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LUCIMÁRIO GOIS DE OLIVEIRA SILVA

ESSAYS ON IMPRECISE REPRESENTATION OF UNCERTAINTY CONSIDERING EVIDENCE THEORY AND FUZZY

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PhD EVALUATION COMMITTEE REPORT ON THE THESIS PRESENTATION OF

LUCIMÁRIO GOIS DE OLIVEIRA SILVA

"ESSAYS ON IMPRECISE REPRESENTATION OF UNCERTAINTY CONSIDERING EVIDENCE THEORY AND FUZZY"

RESEARCH AREA: OPERATIONS RESEARCH

The PhD evaluation committee of with the following examiners, coordinated by the first, considers the PhD candidate LUCIMÁRIO GOIS DE OLIVEIRA SILVA**, APROVADO.**

Recife, 21 de dezembro de 2016.

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ABSTRACT

This paper contains the compendium of three articles on the representation of uncertainty. The two first ones deal with the classical theory of evidence or Dempster - Shafer theory. In these articles, the classification of conflict in the theory of evidence is approached, since bodies of evidence with high level of conflict lead to counterintuitive results when the Dempster Rule of Combination is used. In the first article, the conflict classification is made from class profiles by using the ELECTRE TRI method. In the second article, the classification is made from reference alternatives of each conflict class, the classification of conflict having been obtained through the outranking flow of the PROMETHEE method among the reference alternatives and the alternatives to be classified. In both articles, the parameters of the two methods are obtained by means of disaggregation approaches where the parameters are generated from alternatives pre-classified in conflict in the first article and reference alternatives in the second. Finally, the third article deals with the financial analysis in the development of a new product where a real options model is added by using dynamic programming, the modeling of uncertainty through FUZZY triangular numbers. Thus, in this model it is possible to consider the different types of uncertainty contained in the development of new products.

***Key Words**: Evidence Theory. Conflict. Multicriteria Classification Method. New Product Development. Real option Theory. FUZZY Set Theory.*

RESUMO

Este trabalho contém o compêndio de três artigos sobre representação da incerteza. Os dois primeiros artigos tratam sobre a clássica teoria da evidência ou teoria Dempster - Shafer. Nesses artigos, é abordada a classificação do conflito na teoria da evidência, uma vez que corpos de evidência com alto teor de conflito conduzem a resultados contra intuitivos quando a regra de Dempster é utilizada. No primeiro artigo, a classificação do conflito é feita a partir de perfis de classe, utilizando o método ELECTRE TRI. No segundo artigo, a classificação é feita a partir de alternativas referência de cada classe de conflito, sendo a classificação do conflito obtida por meio do fluxo de sobreclassificação do método PROMETHEE entre as alternativas de referência e as alternativas a serem classificadas. Em ambos os artigos, os parâmetros dos dois métodos são obtidos por meio de abordagens de desagregação onde os parâmetros são gerados a partir de alternativas pré-classificadas em conflito no primeiro artigo e alternativas referência no segundo artigo. Por último, o terceiro artigo trata da análise financeira no desenvolvimento de um novo produto onde é agregado um modelo de opções reais, que utiliza programação dinâmica, a modelagem da incerteza através de números triangulares FUZZY. Assim, nesse modelo é possível considerar os diferentes tipos de incerteza que estão contidos no desenvolvimento de novos produtos.

Palavras Chaves: Teria da Evidência. Conflito. Métodos de Classificação Multicritério. Desenvolvimento de novos produtos. Opções Reais. Teoria dos conjuntos FUZZY.

FIGURE LIST

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1 INTRODUCTION

This thesis contains a compendium of 3 papers written in support of a thesis that forms part of the Post-Graduate Program in Management Engineering at the Universidade Federal de Pernambuco, Brazil. These articles were written to meet one of the requirements for obtaining a Doctorate. The three papers contained in this thesis deal with representations of uncertainty. The first two focus on the Theory of Evidence or Dempster-Shafer Theory (DST), while the third considers Fuzzy Set Theory for financial analysis of the development of a new product.

The first two articles put forward contributions to the Theory of Evidence since combination rules such as Dempster's, for example, produce unsatisfactory results depending on the level of conflict between the bodies of evidence. The study of conflict in DST has become very important after some studies showed that Demspter’s combination rule fails when the two bodies of evidence conflict greatly with each other. Based on this characteristic observed in Dempster's rule, a plethora of rules developed, the main objective of which is to prevent counterintuitive results that invalidate Dempster's rule. However, it should be noted that DRC has some interesting properties, such as associativity and commutativity.

Taking a different tack, some authors prefer to work by using a system for managing conflict in which it is possible to analyze the conflict *a priori* before choosing the rule to be used. In this case, it is necessary to establish a conflict measurement system in which the level of conflict between the bodies of evidence can be identified and, starting with that level of conflict, to choose the appropriate way of dealing with it. Thus, several metrics for identifying conflict have been developed over the years. The first of these metrics to be used was the constant K, which served as the normalization factor in DRC. However, this constant fails to capture all conflict situations, which prompted the emergence of new metrics so that conflict could be quantified.

In general, it is complex to develop an individual metric that can capture all the conflict situations present in DST since the modeling of a problem in this theory can be highly flexible thereby making it difficult to adapt one metric in particular. Given this situation, some authors focus on multidimensional methods for conflict analysis and use more than one metric to identify conflict.

In addition to a multidimensional approach, when the bodies of evidence will be in conflict must be specified. Such a situation, as will become clear in Chapters 2 and 3, is established by using subjective thresholds of conflict. Thus, bodies of evidence will be in conflict when the value of the conflict metric is greater than a pre-set value that depends on the specific application. Taking as a starting point that more than one metric is needed to measure conflict, and then it is also necessary to stipulate more than one threshold of conflict which makes it even more difficult to establish such thresholds.

Given these multidimensional and subjective views of conflict, the first two articles set out to integrate these two views into DST. To do so, the decision of when two bodies are in conflict is modeled as a multicriteria decision problem. In addition, the use of subjective thresholds of conflict that are difficult to explain can be avoided by using disaggregation models that generate the parameters of the model from pre-classified alternatives of conflict. This avoids eliciting thresholds arbitrarily.

Thus, the first article in the second chapter, published on Information Sciences, uses the ELECTRE TRI method to classify conflict in the Theory of Evidence. In order to generate the parameters of the method, a disaggregation model given in the literature is used in which alternatives pre-classified as being in conflict must be generated. In the second article, submitted to Information Fusion and presented in Chapter three, the same premise is analyzed using the PROMETHEE method. However, the pre-classified alternatives used to generate the parameters are also used to classify the conflict. These same alternatives used for classification are also used to generate the parameters by means of a disaggregation method from the literature that this article modifies, resulting in the proposal of a different PROMETHEE classification procedure as another contribution of this thesis.

Finally, we have the third article, submitted to Knowledge-Based Systems, which focuses on the topic with reference to representing uncertainty by using Fuzzy Set Theory. This article is about making a financial analysis of the development of new products. To do so, a model of real options from the literature was used that integrates risk of a technical nature with the risk of a market nature. Given that the availability of historical data is scarce for modeling the uncertainties of developing new products, the innovatory aspect of this article is the fact that the modeling of some of these risks is done by means of fuzzy numbers that are easy to obtain when elicitation procedures are used and integrated with probabilistic information along the project.

1.1 Study Motivation

After the publication of Zadeh's (1986) paper on generating counterintuitive results when the two bodies of evidence present a high level of conflict, a real plethora of combination rules has emerged. Most of these new rules aim at solving the problem of counterintuitive results.

The reason for this high number of rules is directly linked to the broadly open character contained in the theory of evidence. As stressed in the introduction, although this theory aligns well in situations where the classical theory of probability presents difficulty in modeling, some situations of a rather subjective nature may complicate both the application of a combination rule and the measurement of conflict. To exemplify some of these situations, we can mention the required independence of the bodies of evidence in the application of the Dempster Rule of Combination (DRC); hypothesis on updating the framework of discernment; reliability of the bodies of evidence and metric thresholds for conflict determination.

All these raised issues may be decisive for the creation of a new combination rule. Determining the degree of dependence between the bodies of evidence is important depending on the area of application of the combination rule. If the structure or decision issue where the combination rule is to be used does not support the hypothesis of independence, then the DRC should not be used (CHIN & FU, 2015). For instance, in multicriteria decision models many times the hypothesis of independence among the criteria cannot be assumed, so a new rule that adjusts this situation must be used (CHIN & FU, 2015).

Regarding the update of the framework of discernment, two assumptions can be used. The first one would be on the completeness of the framework, i.e., the non-admission of new hypotheses to integrate it. In the second, the possibility of new hypotheses to be added to the frame of discernment is seen by some authors as a source of conflict between bodies of evidence (SMETS, 2007). Thus, the conflict analysis or a new combination rule must conform to one of those two situations.

Another issue widely discussed in the literature which has also raised new rules of combination is the question of the reliability of the bodies of evidence. In this approach, before the bodies of evidence are combined - generally using the DRC -, they are deducted according to a reliability factor. Regarding the origin of the reliability of the bodies of evidence, two sources can be highlighted here, which we will call data external source and data internal source. As far as the external source is concerned, the coefficients of deduction are generated according to the external information about the bodies of evidence, which is extracted according to the analyst's knowledge on the bodies of evidence (SHAFER, 1976). On the other hand, the origin of reliability is obtained by taking into account the information from the data internal source. This analysis may be defined as from Dubois & Prade's (1994) interpretation, where the authors consider that when two bodies are in conflict it is because at least one of the bodies of evidence is not reliable. Considering this interpretation, the coefficients of discounted for each body of evidence can be generated from the deviations of the body of evidence in relation to the others, often having metrics of conflict as a parameter (LEUNG & MA, 2013).

Since the main prerequisite for the continuous emergence of new combination rules is tied to the possibility of counterintuitive results, a conflict measurement approach is needed. Thus, conflict measurement metrics can be structured with two objectives. The first is linked to the administration of conflict where from the results of the metrics the pairs of bodies of evidence can be organized according to the level of conflict they present. For example, Liu (2006) and Destercke & Burger (2013) consider three possibilities according to the result of the metrics having the DRC as parameter. In the second type of approach, the metrics is used to generate the deduction factors of the bodies of evidence.

It is well established in the literature that the constant k used to normalize the DRC alone is not sufficient to determine the conflict, requiring more than one metrics (LIU, 2006), or an interval metrics (DESTERCKE & BURGER, 2013). Regardless of the way in which the conflict is handled, whether it is conflict management or deduction of the bodies of evidence, it is necessary to use more than one metrics to analyze the conflict or to deduct the bodies of evidence.

In addition to the problem of considering a single metrics to quantify the conflict, attention should be paid to the subjective thresholds from which the conflict is set. Thus, when considering more than one metrics, the difficulty to define different thresholds of conflict for the different metrics arises, making the problem even more complex for the analyst. Besides considering those issues, an approach of conflict classification should be as general as possible in order to take the aforementioned issues into account. Thus, in our view, an approach based on more than one metrics and at the same time generated from pre-classified alternatives is more general, since it can better adapt to the situations mentioned here.

In the financial analysis of development of new products, the traditional metrics used to evaluate such projects are criticized in the literature since it does not take into account the flexibility due to the high level of uncertainty present in the early stages of such projects. In this sense, this flexibility can be translated as an opportunity to increase or decrease the losses in the project value throughout its development. Given this characteristic, the literature recognizes the real options approach as an ideal model since it takes into account the flexibility of innovation projects as an opportunity to increase the project value. However, the traditional models used in real options are based on the context of financial options environment, which prevents the correct use of NPD projects due to the nature of the uncertainties present in these projects being different from the uncertainties contained in the financial market.

Another issue which is inherent in NPD projects is the lack of historical data, since these projects are being carried out for the first time. Thus, the nature of uncertainty contained in this project is primarily subjective, having human judgments as its main source of information. Hence, besides choosing the correct model, it is necessary to choose an appropriate uncertainty model to extract human information, taking into account the imperfections present in human judgments.

1.2 Thesis Objectives

1.2.1 General Objective

This study has as main objective the generation of a conflict determination approach in the theory of evidence that takes into account two main aspects pertinent to this theory in a conflict analysis: the quantification of conflict through the integration of different metrics and the expansion of the conflict model by considering the conflict subjective modeling by using disaggregation approaches, avoiding the use of subjective thresholds and increasing the level of information about the conflict. As regards the development of new products, the main objective is developed a financial model that integrated the real option approach and Fuzzy Set Theory for financial analysis of New Product Development.

1.2.2 Specific Objectives

- Determination of a evidence theory conflict measurement approach focused on the main multicriteria classification methods.
- Conflict determination through reference alternatives.
- Evaluation of the efficiency of multicriteria methods in conflict classification.

- Definition of how FUZZY numbers can be integrated in relation to NPD uncertainties.

1.3 Thesis Structure

This thesis is structured in 5 chapters as shown in figure 1.1. In chapter 1, the introduction with the papers produced is presented, along with the motivation and objectives. Chapter 2 presents a conflict classification approach in the theory of evidence using the ELECTRE TRI method. Chapter 3, in turn, presents a conflict classification approach using a variant of the PROMETTHEE method. In Chapter 4, the financial analysis of the development of new products is discussed, integrating the real options approach with the uncertainty modeling by using FUZZY numbers. Finally, the conclusion brings the main results as well as the state of development of research.

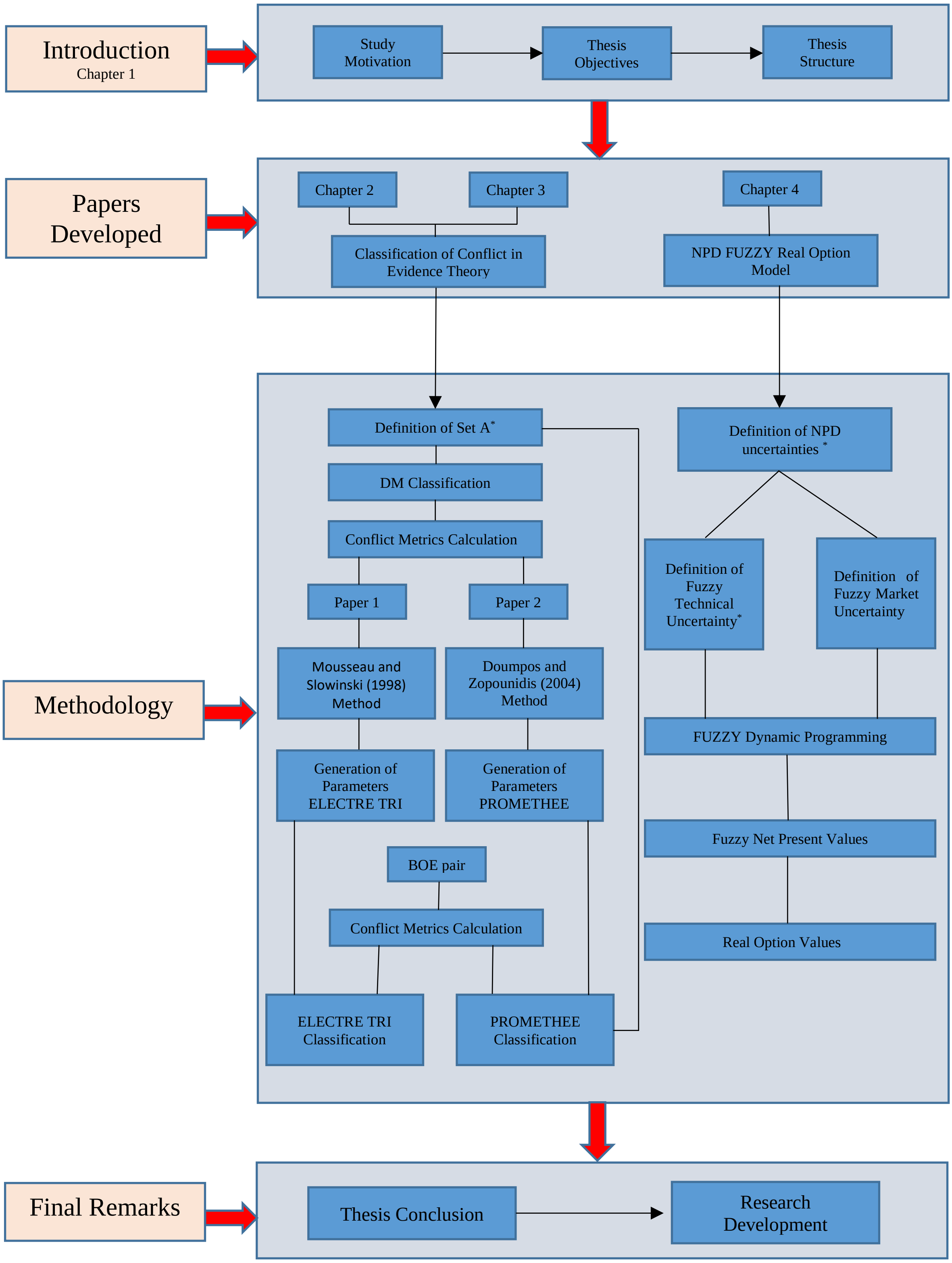

Figure 1.1 – Thesis structure

2 A MULTICRITERIA APPROACH FOR ANALYSIS OF CONFLICTS IN EVIDENCE THEORY

A multicriteria classification problem involves the assertion of a finite set of alternatives $A = \{a_1, a_2, ..., a_m\}$ in pre-defined q groups $C_1, C_2, ..., C_q$, where the alternatives are described by a vector of n criteria $g = \{g_1, g_2, ..., g_n\}$ (ZOPOUNIDIS & DOUMPOS, 2002). In this sense, the general idea is to systematize the classification of the alternatives as from the criteria aggregation.

The analysis carried out by Zopounidis & Doumpos (2002) places the outranking methods as the most used ones for the multicriteria classification. The a_pSa_i outranking relationship states that alternative a_p is at least as good as alternative a_1. In this context, the ELECTRE TRI method (YU, 1992; ROY & BOUYSSOU, 1993), to be dealt with in this chapter, and the PROMETHEE method (BRANS & VINCKE, 1985; VINCKE, 1992), to be used in the next, are included.

2.1 MCDM Analysis of Conflict in Evidence Theory

Since its development in the 60s, Dempster-Shafer Theory (DST) or Evidence Theory (SHAFER, 1976), has been seen as one of the main tools for dealing with situations of uncertainty which classical probability theory has difficulty in modeling. Situations like vagueness, ignorance and others cannot be modeled by classical Probability Theory given their axiomatic premises. As a counterpoint, DST does not require that the axioms of additivity and completeness are adhered to, thus allowing a wider range of situations to be modeled. Therefore, this theory can be used for research studies in very different areas: image processing (LIN, 2010); group decision using multiple criteria (FU & YANG, 2010; SEVASTJANOV & DYMOVA, 2015); maintenance (BARALDI; COMPARE & ZIO, 2013); neural networks (AGGARWAL et al., 2013) etc.

Despite all these characteristics, the main advantage of DST comes from Dempster Rule of Combination (DRC) which allows two belief functions or independent bodies of evidence to be merged. From a practical point of view in DRC, the presence of an *a priori* distribution to establish a merger between two bodies of evidence is not necessary, as required in Bayesian Theory. However, the application of this rule generates counter-intuitive results when the two bodies of evidence involved in the merger conflict with each other to a high degree (ZADEH, 1984).

The combination of bodies of evidence arise in many contexts when aggregating expert's knowledge. There are several works addressing this matter in a fuzzy context for example (MATA et al., 2014; PARREIRAS et al., 2010; PARREIRAS et al., 2012; PARREIRAS et al.; 2012; WU & CHICLANA, 2014). Herrera-Viedma et al. (2014) presents review of fuzzy approaches for aggregating expert's knowledge using group decision making and fuzzy logic through soft consensus, pointing new trends and challenges within this fuzzy context, while Cabrerizo et al. (2010) analyze different consensus approaches in fuzzy group decision making problems, including partial consensus, full consensus and soft consensus.

When considering the DRC there are two main approaches that have been developed over the years in order to overcome the issues due to a high degree of conflict. The first class of approaches is focused on modifying DRC which generated a real jungle of combination rules in the literature (SMETS, 2007). The second focuses on administering the conflict without necessarily modifying DRC (SMETS, 2007; YANG et al., 2011; SCHUBERT, 2011).

As to the first approach, the change in DRC is proportional to some constant that expresses the level of conflict between two bodies of evidence. The first natural metric developed for this is the normalization constant of DRC that some authors associate with the level of conflict between two bodies of evidence. However, as demonstrated by Liu (2006), this constant does not capture all the possible existing conflict situations in this theory, although it may capture them to a certain extent. This impossibility present in the normalization constant caused authors to investigate or develop another way to measure the conflict between two bodies of evidence (JOUSSELME & MAUPIN, 2012; LIU, 2006). Given the computational complexity present in DST, it is complex from the computational point of view to represent all possible conflict situations using a single metric.

Against this background, another approach to identifying conflict becomes necessary. The first is contained in the work of Liu (2006) and is also supported by the results of the work of Jousselme & Maupin (2012), in which they consider measuring conflict by using two metrics plus a numerical threshold of subjective conflict. Following a different line, Destercke & Burger (2013) develop an interval metric based on axioms while Fu et al. (2010) focus on separating the internal conflict from the external one.

Regardless of the method for measuring conflict, two situations are always present when analyzing conflict in DST: More than one metric is needed to measure the conflict in this theory; and, at the same time, there is some degree of subjectivity involved when determining what the conflict is.

An important point to consider is the meaning of the conflict metric and how it would be quantified and aggregated with other types of metrics seeking to capture different types of conflict situations.

Using this prism, the classification of conflict in DST can be seen as a problem of multi-criteria classification. By taking this view, this paper seeks to expand the measurement of conflict in DST by using a multi-criteria method of classification. To this end, the suitability of using such a method when analyzing conflict in DST is ascertained.

Within the multicriteria approaches the kind of methodology that would be more suitable for addressing the issue raised in this paper is a non-compensatory methodology, as it does not consider tradeoffs amongst the criteria. Thus, more than two conflict metrics can be aggregated for dealing with conflict measurement in DST, as proposed in this article.

With this in mind, the ELECTRE TRI method was chosen. This Multi Criteria Decision Making (MCDM) method uses an outranking relationship where each measure of conflict is defined by using a pseudo-criterion in order to integrate the subjective imprecision into assessing what the conflict is.

This article is divided into 6 sections including this Introduction. Section 2 presents the basic elements of DST and DRC. Section 3 discusses conflict and how it is measured in the literature. In Section 4, the ELECTRE TRI method is introduced while section 5 sets out how the problem is structured and a numerical application of the proposed model. Finally, the conclusion discusses the method and what studies could be usefully undertaken in the future.

2.2 Basic Concepts

DST is defined on a non-empty finite, exhaustive and mutually exclusive set, $\boldsymbol{\Theta}$**,** of elementary events. This set is called a "frame of discernment" and the set formed by all possible subsets of $\boldsymbol{\Theta}$ is called a power set, $2^{(|\boldsymbol{\Theta}|)}$. To see how the two sets are related, consider the case where $\boldsymbol{\Theta}$ has three elements, $\boldsymbol{\Theta} = \{\theta_1, \theta_2, \theta_3\}$, in this case $2^{(|\boldsymbol{\Theta}|)}$ will have 2^3 elements defined as follows: $2^{|\Theta|} = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \{\theta_3\}, \{\theta_1, \theta_2\}, \{\theta_1, \theta_3\}, \{\theta_2, \theta_3\}, \boldsymbol{\Theta} \}$. Based on the $2^{|\boldsymbol{\Theta}|}$ set, the basic probability assignment function, *m*, is defined and is given in Equation (2.1) and Equation (2.2).

$$m : 2^{|\boldsymbol{\theta}|} \rightarrow [0,1] \tag{2.1}$$

$$\sum_{\mathbf{A}\in\boldsymbol{\theta}} \mathrm{m}(\mathbf{A}) = 1 \tag{2.2}$$

The function *m (A)* can be interpreted as the degree of belief that the system has in a certain element *A* belonging to the $2^{|\boldsymbol{\theta}|}$ set. If *m (A)* > 0, then set *A* is called the focal element. Using the function *m*, two other functions are defined: The belief function *Bel(A)* and the plausibility function *Pl (A).* The *Bel (A)* function is defined as the total of belief that is attributed to set *A*, which is calculated by the expressions in Equation (2.3) and Equation (2.4).

$$Bel : 2^{|\boldsymbol{\theta}|} \rightarrow [0,1] \tag{2.3}$$

$$Bel(A) = \sum_{B\subseteq A} m(B) \tag{2.4}$$

The *Pl (A)* function measures the maximum amount of belief that can be attributed to set *A*, which is calculated by Equation (2.5) and Equation (2.6).

$$Pl : 2^{|\theta|} \rightarrow [0,1] \tag{2.5}$$

$$Pl(A) = \sum_{B\cap A\neq\emptyset} m(B) \tag{2.6}$$

Given the characteristics of the functions *Bel (A)* and *Pl (A)*, some authors see them as natural limits to the real probability of event *A* occurring.

Given two independent bodies of evidence, m_1 and m_2, Dempster rule of combination is defined by the standard orthonormal sum as may be seen from Equation (2.7) to Equation (2.9).

$$m_{12}(A) = \frac{\sum_{B\cap C=A} m_1(B)m_2(C)}{1-k}, when\ A \neq \emptyset \tag{2.7}$$

$$m_{12}(\emptyset) = 0 \tag{2.8}$$

$$k = \sum_{B \cap C = \emptyset} m_1(B) m_2(C) \tag{2.9}$$

The constant k represents the conflict between m_1 and m_2 while $1\text{-}k$ represents the normalization factor that ensures the condition represented by Equation (2.2). In some combination rules, rather than the presence of the normalization factor, the conflict is completely transferred to the empty set. To demonstrate situations in which this rule can generate counter intuitive results, consider the following pair of bodies of evidence shown in the following situation, where basic probability assignment function to the first belief function is given by $m_1(A) = 0.1$ and $m_1(C) = 0.9$; while basic probability assignment function to the second belief function is $m_2(A)= 0.1$ and $m_2(B)= 0.9$, where $\Theta = \{A, B, C\}$.

The result of the combination of the two bodies of evidence above will result in m_{12} (A) = 1. This is because A is the common element between the two bodies of evidence even if it is the element to which the lowest belief is attributed. Thus the high values attributed to the sets B and C are disregarded.

There is a literature related to the use of belief functions combined with a decision maker preferences, such as presented by Danielson & Ekenberg (2007) when addressing expected utility by considering imprecise statements.

2.3 Measuring Conflict IN DST

Given the problematic issues found in the example given in last section, it is necessary to determine a means to report when two bodies of evidence are in conflict. Thus, the first natural metric used to measure the conflict was the constant k described in (9). As can be seen from (9), this constant represents the sum of beliefs when the intersection of the sets under consideration is empty.

However, as demonstrated by Liu (2006), this metric cannot represent all conflict situations. In order to grasp this situation, now consider the second example when basic probability assignment function for the first belief function is given by: $m_1(\theta_1) = 0.4$; $m_1(\theta_2) = 0.3$; $m_1(\theta_3)= 0.2$; $m_1(\{\theta_1, \theta_3\})= 0.1$. As for second belief function, this is given by: $m_2(\theta_1)=0,2$; $m_2(\theta_2)=0.3$; $m_2(\theta_3)=0.2$; $m_2(\{\theta_1, \theta_2\})$; $m_2(\Theta)=0.1$.

This second example represents a situation where they are normally classified into a low threshold of conflict. However, the value of k=0.49 does not represent this situation. Given situations such as this, a true jungle of conflict factors or distances in order to measure the difference between two bodies evidence sprang up in the literature. The reason for the large number of metrics is linked to the complexity that involves the large number of representations that the evidence of bodies can take on, in this theory. Therefore, authors acknowledge the need to consider at least more than one metric to measure DST conflict .

As main representatives of these metrics, there are betting commitment distances (TESSEM, 1993) and Jousselme's Distance (JOUSSELME, GRENIER & BOSSÉ, 2001). The betting commitment distance is based on what is called a pignistic transformation that is used to assist in decision-making problems which use DST. The formulation of this distance is given by Equation (2.10).

$$d^1\left(m_i, m_j\right) = \max_{A \subseteq \Theta}\left(\left|BetP_i(A) - BetP_j(A)\right|\right) \tag{2.10}$$

Where *BetP(A)* is given by Equation (2.11):

$$BetP(A) = \sum\nolimits_{B \subseteq \Theta} m(B).\left(\frac{|A \cap B|}{|B|}\right) \tag{2.11}$$

The Jousselme distance (JOUSSELME, GRENIER & BOSSÉ, 2001) shown in Equation (2.12), is based on the weighted Euclidean distance, thus allowing a geometrical interpretation of the DST. Apart from the geometrical interpretation, authors also use the Jaccard coefficient, *D*, as a measure of similarity between the focal elements. Thus, in Equation (2.12), m_i and m_j are vectors, the components of which are the basic probability assignment function in the whole set 2^θ. *D* is a matrix 2^θ X 2^θ, the elements of which are defined by$D_{hl} = = -\frac{|A_h \cup A_l|}{|A_h \cap A_l|}$.

$$d^2\left(m_i, m_j\right) = \sqrt{\frac{1}{2}\left(m_i - m_j\right)^T D\left(m_i - m_j\right)} \tag{2.12}$$

Just as in the case of the constant *k*, these two metrics also cannot manage to capture all conflict situations. To see this situation, consider Example 3 where the two functions of basic probability assignment are given as follows: $m_1(\theta_1)=0.8$; $m_1(\Theta)=0.2$ *and* $m_2(\theta_1)=0.1$; $m_2(\{\theta_2, \theta_3\})= 0.2$; $m_2(\Theta)=0.7$.

At first glance, it seems reasonable to assert that the two bodies of evidence in this example conflict greatly, but on analyzing the body of evidence 2, more precisely $m_2(\Theta)=0.7$, it is clear that the two functions cannot contradict each other because part of the mass Θ can be fully or partially moved to θ_1. For this example, there is therefore $d^1 = 0.53$ and $d^2 = 0.57$, while the value of $k=0.16$, which is to say that, in this particular case, it is only by using the constant k that it can be seen that the two bodies of evidence do not contradict each other.

2.4 ELECTRE TRI

The ELECTRE TRI is a method that belongs to a wide class of multi-criteria classification methods. Thus, the problem addressed by this method is that of assigning a group of alternatives $A = \{a_1, a_2, ..., a_m\}$ evaluated in n criteria $g_1, g_2, ...,$ to k categories (WEI, 1992; YU, 1992). The classification of each alternative is made, based on a comparison of each alternative of the set A with the profiles that define the limits of each class.

The first part of the ELECTRE TRI analysis involves defining the following sets:

- The set of criterion indices $F = \{1, 2, ..., m\}$ which represents the total number of criteria of the problem;
- The set of profile indices $B = \{1, 2, ..., p\}$ that represents the profiles that define the upper and lower limits of each class where b_h represents the upper profile of C_h and b_{h-1}, the lower profile of category C_{h-1}.

Therefore, given the definition of the sets above, the profile b_{p+1} represents the maximum profile expected in each criterion while b_0 represents the minimum profile in each criterion. If the decision-maker's evaluation decreases with the increase of the criterion, the situation described above is inverted.

In order to classify each alternative, the method uses a preference relationship known as an out-ranking relationship, *aSb,* which may be translated as: The alternative *a* is at least as good as alternative *b* (ROY & VINCKE, 1984). This relationship is explored analytically by evaluating a criterion measurement function known as a pseudo-criterion that takes two thresholds into consideration: a preference threshold $p_j(b_h)$, and an indifference threshold $q_j(b_h)$. The aim of using these thresholds is to take into account the ever-present vagueness in human judgments.

Thus, the judgment *aSb* is evaluated by two indices:

- A concordance index (C): This index expresses the relation *aSb* will only be accepted if the majority of the criteria are in favor of *a*.

- A Discordance Index (D): This is a cut-off index that points against the assertive *aSb* even if the concordance index points to the contrary. The discordance index is associated with a veto threshold for which $g(b_j)$ cannot be greater than $g(b_j)$.

From the viewpoint of inter-criteria analysis, what is still required is to determine the weights $\{w_1, w_2, ..., w_m\}$ which show the relative importance between the criteria while for the discordance index, a veto vector $\{ v1\ (b_h), v_2\ (b_h), ..., v_m\ (b_h)\}$ must be defined.

In the final evaluation phase of the method, a credibility index of $\sigma\ (a, b)$ is generated, based on the concordance and discordance indices, which indicate by how much alternative *a* outranks alternative *b*.

The $\sigma\ (a, b)$ index is calculated by using the following steps:

1. Calculation of the partial concordance indices $c_j\ (a, b_j)$ for all $j \in F$ is given by Equation (2.13).

$$c_j(a, b_j) = \begin{cases} 0\ if\, g_j(b_h) - g_j(a) \geq p_j(b_j) \\ 1\ if\ g_j(b_h) - g_j(a) \leq q_j(b_j) \\ \dfrac{p_j(b_j) + g_j(a) - g_j(b_h)}{p_j(b_j) - q_j(b_j)}, otherwise \end{cases} \tag{2.13}$$

2. Calculation of the global concordance index $c(b_j)$ for all $j \in F$ is given by Equation (2.14).

$$c(a, b_j) = \frac{\sum_{j \in F} w_j\ . c_j(a, b_j)}{\sum_{j \in F} w_j} \tag{2.14}$$

3. Calculation of the discordance indices $d_j\ (a, b_j)$ is given by Equation (2.15).

$$d_j(a, b_j) = \begin{cases} 0\ if\ g_j(b_h) - g_j(a) \leq p_j(b_j) \\ 1\ if\, g_j(b_h) - g_j(a) \geq v(b_j) \\ \dfrac{g_j(b_j) - g_j(a) - p_j(b_h)}{v_j(b_j) - p_j(b_j)}, otherwise \end{cases} \tag{2.15}$$

4. Calculation of the credibility index $\sigma\ (a, b)$ is given by Equation (2.16).

$$\sigma(a, b) = c(a, b_j) \prod_{j \in \mathrm{C}} \frac{1 - d_j(a, b_j)}{1 - c(a, b_j)} \tag{2.16}$$

Where $C = \{j \in F : d_j(a, b_j) > c(a, b_j)\ \}$.

Finally, an outranking threshold, λ, is defined which defines when alternative *a* outranks alternative *b*.

After defining the indices above, the preference relations will be defined in accordance with the values of the credibility indices σ $(a;b_h)$ and σ $(b_h;a)$ for the following possibilities:

- $\sigma(a, b_h) \geq \lambda$ and $\sigma(b_h, a) \geq \lambda \Rightarrow a\mathrm{S}b_h$ and $b_h\mathrm{S}a \Rightarrow a\mathrm{I}b_h$, i.e. a is indifferent to b_h.
- $\sigma(a, b_h) \geq \lambda$ and $\sigma(b_h, a) < \lambda \Rightarrow a\mathrm{S}b_h$ and not - $b_h\mathrm{S}a \Rightarrow a \gtrsim b_h$, , i.e. a is preferable to b_h.
- $\sigma(a, b_h) < \lambda$ and $\sigma\left(b_h, a\right) \geq \lambda \Rightarrow not\ a\mathrm{S}b_h$ and $b_h\mathrm{S}a \Rightarrow b_h \gtrsim a$, i.e. b_h is preferable to a .
- $\sigma(a, b_h) < \lambda$ e $\sigma\left(b_h, a\right) < \lambda \Rightarrow not\ a\mathrm{S}b_h$ and $not\ b_h\mathrm{S}a \Rightarrow b_hRa$, i.e. b_h is not comparable to a.

Now, the process of classification can follow two procedures:

1. Pessimistic Procedure: Comparing alternative *a* successively with b_i to $i = 1, 2, ..., p-1, p$. Thus, *a* will be classified in the C_{h+1} category when aSb_{h+1}.
2. Optimistic Procedure: Compare the alternative *a* successively with b_i for $i = p, p-1, ..., 2, 1$ .. Thus, *a* will be classified in the category C_h when b_hSa.

2.5 MCDM Conflict Classification IN DST

The purpose of this section is to structure the problem under analysis from the viewpoint of a multi-criteria classification problem. Given a set of bodies of evidence, $G = \{m_1, m_2, ..., m_n\}$, and a set of criteria $F = \{1, 2, ..., k\}$, out of the set G, the set $GxG = \{(m_1, m_2); (m_1, m_3); ...; (m_2, m_3); (m_2, m_4); ...; (m_{k-1}, m_k)\}$ will be generated, the elements of which represent the pairwise combination of all the elements of the set G. Thus, given a set G with cardinality $|G| = k$, the cardinality of GxG is given by $|GxG| = \frac{k!}{(k-2)!.2!}$. Therefore, problems, for which the set G which has low cardinality, may present, in terms of classification of conflict, a high degree of difficulty.

If, for example, a problem is considered with $|G| = 10$, there is a problem that seeks to classify 45 pairs of bodies of evidence. After defining the set G, each pair in the set will be classified as to the conflict, based on a pair-wise comparison with the profiles that represent the classes of conflict. This comparison follows the same principle of the comparison made in the study by Frikha (2014) whose study develops a multi-criteria method for determining the discount coefficients of the bodies of evidence to be used in DRC.

2.5.1 Defining the criteria

Several metrics have been developed over the years in order to measure both the similarity and the conflict in DST. Recently some articles have examined the statistical relationship between different metrics (JOUSSELME & MAUPIN, 2012). It should be noted that these studies are about similarities between two bodies of evidence and not exactly about the conflict in this theory. Although similarity and conflict are related, the two come in the form of different concepts, taking into account the distances of similarities defined in DST. For more information about the distances used in the DST, the reader is encouraged to consult the references (JOUSSELME & MAUPIN, 2012).

In addition to the metrics developed to measure the similarity between the bodies of evidence, other metrics have been developed with a view to capturing the conflict, some examples of which are given in Table 2.1.

Table 2.1 - Metrics of Conflict

Name	Distance	Reference
Conflict Rate	$\lambda = \frac{\sum_{B_i \cap C_i} m_1(B_i)m_2(C_i)}{\sum_{A \subseteq \boldsymbol{\theta}} m_1(A)m_2(A) + \sum_{B_i \cap C_i} m_1(B_i)m_2(C_i)}$	Qu et al. (2009)
Relative coefficient	$r(X,Y) = \frac{\sum m_1 Log(m_1) + \sum m_2 Log(m_2)}{\sum m_1 Log(m_2) + \sum m_2 Log(m_1)}$	Deng et al. (2011)
Similarity	$sim = \frac{\sum m_1{}'(B_i)m_2{}'(B_i)}{(\sum m_1{}'(B_i)^2 m_2{}'(B_i)^2)^{1/2}}$ $m' \ (B \ \ ) = \sum_{A \subseteq \boldsymbol{\theta}, B \in A} \frac{m(A)}{\lvert A \rvert}$	Wen-hao et al. (2011)

2.5.2 Defining the Parameters

The greatest difficulty in ELECTRE TRI is modeling parameters which are not easily obtainable from the decision-maker (DM). This is due to the high number of parameters associated with the problem. Based on this problematic, the study by Mousseau and Slowinski (1998) sets out a method to assist in generating assignment parameters by means of simple assignment judgments. The method considers the selection of a subset $A^* \subset A = \{a_1, a_2, \ldots, a_n\}$, from the set of alternatives of the problem and the DM's assignment of each alternative to a particular class C_h. Thereafter, the parameters are generated using a nonlinear optimization problem. The optimization condition present tries to ensure parameters that best represent the DM´s assignment. The optimization model used has the following mathematical structure (MOUSSEAU & SLOWINSKI, 1998) as described from Equation (2.17) to Equation (2.25):

$$\text{Max } \alpha + \varepsilon \sum_{a_k \in A^*} x_k + y_k \tag{2.17}$$

Subject to:

$$\alpha \leq x_k \tag{2.18}$$

$$\alpha \leq y_k \tag{2.19}$$

$$\frac{\sum_{j=1}^{m} k_j c_j\left(a_k, b_{h_{k-1}}\right)}{\sum_{j=1}^{m} k_j} - x_k - \lambda = 0, such\ that\ \ a_k \in A^* \tag{2.20}$$

$$\lambda - \frac{\sum_{j=1}^{m} k_j c_j\left(a_k, b_{h_k}\right)}{\sum_{j=1}^{m} k_j} - y_k = 0,\ such\ that\ a_k \in A^* \tag{2.21}$$

$$\lambda \in [0.5,1], \tag{2.22}$$

$$g_j(b_{h+1}) \geq g_j(b_h) + p_j(b_h) + p_j(b_h), such\ that\ j \in F\ and\ h \in B \tag{2.23}$$

$$p_j(b_h) \geq q(b_h), such\ that\ j \in F\ and\ h \in B \tag{2.24}$$

$$k_j\ > 0\ , q_j(b_h) \geq 0, such\ that\ j \in F\ and\ h \in B \tag{2.25}$$

In the problem above, $x_{k,}$, y_k and α are the variables related to the pessimistic assignment procedure. The alternative a_k will be designated the C_h class, if the conditions $\sigma\left(a_k, b_{h\,-\,1}\right) \geq \lambda$ and $\sigma\left(a_k, b_h\right) < \lambda$, Equation (2.21) and Equation (2.22), will be satisfied. If the DM did not show inconsistency in the assignment, then any positive value of the objective function, Equation (2.18), guarantees the existence of consistent model parameters. The remaining equations, from Equation (2.23) to Equation (2.15) ensure the consistency of the model. Thus, in computational terms, the problem has *3mp+m+2* variables and *4n+2+3mp* constraints. For further information about the model, the reader is encouraged to consult the reference (MOUSSEAU & SLOWINSKI, 1998).

2.5.3 Defining of the C_h

The classification of conflict in DST is mostly dealt with in binary form, i.e. either there is conflict or there is not. An alternative and more complete form is to consider an intermediate region where the DM or expert system considers the conflict is at an intermediate level. Thus a complete model that takes into account the DM´s own vagueness must consider three regions as shown in Figure 2.1.

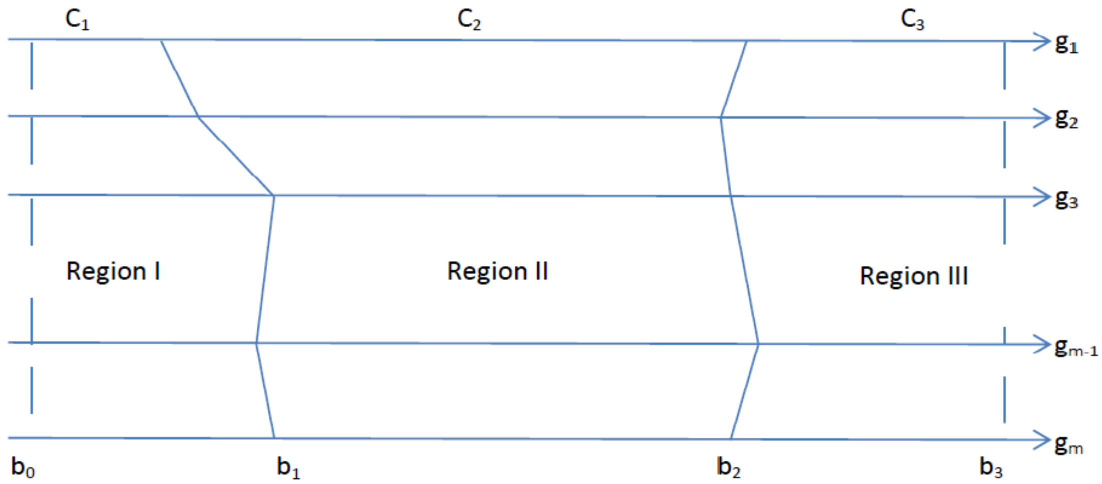

Figure 2.1 - Definition of the regions of conflict in DST

Class C_1 is reserved for the body of evidence pairs which, in the DM´s judgment have a low level of conflict. On the opposite side is class C_3, the region of which is reserved for the pairs that have a high degree of conflict. Class C_2 class is reserved for the pairs, the classification of which the DM is unsure about with regard to the level of conflict, and so is classified as an intermediate region of conflict. The classification described here follows the same reasoning of classification suggested by Liu (2006) who uses DRC as a classification criterion. Therefore, class C_1 is reserved for the pairs for which DRC does not lead to a counterintuitive result while class C_3 class has the opposite characteristic. Finally, C_2 is reserved for the pair of bodies of evidence for which DRC should be observed with precaution.

2.5.4 Defining the set of inference *A**

The definition of the set A^* takes into account the formation structure of the C_h classes. That means, from the DM´s point of view, that this elicitation should follow some rational principles. As an example, consider Table 2.2 which shows examples of pairs of bodies of evidence, the cardinality of which from the frame of discernment is equal to 3. Each pair represents a degree of conflict purposely chosen in accordance with the literature. For example, the pairs (m_1, m_2) and (m_3, m_4) can never be classified in class C_1. On the other hand, the pairs (m_9, m_{10}) and (m_{11}, m_{12}) cannot be classified in the class C_3, i.e., although the classification made by the DM is subjective in nature, this presupposes certain consistency criteria for assignment of the alternatives belonging to $A*$. The advancement of the model comes from creating the C_2 class that has bodies of evidence for which the expert shows indecision regarding conflict classification, i.e. he or she tends to classify it in an intermediate region of conflict.

Table 2.2 – Pairs of classified alternatives.

BOE	Sets							
	$\{\theta_1\}$	$\{\theta_2\}$	$\{\theta_3\}$	$\{\theta_1,\theta_2\}$	$\{\theta_1,\theta_3\}$	$\{\theta_2,\theta_3\}$	Θ	Class
m_1	0.9		0.1					C_3
m_2		0.8	0.1				0.1	
m_3	0.8	0.1					0.1	C_3
m_4		0.1	0.8				0.1	
m_5	0.4	0.4	0.2					C_2
m_6	0.8						0.2	
m_7	0.4	0.4	0.2					C_2
m_8	0.7	0.2					0.1	
m_9	0.4	0.3	0.2		0.1			C_1
m_{10}	0.2	0.3	0.2	0.2			0.1	
m_{11}	0.8						0.2	C_1
m_{12}	0.1					0.2	0.7	

BOE = Body of evidence

2.5.5 Numerical application

In this section, an ELECTRE TRI model for conflict classifying in the DST will be generated. To do so, elements of the set $A*$ used were the bodies of evidence present in Table 2.2 with their respective classifications. As a first approach, classification by means of two criteria was considered. For comparison purposes, the criteria used were those in the two-dimensional approach developed by Liu (2006). In this approach, the conflict is analyzed by using two metrics: The constant k of DRC and the betting commitment distance, d^1. The identification of the conflict is made when two metrics are greater than a certain value ε that, according to the authors, is a subjective value that depends greatly on the problem under consideration. Therefore, it can be seen that this model class is a subcase of the model developed in this study. As a flaw, this approach considers the limit of the conflict ε is the same for both criteria. However, since the information contained in the two criteria is different, this approach may fail to consider different perceptions of conflict about the problem. In order to further expand the model, measuring the conflict using three criteria was considered, by adding a third criterion, the conflict rate, (QU et al., 2009), to the problem.

Table 2.3 shows the decision matrix of the set A^*, considering the three criteria.

Table 2.3 - decision matrix of the set A

Alternatives	Pairs	$g_1 = K$	$g_2 = d^1$	g_3 =Conflict rate	Class
a_1	$(m_1;m_2)$	0.89	0.86	0.99	C_3
a_2	$(m_3;m_4)$	0.8	0.8	0.975	C_3
a_3	$(m_5;m_6)$	0.48	0.46	0.6	C_2
a_4	$(m_7;m_8)$	0.54	0.33	0.6	C_2
a_5	$(m_9;m_{10})$	0.49	0.13	0.7	C_1
a_6	$(m_{11};m_{12})$	0.16	0.53	0.42	C_1

Using Table 2.3 and the mathematical programming model Equation (2.17) to Equation (2.25), the parameters of the model for two criteria (Table 2.4) and for three criteria (Table 2.5) were generated using the Solve that is a supplement of Excel.

Table 2.4 - Parameters generated using Table 2.3.

Parameters	Value
w_1	0.661627712
w_2	0.338372288
$g_1(b_1)$	0.300598816
$g_2(b_1)$	0.283564123
$g_1(b_2)$	0.669400718
$g_2(b_2)$	0.684809286
$q_1(b_1)$	0.000270494
$q_2(b_1)$	0.014296426
$q_1(b_2)$	0.009044307
$q_2(b_2)$	0.009688723
$p_1(b_1)$	0368531408
$p_2(b_1)$	0.071900793
$p_1(b_1)$	0.072271698
$p_2(b_2)$	0.070487823
λ	0.5

Table 2.5 - Parameters generated using Table 2.3.

Parâmetros	Valor
w_1	0.50
w_2	0.25
w_3	0.25
$g_1(b_1)$	0.13
$g_2(b_1)$	0.25
$g_3(b_1)$	0.10
$g_1(b_2)$	0.73
$g_2(b_2)$	0.78
$g_3(b_2)$	0.99
$q_1(b_1)$	0.38
$q_2(b_1)$	0.05
$q_3(b_1)$	0.07
$q_1(b_2)$	0
$q_2(b_2)$	0.18
$q_3(b_2)$	0.02
$p_1(b_1)$	0.57
$p_2(b_1)$	0.11
$p_3(b_1)$	0.71
$p_1(b_2)$	0
$p_2(b_2)$	0.23
$p_3(b_2)$	0.10
λ	0.66

Some aspects should be mentioned in relation to the parameters represented in Tables 2.4 and 2.5, for this numerical application there is the absence of veto thresholds, which are not considered in order to facilitate the construction of the optimization model for this illustrative example. The veto thresholds can be easily included on the model if required.

To illustrate the proposed model application, consider the scenario shown in Table 2.6 that was taken from the article by Frikha (2014), where the combination in the DST is analyzed under the multiple criteria point of view, taking into account the reliability of each body of evidence. On considering 6 bodies of evidence, it is seen that Table 2.6 shows both conflict situations, m_3 in relation to the other ones, such as those of high ignorance m_5 and m_6.

Table 2.6 - Bodies of evidence (FRIKHA, 2014)

	m_1	m_2	m_3	m_4	m_5	m_6
$\{\boldsymbol{\theta_1}\}$	0.75	0.4	0	0.35	0.5	0.05
$\{\boldsymbol{\theta_2}\}$	0.1	0.2	0.9	0.15	0.1	0.1
$\{\boldsymbol{\theta_3}\}$	0.05	0.1	0.1	0.25	0	0
$\{\boldsymbol{\theta_1},\boldsymbol{\theta_2}\}$	0	0.3	0	0.2	0	0.3
$\{\boldsymbol{\theta_1},\boldsymbol{\theta_3}\}$	0	0	0	0	0	0.2
$\{\boldsymbol{\theta_2},\boldsymbol{\theta_3}\}$	0	0	0	0	0.15	0.1
Θ	0.1	0	0	0.05	0.25	0.25

The Table 2.7 contains the classification of conflict considering the number of criteria, 2 and 3, and the classification procedure, whether optimistic or pessimistic.

Table 2.7- Example of classification using ELECTRE TRI (2 and 3 criteria)

Alternative	pair	g_1 = K	g_2= Bet	g_3 = Con.Rate	Classification(OP) 2C	3C	Classification(PE) 2C	3C
a_1	(m_1,m_2)	0.320	0.233333	0.4961	**C_1**	**C_1**	**C_1**	**C_1**
a_2	(m_1,m_3)	0.805	0.783333	0.8888	**C_3**	**C_3**	**C_3**	**C_3**
a_3	(m_1,m_4)	0.395	0.316667	0.5724	**C_1**	**C_1**	**C_1**	**C_1**
a_4	(m_1,m_5)	0.2665	0.2	0.3948	**C_1**	**C_1**	**C_1**	**C_1**
a_5	(m_1,m_6)	0.1975	0.4	0.7315	**C_1**	**C_2**	**C_1**	**C_2**
a_6	(m_2,m_3)	0.547	0.55	0.7397	**C_2**	**C_2**	**C_2**	**C_2**
a_7	(m_2,m_4)	0.425	0.166667	0.625	**C_1**	**C_1**	**C_1**	**C_1**
a_8	(m_2,m_5)	0.260	0.091667	0.5416	**C_1**	**C_1**	**C_1**	**C_1**
a_9	(m_2,m_6)	0.1975	0.166667	0.5737	**C_1**	**C_2**	**C_1**	**C_2**
a_{10}	(m_3,m_4)	0.610	0.633333	0.7922	**C_2**	**C_2**	**C_2**	**C_2**
a_{11}	(m_3,m_5)	0.510	0.641667	0.85	**C_2**	**C_2**	**C_2**	**C_2**
a_{12}	(m_3,m_6)	0.270	0.516667	0.75	**C_1**	**C_2**	**C_1**	**C_2**
a_{13}	(m_4,m_5)	0.3125	0.116667	0.6067	**C_1**	**C_1**	**C_1**	**C_1**
a_{14}	(m_4,m_6)	0.220	0.116667	0.6769	**C_1**	**C_1**	**C_1**	**C_1**
a_{15}	(m_5,m_6)	0.1325	0.116667	0.5408	**C_1**	**C_1**	**C_1**	**C_1**

In Table 2.7 above, some remarks can be made regarding the introduction of the third criterion. On considering the analysis with two criteria, ELECTRE TRI informs that the element with the highest conflict is m_3 since it is classified once in class C_3, three times in class C_2 and only once in C_1. After the introduction of the third criterion, and the element m_6 is also classified as having the highest degree of conflict. Therefore, it is interesting to note that the introduction of criteria Conflict Rate (QU et al., 2009) makes the classification more rigorous.

For example, when two criteria are considered, the pair m_1 and m_6 are classified in the C_1 region. In the case of three criteria, the same pair is classified in the C_2 region. This classification is more appropriate because when we consider the body of evidence m_6, there is doubt as to which element, θ_1 and $\boldsymbol{\theta_2}$, of the frame of discernment frame is true while with respect to m_1 it is noticed that θ_1 is the element with the most evidence. Thus it can be seen that by expanding the number of criteria, the level of information about the conflict can be raised.

In the specific case of the Conflict Rate, as can be seen in Table 2.7, this criterion tends to be a good indicator when there is a high conflict despite not being a good evaluator when the conflict is low. Therefore, the introduction of this metric together with the criteria pointed out by Liu (2006) is indicated when the level of restriction to the conflict of the problem is high. Given this example and the large number of metrics in this theory, new combinations can be made using as a starting point the metric of Liu with the objective to improve the conflict classification within DST.

The advantages of the approach proposed in this article includes that it is not necessary to consider arbitrary conflict limits, for which may be difficult to establish precise values. Such aspect is found in many practical applications, when, due to such difficulty, same limit of conflict ε is considered for both conflict metrics, despite the eventual differences on each conflict metric scale and meaning. Therefore, since the information contained in the conflict metrics is different, it would be expected that the limit of conflict considered for these conflict metrics to be different also.

Another important aspect to highlight is that when using the approach proposed in this article, it is not necessary to define values for the limits of conflict arbitrarily. The parameters used in the MCDM method are estimated based on a holistic evaluation over conflict examples that are easier to classify.

2.6 Conclusion

The main aim of this article was to set out a multi-criteria method for conflict analysis in DST. To this end, the ELECTRE TRI multi-criteria classification method was used in order to integrate the subjectivity in human judgments into the classification of the conflict in DST. Given the high number of parameters that should be elicited, a procedure was established in which the parameters are automatically generated by means of a procedure which involves judging how they are assigned.

The validity of the model was observed by carrying out a numerical study for the case of two criteria that are already well-established in the literature. Another contribution of the presented approach is that the conflict analysis model allows to consider many different conflict measures as criteria for the MCDM model, therefore a broaden conflict analysis can be performed by expanding the number of criteria used. The numerical application presented in this work demonstrated the proposed approach by using two criteria for establishing a comparison with the work presented by Liu (2006) and extended for the case of evaluating three conflict measures, as when considering 3 criteria.

With regard to the conflict analysis and the model developed in this text, some questions should be raised. In terms of conflict in DST, the analysis conducted in this paper takes into account some hypotheses which have been discussed widely and which for a good many years have been the focus of discussion. First of all, Demspter's rule considers that the bodies of evidence were considered independent and with the same level of reliability. Secondly, the analysis discussed here assumes that the frame of discernment is a closed set, i.e. the parameters generated for a problem with $|\Theta|=3$ cannot be used for a problem with $|\Theta|=4$, for example.

3 A NEW PROMETHEE-BASED APPROACH APPLIED TO CONFLICT ANALYSIS IN EVIDENCE THEORY INTEGRATING THREE CONFLICT MEASURES

In this chapter, another classification procedure based on a pairwise comparison model will be presented. As a differential element, the conflict classification is not constructed from class profiles such as the ELECTRE TRI. In this sense, the method developed here establishes the classification as from the pairwise comparison of pre-classified alternatives in a given conflict class. Furthermore, the model uses these same alternatives to generate the model parameters through a disaggregation model. As a starting point, a model in the literature based on the PROMETHEE methods family will be used.

3.1 A Novel PROMETHEE Sorting Procedure Applied to Conflict Analysis in Evidence Theory

Evidence Theory or DST (DEMPSTER, 1967; SHAFER, 1976) is widely used in various areas of knowledge such as neural networks (AGGARWAL et al., 2013); group decision (FU & YANG, 2010); and Maintenance (BARALDI, COMPARE & ZIO, 2013). Two reasons for its widespread use can be identified. The first is directly linked to the possibility of representing situations of judgments of uncertainty which are regularly found in expert systems, namely ignorance, imprecision and vagueness. The second and more important reason results from applying DRC whereby different independent sources of information can be merged (Fusion).

However, in the classic study by Zadeh (1986) it was shown that DRC can deliver counter-intuitive results when the two sources of uncertainty have a high degree of conflict. To overcome this deficiency, several rules were created (YAGER, 1987; DUBOIS & PRADE, 1988, SMETS, 1990; CHIN & FU, 2015). Some of these approaches focus on the normalized DRC whereby prior to applying the rule, the sources of uncertainty are discounted in accordance with some factor of reliability (MARTIN, JOUSSELME & OSSWALD, 2008; JIANG, ZHANG & YANG, 2008; FRIKHA, 2014; MA & AN, 2015; FRIKHA & MOALLA, 2015). There are also approaches that focus on exploring the characteristics of the data, or administering the conflict in these data (SMETS, 2007; SCHUBERT, 2011; YANG et al., 2011). It is claimed that another alternative approach to handling the conflict between sources of imprecise information is that of using Fuzzy approaches (PARREIRAS et al., 2012; PARREIRAS et al., 2012; HERRERA-VIEDMA et al., 2014).

Since the main reason for applying a rule that is different from Dempster's is the presence of the conflict, some of these rules use a factor for measuring conflict. To this end, the first metric used to measure the conflict was the normalizing constant of DRC and subsequently, metrics based on distances (JOUSSELME, GRENIER & BOSSÉ, 2011; BURGER, 2016). In the research line of identifying the conflict, the study by Liu (2006), which is also supported by that of Jousselme & Maupin (2012), signals that at least two metrics are required so as to identify the conflict, since the normalizing constant in DRC fails to capture all conflict situations, and the same applies to distance-based metrics.

Another problem considered when quantifying conflict in the context of Evidence Theory is to determine when two Bodies of Evidence are in conflict. This can be useful to determine the best rule to be used, or rather, by how much a source of uncertainty can be discounted prior to merging (combining) the sources. However, determining conflict in this way presents a subjective character which comes about based on subjective thresholds (LIU. 2006; JOUSSELME, & MAUPIN, 2012, LEUNG, JI & MA, 2013), or else by establishing axiomatic structures of conflict (DESTERCKE & BURGER, 2013; ARNAUD, 2012).

Although these approaches are valid with regard to measuring conflict, they show the extent to which different authors' views on conflict in Evidence Theory can be subjective.

Since it takes more than a metric and subjective thresholds to quantify conflict in the context of Evidence Theory, this shows a multicriteria classification is needed. Thus, this paper presents a multicriteria classification approach based on PROMETHEE methods drawing on the model presented by Silva & Almeida-Filho (2016) who uses the ELECTRE method for classifying conflict in Evidence Theory. Choosing a multicriteria method which constructs outranking relations, pairwise, is justified due to the absence of tradeoff (DE ALMEIDA et al., 2016) among the criteria. As a differential in relation to the approach proposed in Silva & Almeida-Filho (2016), the classification of conflict in this article is undertaken by using pre-classified examples of conflict that generate the parameters of the model and are subsequently used as a reference for the multicriteria classification of conflict in DST.

Besides putting forward a new multicriteria approach for analyzing conflict, the article also sets out to adapt the disaggregation method proposed in Doumpos & Zopounidis (2004) so as to generate the parameters of the classification method. The disaggregation approach proposed, which is one of the contributions of this study, alters the initial formulation presented in Doumpos & Zopounidis (2004) by adding similar slack variables to the ones used by Mousseau & Slowinski (1998) and altering the objective function of the optimization problem so as to obtain the parameters of the model that classifies conflict. The disaggregation approach makes the elicitation process simpler since there is no need to indicate the subjective numerical thresholds, but rather all that is required is to classify pairs of Bodies of Evidence in accordance with the conflict that they present.

Therefore, this article is divided into 7 sections including this Introduction. Section 2 presents the basic concepts of Evidence Theory while Section 3 reviews the discussion on conflict. Section 4 formally introduces the traditional multicriteria method of classification of Doumpos & Zopounidis (2004) which this article enhances, while Section 5 presents a framework for evaluating conflict in the context of Evidence Theory and uses a new procedure for estimating parameters for PROMETHEE, namely a disaggregation procedure. This is described in Section 5.1. Section 6 presents a numerical example from the literature in which the results are analyzed in order to validate the approach proposed. Finally, the conclusion discusses the main benefits and main aspects of using multicriteria support methods to classify conflict in Evidence Theory.

3.2 A Brief Review OF Evidence Theory

Evidence Theory or Dempster-Shafer Theory is defined by a set of elementary and mutually exclusive hypotheses known as a frame of discernment, Θ. By using this set, another set is defined which is called a Power Set, 2^{Θ}, the elements of which represent all the possible combinations of the hypotheses from the frame of discernment. For example, consider a frame of discernment consisting of three hypotheses, $\Theta = \{\theta_1, \theta_2, \theta_3\}$. The Power set for this case will have eight members with the following representation: $2^{\Theta} = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \{\theta_3\}, \{\theta_1, \theta_2\}, \{\theta_1, \theta_3\}, \{\theta_2, \theta_3\}, \Theta\}$.

By using the Power Set, the function of basic probability assignment or Body of Evidence, $m \to 2^{\Theta}: [0,1]$, which represents the support or degree of belief that an expert or expert system assigns to a subset of the Power Set. This assignment should follow the following properties:

$$\sum_{\boldsymbol{A}\subset\Theta} m(\boldsymbol{A}) = 1 \tag{3.1}$$

$$m(\emptyset) = 0 \tag{3.2}$$

When an element belonging to the Power Set has a non-zero mass *m*, this element is called a focal element.

Using the function *m*, two other functions for the decision-making context are defined: The Belief Function, *Bel*, and the Plausibility Function, *Pl*. The Belief Function represents the total belief that can be attributed to a set A which is defined by Equation (3.3) and Equation (3.4).

$$Bel : 2^{|\boldsymbol{\Theta}|} \rightarrow [0,1] \tag{3.3}$$

$$Bel(A) = \sum_{B\subseteq A} m(B) \tag{3.4}$$

As the dual of the Belief Function, there is the Plausibility Function that represents the maximum volume of belief that can be attributed to a particular set A, represented by Equation (3.5) and Equation (3.6).

$$Pl : 2^{|\theta|} \rightarrow [0,1] \tag{3.5}$$

$$Pl(A) = \sum_{B\cap A\neq\emptyset} m(B) \tag{3-6}$$

Given these forms of knowledge about the uncertainty of the system, some authors interpret the interval *[Bel(A),Pl(A)]*, as an interval that contains the real probability of hypothesis *A*. Given these characteristics, some advantages in relation to the models of classic probability can be assigned. First, from the point of view of subjective judgments, there is no need for the judgments about the elements of the Power Set to be complete. Secondly, the function $m(\Theta)$ represents the ignorance of the specialist about the problem. Finally, the amplitude of the interval *[Bel(A),Pl(A)]*, given by *Pl(A)-Bel(A)*, represents the quantification of uncertainty about Event A. All of these possibilities fall within this theory in the field of Theories of Imprecise Probability.

Considering now that there are two Bodies of Evidence m_1 and m_2, the combination of these Bodies of Evidence can be obtained from DRC given from Equation (3.7) to Equation (3.9).

$$m_{12}(A) = \frac{\sum_{B\cap C=A} m_1(B)m_2(C)}{1-k}, when\ A \neq \emptyset \quad (3.7)$$

$$m_{12}(\emptyset) = 0 \quad (3.8)$$

$$k = \sum_{B\cap C=\emptyset} m_1(B)m_2(C) \quad (3.9)$$

The denominator in Equation (3.7), *1-k*, has a normalizing function, the main goal of which is to keep the assumption represented by Equation (3.1). The value represented by *k*, Equation (3.9), is interpreted as the level of conflict between the two Bodies of Evidence. Thus, *k=0* indicates a complete lack of conflict, whereas *k=1* indicates they completely contradict each other. Using DRC presents problems when the Bodies of Evidence are in total conflict or conflict greatly with each other. In the former, DRC cannot be used, while in the latter, using this rule leads to counter-intuitive results. To check this result, consider the following example based on the study by Zadeh (1986).

Example 1. Consider two Bodies of Evidence with the following information: $m_1(A)$ = 0.1 and $m_1(C)$ = 0.9 for the Body of evidence 1 and $m_2(A)= 0.1$ and $m_2(B)= 0.9$ for the Body of Evidence 2 where $\Theta = \{A, B, C\}$.

The result of using DRC to combine these two Bodies of Evidence is m_{12} (A) = 1. The counter-intuitive notion of this result is in the fact of these two Bodies of Evidence assigning least belief to Hypothesis A. However, combining both of them using DRC assigns total belief to Hypothesis A. This example is used as a starting point to show that DRC can generate counter-intuitive results when there is a high degree of conflict between the Bodies of Evidence.

3.3 Measuring Conflict IN DST

Given the possibility of generating counter-intuitive results on using DRC, different approaches were created in an attempt to overcome this problem. Regardless of the path chosen to avoid or soften counter-intuitive results, what first of all must be done is to measure the conflict or to determine when two Bodies of Evidence are in conflict. Given the definition of DRC, first the conflict was quantified by using constant *k*, Equation (3.9). On looking at **Example 1**, it is known intuitively that the two Bodies of Evidence conflict with each other. Therefore, the expected result is that constant *k* has a high value which for this example was k=0.99. Although the constant *k* functions well to classify the two Bodies of Evidence in **Example 1**, as shown by Liu (2006), it may fail in other situations as shown in Example 2 taken from the study by Xu et al. (2007).

Example 2. Consider two Bodies of Evidence with the following information: $m_1(A)$ = 0.55 , $m_1(B)$ = 0.10 and $m_1(C)$ = 0.35 for Body of Evidence 1 and $m_2(A)= 0.56$ *and* $m_2(B)= 0.15$, $m_2(C)=0.29$ for Body of Evidence 2 where $\Theta = \{A, B, C\}$.

Intuitively the two Bodies of Evidence of **Example 2** have a low level of conflict. However, there is a value of k= 0.576 which is a high value for indicating low conflict, since $0<k<1$. Thus, it is clear that *k* may not be ideal for capturing conflict in general. Therefore, different metrics were developed in order to measure the conflict in a complete way. One of these approaches, represented in the study by Liu (2006), uses a two-dimensional metric, i.e., the conflict is defined based on two metrics: The constant *k* and the betting commitment distance (TESSEM, 1991). The latter is given in Equations (3.10) and Equation (3.11).

$$d^1 \underset{A\subseteq\Theta}{\left(m_i, m_j\right)} = \max\left(\left|BetP_i(A) - BetP_j(A)\right|\right) \tag{3.10}$$

Where *BetP(A)* is given by Equation (3.11):

$$BetP(A) = \sum\nolimits_{B\subseteq\Theta} m(B).\left(\frac{|A \cap B|}{|B|}\right) \tag{3.11}$$

In his approach, Liu (2006) adds that two Bodies of Evidence are in conflict when the two metrics are larger than one constant, $(d^1, k) \geq \varepsilon$. The constant ε is regarded as a subjective value, depending on the problem under consideration. Liu also points out that the betting commitment distance alone is not sufficient to quantify the conflict in its entirety since this metric may fail in some situations which involve a degree of ignorance, as can be seen in Example 3.

Example 3. Consider two Bodies of Evidence with the following information: $m_1(\theta_1)=0.8$; $m_1(\Theta)=0.2$ for the Body of Evidence 1 and $m_2(\theta_1)=0.1$; $m_2(\{\theta_2,\theta_3\})= 0.2$; $m_2(\Theta)=0.7$ for the Body of Evidence 2 where $\Theta = \{\theta_1, \theta_2, \theta_3\}$.

Given the high degree of ignorance in the second body of evidence, there are no grounds for claiming that the two bodies conflict greatly with each other. However, there is a value of $d^1 = 0.53$ which demonstrates once again a high value, yet for a low conflict. When the conflict is analyzed from the perspective of constant *k*, *k* has a value of 0.16 which is in agreement with the low conflict expected for Example 3. Though Liu's approach is more general for quantifying conflict, it may fail if it considers that the conflict is quantified in the same way by the two metrics since it considers the same value ε is equal for the two metrics. Example 4 contains a situation where this approach may fail to capture conflict.

Example 4. Consider two Bodies of Evidence with the following information: $m_1(A) = 0.3$ and $m_1(C) = 0.7$ for the Body of Evidence 1 and $m_2(A)= 0.3$ and $m_2(B)= 0.7$ for the Body of Evidence 2 where $\Theta = \{A, B, C\}$.

Using the two-dimensional metric of Liu (2006) for the above example, leads to a result (0.7; 0.91) which shows that the two Bodies of Evidence conflict greatly. However, the choice of ε can lead to different interpretations. For example, a value of $\varepsilon = 0.6$ interprets the two Bodies of Evidence in Example 4 as being in conflict. While a value of $\varepsilon = 0.75$ informs us that they would not be in conflict according to the interpretation of Liu (2006). Thus, arises the need to establish conflict, taking into account the different behaviors of the metrics used.

In addition to these two metrics, other metrics were developed in the literature at different times without, however, ensuring the capture of conflict in a general way. For example, there is a Conflict Rate (QU et al, 2009) in Equation (3.12) , a Relative Coefficient (DENG et al., 2011) in Equation (3.13) and Similarity (WEN-HAO et al. 2013) in Equation (3.14).

$$\lambda = \frac{\sum_{B_i \cap C_i = \emptyset} m_1(B_i) m_2(C_i)}{\sum_{A \subseteq \boldsymbol{\Theta}} m_1(A) m_2(A) + \sum_{B_i \cap C_{i=\emptyset}} m_1(B_i) m_2(C_i)} \quad (3.12)$$

$$r(X,Y) = \frac{\sum m_1 Log(m_1) + \sum m_2 Log(m_2)}{\sum m_1 Log(m_2) + \sum m_2 Log(m_1)} \quad (3.13)$$

$$sim = \frac{\sum m_1{}'(B_i) m_2{}'(B_i)}{(\sum m_1{}'(B_i)^2 m_2{}'(B_i)^2)^{1/2}} \quad (3.14)$$

$$m'\ (B\ \ ) = \sum_{A \subseteq \boldsymbol{\Theta}, B \in A} \frac{1}{|A|}$$

Where $m'\ (B\ \ ) = \sum_{A \subseteq \boldsymbol{\Theta}, B \in A} \frac{m(A)}{|A|}$ in 3-14.

3.4 PROMETHEE Classification Procedures

A problem of multicriteria classification involves assigning a finite set of alternatives $A = \{a_1, a_2, ..., a_m\}$ in pre-defined q groups C_1, $C_2, ..., C_q$ where the alternatives are described by a vector of n criteria $g = \{g_1, g_2, ..., g_n\}$. Thus, the general idea is to systematize classifying the alternatives based on aggregating the criteria.

The analysis by Zopounidis & Doumpos (2002) considers that outranking methods are the ones most widely used for multicriteria classification. The outranking relationship a_pSa_i regards the alternative a_p as at least as good as alternative a_i. In this context, the ELECTRE TRI (YU, 1992; ROY & BOUYSSOU, 1993) and the PROMETHEE methods (BRANS &VINCKE, 1985; VINCKE, 1992), which this paper discusses, stand out.

The PROMETHEE method uses a flow generated by the evaluations of the criteria so as to represent the intensity of preference by which one alternative outranks another. In the case of the classification problem, most approaches use the variant known as PROMETHE II (DOUMPOS, & ZOPOUNIDIS, 2004; NEMERY & LAMBORAY, 2008; HU & CHEN, 2011). This paper will also use this variant as a reference where the methodology is the same as that described in the study by Doumpos & Zopounidis (2004). In this case, the classification is not conducted by means of reference alternatives, but by a set of alternatives that represent a given class, as can be seen in Figure 3.1 below. In this representation, the classification of a_i is defined by a pair-wise comparison with of alternative a_i in which each of the alternatives is a representative of a particular class. Thus, the arrows coming out of the alternatives of the C_1

classes and heading towards alternative a_i indicate a strong preference for alternative a_i in relation to the alternatives b_i of class C_1. The contrary to this analysis is seen in relation to the C_2 class where the arrows point towards the alternatives b_i belonging to the class C_2.

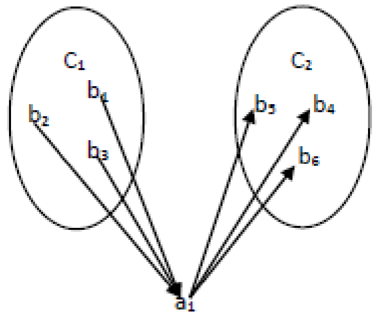

Figure 3.1 - Classification procedure using pair-wise comparison (adapted from Doumpos & Zopounidis, 2004)

The intensity of each of the arrows (preference) is defined by a criteria aggregation index $P(a_i,b_k)$ if the arrow is heading for a_i or, $P(b_k,a_i)$ if the arrow is leaving a_i where $P(a_i,b_k)$ is defined by Equation (3.15).

$$P(a_i, b_k) = w_j p_j(a_i, b_k) \tag{3.15}$$

In Equation (3.15) $p_j(a_i, b_k)$ is defined by the difference between the performance of the alternatives in the criteria, $d_j(a_i, b_k) = g_j(a_i) - g_j(b_k)$, when the preference is increasing in criterion j. and in $d_j(a_i, b_k) = g_j(b_k) - g_j(a_i)$ when the preference is decreasing in relation to criterion j. After defining $d_j(a_i, b_k)$, $p_j(a_i, b_k)$ is defined by Equation 3-16.

$$p_j(a_i, b_k) = \begin{cases} 0 & if\ d_j(a_i, b_k) < 0 \\ h_j\left(d_j(a_i, b_k)\right) & if d_j(a_i, b_k) \geq 0 \end{cases} \tag{3.16}$$

Where $p_j(a_i, b_k)$ is a function limited between 0 and 1. The $h_j\left(d_j(a_i, b_k)\right)$ function may take different forms that depend on the DM's preference behavior. For example, the study by Brans & Vincke (1985) defines six different functions for the DM's behavior.

After this explanation, the preference index, $P(a_i, b_k)$, will serve as the basis for classifying alternative a_i into a particular class. To do so, the classification rule is defined based on a classification flow of alternative a_i in relation to the alternatives b_i belonging to the classes. For example, on considering the classification with two classes, there is the one that is structured in Equation (3.17) below (DOUMPOS & ZOPOUNIDIS, 2004).

$$f_i = \frac{1}{m_2} f_i^+ - \frac{1}{m_1} f_i^- = \frac{1}{m_2} \sum\nolimits_{b_k \in C_2} P(a_i, b_k) - \frac{1}{m_1} \sum\nolimits_{b_k \in C_1} P(b_k, a_i) \quad (3.17)$$

In this case, it is considered that C_1 is preferable to class C_2. The f_i^+ flow represents the amount by which alternative a_i outranks all alternatives belonging to the C_2 class while the f_i^- flow represents the intensity at which alternative a_i is outranked by all the alternatives belonging to the C_1 class. The values m_1 and m_2 represent the number of alternatives belonging to class C_1 and C_2 respectively. The higher the f value, the more prone is the classification of a_i into Class C_1. After defining the f_i flow, the classification of a_i is established based on a cut-off threshold b as shown in Equation (3.18) (DOUMPOS & ZOPOUNIDIS, 2004).

$$\begin{cases} f_i > b \rightarrow a_i \in C_1 \\ f_i < b \rightarrow a_i \in C_2 \end{cases} \quad (3.18)$$

For the general case involving the classification of an alternative into q classes, the authors suggest that the problem be broken into subproblems like those described in Equation (3.17) and (3.18). Therefore, if the classes have the following preference relation $C_1 \prec C_2 \ldots \prec \cdots \prec C_q$, then the f_{ir}^+ flow is created which represents the outflow of alternative a_i in relation to the alternatives that belong to the set of classes $\{C_{r+1}, C_{r+2,\ldots,} C_q\}$ and the inflow flow f_{ri}^- which represents the inflow into alternative a_i that arises from the alternatives belonging to the set of classes $\{C_1, C_{2,\ldots,} C_r\}$. Therefore, Equation (3.18) should be modified to adapt the general case by using Equation (3.19) and Equation (3.20) (DOUMPOS & ZOPOUNIDIS, 2004).

$$f_{ir} = \frac{1}{m_2} f_{ir}^{+} - \frac{1}{m_1} f_{ri}^{-} \tag{3.19}$$

$$= \frac{1}{m_{ir}} \sum\nolimits_{b_k \in C_{ir}} P(a_i, b_k) - \frac{1}{m_{ri}} \sum\nolimits_{b_k \in C_{ri}} P(b_k, a_i)$$

$$\begin{cases} f_{i1} > b_1 \rightarrow a_i \in C_1 \\ \quad else\ f_{i2} > b_2 \rightarrow a_i \in C_2 \\ \quad\quad \dots \\ \quad\quad else\ f_{iq-1} > b_{q-1} \rightarrow a_i \in C_{q-1} \\ \quad\quad\quad else\ \ a_i \in C_q \end{cases} \tag{3.20}$$

3.5 A Proposed Framework for THE Multicriteria Classification of Conflict IN DST Based ON THE PROMETHEE Method

As seen in Section 3, a classification of conflict within DST should involve at least more than one criterion and at the same time should aggregate them while taking into account the different behaviors of the criteria and characteristics of the data which to some extent are subjective as explained by Liu (2006). Given these characteristics, a multicriteria classification method makes it ideal for analyzing conflict in DST. Thus, the alternatives in question to be classified will be all the possible pairs of Bodies of Evidence under analysis. The second part consists of determining what the classes of conflict are. Therefore, it was considered that there are three classes $\{C_1, C_{2,} C_3\}$. Where C_1 is the class of low conflict, C_3 represents the class of high conflict and C_2 is the class of alternatives with moderate conflict. From the point of view of a decision problem, it is logical that a DM´s preference structure is $C_3 \prec C_2 \prec C_1$, since, if all alternatives were to be classified in class C_1, DRC can be used without the repercussion of generating counter-intuitive results.

The aim of choosing three classes is to avoid the movement between the absence of conflict to the state of conflict being abrupt. This classification is also based on using DRC. Thus, the standard that Liu (2006) describes can be followed. It seeks to classify pairs of Bodies of Evidence in three situations: Pairs where DRC should be avoided (high conflict), pairs where

DRC can be applied (low conflict) and pairs where the rule should be used with caution (moderate conflict).

3.5.1 Procedure approach for generating parameters by disaggregation for classification using PROMETHEE

A known problem in the methods discussed in Section 4 is the high number of parameters that should be compiled based on the DM´s judgment. One way to reduce the cognitive effort required in the elicitation of parameters phase is to use a disaggregation approach. In this approach, the DM classifies a group of alternatives, *A**, *a priori* into a particular class and from this classification, parameters are generated which best fit the DM's choice. Thus, on having as a parameter the study by Silva and Almeida-Filho (2016), which uses a disaggregation model for ELECTRE TRI developed by Mousseau & Slowinski (1998) to classify conflict in Evidence Theory, a disaggregation model for the PROMETHEE method was used.

This section presents a method of disaggregation based on the approach contained in the study by Doumpos & Zopounidis (2004) who use the idea contained in the paper by Siskos & Yannacopoulos (1985) in which the $p_j(a_i, a_l)$ function is considered a pair-wise linear function. Thus, the maximum difference is defined between the alternatives contained in set A^* in each j criterion given by $d_j^{max} = \max\{a_{ij} - a_{lj}\}$ where the alternatives belong to class A^*. Then, the interval $[0, d_j^{max}]$ is set at s_j subintervals $[0, d_j^1]$, $(d_j^1, d_j^2]$, ... , $\left(d_j^{s_{j-1}}, d_j^{max}\right]$. If $d_j^{il} \in (d_j^{t-1}, d_j^t)$, the $p_j(a_i, a_l)$ preference function is given by:

$$p_j(a_i, a_l) = \sum_{v=1}^{t-1} h_{jv} + \frac{d_j^{il} - d_j^{t-1}}{d_j^t - d_j^{t-1}} h_{jt} \tag{3.21}$$

Where $h_{jt} = h_j(d_j^t) - h_j(d_j^{t-1}) \geq 0$. The problem is then summarized in the full estimation of the h_{jv} functions that defined the preference relation p_j. However, this problem presents a non-linear dimension since the weights w_j must also be estimated. In order to reduce the computational effort to solve the problem, the authors use the following change of variable $h'_{jt} = w_j h_{jt}$. Thus, the problem can be rewritten as given in Equation (3.22).

$$p'_j(a_i, a_l) = w_j p_j = \sum_{v=1}^{t-1} h'_{jv} + \frac{d_j^{il} - d_j^{t-1}}{d_j^t - d_j^{t-1}} h'_{jt} \tag{3.22}$$

Now the h'_{jt} values can be determined by an approach using a linear programming model. Given the alternatives pre-classified by the DM, a linear programming model is used to generate parameters – see Expressions from Equation (3.23) to Equation (3.32) below:

$$\max \alpha \tag{3.23}$$

Subject to:

$$\alpha \leq e_i^{b_j} \tag{3.24}$$

$$f_i - e_i^{b_1} = b_1 \; \forall a_i \in A^* \cap C_1 \tag{3.25}$$

$$f_i + e_i^{b_1} = b_1 \; \forall a_i \in A^* \cap C_2 \tag{3.26}$$

$$f_i - e_i^{b_2} = b_2 \; \forall a_i \in A^* \cap C_2 \tag{3.27}$$

$$f_i + e_i^{b_2} = b_2 \; \forall a_i \in A^* \cap C_3 \tag{3.28}$$

$$f_i - e_i^{b_1} = b_1 \ \forall a_i \ \in A^* \cap C_3 \tag{3.29}$$

$$\sum_{j=1}^{n} \sum_{t=1}^{s_j} h'_{jt} = 1 \tag{3.30}$$

$$w_i \geq 0 \ \forall a_i \ \in A^* \tag{3.31}$$

Where b_i is unrestricted with regard to sign.

The difference in the approach contained from Equation (3.20) to Equation (3.29) in relation to the approach described in Doumpos & Zopounidis (2004) is in the $e_i^{b_j}$ variable which represents the slack variables in the set of constraints in Equation (3.24) to Equation (3.29). The idea of using these variables follows the same reasoning described in Mousseau & Slowinski (1998), that is, the higher the values are, the greater the adjustment of the model to the parameters. This optimization condition is guaranteed by Equation (3.23) and Equation (3.24). Equations from Equation (3.30) to Equation (3.32) represent the conditions for the model to exist.

3.5.2 Framework for conflict classification within DST with multiple conflict measures

The framework proposed in this paper for conflict considers it possible classification using examples of known conflict to determine conflict levels in advance and use this information to provide the parameter of a multicriteria model considering various metrics. Thus, it is possible to eliminate arbitrariness inherent in the definition of certain parameters in situations in which the decision maker does not have a perception of the scale, as occurs in some situations when analyzing metrics conflict in DST.

After defining the disaggregation method for setting the parameters as presented in last Section, Figure 3.2 summarizes the steps defined for the application framework approach proposed.

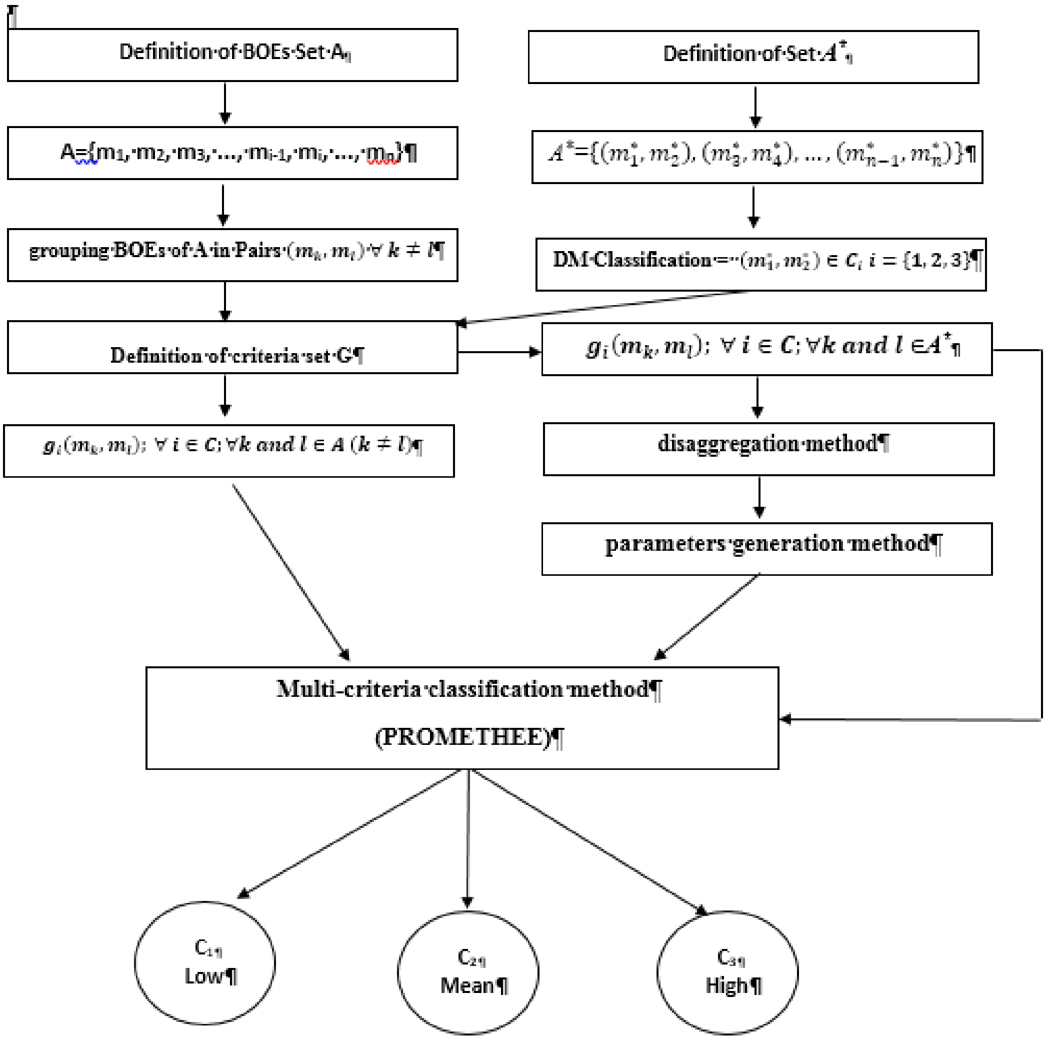

Figure 3.2 - A Multicriteria Conflict Classification Framework for Evidence theory

3.6 Numerical Application

In this Section the framework proposed in this paper is applied to a numerical example in the literature. On using this example, the procedure proposed is applied so as to obtain the parameters of the multicriteria model for classifying conflict. Therefore, it is shown how the method developed in this paper can be used while having as a base examples of conflict which have been assigned *a priori* according to the DM's classification so as to obtain the parameters of the method of classification and subsequent classification of the conflicts between the alternatives that may come to be considered.

The first step of the proposed framework (Figure 3.2) is to define the BOEs referring to set $A*$ in view of the need to generate parameters which will be obtained from a pre-classification of conflict combined with the procedure described in Section 5. Therefore, the data in Table 3.1 show 6 pairs of Bodies of Evidence which were classified *a priori* as to conflict by a hypothetical DM (Table 3.1), which, although they are different data, were classified according to the proposal by Liu (2006) as seen in the last column of Table 3.1.

Table 3.1 - BOE of set A^* (Silva & Almeida-Filho, 2016)

BOE	Sets							
	$\{\boldsymbol{\theta_1}\}$	$\{\boldsymbol{\theta_2}\}$	$\{\boldsymbol{\theta_3}\}$	$\{\boldsymbol{\theta_1},\boldsymbol{\theta_2}\}$	$\{\boldsymbol{\theta_1},\boldsymbol{\theta_3}\}$	$\{\boldsymbol{\theta_2},\boldsymbol{\theta_3}\}$	$\boldsymbol{\Theta}$	Class
m_1	0.9		0.1					C_3
m_2		0.8	0.1				0.1	
m_3	0.8	0.1					0.1	C_3
m_4		0.1	0.8				0.1	
m_5	0.4	0.4	0.2					C_2
m_6	0.8						0.2	
m_7	0.4	0.4	0.2					C_2
m_8	0.7	0.2					0.1	
m_9	0.4	0.3	0.2		0.1			C_1
m_{10}	0.2	0.3	0.2	0.2			0.1	
m_{11}	0.8						0.2	C_1
m_{12}	0.1					0.2	0.7	

It is from Tables 3.1 and 3.2 that the parameters are generated for the PROMETHEE method using the disaggregation methods which were described in Section 6. Table 3.3 shows the parameters generated for the PROMETHEE method, while the criteria functions adopted can be seen in Figure 3.3.

Table 3.2 - Decision matrix of set A^* (Silva & Almeida-Filho, 2016)

Alternatives	Pairs	g_1 = K	g_2= d^1	g_3 =Conflict rate	Class
a_1	$(m_1;m_2)$	0.89	0.86	0.99	C_3
a_2	$(m_3;m_4)$	0.8	0.8	0.975	C_3
a_3	$(m_5;m_6)$	0.48	0.46	0.6	C_2
a_4	$(m_7;m_8)$	0.54	0.33	0.6	C_2
a_5	$(m_9;m_{10})$	0.49	0.13	0.7	C_1
a_6	$(m_{11};m_{12})$	0.16	0.53	0.42	C_1

Table 3.3 - Parameters generated in accordance with Tables 3.1 and 3.2

w_1	w_2	w_3	b_1	b_2
0.2539	0.5	0.2461	0.125	-0.125

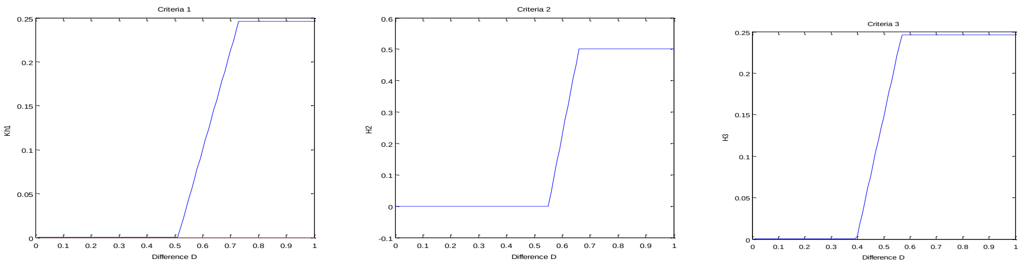

Figure 3.3 - The PROMETHEE Elicitation Criteria.

To classify conflict, the analysis using three criteria used in Silva & Almeida - Filho (2016) was considered, namely, the constant k of DRC; the betting commitment distance and the conflict rate. It should be noted that to allow comparison with the results obtained in Silva & Almeida-Filho (2016), what was considered was that the weights of the criteria used in this application of the PROMETHEE method were fixed and equal to those obtained for the ELECTRE TRI method (SILVA & ALMEIDA - FILHO, 2016) and the data of Table 3.4 were used in accordance with Frikha (2014).

Table 3.4 - Bodies of Evidence (Frikha, 2014)

$2^{\boldsymbol{\Theta}}$	m_1	m_2	m_3	m_4	m_5	m_6
$\{\theta_1\}$	0.75	0.4	0	0.35	0.5	0.05
$\{\theta_2\}$	0.1	0.2	0.9	0.15	0.1	0.1
$\{\theta_3\}$	0.05	0.1	0.1	0.25	0	0
$\{\theta_1, \theta_2\}$	0	0.3	0	0.2	0	0.3
$\{\theta_1, \theta_3\}$	0	0	0	0	0	0.2
$\{\theta_2, \theta_3\}$	0	0	0	0	0.15	0.1
$\boldsymbol{\Theta}$	0.1	0	0	0.05	0.25	0.25

This numerical example is interesting for this analysis because it contains types of Bodies of Evidence with different conflict properties that would not be classified adequately whenever only one metric of conflict in DST was applied. The result of the classification obtained by the proposed procedure is shown in Table 3.5.

Table 3-5 - Conflict classification for the example

Alternative	pair	g_1 = K	g_2= Bet	g_3 = Con.Rate	PROMETHEE CLASSIFICATION
a_1	(m_1,m_2)	0.320	0.233333	0.4961	**C_1**
a_2	(m_1,m_3)	0.805	0.783333	0.8888	**C_3**
a_3	(m_1,m_4)	0.395	0.316667	0.5724	**C_2**
a_4	(m_1,m_5)	0.2665	0.2	0.3948	**C_1**
a_5	(m_1,m_6)	0.1975	0.4	0.7315	**C_2**
a_6	(m_2,m_3)	0.547	0.55	0.7397	**C_2**
a_7	(m_2,m_4)	0.425	0.166667	0.625	**C_1**
a_8	(m_2,m_5)	0.260	0.091667	0.5416	**C_1**
a_9	(m_2,m_6)	0.1975	0.166667	0.5737	**C_1**
a_{10}	(m_3,m_4)	0.610	0.633333	0.7922	**C_2**
a_{11}	(m_3,m_5)	0.510	0.641667	0.85	**C_2**
a_{12}	(m_3,m_6)	0.270	0.516667	0.75	**C_2**
a_{13}	(m_4,m_5)	0.3125	0.116667	0.6067	**C_1**
a_{14}	(m_4,m_6)	0.220	0.116667	0.6769	**C_1**
a_{15}	(m_5,m_6)	0.1325	0,116667	0.5408	**C_1**

Based on this classification, it is seen that the method presents the Body of Evidence m_3 as the body of evidence of greatest conflict in relation to the others. With regard to validating the proposed procedure, the classification obtained is compatible with the classification proposed in Silva & Almeida-Filho (2016).

3.7 Conclusion

This article discussed how to determine conflict in Evidence Theory from the perspective of multicriteria classification. As an expansion in relation to the method developed in Silva & Almeida-Filho (2016), a multicriteria classification approach to conflict in Evidence Theory was implemented which considered multiple metrics of conflict.

Besides the advantage of considering the possibility of integrating more than one criterion, which already means a greater capture of the conflict in Evidence Theory, this study also deals with the subjective aspect of classifying conflict. Accordingly, and instead of considering the direct elicitation of parameters, which very often makes this task prohibitive from the standpoint of cognitive effort, disaggregation techniques are used where the parameters of the model are generated indirectly by pre-judging conflict, and three classes of conflict are considered. Even as a differential, the same alternatives obtained in the pre-classification of conflict are used in the multicriteria classification procedure.

By means of an example considered in the literature, the validity of the classification using the PROMETHEE method was demonstrated. On using a numerical application, it was noted that the classification procedure proposed in this paper is compatible with the classification obtained by using the ELECTRE TRI method of Silva & Almeida-Filho (2016), which shows the consistency of this proposal.

As a further contribution of this research, it is important to note that the method of disaggregating parameters for PROMETHE proposed in this article has the advantage of being based on linear programming, since the reference Silva & Almeida-Filho (2016), uses a procedure based on non-linear programming. Moreover, the flexibility of representation of the criterion function in the proposed method is greater than the method developed in Silva & Almeida-Filho (2016).

4 A POSSIBILISTIC-PROBABILISTIC KNOWLEDGE-BASED REAL OPTIONS MODEL FOR NPD PROJECT FINANCIAL EVALUATION

In this chapter we will present a fuzzy model of real options so as to make a financial evaluation of NPD in order to overcome shortcomings in existing models, considering the main uncertainties involved in the development of new products, in which technical uncertainty and market uncertainty are analyzed separately. Given the imprecision and subjectivity involved in such projects, fuzzy numbers together with probabilistic approaches are used to model uncertainty. A new visualization is presented to illustrate the results from the combination of fuzzy numbers and the probabilistic approach used. A case study is presented where the technical and market flexibility are evaluated.

4.1 Knowledge-Based Real Options Model for NPD Evaluation

The financial analysis of New Product Development (NPD) is a major challenge for financial engineering analysts. In the first instance, traditional metrics that involve the analysis of discounted cash flow tend to underestimate the present value of a project because this does not take into account the internal flexibility of the project in the context of uncertainty.

To remedy these shortcomings, the literature recommends the use of real options as an ideal approach for analyzing such projects, since this places decision-makers in a setting of active performance, thereby enabling them to be in command of the value of a project to the extent that the uncertainty related to it decreases (LINT & PENNINGS, 1998; FAULKNER, 1996; DIXIT & PINDYCK, 1995).

However, the strong level of flexibility is not the only factor of difficulty when analyzing NPD projects. The different types of uncertainties linked to choosing a model also represent a complicating factor which, if not dealt thoroughly with, can lead the analyst to design a model with a low level of information (PERMINOVA et al., 2008).

Therefore, what is needed first of all is to stipulate what the risks are that affect the project, which from point of view of NPD are basically derived from two types of uncertainty (HUCHZERMEIER & LOCH, 2001; SANTIAGO & BIFANO, 2005; WANG & YANG, 2011; WANG, WANG & WATADA, 2015): The technical uncertainty that can be controlled and decreased by designers throughout the period of new product development and the market uncertainty that can be taken on board as an opportunity when launching the product.

As a final step, there is modeling of uncertainty where its nature must be taken into consideration. In the specific case of NPD, these uncertainties are mostly subjective and elicited by using experts since the analysis takes place in the early stages of the project. Many techniques may support the modeling of uncertainty and the experts' knowledge assessment, which for instance includes evidence theory approaches, such as the one presented by AbuDahab, Xu & Chen (2016) or the one exemplified by Yang et al (2016) for an NPD context. Hence probabilistic traditional approaches can also be criticized as to different aspects such as ambiguity, the lack of a priori knowledge and the limited capacity of human beings to process information (Pender, 2008), thus, fuzzy approaches may be widely used within NPD context (Zaim et al., 2014).

Traditional models of real options used to value projects are derived from financial options or more precisely the Black and Scholes (1973), Gesk (1979) and Cox, Ross & Rubinstein (1979) models. Such models tend not to be very informative with regard to NPD because, since they are derived from financial models, they are more suitable for projects that have random uncertainties, which do not consider the influence of the management and development of a project with respect to its performance.

Therefore, a possibilistic-probabilistic model will be used to model the uncertainty built into a real options model that incorporates the technical uncertainty by means of a binomial tree representing the development of the product over time while market uncertainty is incorporated via a payoff function that incorporates the total amount paid by the market. One way to model possibilistic uncertainty is by using Fuzzy Set Theory (Zadeh, 1999). Thus, project variables, which have an imprecise and ambiguous level of uncertainty, are modeled as triangular fuzzy variables.

This work addresses the issues raised by Wang, Wang & Wu (2015) regarding the integration of fuzzy sets theory with real options analysis for evaluating NPD projects. Another contribution of this work, besides the fuzzy real options approach for NPD evaluation, is a new visualization approach to illustrate the results from the combination of fuzzy and probabilistic approaches used in the proposed model.

Besides this introduction, this paper is divided into 5 sections. In Section 2, a review of the literature on models that combine real options with fuzzy uncertainty modeling is made.

Section 3 contains a description of the proposed model of fuzzy real options that integrates the main sources of uncertainty regarding NPD projects while Section 4 contains a case study on the development of a software program. Finally, there is the conclusion where the main results of this research study are given and future lines of research are suggested.

4.2 Recent Development ON FUZZY Real Options Analysis

Recently the number of articles using Fuzzy Set Theory in the context of real options has increased. In this area, the first basis for such studies has been research by Carlsson & Fullér (2003) in which they develop a model for determining the optimal exercise time of an option, which includes the expected present value of the cash flow and costs using fuzzy trapezoidal numbers. To estimate the value of the real option, Carlsson & Fullér (2003) used a modified Black-Scholes equation (Black-Scholes, 1973). Similarly, Lee, Tzeng & Wang (2005) proposed to the Black and Scholes option pricing model a fuzzy decision theory and Bayes' rule throughout a fuzzy decision space based on fuzzy states, fuzzy sample information and fuzzy actions. Bi & Wang (2009) adapted the Black-Scholes model to evaluate a BOT infrastructure project which included the expected return of the project by using triangular fuzzy variables, the expected cost and the risk-free rate. Based on this type of fuzzy real options approach, Carlsson et al (2007) put forward a mixed integer programming model for selecting a portfolio of R&D projects.

Using a similar approach, studies by Wang & Hwang (2007) also develop a model for selecting a portfolio of R&D projects. Given the sequential aspect of the model, a fuzzy compound options model is used to evaluate R&D projects in which a qualitative approach is used to convert the portfolio into a crisp number. Also within the fuzzy model compound options model, studies by Bednyagin & Gnansounou (2011) analyze an R&D program linked to the fusion of energy area.

Some authors use the same Carlsson & Fullér (2003) equation to analyze opportunities in different types of projects in a fuzzy environment. Cheng & Lee (2007) develop a real options model that combines fuzzy real options, weighted real fuzzy options and fuzzy decision space in order to determine the optimal exercise price of the option related to product outsourcing.

The formulations using the Black-Scholes model make reference to a continuous decision model. Given that many projects related to real assets involve sequential decision models, the literature has also explored other approaches that incorporate this feature in the model.

In the sequential context, Zmeškal (2010) develops a stochastic model in a fuzzy environment for pricing American options. The input data (up index, down index, growth rate, initial underlying asset price, exercise price and risk-free rate) are modeled as fuzzy numbers and the result, the possibility-expected option value, is determined as a fuzzy set.

Kahraman (2008) develop a multi-criteria model for evaluating R&D projects where the risk aspect of the project is evaluated by means of a fuzzy real options model using a trinomial lattice solution method which is extension of the binomial model while the multidimensional aspect of the project is analyzed by fuzzy AHP method.

Still in the context of binary options, the model by Thavaneswaran et al. (2013) considers the fuzzy uncertainty relating to the maturity value of the stock price. To test the approach, the authors use three different types of fuzzy numbers: trapezoidal, parabolic, and the adaptive fuzzy number.

In the article by Yoshida et al. (2006), a new approach is developed for calculating the average value of a fuzzy number where the mean that they develop may also be extended to fuzzy random numbers. Armed with this new measure, an American option is analyzed in which the optimal price of the option is given by the average value linked to a subjective parameter which depends on the decision-maker.

Also in the context of sequential decision, study by Ho & Liao (2011) develop a sequential model based on the Cox et al binomial model (COX, ROSS & RUBINSTEIN, 1979) where the volatility of the project and the cost are dealt with as fuzzy triangular numbers. As a contribution, the methods further develop a metric for evaluating the options by using the concept of the fuzzy average of the present value of the project.

In terms of the uncertainty of the project with regard to randomness and fuzziness, Wang, Wang & Watada (2009) developed a hybrid model to deal with these two types of uncertainty. Therefore, they used fuzzy random numbers to model the cash flow as input for a new model that uses a binomial lattice-based model with fuzzy random variables, called Fuzzy Random Real Options Analysis (FR-ROA).

In a different context from the Black and Scholes approaches and the binary model, Collan, Fullér & Mezei (2009) developed a method known as the Payoff method. This considers the calculation of the real option valuation of the present value associated with a cash flow, taking into account the ratio between the positive area of the fuzzy NPV and the total area of the fuzzy number.

As applications Payoff method, one can cite the study by Favato et al (2013) where the payoff method is used to assess the value of the option of 4 vaccines used in the treatment of HPV. The paper by Hassanzadeh, Collan & Modarres. (2012) develops a method for selecting a portfolio of R&D projects where each project is assessed individually by the Payoff method.

Generally, approaches that exploit real options in the fuzzy context normally deal with numbers that have a closed form. In this regard, Wang, Kilgour & Hipel (2011) developed a numerical model to evaluate projects using fuzzy real options that include random fuzzy numbers. To generate fuzzy numbers, they use the least squares approach of the Monte-Carlo simulation.

4.3 Fuzzy Real Options Model TO Evaluate NPD

The model proposed by Huchzermeier & Loch (2001) and improved by Santiago and Vakili (2005) considers the use of real options for the financial analysis of research and development projects that take into account the operational risks (technical uncertainty) and market risk. The model proposed in this paper features a fuzzy real options approach to improve these models (Huchzermeier & Loch, 2001; Santiago & Vakili, 2005) by allowing to consider the imprecision and vagueness inherent to NPD project evaluation.

The main advantage of this model compared to those presented in section 2 is that the approach considered evaluate the main uncertainties present in innovation projects as the development of new products, while the models presented in section 2 mainly consider uncertainties as financial options.

Due to the subjective and imperfect knowledge on the uncertainties associated to NPD projects, fuzzy is a suitable approach for dealing with such context. Therefore, a fuzzy approach is more appropriate to deal with those uncertainties than traditional probabilistic approaches, as within the fuzzy approach is possible to consider the imprecision and vagueness inherent to NPD.

The fuzzy real options approach for NPD evaluation proposed in this work takes into account previous models from the literature and adds new features to enhance its applicability by considering practical aspects that cannot be ignored by considering traditional probabilistic approaches, which are incompatible with such aspects that lead to imprecision and vagueness. Thus, the proposition hereby presented considers the integration of fuzzy random numbers within a probabilistic approach to evaluate the uncertainty in the framework initially proposed by Huchzermeier & Loch (2001). By assuming that the knowledge associated with the payoffs and technical uncertainty have a higher degree of imprecision and vagueness, a fuzzy approach is used to model these aspects with a possibilistic methodology.

4.3.1 Fuzzy technical Uncertainty

The performance variable is one of the main variables affecting the value of a NPD project. To illustrate the impact of this variable in such context, consider the development of a new computer processor. In this specific case, the variable performance will be the speed of processor which at each stage, this variable suffers successive increases.

Thus, the technical uncertainty is associated with performance variable $\tilde{X}_t$ and probability of success p at stage t. In that sense, $\tilde{X}_t$ will be a fuzzy random number, as can be seen in Equation (4.1):

$$\tilde{X}_{t+1} = \tilde{X}_t + \tilde{w}_t \tag{4.1}$$

The variable $\tilde{w}_t$ is called the developed performance of product at stage t. This variable follow a binomial fuzzy path as can be seen in Equation (4.2) and Equation (4.3):

$$\tilde{w}_t(continue) = \begin{cases} \tilde{l}_t \; with\, probability\, p \\ \tilde{q}_t \;\; with\, probability 1 - p \end{cases} \tag{4.2}$$

$$\tilde{w}_t(improve) = \begin{cases} \tilde{l}_t \; with\, probability\; p + \beta \\ \tilde{q}_t \; with\, probability\; 1 - (p + \beta\ ) \end{cases} \tag{4.3}$$

Where $\tilde{l_t}$ and $\tilde{q_t}$ are represented by triangular fuzzy numbers. $\tilde{l_t}$ represents the maximum development that can be achieved by the product when all uncertainties in *t* are resolved while $\tilde{q_t}$ represents the minimal development that the specialist believes that the product can achieve during the time interval *t*. The modification of the model is contained in the variable β which represents an increase in the probability of success. In traditional approaches, a deterministic physical addition, *I,* in performance variable is considered. In our view, this approach is closer to reality, as an additional investment in the project is not deterministic for the full success of the project in phase *t*, but only for the increase in the probability of success since other non-financial management factors are involved in the success of the product. Therefore, the fuzzy cost is defined by Equation (4.4):

$$\tilde{c}_t(u_t) = \begin{cases} \widetilde{K}_t, & if\ u_t = Continue \\ \widetilde{K}_t + \tilde{a}_t, & if\ u_t = Improve \end{cases} \tag{4.4}$$

Where $\widetilde{K}_t$ represents the fuzzy cost to continue the project, while $\tilde{a}_t$ represents the fuzzy cost to improve the project.

4.3.2 Fuzzy Market Uncertainty

The product value or market payoff will be defined through the performance variable achieved by launching the product in *T*, the price that may be paid by the market ($\widetilde{Pr}$), and finally the market requirement function $f(\tilde{X})$, which may be defined as the market share achieved to a certain performance level, $\tilde{X}$.

In their model, Huchzermeier & Loch (2001) consider that the market requirement that represents the fraction of the product on the market is represented by a normal distribution with a mean of μ and a standard deviation of σ. However, these variables are not easily obtained from human judgments. Since there are no historical data to model these variables, a way to analyze the market demand is by using the opinion of market experts whose imprecision should be taken into consideration. Therefore, in the proposed model it is assumed that the market requirement can be represented by a triangular probability distribution that is obtained by means of three parameters: the minimum demand value, c_1 , the average demand value, c_2, and the maximum value c_3. The choice of this approach is also linked to the fact that other variables are also elicited based on estimating triangular fuzzy numbers. Thus, *f(X)* represents the triangular probability distribution, the parameters of which can be accessed directly using the knowledge of the experts involved in the Marketing area of the project. To complete the analysis, the price paid by market will be a fuzzy triangular number $\widetilde{Pr}$ = [*m*,*S*,*M*] where *m* represents the lowest price paid by the market; *M* the highest price paid by the market and, finally, *S* will represent the most likely value paid by the market. Thus, the market Payoff function, $\widetilde{\Pi}(\tilde{X})$, is given by Equation (4.5).

$$\tilde{\Pi}(\tilde{X}) = \widetilde{Pr}.f(\tilde{X}) \tag{4.5}$$

Thus, the current project value is reached through the following dynamic programming model:

$$\tilde{V}(\widetilde{X_T}) = \max_{u_t}\left(-\tilde{K}_t + \frac{p.\tilde{\Pi}(\widetilde{X_T}+\tilde{l_t})+(1-p).\tilde{\Pi}(\widetilde{X_T}+\widetilde{q_t})}{1+r} ; -\tilde{K}_t - \tilde{a}_t + \frac{(p+\beta).\tilde{\Pi}(\widetilde{X_T}+\tilde{l_t})+(1-p-\beta).\tilde{\Pi}(\widetilde{X_T}+\widetilde{q_t})}{1+r}\right) \tag{4.6}$$

Equation (4.6) represents the product launch on the market when the expected payoff is achieved. The other values of the binomial tree are then obtained by recursive equation through equation (4.7).

$$\tilde{V}(\widetilde{X_t}) = \max_{u_t}\left(-\tilde{K}_t + \frac{p.\tilde{V}(\widetilde{X_t}+\tilde{l_t})+(1-p).\tilde{V}(\widetilde{X_t}+\widetilde{q_t})}{1+r} ; -\tilde{K}_t - \tilde{a}_t + \frac{(p+\beta).\tilde{V}(\widetilde{X_t}+\tilde{l_t})+(1-p-\beta).\tilde{V}(\widetilde{X_t}+\widetilde{q_t})}{1+r}\right) \tag{4.7}$$

4.4 Case Study

In this section, a numerical application of the proposed model is presented and uses a case study about the development of a design of a new software for a management platform and the development of innovative corporate projects and for improving processes using suggestions that can be inserted into system by the employees themselves and structured into improvement projects to be prioritized, implemented and monitored by using this platform.

Thus, the design of this new product which is being evaluated allows various features to be included that may or may not be implemented, after considering several dimensions such as: assessment modules to reward the best ideas, communication channels between the managers taking part, the presentation of the panorama of ongoing projects, the monitoring of projects, resource management, etc.

The development of this product was structured into four phases, the length of each being estimated as four months. At the end of each phase, the progress of the project is passed to the managers and some actions can be corrected to increase its performance. Therefore, each phase will be considered as a percentage of the number of goals reached that differentiate this product from the others on the market. Thus, $\tilde{X}$ ranges from 0 to 100% where 0 represents the initial stage of the project while 100% represents the stage where all objectives were fully achieved.

During the survey phase, the managers considered the four identical stages which sets the following hypotheses: $\widetilde{l_1} = \widetilde{l_2} = \widetilde{l_3} = \widetilde{l_4}$, $\tilde{q}_1 = \tilde{q}_2 = \tilde{q}_3 = \tilde{q}_4$ and $p_1 = p_2 = p_3 = p_4$. Thus, the shift in the technical performance of the product can be described by the binomial tree shown in Figure 4.1.

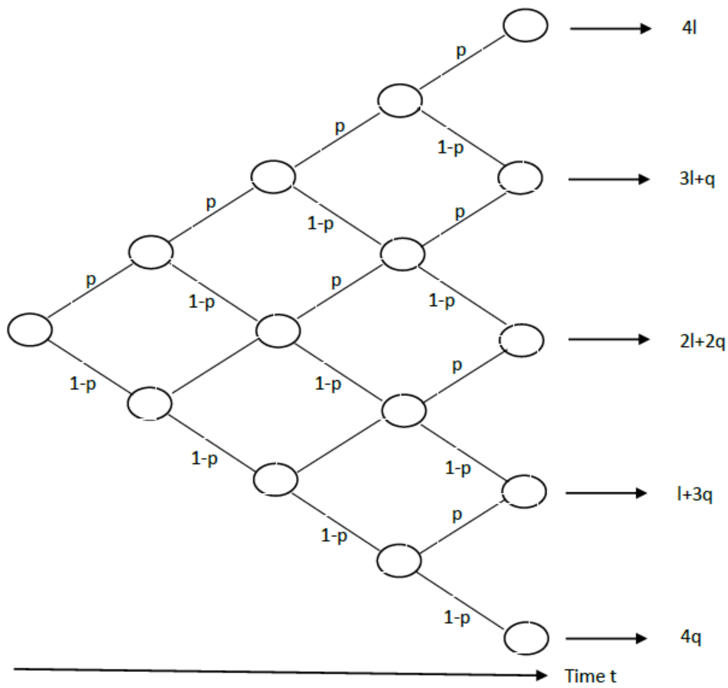

Figure 4.1 - Binomial Tree of project

In Table 4.1, the data elicited from experts are presented after a linear transformation. The variables $\tilde{l}$ and $\tilde{q}$ represent the expected performance of the project and are modeled as a percentage based on the number of criteria developed in each phase. The maximum performance of the variable $\tilde{l}$ is defined as *100/T=100/4* = 25% where *T* represents the phase number of the project.

Table 4.1 – NPD Project Data

Time	Cost (continue)	Cost (Improve)	l_t %	q_t %	p %	β %
0	[70, 90, 180]	[5, 10, 20]	x	x	x	x
1	[20, 25, 50]	[10,15,30]	[12,18,25]	[5,10,15]	0.5	0.2
2	[15, 20, 30]	[15,20,45]	[12,18,25]	[5,10,15]	0.5	0.2
3	[10, 15, 25]	[25,30,50]	[12,18,25]	[5,10,15]	0.5	0.2
4	[5, 10, 15]	[30,45,60]	[12,18,25]	[5,10,15]	0.5	0.2

After defining the variables that impact the technical project performance, the binary tree of project development can be formed. Figure 4.1 shows the expected performance of the project over time. Thus, on the product launch date, the performance scenarios expected are: $4\tilde{q}$; $\tilde{l}+3\tilde{q}$; $2\tilde{l}+2\tilde{q}$; $3\tilde{l}+\tilde{q}$ and $4\tilde{l}$.

As to market uncertainty, first of all, the total value paid by the *Pr* market was considered. This was modeled as a triangular fuzzy variable, $\tilde{P}r = [100, 400, 800]$. In the case of the function of the market distribution requirement, *f*, this was modeled as a random variable that follows a triangular probability distribution $R= [a,b,c]$, with the following parameters $a = 0$, $b = 60$ and $c = 100$.

From these variables and the binomial tree shown in Figure 4.1, the scenarios associated with the market can be generated. At each expected market performance $\tilde{X}_4$ is associated with a market payoff which represents the combination of the market performance, requirement and the total value paid by the market. Given that this combination involves a triangular probability distribution and a triangular fuzzy number, the result of the payoff function does not necessarily represent a triangular fuzzy number as can be seen in Figure 4.2, which represents the expected fuzzy market payoff when the product is launched on the market. The expected fuzzy payoff is obtained using alpha-cut method through algebraic operations amongst the vectors that represent the triangular fuzzy numbers in terms of alpha-cuts and monetary values and the triangular probability distribution.

The result for the expected fuzzy payoff is a similar vector represented in Figure4.2 for *t* = 4, where Figure 4.2 (a), (b), (c), (d) and (e) presents the expected fuzzy payoff respectively for $\tilde{\Pi}(4l)$, $\tilde{\Pi}(3l+q)$, $\tilde{\Pi}(2l+2q)$, $\tilde{\Pi}(l+3q)$ and $\tilde{\Pi}(4q)$. The visualization presented in Figure 4.2 illustrates the results obtained from the combination of the possibilistic and the probabilistic approach used in the proposed model. Such visualization allows to understand the joint possibilistic and probabilistic features of the results, as illustrated in Figure 4.2 and in the following figures presenting the FENPV.

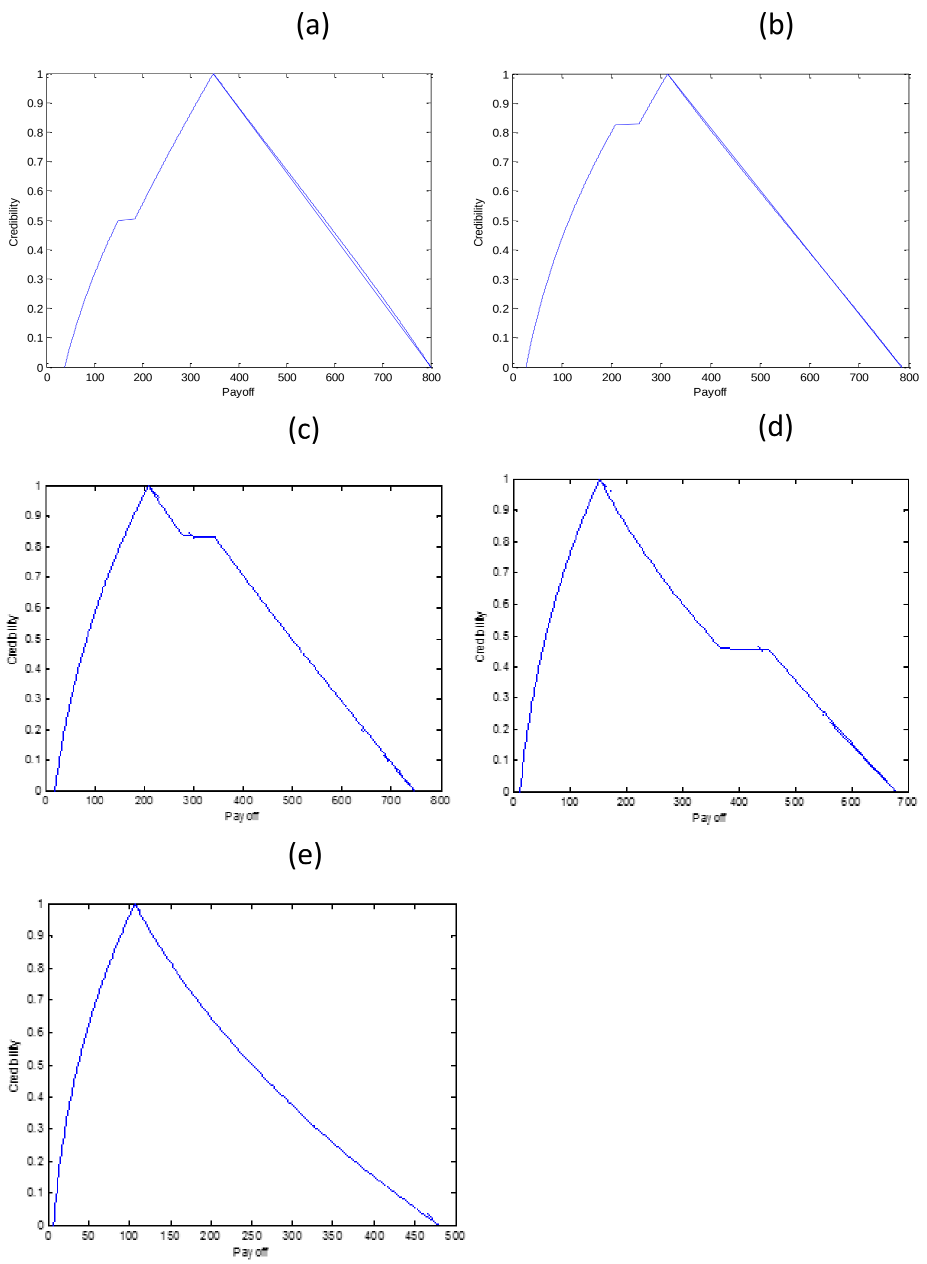

Figure 4.2 - Expected payoff in t = 4 for the binomial tree

4.4.1 Option of Improving the project

After defining the market payoff, the fuzzy triangular number is derived by using the concept of the Fuzzy Expected Net Present Value (FENPV) (Liao & Ho, 2010; Ho & Liao, 2011). Initially, the flexibility of the project was evaluated by taking into consideration only the option of improving the design by means of an additional cost which increases the probability of success at a fixed value, β.

Thus, the FENPV of the project was obtained by means of equations (4.6) and (4.7), considering a risk-free rate of 4,4% over 4 months. For comparison purposes, two fuzzy numbers were generated. Figure 4.3 presents the comparison for the FENPV of the project with (a) and without flexibility (b).

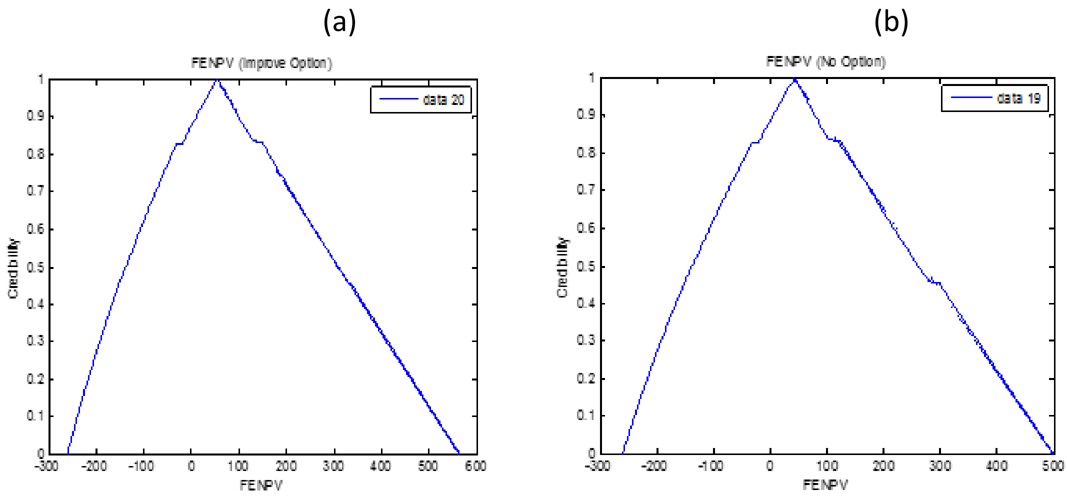

Figure 4.3 - Results of FENPV with improve option and without flexibility

4.4.2 Option of to Expand the Project

Besides the option of to improve the project, another opportunity that projects of this nature present and was not contemplated in the original model by Huchzermeier & Loch (2001), is linked to the option of to expand the business, such as by expanding the scope of the product or adapting it to permit its being marketed in another region based on this additional cost. In this case, the decision is made only at the end of node *T=4*.

Taking as an example the software project, consider the case in which the company has the option to market the software in another country, which although it has a smaller market, has the competitive advantage of there being less competition. Thus, experts are able to visualize the total paid by this new market $\tilde{Pr}^{*}$ = *[100, 300, 400]* at an additional cost of investment, $\tilde{C}^{Ext}$= *[30, 35, 50]*, while of course maintaining the same market requirement. Thus, the optimal decision is made according to Equation (4.8) at T=4

$$V_4 = \max \begin{cases} \widetilde{Pr}.f(\tilde{X}) & \text{if not Expanded} \\ (\widetilde{Pr} + \widetilde{Pr}^{*} - \tilde{C}^{Ext})f(\tilde{X}) & \text{if Expanded} \end{cases} \qquad (4.8)$$

On considering Equation (4.8), the FENPV of the option to expand can be generated and compared to the value of the project without flexibility, as shown in Figure 4.4 (a) and (b) respectively.

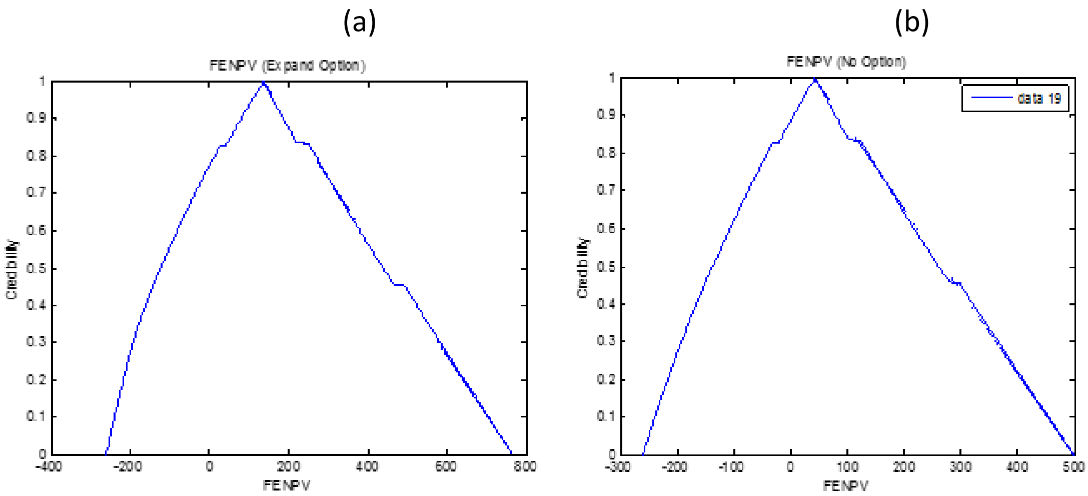

Figure 4.4 - Results of FENPV with expand option and without flexibility

4.4.3 Multiple Options

Finally, there is the combined analysis of multiple options that considers the joint use of the option to improve with the option to expand the project. Thus, the decision-maker can either change the course of the project, considering an additional cost that represents an increase in the probability of the technical success of the project or the option of trying to increase the profit of the project, considering the option of marketing the product in other markets at an extra logistical cost.

The result of FENPV for the multiple options and without flexibility is presented in Figure 4.5 (a) and (b), respectively.

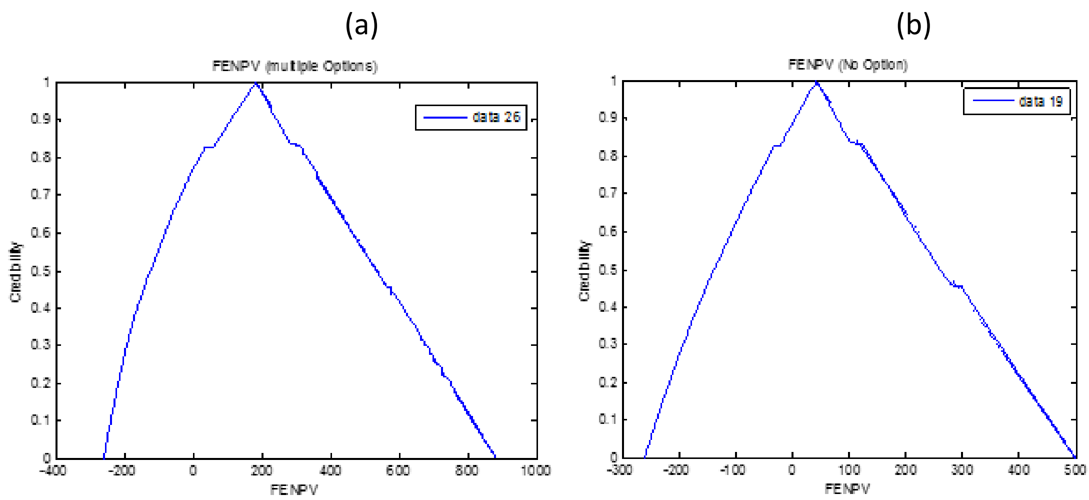

Figure 4.5 - Result of FENPV for the multiple options and without flexibility

4.4.4 Mean Fuzzy Expected Net Present Value

Although the graphic display is valid to analyze the results, quantifying the value of the option is also needed and this is done by using the mean value in the case of the probabilistic approaches. Thus, this article uses the concept of the fuzzy mean value obtained from Equation (4.9) (Yoshida et al., 2006; Liao & Ho, 2010; Ho & Liao, 2011):

$$FENPV = \int_{0}^{1} [(1-\lambda).A_1(\alpha)d\alpha + \lambda.A_2(\alpha)]d\alpha \qquad (4.9)$$

Where $\lambda \in [0,1]$ represents the optimism-pessimism index and is linked to the investors' perception of risk and A_1 and A_2 represent the alpha-cuts of the fuzzy number. For didactic purposes, in this paper, the investor was considered neutral, that is, $\lambda = 0.5$. The fuzzy average values can be seen in Table 4.2 in which the option value is the difference between the average value of the project with the option and the average value in the absence of the option.

Table 4.2 – Mean FENPV of Project in different options

Project flexibility	Mean-FENPV	Option Value
No Options	65,117	X
Improve	91,292	26,175
Expand	174,367	109,25
Multiple Options	220,514	155,397

4.4.5 Analysis of The Results

The results obtained enable the NPD project to be evaluated considering the particulars of a project with these features. By applying the proposed model, it can be seen that vis-à-vis the nature of the level of observation which it has for these types of projects, the use of a fuzzy approach enables the experts' knowledge to be better represented and better portrays the imprecision that exists in the information available.

From the results of Figures 4.3 and 4.4, which are summarized in Figure 4.5 to represent the possibility of exercising the multiple options, the possibility of considering the impacts of the technical and market uncertainty are portrayed. As can be seen from Table 4.2, the advantage in using multiple options (of to improve and to expand) can be analyzed when it is verified that the individual sums of the option value of to improve and to expand are less than the value of the option when the two options are combined in the model.

By comparing with the FENPV, a consolidated metric 4-9 is obtained which represents the value of each of the options and the opportunities that the model is able to capture in such a way that it is possible to provide a decision-maker with a consolidated analysis - similar to that offered by other models – but which is more robust since it considers the imprecision inherent in this context by using a fuzzy approach.

4.5 Conclusions

This article set out to put forward a financial evaluation model for NPD projects with a view to integrating experts' subjective knowledge by using a probabilistic-fuzzy approach in a real options model that takes into account the uncertainties normally present in projects of this nature.

The main advantage of the proposed model is the separation of the technical flexibility of the project in relation to the market flexibility in the context of a fuzzy environment, since the existing approaches do not differentiate these uncertainties in terms of flexibility with a fuzzy approach. On incorporating a fuzzy approach, inconsistencies can be avoided that arise due to the lack of knowledge and precision some of these variables.

Through a case study that makes a financial evaluation of an NPD project, it was possible to illustrate the application of the proposed model, and given the results, it can be seen that the flexibility on combining the two options is greater than the individual flexibilities regarding the technical and market uncertainty which gives evidence for there being greater integration between the technical team and the marketing team while the project is being drawn up and carried out. This tends to increase the value of the option.

Drawing on this article, new models can be constructed which may seek to integrate other sources of flexibility in NPD projects, such as, time to market the product which has an impact from both the technical and market point of view.

Furthermore, since an NPD project generally involves a large number of participants, there is the possibility of integrating and proposing models that incorporate aggregation methods and combinations of experts' different opinions using fuzzy logic (Kokshenev et al., 2015) or rough group ANP (CAO & SONG, 2016). Another development opportunity is that of a model of real options for NPD projects in the context of group decision and negotiation (PEDRYCZ & SONG, 2011; PARREIRAS et al., 2012; PELTA & YAGER, 2010; WANG, KILGOUR & HIPEL 2015) since different areas may be involved in drawing up and evaluating this type of project.

5 FINAL REMARKS

5.1 Thesis Conclusions

This thesis has presented three articles dealing with the representation of uncertainty. In relation to the first three articles that deal with conflict analysis in Dempster - Shafer Theory (DST) using multicriteria decision models, it is seen that the articles have complementary points. In the first article, class boundaries are defined based on class profiles that are directly obtained from the proposed disaggregation model while in the second article these boundaries are defined based on the pairs of pre-classified bodies of evidence pre-classified in accordance with the conflict.

In the third article, which integrated a real options model with Fuzzy set logic, the measurement of risk in NPD projects was expanded. To do so, the traditional notion of NPV was expanded to take into account the dimensions of the technical risk and the market risk by using triangular fuzzy numbers. Thus, risk analysis can be obtained by using the FNPV notion while the flexibility value of the project is understood by using the FROV value. It should also be emphasized that in the case of NPD projects, the uncertainty modeling present in this type of project requires a subjective analysis. Thus, the use of a model approach that takes into account the vagueness present in subjective judgments facilitates the elicitation of those who possess knowledge about the project.

5.2 Research Developed

Regarding the current state of the articles, the first article, Chapter 2, was published in *Information Sciences* while the other articles are being peer reviewed. The second article has been submitted to *Information Fusion*, the third to *Knowledge-Based Systems.*

In addition to the articles presented in this thesis, other studies were undertaken during the PhD program, including an article published in *IEEE Power Delivery* (SILVA, da SILVA & ALMEIDA-FILHO, 2016) and another in *IEEE Latin America* (da SILVA JUNIOR, de OLIVEIRA SILVA & de ALMEIDA-FILHO, 2016) which address proposals for allocating measurement sensors in an electricity distribution network to identify non-technical losses, which were not included in this thesis because they are not integrated into this topic.

The future work following this research shall consider a model for checking the consistency of a set of conflict metrics based on pairs of bodies of evidence pre-classified into classes of conflict. Therefore, such model can be used as input for applying both what the first and second articles proposed since by using the DRSA, reducts of metrics can be generated that have the same consistency as the total set of metrics.

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