixed price $C_s^{(1)}$ for each repair; during this period (after expiration of base warranty), no penalty is incurred due to delays in repairs. Only class 1 customers hire this type of service;
- A_4: Buying a device with nonpriority service, and nonpriority on-call service after warranty expiration. Device has price C_b and includes a base nonpriority warranty with duration T_1; after expiration of base warranty, the customer receives on-call service for a duration T_2, paying $C_s^{(2)}$ for each repair. During base warranty, every

failure is repaired by the OEM without additional cost to the customer. During base warranty, the OEM must return the device to operational state within a time $\tau^{(2)}$, otherwise a penalty must be paid to the customer, equal to $\theta_1^{(2)}(y - \tau^{(2)})$. After base warranty expiration, device is covered by nonpriority on-call service, where the customer must pay a fixed price $C_s^{(2)}$ for each repair; during this period, no penalty is incurred due to delays in repairs. Only class 2 customers hire this type of service.

Regarding each agent's decision problem, which is explained in more detail in Sections 3.3 and 3.4, it can be summarized as follows. OEM must decide how many customers to serve ($M^{(1)}$ and $M^{(2)}$), as well as each service price to be charged ($P_w^{(c)}$ and $C_s^{(c)}$, with $c \in \{1,2\}$). Customers of both classes, given the prices of each service, must decide whether to buy a device and which service type to hire. Each customer buys at most one device, and customers are considered homogeneous inside each class, so that all customers in each priority class choose the same strategy. Solution to this problem is reached by backward induction (Osborne & Rubinstein [24]), where the customers' decision problem is solved first, then the OEM's.

Notice that, due to their preferences, class 1 customers choose among strategies A_0, A_1 and A_3, while class 2 customers choose among A_0, A_2 and A_4. Device generates revenue when operational. Class 1 customers can generate revenue of $R^{(1)}$ per operational hour, while class 2 customers generate $R^{(2)}$ per operational hour. Notice that when failed, equipment stops generating revenue, and thus the efficiency of maintenance services is extremely important for OEM and customers. When equipment spends too much time in failed state, customers generate less revenue, and OEM must pay more penalties.

OEM is risk-neutral and seeks to maximize its expected profit. Customers are risk-averse, thus aim to maximize their expected utility. Customer's utility function for a wealth w is given by Equation (3.1) (Varian [37]), where γ is the customer's risk aversion parameter. When γ is high, customers tend to avoid strategies with high uncertainty, seeking more predictable strategies to diminish eventual losses.

$$U(w) = \frac{1 - e^{-\gamma w}}{\gamma} \tag{3.1}$$

3.2 Failure-Repair Behavior

Each device sold by the OEM follows the failure behavior described in Section 2.2, thus the set of equipment sold for all customers forms a two-class priority GRP queue, a GRP/Markovian/m/∞/$M^{(1)} + M^{(2)}$/2-class-FCFS. Thus, failure times for each device follows a GRP-Weibull distribution with scale parameter α, shape parameter β, and rejuvenation parameter q. Repair times follow exponential distribution with rate μ, and may be imperfect, perfect or minimal, depending on the value of q.

Figure 3.1 shows a simplified representation of a system with the description above, considering one service crew. Each line represents a timeline for a device, showing occurrences of failures, queue, repairs and overtime. All equipment is new at initial time. Device 1 belongs to the priority class. The first failure happens to device 2, which starts being repaired immediately, since there was a service crew available. Before device 2 finishes being repaired, device 3 fails, followed by device 1. Notice, however, that when the service crew becomes free after repairing device 2, device 1 starts being repaired, due to it having priority service.

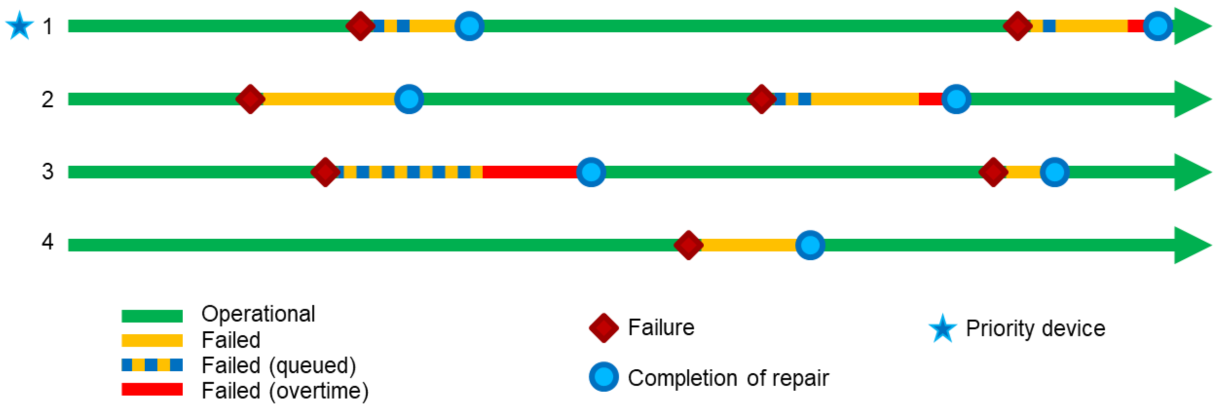

Figure 3.1 – Simplified representation of the two-class priority GRP queue model
Source: This Research (2017)

3.3 Customer's Decision Problem

As described in Section 3.1, customers choose among the five possible decision alternatives, aiming to maximize their expected utility. In order to find the expected utility, we first obtain customer's wealth (return) for each alternative. For strategy A_0, customer's wealth is equal to zero, as in Equation (3.2); for strategies A_1 and A_2, customer's wealth is given in Equations (3.3) and (3.4), respectively; and for strategies A_3 and A_4, Equations (3.5) and (3.6) respectively indicate customer's wealth. For customer j of class c: $T_j^{(c)}$ is the total operational time during coverage time T ($T = T_1 + T_2$); $O_{i,j}^{(c)}$ is the overtime during coverage period i; and $N_{i,j}^{(c)}$ is the number of failures during coverage period i.

$$w_{A_0} = 0 \tag{3.2}$$

$$w_{A_1} = R^{(1)}T_j^{(1)} + \theta_1^{(1)}O_{1,j}^{(1)} + \theta_2^{(1)}O_{2,j}^{(1)} - P_w^{(1)} - C_b \tag{3.3}$$

$$w_{A_2} = R^{(2)}T_j^{(2)} + \theta_1^{(2)}O_{1,j}^{(2)} + \theta_2^{(2)}O_{2,j}^{(2)} - P_w^{(2)} - C_b \tag{3.4}$$

$$w_{A_3} = R^{(1)}T_j^{(1)} + \theta_1^{(1)}O_{1,j}^{(1)} - N_{2,j}^{(1)}\, C_s^{(1)} - C_b \tag{3.5}$$

$$w_{A_4} = R^{(2)}T_j^{(2)} + \theta_1^{(2)}O_{1,j}^{(2)} - N_{2,j}^{(2)}\, C_s^{(2)} - C_b \tag{3.6}$$

By substituting Equations (3.2)-(3.6) into Equation (3.1), it is possible to obtain the equations for the utility for each strategy alternative. The respective utilities for strategies A_0-A_4 are given in Equations (3.7)-(3.11).

$$U\left(w_{A_0}\right) = 0 \tag{3.7}$$

$$U\left(w_{A_1}\right) = \frac{1}{\gamma}\left\{1 - \exp\left[-\gamma\left(R^{(1)}\, T_j^{(1)} + \theta_1^{(1)}\, O_{1,j}^{(1)} + \theta_2^{(1)}\, O_{2,j}^{(1)} - P_w^{(1)} - C_b\right)\right]\right\} \tag{3.8}$$

$$U\left(w_{A_2}\right) = \frac{1}{\gamma}\left\{1 - \exp\left[-\gamma\left(R^{(2)}\, T_j^{(2)} + \theta_1^{(2)}\, O_{1,j}^{(2)} + \theta_2^{(2)}\, O_{2,j}^{(2)} - P_w^{(2)} - C_b\right)\right]\right\} \tag{3.9}$$

$$U\left(w_{A_3}\right) = \frac{1}{\gamma}\left\{1 - \exp\left[-\gamma\left(R^{(1)}\, T_j^{(1)} + \theta_1^{(1)}\, O_{1,j}^{(1)} - N_{2,j}^{(1)}\, C_s^{(1)} - C_b\right)\right]\right\} \tag{3.10}$$

$$U\left(w_{A_4}\right) = \frac{1}{\gamma}\left\{1 - \exp\left[-\gamma\left(R^{(2)}\, T_j^{(2)} + \theta_1^{(2)}\, O_{1,j}^{(2)} - N_{2,j}^{(2)}\, C_s^{(2)} - C_b\right)\right]\right\} \tag{3.11}$$

Equations (3.12)-(3.16) show the expected utilities for strategies A_0-A_4, respectively, which are obtained by evaluating the expected values for Equations (3.7)-(3.11) respectively.

$$E\left[U\left(w_{A_0}\right)\right] = 0 \tag{3.12}$$

$$E\left[U\left(w_{A_1}\right)\right] = \frac{1}{\gamma}\left\{1 - \exp\left[\gamma\left(P_w^{(1)} + C_b\right)\right] E\left[\exp\left[-\gamma\left(R^{(1)}\, T_j^{(1)} + \theta_1^{(1)}\, O_{1,j}^{(1)} + \theta_2^{(1)}\, O_{2,j}^{(1)}\right)\right]\right]\right\} \tag{3.13}$$

$$E\left[U\left(w_{A_2}\right)\right] = \frac{1}{\gamma}\left\{1 - \exp\left[\gamma\left(P_w^{(2)} + C_b\right)\right] E\left[\exp\left[-\gamma\left(R^{(2)}\, T_j^{(2)} + \theta_1^{(2)}\, O_{1,j}^{(2)} + \theta_2^{(2)}\, O_{2,j}^{(2)}\right)\right]\right]\right\} \tag{3.14}$$

$$E\left[U\left(w_{A_3}\right)\right] = \frac{1}{\gamma}\left\{1 - \exp[\gamma\, C_b]\, E\left[\exp\left[-\gamma\left(R^{(1)}\, T_j^{(1)} + \theta_1^{(1)}\, O_{1,j}^{(1)} - N_{2,j}^{(1)}\, C_s^{(1)}\right)\right]\right]\right\} \tag{3.15}$$

$$E\left[U\left(w_{A_4}\right)\right] = \frac{1}{\gamma}\left\{1 - \exp[\gamma\, C_b]\, E\left[\exp\left[-\gamma\left(R^{(2)}\, T_j^{(2)} + \theta_1^{(2)}\, O_{1,j}^{(2)} - N_{2,j}^{(2)}\, C_s^{(2)}\right)\right]\right]\right\} \tag{3.16}$$

Due to SG's assumption of complete and perfect information, after the OEM defines how many customers to serve ($M^{(c)}$) and service prices ($P_w^{(c)}$ and $C_s^{(c)}$), customers of both classes can estimate their expected utilities, since customers and OEM can estimate $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$, which are the stochastic components of these equations. However, the exact analytical estimation of these three metrics is intractable due to the model's characteristics, such as the assumption of a two-class priority GRP queue (Moura *et al.* [8]), and the assumption of risk-averse customers (Ashgarizadeh & Murthy [18]). Given the difficulties imposed by these assumptions, the proposed model resorts to a discrete event simulation (DES) approach, as described in more detail in Chapter 4. Using the DES approach, the values of $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$ can be obtained, making possible evaluating the present model's equations.

3.4 Manufacturer's Decision Problem

The OEM seeks to maximize its expected utility, which depends on customers' decisions. For each customer decision alternative, the OEM's profit can be found by using Equations (3.17)-(3.21), where C_r is the average cost spent by the OEM on execution of each repair. For each customer of class c who chooses extended warranty (strategy A_1 or A_2), the OEM receives $P_w^{(c)}$ for selling the extended warranty, but has to pay for all repairs during coverage period T, as well as for penalties during the base and extended warranty periods. When a customer of class c choses not to extend the warranty (strategy A_3 or A_4), OEM has to pay for failures and penalties during base warranty, but receive a payment of $C_s^{(c)}$ for every failure after base warranty expiration (although repair costs of C_r are still incurred to the OEM).

$$\pi_{A_0} = 0 \tag{3.17}$$

$$\pi_{A_1}\left(P_w^{(1)}, M^{(1)}\right) = M^{(1)}P_w^{(1)} - C_r \sum_{j=1}^{M^{(1)}} N_j^{(1)} - \theta_1^{(1)} \sum_{j=1}^{M^{(1)}} O_{1,j}^{(1)} - \theta_2^{(1)} \sum_{j=1}^{M^{(1)}} O_{2,j}^{(1)} \tag{3.18}$$

$$\pi_{A_2}\left(P_w^{(2)}, M^{(2)}\right) = M^{(2)}P_w^{(2)} - C_r \sum_{j=1}^{M^{(2)}} N_j^{(2)} - \theta_1^{(2)} \sum_{j=1}^{M^{(2)}} O_{1,j}^{(2)} - \theta_2^{(2)} \sum_{j=1}^{M^{(2)}} O_{2,j}^{(2)} \tag{3.19}$$

$$\pi_{A_3}\left(C_s^{(1)}, M^{(1)}\right) = \left(C_s^{(1)} - C_r\right) \sum_{j=1}^{M^{(1)}} N_{2,j}^{(1)} - C_r \sum_{j=1}^{M^{(1)}} N_{1,j}^{(1)} - \theta_1^{(1)} \sum_{j=1}^{M^{(1)}} O_{1,j}^{(1)} \tag{3.20}$$

$$\pi_{A_4}\left(C_s^{(2)}, M^{(2)}\right) = \left(C_s^{(2)} - C_r\right) \sum_{j=1}^{M^{(2)}} N_{2,j}^{(2)} - C_r \sum_{j=1}^{M^{(2)}} N_{1,j}^{(2)} - \theta_1^{(2)} \sum_{j=1}^{M^{(2)}} O_{1,j}^{(2)} \tag{3.21}$$

However, since customers decide based on service prices defined by the OEM ($P_w^{(c)}$ and $C_s^{(c)}$), as well as the number of customers served ($M^{(c)}$), the OEM can influence customer's decision. Therefore, OEM's total profit π is the sum of the profits π_{A_k} which result from each customer class. Given the values of $P_w^{(c)}$, $C_s^{(c)}$ and $M^{(c)}$ for each class c, the OEM estimates its expected profit for each possible customer strategy by evaluating the expected values of Equations (3.17)-(3.21). Thus, in order to maximize its profit, the OEM needs to choose adequate values for $P_w^{(c)}$, $C_s^{(c)}$ and $M^{(c)}$, using backward induction. This process is described in Chapter 4. Also, for the OEM's decision problem to be solved, the values of $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$ are also needed. As already mentioned, these are obtained through a DES approach, also described in Chapter 4.

4 MODEL SOLUTION

4.1 Estimation of Service Prices

Since the model here developed considers complete and perfect information, the OEM can predict the decision of customers, which, in turn, decide based on the observed service prices and total number of customers chosen by OEM. Thus, the OEM acts first, having to choose values for $P_w^{(c)}$, $C_s^{(c)}$ and $M^{(c)}$ to maximize its own expected profit $E[\pi]$. In order to do this, the OEM analyzes customers' decision, by applying the process of backward induction.

Given a number of customers $M^{(c)}$, the OEM needs to find $P_w^{(c)*}$ and $C_s^{(c)*}$, which are the respective values of $P_w^{(c)}$ and $C_s^{(c)}$ that make customers of class c indifferent to all strategies, i.e., by making $P_w^{(c)} = P_w^{(c)*}$ and $C_s^{(c)} = C_s^{(c)*}$, customers would expect the same value of utility for every strategy. Since the expected utility of strategy A_0 is zero, this means that when $P_w^{(c)} = P_w^{(c)*}$ and $C_s^{(c)} = C_s^{(c)*}$, customers' expected utilities are all equal to zero, so that Equations (4.1) and (4.2) are true. Note that $P_w^{(c)*}$ and $C_s^{(c)*}$ are the respective maximum prices a customer of class c is willing to pay for extended warranty and on-call repairs.

$$E\left[U\left(w_{A_0}\right)\right] = E\left[U\left(w_{A_1}\right)\right] = E\left[U\left(w_{A_3}\right)\right] = 0 \tag{4.1}$$

$$E\left[U\left(w_{A_0}\right)\right] = E\left[U\left(w_{A_2}\right)\right] = E\left[U\left(w_{A_4}\right)\right] = 0 \tag{4.2}$$

In order to obtain $P_w^{(c)*}$ and $C_s^{(c)*}$, we first equate expected utilities among each customer class to zero, that is, equate each of Equations (3.12)-(3.16) to zero. This allows us for obtaining relations for estimating $P_w^{(c)*}$ and $C_s^{(c)*}$. $P_w^{(c)*}$ is found by equating Equations (3.13) or (3.14) to zero, and then solving to $P_w^{(c)}$, which results in Equation (4.3). Analogously, Equations (3.15) and (3.16) are equated to zero for finding $C_s^{(c)*}$. Even though a closed form equation cannot be found, we obtain Equation (4.4), which yields $C_s^{(c)*}$ when solved under numerical methods. Also, notice that once more the stochastic values of $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$ become necessary for evaluation of equations.

$$P_w^{(c)*} = -C_b - \frac{1}{\gamma} \ln E\left[\exp\left[-\gamma\left(R^{(c)}\, T_j^{(c)} + \theta_1^{(c)}\, O_{1,j}^{(c)} + \theta_2^{(c)}\, O_{2,j}^{(c)}\right)\right]\right] \tag{4.3}$$

$$\gamma\, C_b + \ln E\left[\exp\left[-\gamma\left(R^{(c)}\, T_j^{(c)} + \theta_1^{(c)}\, O_{1,j}^{(c)} - N_{2,j}^{(c)}\, C_s^{(c)*}\right)\right]\right] = 0 \tag{4.4}$$

4.2 Optimal Solution

Note that, given the number of customers in each class ($M^{(1)}$ and $M^{(2)}$), the OEM finds each customer's maximum willingness to pay for extended warranty and on-call repairs, as shown above. Thus, so that OEM's profit is maximized, it is possible to substitute the values of $P_w^{(c)*}$ and $C_s^{(c)*}$ into the equations for π_{A_k}, resulting in the profit each possible customer decision alternative would yield. Then, the OEM compares these profits, finding which set of customer decisions (combination of strategies of class 1 and 2) is the best. Then, it is possible to define $P_w^{(c)}$ and $C_s^{(c)}$, inducing customers to choose the best strategies for the OEM. This is done as listed below, also given in summarized form in Table 4.1 and Figure 4.1.

- $P_w^{(c)} > P_w^{(c)*}$ and $C_s^{(c)} > C_s^{(c)*}$: customers of class c choose strategy A_0, deciding not to buy a device, since both prices charged are higher than their maximum willingness to pay;
- $P_w^{(c)} = P_w^{(c)*}$ and $C_s^{(c)} > C_s^{(c)*}$: customers of class c choose strategy A_1 (when $c = 1$) or A_2 (when $c = 2$), extending their warranty, since the price for on-call repairs are higher than what they accept to pay;
- $P_w^{(c)} > P_w^{(c)*}$ and $C_s^{(c)} = C_s^{(c)*}$: customers of class c choose strategy A_3 (when $c = 1$) or A_4 (when $c = 2$), not extending their warranty, since the price for extending the warranty is higher than their maximum willingness.

Table 4.1 – Customer strategy for possible price combinations

Prices	$P_w^{(c)} > P_w^{(c)*}$	$P_w^{(c)} = P_w^{(c)*}$
$C_s^{(c)} > C_s^{(c)*}$	A_0	A_1 or A_2
$C_s^{(c)} = C_s^{(c)*}$	A_3 or A_4	A_1/A_2 or A_3/A_4

Source: This Research (2017)

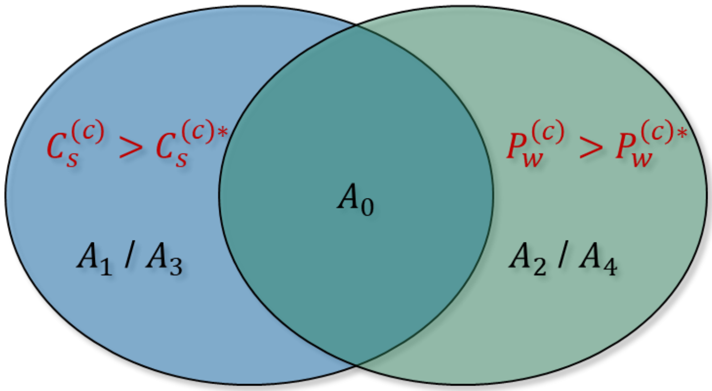

Figure 4.1 – Customer strategy for possible price combinations
Source: This Research (2017)

The number of customers to be served in each class is also defined by the OEM for maximization of its profit. The procedure shown above for definition of service prices assumes that these numbers have already been defined. For the optimal number of customers in each class, the OEM evaluates their expected profit for every possible combination of numbers of customers in the two classes. Finally, the combination of $M^{(1)}$ and $M^{(2)}$ that results in the greatest OEM's profit is chosen.

4.3 Simulation Approach

Discrete event simulation (DES) is the process of replicating stochastic processes by employing modeling with variables and discrete events (Ross [38]). DES makes possible the replication of real complex processes, allowing for better understanding, easier and more robust decision making. When dealing with models, where analytical solution is not possible or is too complex, DES becomes especially useful (Zio [39]). In our case, DES is used for finding the values of the stochastic variables $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$, needed for obtaining a model solution.

4.3.1 Priority GRP Queue Simulation Algorithm

As previously stated, obtainment of values for $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$, which result as metrics of the GRP/Markovian/$m/\infty/M^{(1)} + M^{(2)}$/2-class-FCFS queue, requires the application of simulation methods, due to analytical limitations in face of the model's complexity. Thus, a DES algorithm for simulating such model was developed, and is presented in Figure 4.2. A description of the algorithm is given in the following paragraphs.

Initially, in step 1 of the algorithm, simulation time is set to $t = 0$, and, the time of next completion of repair is initially set to $t_d = \infty$ so that the first event is never a repair. First, failure times for every device are generated using the GRP-Weibull distribution; t_{a_1} and t_{a_2} are respectively set to the earliest times of failures among devices of each class. Then, t_{a_1}, t_{a_2} and t_d are compared, so that the next event can be identified. When $t_{a_c} < t_d$, the next event is a failure of a device of class c; if $t_d < t_{a_1}$ and $t_d < t_{a_2}$, then the next event is a completion of repair. When an event occurs, simulation time is updated to the event time, and necessary actions are executed, updating variables and saving results whenever necessary.

When a failure occurs, step 2.1.1 in the algorithm, the number of failed devices is incremented; the respective device's number of failure for the current coverage period is also incremented; failure time is saved, so that downtime and overtime can later be calculated. If there is a free server at failure time, then the device is immediately repaired; thus, a repair time is generated, allowing for the calculation of repair completion time t_d. The difference between repair completion time and failure time is used for calculating downtime and overtime, which are then added to the respective device's variables for these values. Finally, a new failure time for class c is set, which is the value of the earliest failure time among the remaining operational equipment in that class; if all equipment of class c is in failed state, then $t_{a_c} = \infty$.

Inputs: α, β, q, μ, m, $\tau^{(1)}$, $\tau^{(2)}$, T_1, T_2, $M^{(1)}$, $M^{(2)}$

1. Initialization

1.1. Generate first failure time for each device and set t_{a_1} and t_{a_2} equal to the earliest failure time of each customer class

1.2. For convenience, set $t_d = \infty$

1.3. Set initial time: $t = 0$

2. Simulation

2.1. Is the next event a failure ($t_{a_c} \leq t_d$) or a completion of repair ($t_d < t_{a_c}$)?

2.1.1. Failure of device j of class c (only if $t_{a_c} < T$)

- Update current time $t = t_{a_c}$
- Store failure information
  - Increment number of failures for device j for current warranty period i ($N_{i,j}^{(c)}$)
  - Increment number of failed devices
  - Store failure time
- If there is at least one free server: begin repair immediately
  - Generate repair duration r and set $y = r$
  - $t_d = t + y$
  - Increase downtime by y for device j for current warranty period i ($D_{i,j}^{(c)}$)
  - Store repair duration and departure time
  - If $y > \tau$, increase overtime by $y - \tau^{(c)}$ for device j for current warranty period i ($O_{i,j}^{(c)}$)
- Set t_{a_c} equal to the earliest failure time among the remaining operational equipment in class c

2.1.2. Completion of repair on device j in class c

- Update current time $t = t_d$
- Store repair information
  - Decrement number of failed devices
  - Generate a failure time t'_{a_c} for this device
  - If $t'_{a_c} < t_{a_c}$, then set $t_{a_c} = t'_{a_c}$
- Are there any remaining failed devices?
  - Yes
    - Let u be the time spent in queue
    - Generate repair duration r and set $y = u + r$
    - $t_d = t + y$
    - Increase downtime by y for device j for current warranty period i ($D_{i,j}^{(c)}$)
    - Store repair duration and departure time
    - If $y > \tau$, increase overtime by $y - \tau^{(c)}$ for device j for current warranty period i ($O_{i,j}^{(c)}$)
  - No
    - Set $t_d = \infty$

2.2. Is $t_{a_c} > T$ and the system empty?

2.2.1. Yes: go to step 3.

2.2.2. No: repeat step 2.

3. Output generation: for each device, during each warranty period, return the following measures:

3.1. Number of failures ($N_{1,j}^{(c)}$, $N_{2,j}^{(c)}$)

3.2. Downtime ($D_{1,j}^{(c)}$, $D_{2,j}^{(c)}$)

3.3. Overtime ($O_{1,j}^{(c)}$, $O_{2,j}^{(c)}$)

Figure 4.2 – Priority GRP Queue DES algorithm
Source: This Research (2017)

If a repair is completed, step 2.1.2 in the algorithm, the number of failed devices is decremented. A failure time for the device just repaired is generated and compared to the earliest failure time among the remaining operational equipment of class c; if the generated time is earlier than the others, t_{a_c} is set to that time. If there is any failed priority device in queue, the next priority device in queue begins its repair; if there are only nonpriority devices in queue, the next nonpriority device in queue begins its repair; if there are no failed equipment in queue, then t_d is set to infinity. If a device from the queue is chosen to begin its repair, its repair time is generated, t_d is calculated, and its downtime and overtime registers are updated, similarly as in the previous paragraph.

When $t_{a_c} > T$, no more failures will be simulated, since the next failure would occur after the coverage period. However, remaining failed devices are repaired, until the queue is empty. When the condition that $t_{a_c} > T$ and the queue is empty is met, simulation can be stopped. The outputs of the simulation are, for each customer in each class, the number of failures during each coverage period ($N_{i,j}^{(c)}$), downtime for each coverage period ($D_{i,j}^{(c)}$), and overtime for each coverage period ($O_{i,j}^{(c)}$). Notice that, operational time can be calculated from downtime by doing $T_{i,j}^{(c)} = T_i - D_{i,j}^{(c)}$ or $T_j^{(c)} = T - D_{1,j}^{(c)} - D_{2,j}^{(c)}$.

4.3.2 Optimization Algorithm

Figure 4.3 contains the algorithm used for solving the optimization problem presented in this text. At first, population size is set to $z = 1$, and customer class size is then defined as $M^{(1)} = z$ and $M^{(2)} = 0$.

Then, step 2.1 is executed, which is actually the DES algorithm proposed previously in Figure 4.2; simulation is run for n_{rep} Monte Carlo replications. Simulation results are obtained, and the best strategies and OEM's profit are saved. Next, it is checked whether all possible combinations of $M^{(1)}$ and $M^{(2)}$ with $M^{(1)} + M^{(2)} = z$ are simulated. If not, $M^{(1)}$ is decremented, $M^{(2)}$ is incremented, and step 2 is repeated. After all combinations are simulated, the population size z itself is incremented, and step 2 is repeated. When the population size z becomes too large, the available number of service crews may not be sufficient to maintain all devices at acceptable availability levels, resulting in too much wait in queues and, consequently, too frequent occurrence of penalties. Thus, there comes a point where further incrementing z results in decreased OEM's profit. This behavior can be used for defining a stopping condition for the optimization algorithm; as soon as all customer alternatives result in decreased OEM's profit for any class combination for a given z when compared with $z-1$, the algorithm can stop. Then, after all possible population sizes and combinations were tested, it is possible to define the optimal solution, which is the number of customers and strategies that result in the greatest OEM's profit. Based on these results, $P_w^{(c)}$ and $C_s^{(c)}$ can be defined.

Inputs: α, β, q, μ, m, $\theta_1^{(1)}$, $\theta_2^{(1)}$, $\theta_1^{(2)}$, $\theta_2^{(2)}$, γ, $\tau^{(1)}$, $\tau^{(2)}$, $R^{(1)}$, $R^{(2)}$, C_b, C_r, T_1, T_2, $M^{(1)}$, $M^{(2)}$, n_{rep}

1. **Initialization**:
   - 1.1. Set initial population size $z = 1$
   - 1.2. Set initial class combination
     - 1.2.1. $M^{(1)} = z$
     - 1.2.2. $M^{(2)} = 0$
2. **Model execution**
   - 2.1. Run priority GRP queue simulation for the current population $M = M^{(1)} + M^{(2)}$ for the required number of replications n_{rep}
   - 2.2. Get queue measures from simulation
     - 2.2.1. Estimate $C_s^{(c)*}$ and $P_w^{(c)*}$
     - 2.2.2. Choose best strategy for each class
     - 2.2.3. Store OEM's profit and strategies
   - 2.3. Have all combinations of $M^{(1)}$ and $M^{(2)}$ been simulated?
     - 2.3.1. No:
       - 2.3.1.1. $M^{(1)} = M^{(1)} - 1$, $M^{(2)} = M^{(2)} + 1$
       - 2.3.1.2. Repeat step 2
     - 2.3.2. Yes:
       - Did OEM's profits decrease for both repair service options?
         - No
           - Increment population size $z = z + 1$
           - $M^{(1)} = z$, $M^{(2)} = 0$
           - Repeat step 2
         - Yes: go to step 3
3. **Results definition**
   - 3.1. Choose population size and strategies which resulted in highest OEM profit
   - 3.2. Define $C_s^{(c)}$ and $P_w^{(c)}$ accordingly

Figure 4.3 – Model optimization algorithm
Source: This Research (2017)

5 APPLICATION EXAMPLE

5.1 Example Description

For demonstrating the present methodology, an application example is given in this Chapter. Actual failure data from a technology-intensive medical device, fitting the general characteristics covered by the methodology, are here presented. The data are from an angiography device, used for imaging examinations with help of a contrast agent, allowing doctors to visualize blood vessels and blood flow (Dyro [40]; Suri & Laxminarayan [41]). Failures might cause diagnostic errors, imprecise readings and even prevent it from functioning at all. In this context, the customers are hospitals that intend to buy an angiography device. Class 1 customers are big hospitals, with a higher number of patients, while class 2 customers are smaller hospitals and clinics.

In order to simulate equipment behavior, we first estimate GRP parameters α, β and q by employing the maximum likelihood estimators (MLEs)-based method proposed by Yañez, Joglar & Modarres [20]. Estimated values were $\hat{\alpha} = 1{,}351.83$ h, $\hat{\beta} = 1.658$, and $\hat{q} = 0.097$. These parameters were used as inputs for the application example. The remaining model parameters, although not mathematically estimated, were defined based on available data about the device under analysis, such as number of examinations performed over previous years, as well as the revenue received per examination . The GRP MLEs, and the remaining model parameters used in the application example are shown in Table 5.1.

Table 5.1 – Parameters used for the application example

Description	Value
GRP-Weibull scale parameter (α)	1,351.83 h
GRP-Weibull shape parameter (β)	1.658
Repair parameter q	0.097
Repair rate (μ)	0.1 h^{-1}
Customer risk-aversion (γ)	0.1
Equipment price (C_b)	\$ 1,476.49 (10^3)
Class 1 operational revenue ($R^{(1)}$)	\$ 0.1 (10^3) / h
Class 2 operational revenue ($R^{(2)}$)	\$ 0.096 (10^3) / h
Base warranty duration (T_1)	8,760 h (1 year)
Extended warranty duration (T_2)	8,760 h (1 year)
Base penalty rate for class 1 ($\theta_1^{(1)}$)	\$ 1 (10^3) / h

Description	Value
Extended penalty rate for class 1 ($\theta_2^{(1)}$)	\$ 3 (10^3) / h
Base penalty rate for class 2 ($\theta_1^{(2)}$)	\$ 0.5 (10^3) / h
Extended penalty rate for class 2 ($\theta_2^{(2)}$)	\$ 2 (10^3) / h
Unpenalized time for class 1 ($\tau^{(1)}$)	24 h
Unpenalized time for class 2 ($\tau^{(2)}$)	36 h
OEM cost per repair (C_r)	\$ 2.5 (10^3)
Number of replications (n_{rep})	1,000,000

Source: This Research (2017)

Then, the methodology described throughout this text was applied, following the steps in Figure 4.3. Figure 5.1 shows the expected number of failure over time, using simulated data by considering $\hat{\alpha}$, $\hat{\beta}$ and $\hat{q}$, demonstrating close agreement with real data. The models' results are shown in the following Section.

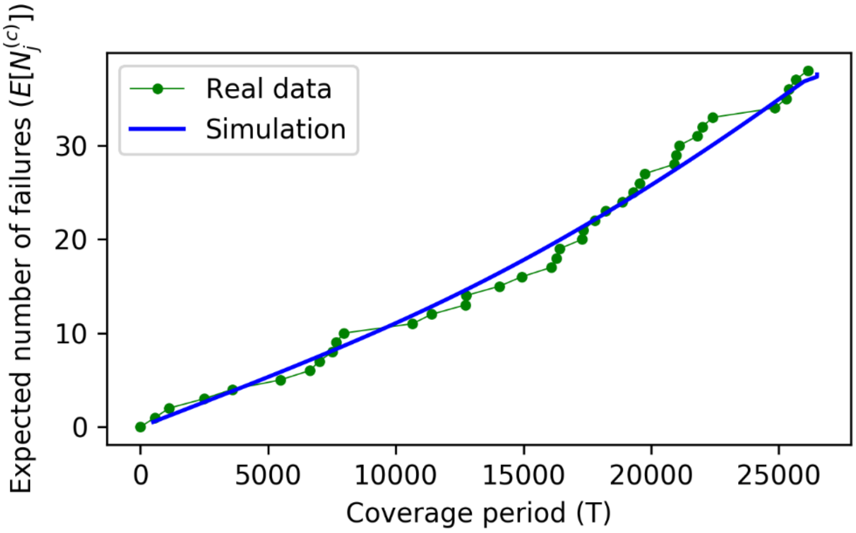

Figure 5.1 – Simulated expected number of failures over time, along with real failure data
Source: This Research (2017)

5.2 Results

Model's main results for the application example are given in Table 5.2. On the optimal solution, the OEM sells devices to $M^{(1)} = 3$ class 1 customers and $M^{(2)} = 26$ class 2 customers, with a total of $M = 29$. Both class 1 and class 2 customers choose the extended warranty; thus, class 1 customers choose strategy A_1, while class 2 customers choose strategy A_2. Note that priority services have considerably higher prices, since priority customers suffer less downtime, and receive more compensation in the form of penalties when there is overtime. OEM's resulting expected profit is $E[\pi] = \$\ 2{,}584{,}038$.

Table 5.2 – Main results for the application example

Metric	Value
Number of priority customers ($M^{(1)}$)	3
Number of nonpriority customers ($M^{(2)}$)	26
Price of priority extended warranty ($P_w^{(1)*}$)	\$ 280,301
Price of nonpriority extended warranty ($P_w^{(2)*}$)	\$ 12,121
Price of priority on-call repair ($C_s^{(1)*}$)	\$ 192,846
Price of nonpriority on-call repair ($C_s^{(2)*}$)	\$ 8,833
Class 1 customers' strategy ($A^{(1)}$)	A_1
Class 2 customers' strategy ($A^{(2)}$)	A_2
OEM's expected profit ($E[\pi]$)	\$ 2,584,038

Source: This Research (2017)

It is also interesting to analyze some queue indicators and performance measures; these are given in Table 5.3. Notice that priority devices spend 12.75% less time in failed state than nonpriority ones. Also, expected overtime for priority customers is higher than for nonpriority customers. This occurs because the unpenalized time $\tau^{(c)}$ is much stricter for priority customers, therefore, even though they receive better service, the time limit for the OEM to finish repairing priority equipment is much shorter, resulting in higher likelihood of overtime occurrence.

Table 5.3 – Main metrics and performance measures

Metric	Value
Total expected number of failures for a priority device ($E\left[N_j^{(1)}\right]$)	21.22
Total expected number of failures for a nonpriority device ($E\left[N_j^{(2)}\right]$)	21.21
Total expected downtime for a priority device ($E\left[D_j^{(1)}\right]$)	290.76 h
Total expected downtime for a nonpriority device ($E\left[D_j^{(2)}\right]$)	333.25 h
Total expected overtime for a priority device ($E\left[O_j^{(1)}\right]$)	43.79 h
Total expected overtime for a nonpriority device ($E\left[O_j^{(2)}\right]$)	35.05 h

Source: This Research (2017)

It is possible to notice that, although priority service is considerably more expensive, it offers a better situation for class 1 customers, since downtime is lower than for nonpriority customers, and yet compensation with penalties is higher. This is also a reflection of the characteristics of each customer class, since class 1 customers want high equipment availability, even if it costs higher, while class 2 customers prefer to pay lower prices at the cost of worse equipment availability. Also, class 1 customers can generate more hourly revenue with their devices, which increases their willingness to pay for priority services.

5.3 Sensitivity Analysis

In addition to the results of the original case given above, this Section presents a sensitivity analysis on several of the model's parameters. Results for different values of these parameters are shown, along with some comments regarding model behavior due to parameter changes. Throughout the tables in this analysis, original case results are marked in grey.

Figure 5.2 – Number of customers and OEM's expected profit for different values of scale parameter
Source: This Research (2017)

Table 5.4 and Figure 5.2 show results for different values of α, scale parameter for the Weibull-GRP distribution. Higher values for this parameter cause longer times until failures, causing higher equipment availability. Thus, devices tend to suffer fewer failures over the same amount of time when α increases. As for customer behavior, these effects translate into willingness to pay higher values of $P_w^{(c)}$ (higher availability allows for more revenue generation) and $C_s^{(c)}$ (also since fewer failures occur). Since customers pay more for services and equipment tends to suffer fewer failures, the OEM can serve more customers when α is higher; OEM's profit also benefits greatly from higher α. Notice that, with greater α, more priority customers are served, since the OEM can provide better service for these customers when equipment fails less. On the other hand, when $\alpha = 1{,}200$ h, the OEM only serves nonpriority customers, since equipment fails too much, causing penalties to class 1 customers to rise, lowering OEM's profit.

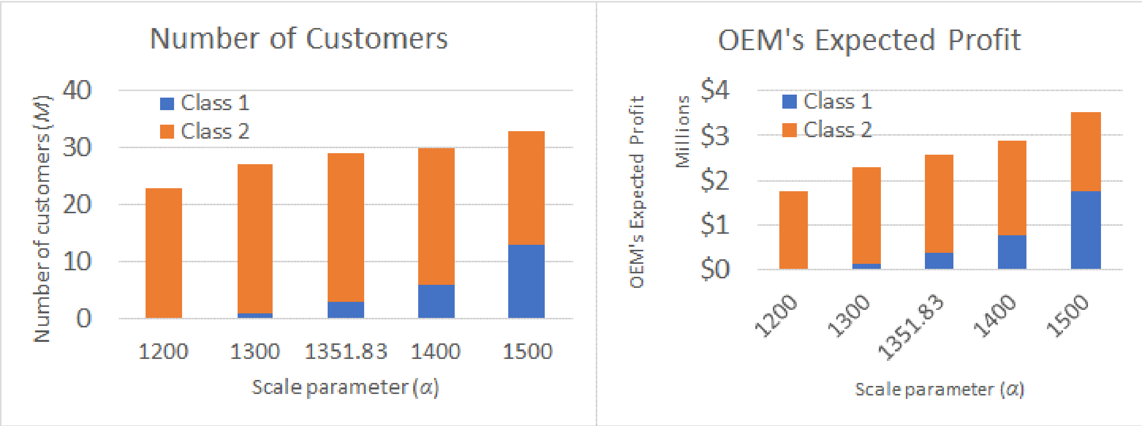

Figure 5.2 – Number of customers and OEM's expected profit for different values of scale parameter
Source: This Research (2017)

Table 5.4 – Results for different values of scale parameter

α (h)	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
1,200	23	0	–	–	A_0	23	188,628	7,652	A_2	1,750,658
1,300	27	1	279,861	11,482	A_1	26	191,307	8,440	A_2	2,289,127
1,351.83	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
1,400	30	6	280,587	12,554	A_1	24	193,971	9,194	A_2	2,878,761
1,500	33	13	281,548	13,368	A_1	20	196,475	9,924	A_2	3,515,986

Source: This Research (2017)

Results for different values of β, Weibull-GRP shape parameter, are shown in Table 5.5 and Figure 5.3. β drives failure rate variation over time; with $\beta > 1$, failure rate increases over time, while lower values of β indicate more reliable equipment. Thus, similarly as seen for α, when devices suffer fewer failures, more customers can be served, especially from priority class, since they are willing to pay more for services in relation to nonpriority customers. Consequently, OEM's profit increases with lower β. When $\beta = 1.8$, class 1 customers choose strategy A_3, while class 2 is not served at all. Since penalties become higher for this value of β, the OEM prefers not to cover EWs; also, only class 1 customers are served, since they are more lucrative to the OEM.

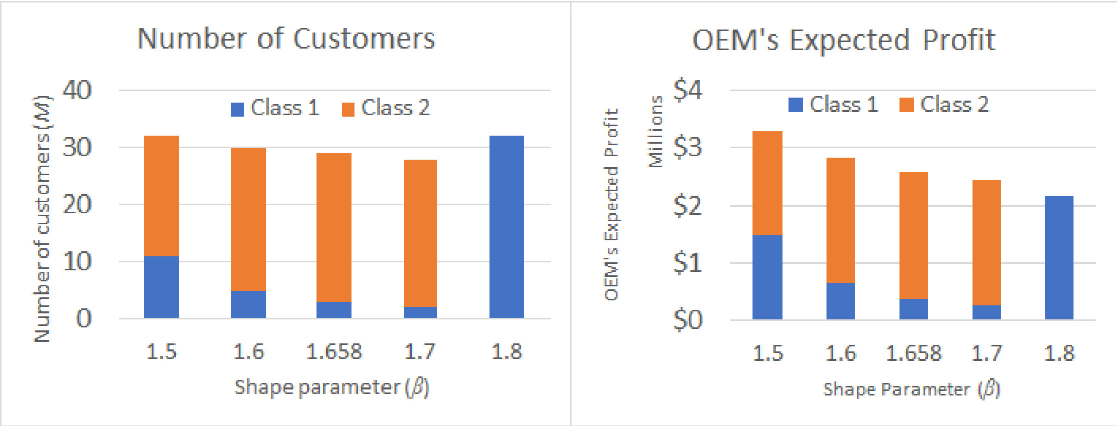

Figure 5.3 – Number of customers and OEM's expected profit for different values of shape parameter
Source: This Research (2017)

Table 5.5 – Results for different values of shape parameter

β	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
1.5	32	11	281,170	13,016	A_1	21	195,209	9,457	A_2	3,300,592
1.6	30	5	280,499	12,435	A_1	25	193,515	9,128	A_2	2,821,818
1.658	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
1.7	28	2	280,127	11,823	A_1	26	192,247	8,685	A_2	2,431,375
1.8	32	32	299,249	10,995	A_3	0	–	–	A_0	2,153,831

Source: This Research (2017)

Table 5.6 and Figure 5.4 show results for different values of q, repair quality parameter. Since failure rate increases over time ($\beta > 1$), when q is close to zero and a failure occurs, failure rate is reduced to a condition close to that of a new device. Effectively, lower values of q indicate better condition recovery after each repair, resulting in better equipment availability in the long run. Thus, more failures tend to occur for higher values of q. Thus, it is possible to notice that, for lower values of q, more customers can be served, since equipment suffers fewer failures.

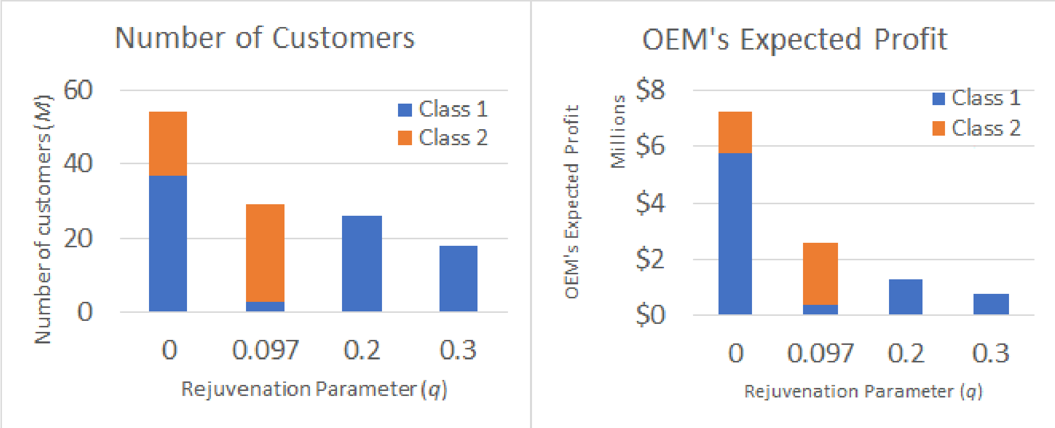

Figure 5.4 – Number of customers and OEM's expected profit for different values of repair quality parameter
Source: This Research (2017)

Table 5.6 – Results for different values of repair quality parameter

q	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
0	54	37	284,121	21,611	A_1	17	202,800	16,903	A_2	7,235,889
0.097	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
0.2	26	26	304,405	8,604	A_3	0	–	–	A_0	1,289,499
0.3	18	18	298,970	7,187	A_3	0	–	–	A_0	788,942

Source: This Research (2017)

Table 5.7 and Figure 5.5 show results for different values of repair rate μ. Although this parameter does not directly influence failure rate, it does affect downtime due to each failure; higher values of μ result in faster repairs, which in turn reduce queues and time lost due to failures. On the other hand, lower repair rates tend to increase occurrence of overtime, resulting in more penalties paid by the OEM. Therefore, the number of customers and OEM's profit increase as μ rises. For $\mu = 0.0875$, only nonpriority customers are served, choosing extended warranty.

With $\mu = 0.075$ strategies of both customer classes are not to extend warranty. Notice that, since there is no penalty after the base warranty, the OEM can serve 20 priority customers and only 2 nonpriority customers; in this situation, nonpriority customers suffer from much longer waits to repair than priority customers, due to the presence of a high number of priority customers. Yet, this a feasible (and optimal) solution for the OEM, since nonpriority customers allow for longer repairs before incurrence of penalties during the base warranty, and no penalty is incurred after the base warranty expires.

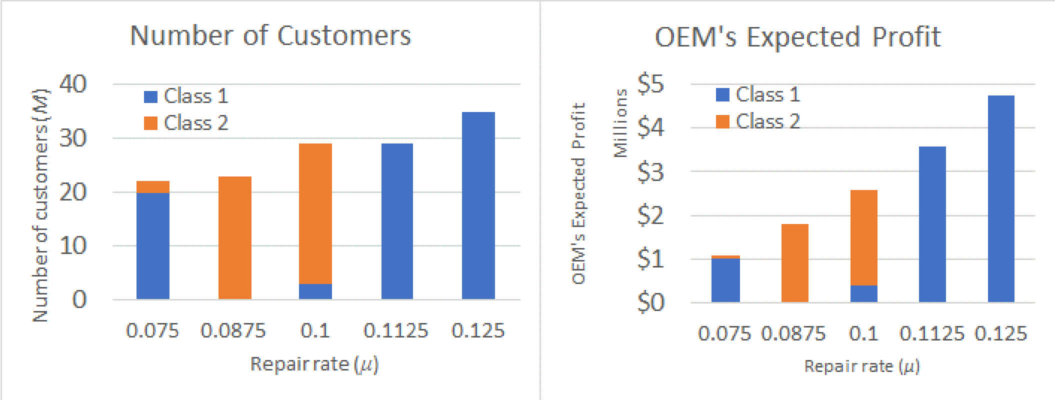

Figure 5.5 – Number of customers and OEM's expected profit for different values of repair rate
Source: This Research (2017)

Table 5.7 – Results for different values of repair rate

μ (h^{-1})	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
0.075	22	20	307,176	11,751	A_3	2	208,645	8,223	A_4	1,068,517
0.0875	23	0	–	–	A_0	23	193,767	8,741	A_2	1,804,579
0.1	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
0.1125	29	29	280,067	11,819	A_1	0	–	–	A_0	3,587,070
0.125	35	35	277,605	11,891	A_1	0	–	–	A_0	4,758,634

Source: This Research (2017)

Model behavior for different values of customer risk aversion parameter γ is given in Table 5.8 and Figure 5.6. When customers are more risk-averse (higher γ) they tend to be more conservative, avoiding uncertainties. Thus, notice that for $\gamma = 0.05$ customers choose not to extend warranty, since they become more likely to accept variable costs of paying for each repair intervention; in this situation, only priority customers buy the equipment, since they are able to pay more for the services, thus being more profitable for the OEM. When $\gamma \geq 0.15$ only class 2 customers buy the equipment, due to their less strict unpenalized period. Their revenue is more variable than that of nonpriority customers since penalties may occur more often for priority customers. In this way, nonpriority customers are more lucrative for the OEM.

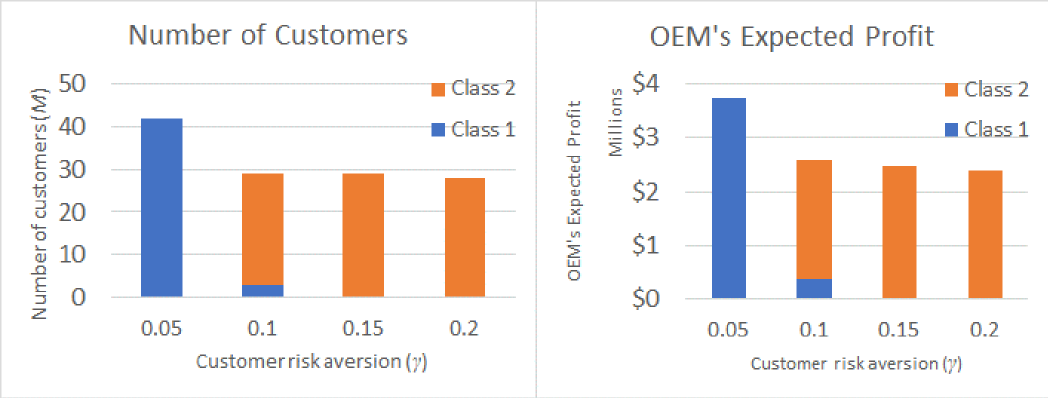

Figure 5.6 – Number of customers and OEM's expected profit for different values of customer risk aversion
Source: This Research (2017)

Table 5.8 – Results for different values of customer risk aversion parameter

γ	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
0.05	42	42	345,083	14,411	A_3	0	–	–	A_0	3,738,898
0.1	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
0.15	29	0	–	–	A_0	29	188,212	7,598	A_2	2,473,487
0.2	28	0	–	–	A_0	28	185,346	7,228	A_2	2,398,579

Source: This Research (2017)

Table 5.9 and Figure 5.7 contain results for $\theta_1^{(1)}$, the priority base warranty penalty rate. θ defines how much each hour of overtime costs to the OEM; notice that it does not directly affect the likelihood of occurrence of overtime. When $\theta_1^{(1)}$ assumes low values, more priority customers are served, since the OEM has lower costs with penalties for them. For instance, when $\theta_1^{(1)} = 0.5$, only class 1 customers are served, and their strategy is not to extend warranty. This occurs because the OEM can serve more priority customers due to the lower penalty rate during base warranty, and due to the increased number of customers, the OEM would have to pay too much in penalties if these customers extended their warranties. When $\theta_1^{(1)} \geq 1.25$ no priority customers are served, since penalties would become too expensive to the OEM.

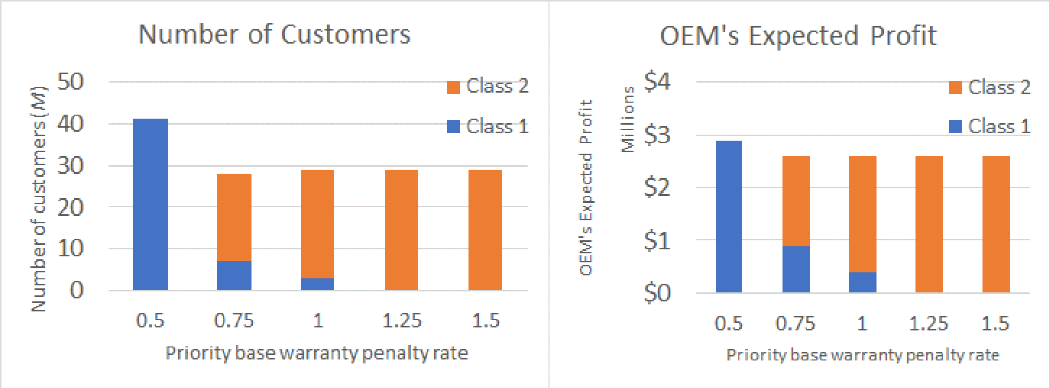

Figure 5.7 – Number of customers and OEM's expected profit for different values of priority base warranty penalty rate
Source: This Research (2017)

Table 5.9 – Results for different values of priority base warranty penalty rate

$\theta_1^{(1)}$ (10^3 \$/h)	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
0.5	41	41	302,030	11,367	A_3	0	–	–	A_0	2,895,731
0.75	28	7	279,491	11,921	A_1	21	193,407	8,831	A_2	2,600,313
1	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
1.25	29	0	–	–	A_0	29	192,021	8,814	A_2	2,582,668
1.5	29	0	–	–	A_0	29	192,021	8,814	A_2	2,582,668

Source: This Research (2017)

Results for different values of $\theta_2^{(1)}$, the extended warranty priority penalty rate, are given in Table 5.10 and Figure 5.8. A similar effect to variations on the previous parameter ($\theta_1^{(1)}$) are observed; notice, however, that for lower values of $\theta_2^{(1)}$ ($\theta_2^{(1)} \leq 2$) only priority customers are served, and choose to extend their warranties, since the penalties to the OEM are reduced during the extended warranty period. Also, OEM's profit increases for lower values of $\theta_2^{(1)}$, since it pays less in penalties. For $\theta_2^{(1)} \geq 4$, only nonpriority customers buy the equipment, since penalties due to delays for priority customers are increased.

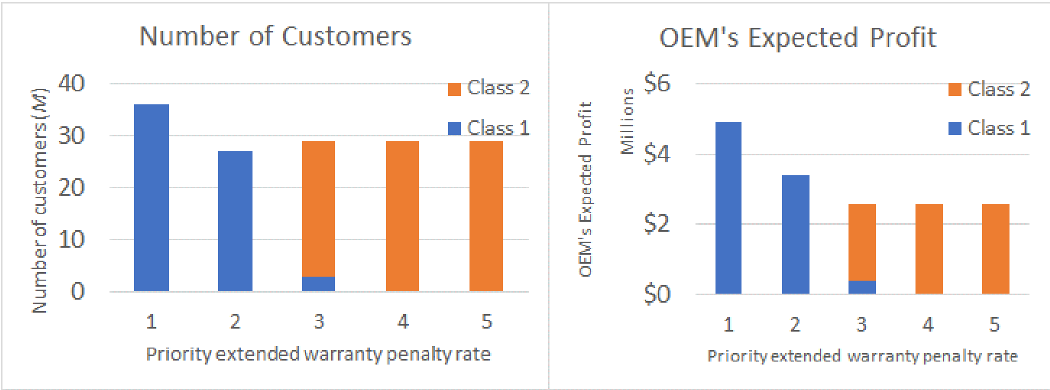

Figure 5.8 – Number of customers and OEM's expected profit for different values of priority extended warranty penalty rate
Source: This Research (2017)

Table 5.10 – Results for different values of priority extended warranty penalty rate

$\theta_2^{(1)}$ (10^3 \$/h)	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
1	36	36	288,339	11,528	A_1	0	–	–	A_0	4,898,665
2	27	27	284,492	11,745	A_1	0	–	–	A_0	3,387,966
3	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
4	29	0	–	–	A_0	29	192,021	8,814	A_2	2,582,668
5	29	0	–	–	A_0	29	192,021	8,814	A_2	2,582,668

Source: This Research (2017)

Table 5.11 and Figure 5.9 show the model behavior for changes in $\theta_1^{(2)}$, the nonpriority base warranty penalty rate. Values of $\theta_1^{(2)}$ lower than that of the original result cause little difference in the results; this indicates that nonpriority customers already receive low values in penalties, i.e., $\tau^{(2)}$ is permissible enough for the OEM to repair class 2 equipment without incurring too much penalty. However, an interesting effect can be observed: notice that as $\theta_1^{(2)}$ = 0.3, OEM's profit rises; yet, when $\theta_1^{(2)} = 0.1$, a slight decrease in OEM's profit is observed. Although the OEM pays less in penalties for lower values of $\theta_1^{(2)}$, nonpriority customers are also willing to pay less for the extended warranty, since they receive less compensation for excessive downtime, which results in overall decrease in OEM's profit. When $\theta_1^{(2)}$ is higher than in the original case, OEM pays more in penalties to nonpriority customers, causing its profit to decrease.

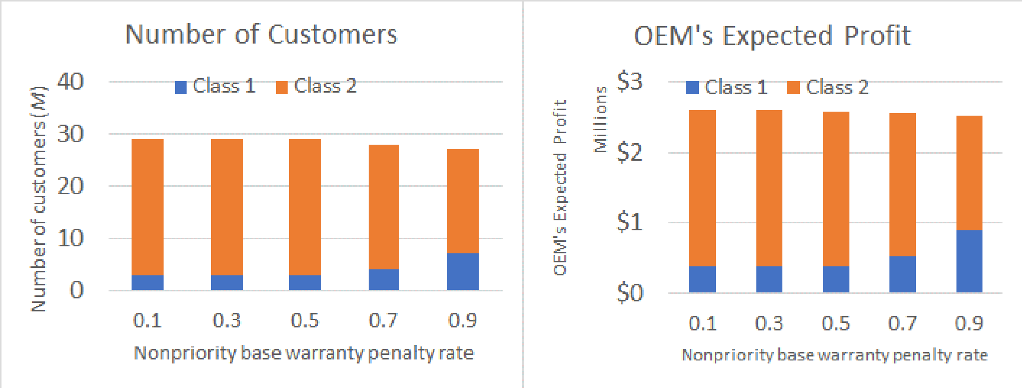

Figure 5.9 – Number of customers and OEM's expected profit for different values of nonpriority base warranty penalty rate
Source: This Research (2017)

Table 5.11 – Results for different values of nonpriority base warranty penalty rate

$\theta_1^{(2)}$ (10^3 \$/h)	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
0.1	29	3	280,290	12,080	A_1	26	189,512	8,721	A_2	2,600,438
0.3	29	3	280,281	12,074	A_1	26	191,536	8,806	A_2	2,602,236
0.5	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
0.7	28	4	280,137	12,117	A_1	24	193,475	8,889	A_2	2,558,141
0.9	27	7	280,520	12,032	A_1	20	194,413	8,922	A_2	2,529,784

Source: This Research (2017)

Table 5.12 and Figure 5.10 contain results for different $\theta_2^{(2)}$, the nonpriority extended warranty penalty rate. In relation to the original results, when $\theta_2^{(2)}$ is raised only priority customers buy the equipment, since penalties become too great for nonpriority customers; when $\theta_2^{(2)}$ decreases, penalties for nonpriority customers decrease, causing the OEM to serve only this class. As expected, OEM's profit increases when penalties decrease, while nonpriority customers pay more for extended warranty when they receive more compensation for downtime.

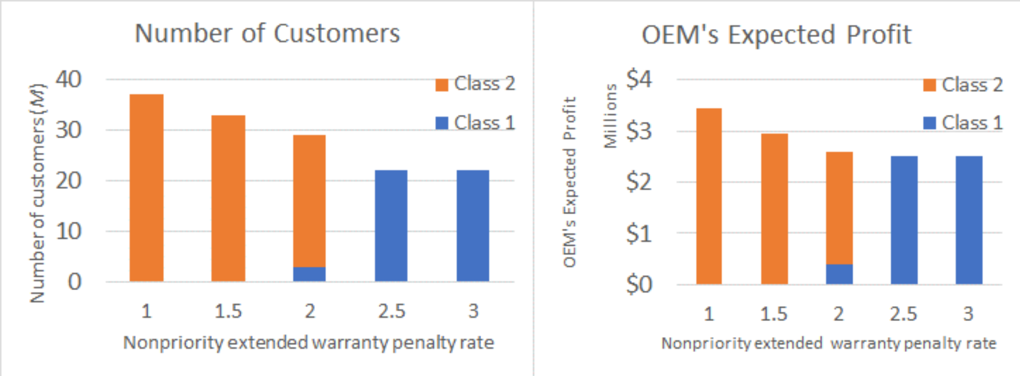

Figure 5.10 – Number of customers and OEM's expected profit for different values of nonpriority extended warranty penalty rate
Source: This Research (2017)

Table 5.12 – Results for different values of nonpriority extended warranty penalty rate

$\theta_2^{(2)}$ (10^3 \$/h)	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
1	37	0	–	–	A_0	37	192,900	8,518	A_2	3,437,487
1.5	33	0	–	–	A_0	33	192,856	8,758	A_2	2,942,931
2	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
2.5	22	22	282,147	11,907	A_1	0	–	–	A_0	2,501,298
3	22	22	282,147	11,907	A_1	0	–	–	A_0	2,501,298

Source: This Research (2017)

Table 5.13 and Figure 5.11 present results for different values of $\tau^{(1)}$, the priority unpenalized period. When this parameter is reduced, penalties for delays when repairing priority devices occur more often, since there is less unpenalized time for returning units to operational state, resulting in more penalty paid by the OEM. Thus, for low values of $\tau^{(1)}$, only class 2 customers buy the equipment, since class 1 customers receive too much penalty. On the other hand, when $\tau^{(1)}$ is higher than in the original case, only priority customers are served, since the OEM benefits from having to pay fewer penalties for that class of customers. OEM's profit increases along with $\tau^{(1)}$.

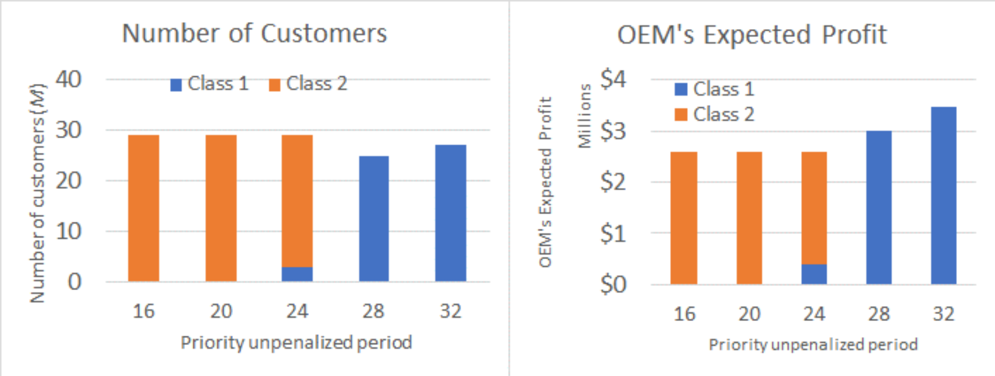

Figure 5.11 – Number of customers and OEM's expected profit for different values of priority unpenalized period
Source: This Research (2017)

Table 5.13 – Results for different values of priority unpenalized period

$\tau^{(1)}$ (h)	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
16	29	0	–	–	A_0	29	192,021	8,814	A_2	2,582,668
20	29	0	–	–	A_0	29	192,021	8,814	A_2	2,582,668
24	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
28	25	25	275,392	11,771	A_1	0	–	–	A_0	3,001,784
32	27	27	269,195	11,695	A_1	0	–	–	A_0	3,464,294

Source: This Research (2017)

Table 5.14 and Figure 5.12 show model results for different values of nonpriority unpenalized period $\tau^{(2)}$, and analogous effect to the one of $\tau^{(1)}$ is observed. When $\tau^{(2)}$ is reduced, the optimal solution is to serve only priority customers, since penalties due to delays in repairing nonpriority equipment becomes more frequent. As $\tau^{(2)}$ increases, only nonpriority customers are served in the optimal solution, due to the reduction in penalties.

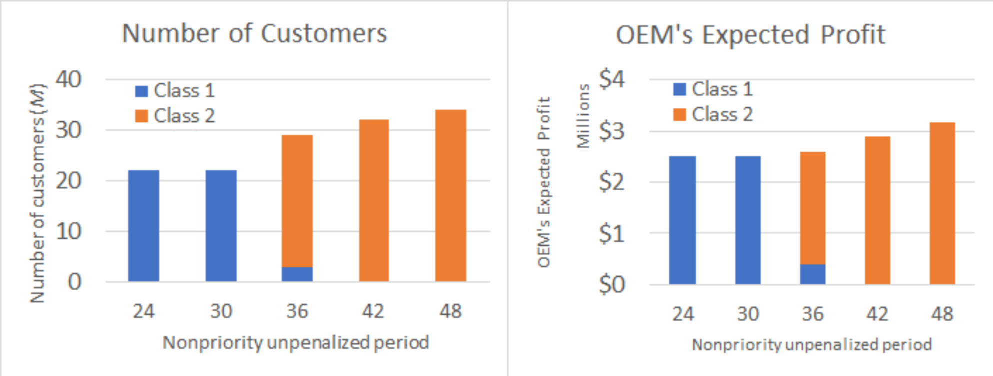

Figure 5.12 – Number of customers and OEM's expected profit for different values of nonpriority unpenalized period
Source: This Research (2017)

Table 5.14 – Results for different values of nonpriority unpenalized period

$\tau^{(2)}$ (h)	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
24	22	22	282,147	11,907	A_1	0	–	–	A_0	2,501,298
30	22	22	282,147	11,907	A_1	0	–	–	A_0	2,501,298
36	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
42	32	0	–	–	A_0	32	186,547	8,646	A_2	2,897,049
48	34	0	–	–	A_0	34	181,653	8,634	A_2	3,162,877

Source: This Research (2017)

Results for different values of priority revenue per operational hour $R^{(1)}$ are given in Table 5.15 and Figure 5.13. Customers' revenue directly influences their willingness to pay for services, since their revenue defines how much can be spent for purchasing and maintaining equipment. Thus, when $R^{(1)}$ is high, priority customers have higher willingness to pay for services, making the OEM serve only that class. However, when $R^{(1)}$ is low, only class 2 customers are served. OEM's profit increases along with $R^{(1)}$.

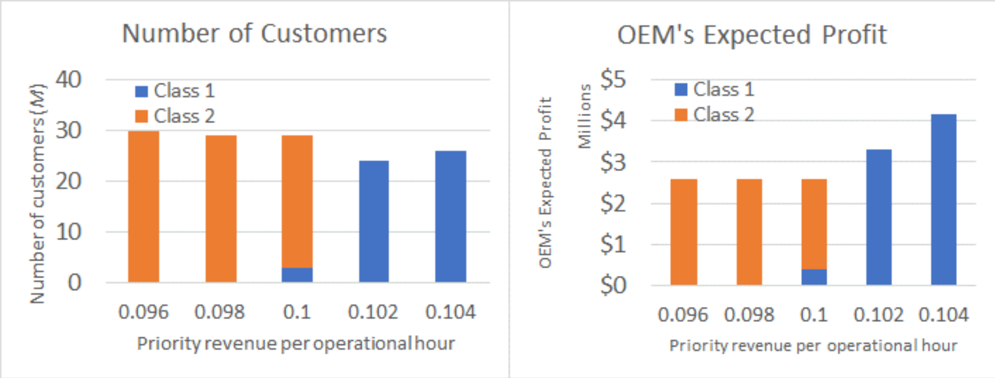

Figure 5.13 – Number of customers and OEM's expected profit for different values of priority revenue per operational hour
Source: This Research (2017)

Table 5.15 – Results for different values of priority revenue per operational hour

$R^{(1)}$ (10^3 \$/h)	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
0.096	29	0	–	–	A_0	29	192,021	8,814	A_2	2,582,668
0.098	29	0	–	–	A_0	29	192,021	8,814	A_2	2,582,668
0.1	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
0.102	24	24	318,637	13,175	A_1	0	–	–	A_0	3,313,519
0.104	26	26	355,176	14,245	A_1	0	–	–	A_0	4,177,372

Source: This Research (2017)

Table 5.16 contains results for different values of nonpriority revenue per operational hour $R^{(2)}$. Analogous results to variations in $R^{(1)}$ are observed. When $R^{(2)}$ increases, nonpriority customers pay more for services, causing the OEM to serve only these customers. When $R^{(2)}$ decreases, only class 1 is served. OEM's profit increases along with $R^{(2)}$.

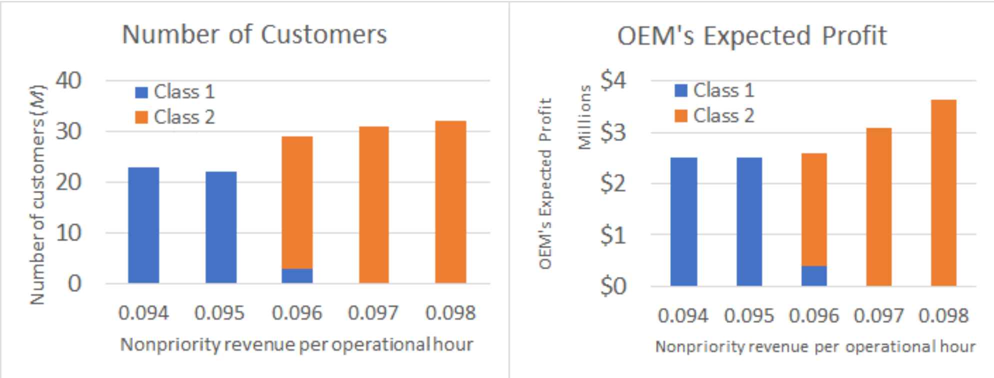

Figure 5.14 – Number of customers and OEM's expected profit for different values of nonpriority revenue per operational hour
Source: This Research (2017)

Table 5.16 – Results for different values of nonpriority revenue per operational hour

$R^{(2)}$ (10^3 \$/h)	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
0.094	22	22	282,147	11,907	A_1	0	–	–	A_0	2,501,298
0.095	22	22	282,147	11,907	A_1	0	–	–	A_0	2,501,298
0.096	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
0.097	31	0	–	–	A_0	31	210,266	9,297	A_2	3,097,486
0.098	32	0	–	–	A_0	32	228,071	10,044	A_2	3,634,013

Source: This Research (2017)

Finally, results when different numbers of service crews are available are shown in Table 5.17 and Figure 5.15. When there are more service crews available, queues are greatly reduced, since more simultaneous repairs can occur, and also the effect of long repairs is reduced. For instance, when a service crew takes longer than usual to complete a repair, the remaining crews can continue repairing other failed units, instead of blocking the whole system, as is the case with a single service channel. When $m = 2$, $M^{(1)}$ becomes considerably greater than $M^{(2)}$; when $m = 3$, only class 1 customers are served. Since the OEM has greater service capability when m increases, penalties are significantly reduced. Given that class 1 customers are more lucrative to the OEM, with more service channels, more class 1 customers are served.

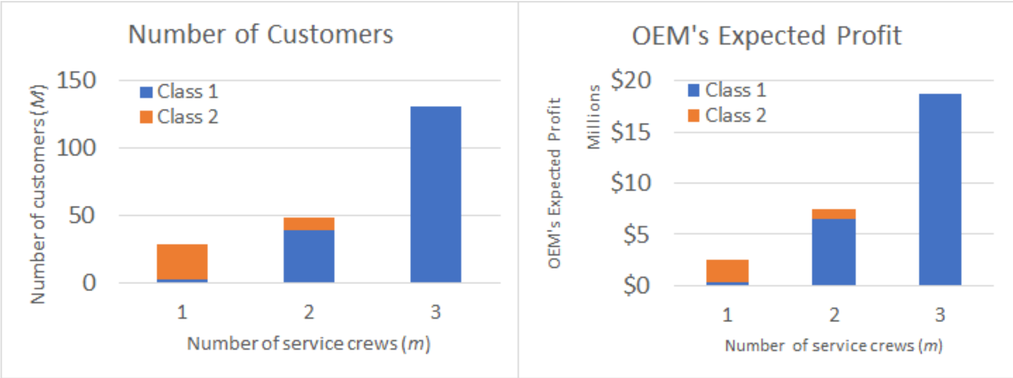

Figure 5.15 – Number of customers and OEM's expected profit for different numbers of service crews
Source: This Research (2017)

Table 5.17 – Results for different numbers of service crews

m	M	$M^{(1)}$	$P_w^{(1)*}$	$C_s^{(1)*}$	$A^{(1)*}$	$M^{(2)}$	$P_w^{(2)*}$	$C_s^{(2)*}$	$A^{(2)*}$	$E[\pi]$
1	29	3	280,301	12,121	A_1	26	192,846	8,833	A_2	2,584,038
2	48	39	271,808	11,918	A_1	9	188,227	9,232	A_2	7,510,107
3	131	131	276,175	11,588	A_1	0	–	–	A_0	18,780,080

Source: This Research (2017)

6 CONCLUDING REMARKS

This text presented an approach for modeling MSC of technology-intensive equipment, considering a market with customers in two different classes, one formed by large organizations that require high equipment availability, and the other with smaller organizations that prefer lower service prices. The OEM was considered risk-neutral and intended to maximize its expected profit, while customers were considered risk-averse and aimed to maximize their expected utilities.

Game theory was employed for modeling the interaction among service provider (OEM) and customers. Equipment followed a failure-repair behavior governed by a GRP, with virtual age conditioned Weibull distributed times until failures, and imperfect repairs; however, due to the existence of multiple devices, a priority queue system was formed. This required the usage of a discrete event simulation approach for finding the model's metrics and solutions.

Following a Stackelberg game formulation, perfect and complete information was assumed, so that OEM and customers have complete and perfect knowledge about each other's behavior and decisions, as well as perfectly knowing equipment reliability behavior. For instance, OEM could predict customers' decisions, using this information to maximize its own profit by backward induction. All customers of a given class were homogeneous, meaning that they make the same decision, whereas customers of different classes may decide differently.

An application example was presented, using real failure data from a technology-intensive medical equipment, along with some comments about model behavior. Other possible application scenarios include (but are not limited to) wind turbines, mining trucks (Dymasius, Wangsaputra & Iskandar [42]), and industrial robotic tools. In addition, a sensitivity analysis was performed for better understanding about the effects of each parameter in the model's results.

Although presented model overcame limitations of some existing methods, there is still room for improvement, which is concern of our ongoing research. Some possibilities for future modifications or extensions are listed below

- Inclusion of preventive maintenance actions (Moura *et al.* [28]);
- Consideration of information asymmetry (Jin, Tian & Xie [43]), so that OEM and customers cannot precisely predict each other's decisions and equipment behavior;
- Incorporation of two-dimensional warranty, considering effects such as time and usage into equipment reliability (Samatli-Paç & Taner [44]);

- Allowing for renewal of warranty and/or extension of warranty after the moment of purchase.

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APPENDIX 1 – GENERAL STEADY STATE PROBABILITIES

The list of equations below relates present model's steady state probabilities.

s_0: $\left(M^{(1)}\,\lambda^{(1)} + M^{(2)}\,\lambda^{(2)}\right) p_0 = \mu^{(1)}\,p_{1,0,1} + \mu^{(2)}\,p_{0,1,2}$;

$s_{1,0,1}$: $\left(\left(M^{(1)} - 1\right)\lambda^{(1)} + M^{(2)}\,\lambda^{(2)} + \mu^{(1)}\right) p_{1,0,1} = M^{(1)}\,\lambda^{(1)}\,p_0 + \mu^{(1)}\,p_{2,0,1} + \mu^{(2)}\,p_{1,1,2}$;

$s_{0,1,2}$: $\left(M^{(1)}\,\lambda^{(1)} + \left(M^{(2)} - 1\right)\lambda^{(2)} + \mu^{(2)}\right) p_{0,1,2} = M^{(2)}\,\lambda^{(2)}\,p_0 + \mu^{(1)}\,p_{1,1,1} + \mu^{(2)}\,p_{0,2,2}$;

$s_{a,0,1}$: $\left(\left(M^{(1)} - a\right)\lambda^{(1)} + M^{(2)}\,\lambda^{(2)} + \mu^{(1)}\right) p_{a,0,1} = \left(M^{(1)} - a + 1\right)\lambda^{(1)}\,p_{a-1,0,1} + \mu^{(1)}\,p_{a+1,0,1} + \mu^{(2)}\,p_{a,1,2}\ \left(1 < a < M^{(1)}\right)$;

$s_{0,b,2}$: $\left(M^{(1)}\,\lambda^{(1)} + \left(M^{(2)} - b\right)\lambda^{(2)} + \mu^{(1)}\right) p_{0,b,2} = \left(M^{(2)} - b + 1\right)\lambda^{(2)}\,p_{0,b-1,2} + \mu^{(1)}\,p_{1,b,1} + \mu^{(2)}\,p_{0,b+1,2}\ \left(1 < b < M^{(2)}\right)$;

$s_{M^{(1)},0,1}$: $\left(M^{(2)}\,\lambda^{(2)} + \mu^{(1)}\right) p_{M^{(1)},0,1} = \lambda^{(1)}\,p_{M^{(1)}-1,0,1} + \mu^{(2)}\,p_{M^{(1)},1,2}$;

$s_{0,M^{(2)},2}$: $\left(M^{(1)}\,\lambda^{(1)} + \mu^{(2)}\right) p_{0,M^{(2)},2} = \lambda^{(2)}\,p_{0,M^{(2)}-1,2} + \mu^{(1)}\,p_{1,M^{(2)},1}$;

$s_{1,1,1}$: $\left(\left(M^{(1)} - 1\right)\lambda^{(1)} + \left(M^{(2)} - 1\right)\lambda^{(2)} + \mu^{(1)}\right) p_{1,1,1} = M^{(2)}\,\lambda^{(2)}\,p_{1,0,1} + \mu^{(1)}\,p_{2,0,1} + \mu^{(2)}\,p_{1,2,2}$;

$s_{1,1,2}$: $\left(\left(M^{(1)} - 1\right)\lambda^{(1)} + \left(M^{(2)} - 1\right)\lambda^{(2)} + \mu^{(2)}\right) p_{1,1,2} = M^{(1)}\,\lambda^{(1)}\,p_{0,1,2}$;

$s_{1,b,1}$: $\left(\left(M^{(1)} - 1\right)\lambda^{(1)} + \left(M^{(2)} - b\right)\lambda^{(2)} + \mu^{(1)}\right) p_{1,b,1} = \left(M^{(2)} - b + 1\right)\lambda^{(2)}\,p_{1,b-1,1} + \mu^{(1)}\,p_{2,b,1} + \mu^{(2)}\,p_{1,b+1,2}\ \left(1 < b < M^{(2)}\right)$;

$s_{1,b,2}$: $\left(\left(M^{(1)} - 1\right)\lambda^{(1)} + \left(M^{(2)} - b\right)\lambda^{(2)} + \mu^{(2)}\right) p_{1,b,2} = M^{(1)}\,\lambda^{(1)}\,p_{0,b,2} + \left(M^{(2)} - b + 1\right)\lambda^{(2)}\,p_{1,b-1,2}\ \left(1 < b < M^{(2)}\right)$;

$s_{a,1,1}$: $\left(\left(M^{(1)} - a\right)\lambda^{(1)} + \left(M^{(2)} - 1\right)\lambda^{(2)} + \mu^{(1)}\right) p_{a,1,1} = \left(M^{(1)} - a + 1\right)\lambda^{(1)}\,p_{a-1,1,1} + M^{(2)}\,\lambda^{(2)}\,p_{a,0,1} + \mu^{(1)}\,p_{a+1,1,1} + \mu^{(2)}\,p_{a,2,2}\ \left(1 < a < M^{(1)}\right)$;

$s_{a,1,2}$: $\left(\left(M^{(1)} - a\right)\lambda^{(1)} + \left(M^{(2)} - 1\right)\lambda^{(2)} + \mu^{(2)}\right) p_{a,1,2} = \left(M^{(1)} - a + 1\right)\lambda^{(1)}\,p_{a-1,1,2}\ \left(1 < a < M^{(1)}\right)$;

$s_{a,b,2}$: $\left(\left(M^{(1)} - a\right)\lambda^{(1)} + \left(M^{(2)} - b\right)\lambda^{(2)} + \mu^{(2)}\right) p_{a,b,2} = \left(M^{(1)} - a + 1\right)\lambda^{(1)}\,p_{a-1,b,2} + \left(M^{(2)} - b + 1\right)\lambda^{(2)}\,p_{a,b-1,2}\ \left(1 < a < M^{(1)},\ 1 < b < M^{(2)}\right)$;

$s_{a,b,1}$: $\left(\left(M^{(1)} - a\right)\lambda^{(1)} + \left(M^{(2)} - b\right)\lambda^{(2)} + \mu^{(1)}\right) p_{a,b,1} = \left(M^{(1)} - a + 1\right)\lambda^{(1)}\,p_{a-1,b,1} + \left(M^{(2)} - b + 1\right)\lambda^{(2)}\,p_{a,b-1,1} + \mu^{(1)}\,p_{a+1,b,1} + \mu^{(2)}\,p_{a,b+1,2}\ \left(1 < a < M^{(1)},\ 1 < b < M^{(2)}\right)$;

$s_{M^{(1)},b,1}$: $\left(\left(M^{(2)}-b\right)\lambda^{(2)}+\mu^{(1)}\right) p_{M^{(1)},b,1} = \lambda^{(1)} p_{M^{(1)}-1,b,1} + \left(M^{(2)}-b+1\right)\lambda^{(2)} p_{M^{(1)},b-1,1} + \mu^{(2)} p_{M^{(1)},b+1,2}$ $\left(1 < b < M^{(2)}\right)$;

$s_{M^{(1)},b,2}$: $\left((M^2-b)\lambda^2+\mu^2\right) p_{M^{(1)},b,2} = \lambda^{(1)} p_{M^{(1)}-1,b,2} + (M^2-b+1)\lambda^2 p_{M^{(1)},b-1,2}$ $(1 < b < M^2)$;

$s_{a,M^{(2)},1}$: $\left(\left(M^{(1)}-a\right)\lambda^{(1)}+\mu^{(1)}\right) p_{a,M^{(2)},1} = \left(M^{(1)}-a+1\right)\lambda^{(1)} p_{a-1,M^{(2)},1} + \lambda^{(2)} p_{a,M^{(2)}-1,1} + \mu^{(1)} p_{a+1,M^{(2)},1}$ $\left(1 < a < M^{(1)}\right)$;

$s_{a,M^{(2)},2}$: $\left(\left(M^{(1)}-a\right)\lambda^{(1)}+\mu^{(2)}\right) p_{a,M^{(2)},2} = \left(M^{(1)}-a+1\right)\lambda^{(1)} p_{a-1,M^{(2)},2} + \lambda^{(2)} p_{a,M^{(2)}-1,2}$ $\left(1 < a < M^{(1)}\right)$;

$s_{M^{(1)},M^{(2)},1}$: $\mu^{(1)} p_{M^{(1)},M^{(2)},1} = \lambda^{(1)} p_{M^{(1)}-1,M^{(2)},1} + \lambda^2 p_{M^{(1)},M^{(2)}-1,1}$;

$s_{M^{(1)},M^{(2)},2}$: $\mu^{(2)} p_{M^{(1)},M^{(2)},2} = \lambda^{(1)} p_{M^{(1)}-1,M^{(2)},2} + \lambda^2 p_{M^{(1)},M^{(2)}-1,2}$.

$q_0 + \sum_{a=1}^{M^{(1)}} q_{a,0,1} + \sum_{b=1}^{M^{(2)}} q_{0,b,2} + \sum_{a=1}^{M^{(1)}} \sum_{b=1}^{M^{(2)}} \sum_{c=1}^{2} q_{a,b,c} = 1$

APPENDIX 2 – EXAMPLE STEADY STATE PROBABILITIES

The following set of equations can be employed when relating steady state probabilities for a case with two class 1 and two class 2 customers.

s_0: $\left(2\lambda^{(1)} + 2\lambda^{(2)}\right) p_0 = \mu^{(1)}\, p_{1,0,1} + \mu^{(2)}\, p_{0,1,2}$;

$s_{0,1,2}$: $\left(2\lambda^{(1)} + \lambda^{(2)} + \mu^{(2)}\right) p_{0,1,2} = 2\,\lambda^{(2)}\, p_0 + \mu^{(2)}\, p_{0,2,2} + \mu^{(1)}\, p_{1,1,1}$;

$s_{0,2,2}$: $\left(2\lambda^{(1)} + \mu^{(2)}\right) p_{0,2,2} = \lambda^{(2)}\, p_{0,1,2} + \mu^{(1)}\, p_{1,2,1}$;

$s_{1,0,1}$: $\left(\lambda^{(1)} + 2\lambda^{(2)} + \mu^{(1)}\right) p_{1,0,1} = 2\,\lambda^{(1)}\, p_0 + \mu^{(2)}\, p_{1,1,2} + \mu^{(1)}\, p_{2,0,1}$;

$s_{1,1,1}$: $\left(\lambda^{(1)} + \lambda^{(2)} + \mu^{(1)}\right) p_{1,1,1} = 2\,\lambda^{(2)}\, p_{1,0,1} + \mu^{(2)}\, p_{1,2,2} + \mu^{(1)}\, p_{2,1,1}$;

$s_{1,1,2}$: $\left(\lambda^{(1)} + \lambda^{(2)} + \mu^{(2)}\right) p_{1,1,2} = 2\,\lambda^{(1)}\, p_{0,1,2}$;

$s_{1,2,1}$: $\left(\lambda^{(1)} + \mu^{(1)}\right) p_{1,2,1} = \lambda^{(2)}\, p_{1,1,1} + \mu^{(1)}\, p_{2,2,1}$;

$s_{1,2,2}$: $\left(\lambda^{(1)} + \mu^{(2)}\right) p_{1,2,2} = 2\,\lambda^{(1)}\, p_{0,2,2} + \lambda^{(2)}\, p_{1,1,2}$;

$s_{2,0,1}$: $\left(2\lambda^{(2)} + \mu^{(1)}\right) p_{2,0,1} = \lambda^{(1)}\, p_{1,0,1} + \mu^{(2)}\, p_{2,1,2}$;

$s_{2,1,1}$: $\left(\lambda^{(2)} + \mu^{(1)}\right) p_{2,1,1} = \lambda^{(1)}\, p_{1,1,1} + 2\,\lambda^{(2)}\, p_{2,0,1} + \mu_2\, p_{2,2,2}$;

$s_{2,1,2}$: $\left(\lambda^{(2)} + \mu^{(2)}\right) p_{2,1,2} = \lambda^{(1)}\, p_{1,1,2}$;

$s_{2,2,1}$: $\mu^{(1)}\, q_{2,1,1} = \lambda^{(1)}\, p_{1,2,1} + \lambda^{(2)}\, p_{2,2,1}$;

$s_{2,2,2}$: $\mu^{(2)}\, p_{2,2,2} = \lambda^{(1)}\, p_{1,2,2} + \lambda^{(2)}\, p_{2,1,2}$;

$p_0 + p_{1,0,1} + p_{0,1,2} + p_{2,0,1} + p_{1,1,1} + p_{1,1,2} + p_{0,2,2} + p_{2,1,1} + p_{2,1,2} + p_{1,2,1} + p_{1,2,2} +$
$p_{2,2,1} + p_{2,2,2} = 1.$