BRUNO NUNES GUEDES

Analysis of Extended Warranties for Medical Equipment: A Game Theory Based Approach Using Priority Queues

Recife

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Dissertação de Mestrado apresentada à UFPE para a obtenção de Mestre como parte das exigências do Programa de Pós-Graduação em Engenharia de Produção (Área de Pesquisa Operacional).

Orientador: Márcio José das Chagas Moura, Dsc

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ÁREA DE CONCENTRAÇÃO: PESQUISA OPERACIONAL

A comissão examinadora composta pelos professores abaixo, sob a presidência do primeiro, considera o candidato BRUNO NUNES GUEDES **APROVADO**.

Recife, 19 de Fevereiro de 2016.

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Prof. MÁRCIO JOSÉ DAS CHAGAS MOURA, Doutor (UFPE)

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Prof. ISIS DIDIER LINS, Doutora (UFPE)

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Prof. ALEXANDRE STAMFORD DA SILVA, Doutor (UFPE)

DEDICATION

To my parents, Antonio and Madalena.

To my aunt Socorro.

To my brother Rodrigo.

To my nephew João.

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Agradeço a todos que de alguma forma contribuíram ao longo de toda minha trajetória discente para a concretização deste trabalho. Agradeço em especial:

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EPIGRAPH

"It always seems impossible until it's done"

Nelson Mandela

ABSTRACT

A growing trend in hiring maintenance services has been observed in companies in general in order to enhance competition and reduce costs. This practice becomes even more evident in the context of health institutions, as they strongly employ technology-intensive equipment that must follow tight quality standards that intend to ensure the continuity of the service and the safety of patients. These characteristics contribute in allowing the maintenance to be executed by the Original Equipment Manufacturer (OEM), since several pre-established procedures must be attended during maintenance. Thus, it becomes relevant to analyze the interaction among customers (hospitals) and the equipment manufacturer in this particular maintenance services market. In the developed model the customers are divided into 2 classes, great size hospitals belong to class 1 and small hospitals belong to class 2 and class 1 customers have priority over class 2 customers. Class 1 customers have the option of hiring an Extended Warranty (EW) with priority or of paying for each maintenance intervention on demand, while class 2 customers have the option of hiring an standard EW (with no priority) or of paying for each maintenance intervention on demand. To model such dynamics a 2-class priority queuing system is implemented. The customers select the option that maximize their expected utilities, as they are risk averse, while the manufacturer needs to set the EW and maintenance intervention prices and select the optimal number of customers of each class to service in order to maximize their expected profit. A Stackelberg Game is used to model the interaction among players, in which the OEM is the leader and the customer is the follower. In the numerical example it has been found that the customers of class 1 decide to hire EW with priority, while class 2 customers decide to pay for maintenance services on demand. Also the OEM decides to service 3 customers of class 1 and 100 customers of class 2, which yields an expected profit of $ 3,204,450. A sensitivity analysis is also performed to analyze how the optimal solution changes due to parameters variations.

Keywords: Maintenance services, Medical equipment, Extended warranty, Priority queues, Stackelberg game.

RESUMO

Uma tendência crescente para a contratação de serviços de manutenção tem sido observada em empresas em geral com o objetivo de aumentar sua competitividade e reduzir custos. Tal prática se torna ainda mais evidente no contexto de instituições de saúde, já que elas utilizam diversos equipamentos intensivos em tecnologia que precisam se adequar a rígidos padrões de qualidade de forma a garantir a continuidade do serviço e a segurança dos pacientes. Essas características contribuem para que os serviços de manutenção sejam executados pelo fabricante do equipamento, já que diversos procedimentos específicos precisam ser seguidos durante a manutenção. Assim, torna-se relevante analisar a interação entre clientes (hospitais) e o fabricante do equipamento neste mercado particular. No modelo desenvolvido, os clientes foram divididos em 2 classes, hospitais de grande porte pertencem à classe 1 e hospitais pequenos pertencem à classe 2 e os clientes da classe 1 têm prioridade em relação aos clientes da classe 2. Os clientes da classe 1 têm a opção de contratar uma garantia estendida com prioridade ou de pagar por cada intervenção de manutenção sob demanda, já os clientes da classe 2 têm a opção de contratar uma garantia estendida padrão (sem prioridade) ou de pagar por cada intervenção de manutenção sob demanda. Para modelar esta dinâmica um sistema de filas com 2 classes de prioridade foi implementado. Os clientes escolhem a opção que maximiza suas utilidades esperadas, já que são avessos ao risco, enquanto o fabricante deve determinar os preços das garantias estendidas e das intervenções avulsas além do número ótimo de clientes que ele deve atender de forma a maximizar o seu lucro esperado. Para modelar a interação entre os jogadores foi utilizado um Jogo de Stackelberg em que o fabricante é o líder e o cliente, o seguidor. O exemplo numérico apresentado mostra que a decisão ótima para os clientes da classe 1 é adquirir a garantia estendida com prioridade, enquanto para os clientes da classe 2 a decisão-ótima é pagar pelos serviços de manutenção sob demanda. O fabricante decide atender 3 clientes da classe 1 e 100 clientes da classe 2, o que lhe gera um lucro de $ 3,204,450. Uma análise de sensibilidade é apresentada em seguida para investigar como a solução ótima muda em decorrência de variações nos parâmetros.

Palavras-chave: Serviços de manutenção, Equipamento hospitalar, Garantia estendida, Filas com prioridade, Jogo de Stackelberg.

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LIST OF ACRONYMS

BW – Base Warranty;

CCU – Camera Control Unit;

CDF – Cumulative Distribution Function;

EW – Extended Warranty;

FCFS – First come, first served;

GRP – Generalized Renewal Process;

HD – Hard disk;

HPP – Homogeneous Poisson Process;

ICU – Intensive Care Unit;

LCFS – Last come, first served;

MSC – Maintenance Service Contract;

NHPP – Non-homogeneous Poisson Process;

OEM – Original Equipment Manufacturer;

ONA – National Accreditation Organization;

RP – Renewal Process.

LIST OF NOTATION

P_{wi}: Extended warranty price a class i customer would be charged by the manufacturer;
C_{si}: Maintenance service price a class i customer would be charged by the manufacturer;
$P_{wi}(max)$: Maximal price a class i customer would accept to pay for an EW;
$C_{si}(max)$: Maximal price a class i customer would accept to pay for a maintenance service;
M_i: Number of customers of class i serviced by the manufacturer;
M: Total number of customers serviced;
A_i: Customers' decision variable;
U(w): Utility associated to the wealth w;
R: Revenue generated by the equipment while in operational state per unit time;
L: Time horizon considered;
C_b: Equipment sale price;
λ: Equipment failure rate;
C_r: Maintenance service cost incurred by the manufacturer;
y: Time to return a failed unit to operational state;
α_i: Penalty parameter for class i customers;
τ_i: Maximal time to repair a unit under extended warranty for class i customers;
β_i: Risk aversion parameter of class i customers;
N_j: Number of failures occurred to customer j's equipment over its useful life;
X_{ji}: Time to the i^{th} failure after the $(i - 1)^{th}$ repair;
$\tilde{X}_j$: Time for which the equipment was in operational state at the end of its useful life after being returned to operational state after the last repair;
y_{ji}: Time to return the equipment to operational state after the i^{th} failure (waiting time + repair time);
μ: Service rate;
$\tilde{N}_j$: Number of times the manufacturer incurs in penalty;
T_{a1}: Time of arrival of class 1 customer;
T_{a2}: Time of arrival of class 2 customer;
T_d: Time of departure of a customer.

1. INTRODUCTION

1.1. Problem Description

A growing trend in outsourcing maintenance services has been observed in recent years, aiming to accomplish lower costs, higher service quality, and thus to turn a company more competitive in its market. As pointed out by (Jackson & Pascual, 2008), the following characteristics contribute to the decision of hiring a service rather than performing it in-house: 1) access to high-level specialists; 2) better services due to expertise of the service provider; 3) fixed price contracts, which control risks of extremely high costs. Those advantages in outsourcing make the company search for service providers, which are able to solve problems at hand.

Outsourcing is naturally related to the establishment of a legal instrument to guide what has been agreed. According to (Staub, 2009), on the specific context of maintenance service contracts (MSCs) or Extended Warranties (EWs), the following steps should be considered: 1) specification of the service to be executed and of the equipment considered; 2) selection of possible service provider in accordance with previously established criteria; 3) hiring the selected company; 4) preparation, supervision and evaluation of the service, and 5) after-care, i.e., attention provided by the maintenance company after the execution of services. Each of those steps has an associated cost and, consequentially a certain degree of risk, which, according to (Tan et al., 2011), may be defined as a combination of consequences arising from undesirable events with the proper probabilities related to each one of them.

As indicated by (Damnjanovic & Zhang, 2008), some of the risks to which customers are exposed to are equipment failures and maintenance intervention of bad quality; each of them is subjected to a determined loss. In contrast, the maintenance agent is subject to the risks of a very long effort to complete a repair or of an early equipment replacement. Both cases may be associated to considerably high costs, which depending on the MSC or EW terms, may be completely transferred to the service agent.

In the medical context due to the emergence of technology-intensive equipment and the necessity of fast and high-class maintenance interventions, outsourcing became common. In this specific case, it is important to consider some particular aspects such as the importance of the service to be executed and the best method of managing it as the use of many of those equipment is crucial to serve the hospital patients. Therefore, the maintenance team must not only repair the failed equipment, but also be aware of its level of importance to serve patients, and also of the impact generated by unavailability and adjustment problems (DYRO, 2004).

For such a purpose it is important to know the equipment's failure record, useful-life, obsolescence level, building characteristics and replacement possibilities during a maintenance intervention; in brief, all information that in some way support a safer and higher quality maintenance (Calil & Teixeira, 1998). That information will be important to improve the failure analysis and to determine the service urgency level, define an effective preventive policy and to achieve the required reliability level.

Hospital maintenance is characterized by being a diverse field, as it encompasses equipment, infrastructure and medical instruments maintenance (Calil & Teixeira, 1998). Depending on the equipment or on the department that requests maintenance, the service may acquire a higher criticality as it may be essential for emergencies, such as equipment used in the Intensive Care Unit (ICU).

Finally, it is necessary to be aware of the importance of the maintenance function inside a hospital, due to services executed in-house and by external agents. That importance goes far beyond failure repairs, passing through an extensive maintenance policy planning, scheduling of preventive interventions, condition monitoring and quality control, to ensure that the equipment will be available when necessary.

Due to all above-mentioned factors, it may be realized that the level of demand over maintenance services in a hospital is high. Tied with that, the growing technology complexity of some medical equipment is making it difficult to execute maintenance services in-house. When one decides to outsource maintenance services, two main options emerge: 1) hiring the Original Equipment Manufacturer (OEM), with whom an EW would be established, or 2) hiring an external agent, with whom an MSC would be established.

Nevertheless, in order to guard the market for himself, the OEM may adopt some protectionist measures such as, not providing training to possible external maintenance service providers and not providing sufficient spare parts to the market, which makes it difficult for external maintenance providers to repair such failed units, as mentioned by (DE VIVO et al., 2004). Therefore, the OEM becomes the only feasible agent to execute maintenance services to the equipment manufactured by him, granting him monopoly power in the maintenance services market and a higher bargain power than the customer, and thus some advantages when negotiating the terms of the EW to be celebrated by both parties.

The proposed model deals specifically with the problem of analyzing EWs for medical equipment, aiming to maximize the objective functions of both players involved, manufacturer and customer, i.e., the former intends to maximize his expected economic profit and the latter intends to maximize his expected utility. Their interaction will be modeled according to a Stackelberg Game formulation (Osborne & Rubinstein, 1994), with perfect and complete information, in which the former acts as a leader and the latter as a follower. Game Theory is commonly employed to model negotiation and conflicts. The Stackelberg Game is used because it fits well situations in which one of the players is a monopolist, i.e., has more bargain power than the other, as occurs in the context of medical equipment maintenance.

The manufacturer will be considered risk neutral and will pursue profit maximization through the sale of the medical equipment, analysis of EWs and the performance of maintenance interventions on demand. The customer will be considered risk averse and may belong to two different classes of customers: class 1, composed by hospitals which are likely to pay more for shorter repair times, normally big hospitals, and class 2, composed by hospitals that are more likely to wait for longer repair times, normally small hospitals, where class 1 customers have priority over class 2 customers. To model risk aversion, the utility function is used. Utility is a function of the profit obtained by the use of the medical equipment. As it is a game of perfect and complete information the manufacturer knows when he is dealing with a class 1 or a class 2 customer and is able to charge them different prices.

The game develops in three stages: first, the OEM observers if he is dealing with a class 1 or class 2 customer, second the leader (manufacturer) defines the EW's and maintenance intervention prices, and third the follower (customer) decides whether or not to buy the equipment and whether or not to hire the EW. Such process occurs for each customer and is repeated as many times as the number of customers to be serviced. Class 1 customers are offered an EW with priority, which means that they will be serviced first even when there are other class 2 customers in queue; while class 2 customers are offered an standard EW (with no priority). The EW declares that every failure that occurs during the equipment's useful life should be repaired at no additional cost for the customer. If no EW is hired, the customer will pay a fixed price for each intervention when necessary. In this case, the OEM is able to charge different prices for class 1 and class 2 customers, although there is no priority associated.

In order to model the equipment's failures and repairs, a stochastic process will be employed, it is supposed that the equipment is subject to perfect repair, which is modeled according to a Homogeneous Poisson Process (HPP). Furthermore, as we consider multiple customers, and consequently multiple equipment to be repaired, we will use Queueing Theory (Gross et al., 2008) to model the dynamics of equipment failing, waiting for repair and being restored to the operational state. More specifically, a 2-class priority queueing system will be implemented using discrete-event simulation.

1.2. Justification

In the 1950 decade, maintenance was often understated by companies, as it was considered only a source for expenses. However, it started to change over the years, and maintenance became essential for costs reduction, as great amounts of resources were spent to provide those services, and that it was vital to increase market competitiveness, as constant failures or unregulated equipment lead to losses in the short-term and to market share losses in the long-term. (Dekker, 1996) still adds that the growing complexity of equipment led to even higher investment in maintenance, coupled with the need of rationalizing the expenses through mathematical models. This trend, which may be observed in companies in general is also common among hospitals and healthcare institutions, where the sense of urgency and the requirement for quality and availability of equipment is even higher, as mentioned by (MUTIA; KIHIU & MARANGA, 2012).

The growing need to reduce costs and increase maintenance agility and quality led to a trend of outsourcing maintenance activities, as mentioned by (ASHGARIZADEH; MURTHY, 2000; ESMAEILI; SHAMSI GAMCHI; ASGHARIZADEH, 2014; JACKSON; PASCUAL, 2008; MURTHY; ASGHARIZADEH, 1999; MURTHY; YEUNG, 1995). Outsourcing allows cutting costs with hiring and training, which for technology-intensive equipment may be very substantial. It also gives companies access to professionals specialized in each equipment that needs maintenance. This fact is especially relevant in the medical context as each sector of the hospital may be equipped with quite different technologies.

Besides the generalized trend to outsourcing due to the reasons above-mentioned, some specific characteristics inherent to hospitals support this practice even more. As pointed out by (De Vivo et al., 2004), a significant amount of medical technologies is so complex that performing in-house maintenance becomes uneconomical and hiring independent service agents is difficult as they may not be familiar enough with the equipment to be repaired and may not have access to all necessary information, which should be provided by the equipment manufacturer, as manuals or complementary documentation, to allow the execution of a high-class maintenance service. Thus, the OEM emerges as the only feasible option to provide maintenance interventions on the technology-intensive medical equipment manufactured by him. A comparative table with advantages and disadvantages of performing in-house maintenance, hiring the OEM or a maintenance agent may be seen in Table 1.1.

Table 1.1 – Comparing in-house, manufacturer's and third party servicing organization

Maintenance organization	Advantages	Disadvantages
In-house	Fast response possible for breakdown	Special tools and test equipment may be not available or may need additional costs (e.g. calibration by the manufacturer)
	Technical staff can work closely with professional users	Hard to maintain adequate stocks of spare parts across a wide range of devices
	On-site repairs can lead to short down times	Training costs high and manufacturers are sometimes reluctant to provide it
	Often less costly than an outside organization for a given level of service	In-house staff is typically generalist rather than specialist
Manufacturer	Predictable costs	Contracts with many separate manufacturers need to be negotiated and updated
	Same build standard as original device with modification and updates incorporated	Staff needed to administer the maintenance system
	Assures access to spare parts	Quality control must be monitored
	Remote diagnostics via computer network sometimes available	Response time may be long, depending on the contract
	No problems with warranty/liability	
	Availability of training for professional users	
Third party	Often cheaper than manufacturer	May only be available for certain devices
	Possible to have on-site engineer	Manufacturers are usually reluctant to train personnel
	Fewer external organizations to deal with	Possible liability problems

Source: (De Vivo et al., 2004)

Table 1.1 supports what has been mentioned before and justifies the choosing of the OEM to perform maintenance interventions, as the advantages are much stronger than the disadvantages for this service provider when compared to the other options. This argument is also used by (Bollapragada; Gupta & Lawsirirat, 2007) who indicate that long-term EWs negotiated by customers and manufacturers of high-tech industries, such as aviation and medical, as a positive trend for both sides. They also suggest that the OEM's possibility of negotiating with multiple customers allows scale gains that lead to lower costs. This cost reduction may be partially transferred to the customers and it is one of the reasons why these legal instruments may be beneficial for both sides. Furthermore, customers also benefit of the risk transferred to the manufacturer after an EW is celebrated.

For all the above exposed, this master thesis relevance for healthcare institutions, medical equipment manufacturers and society is supported. Additionally, although the MSCs and EWs have been widely discussed in literature, their study in the specific context of healthcare institutions is still scarce and, at the best of author's knowledge, has only been found in (DE VIVO et al., 2004). Then, a new contribution in this topic using more recent approaches becomes quite relevant. Moreover, the offer of priority services to bigger hospitals has been observed as a common practice in the real world and, at best of author's notice, has not been studied previously.

Thus, using similar approaches to (Esmaeili; Shamsi Gamchi & Asgharizadeh, 2014; Jackson & Pascual, 2008; Murthy & Asgharizadeh, 1999; Murthy & Yeung 1995), which have used Game Theory to model maintenance service outsourcing, specifically the model proposed by (Ashgarizadeh & Murthy, 2000) is extended by considering 2 classes of customers, where class 1 customers have priority over class 2 customers, applying the proposed model to the field of hospital maintenance, which is carefully characterized and lacks of decision support models, mainly mathematical models (Cruz & Rincon, 2012). However, the present master thesis will focus on technology-intensive clinical equipment used for imaging exams, whose maintenance services market is characterized as monopolistic, as the necessary knowledge, tools and spare-parts are concentrated in the hands of the OEM (De Vivo et al., 2004). None of these extensions has been proposed nor applied in the cited papers.

In order to model the negotiation process between OEM and each customer, Game Theory is used, as it is a tool which allows each player to maximize his own expected profit or utility taking into account the actions of the others. A Stackelberg Game formulation is proposed as it is commonly used to model monopolistic markets and fits well the maintenance services outsourcing of technology-intensive equipment. Due to these characteristics, other problem solving approaches such as multicriteria analysis, used by (Sundarraj, 2004), and profit maximization, used by (Huber & Spinler, 2012; Shamsi Gamchi; Esmaeili & Saniee Monfared, 2013) were neglected.

1.3. Objectives

1.3.1. General Objective

Formulate and solve a mathematical model, which deals with the analysis of extended warranties for technology-intensive medical equipment, using a stochastic process in its formulation, a 2-class priority queueing system and a Stackelberg Game with perfect and complete information.

1.3.2. Specific Objectives

i. Literature review and theoretical background study about maintenance, its policies, repair types and technology-intensive equipment for healthcare. Outsourcing, extended warranties and maintenance service contracts, game theory, its objectives and game types;
ii. To assess the mentioned problem mathematically and model it considering that the equipment is repairable and fails according to a stochastic process and that the customers are divided into 2 classes, where class 1 customers have priority over class 2 customers;
iii. To determine the manufacturer's and customers' optimal strategies in a game of perfect information;
iv. To apply the model formulated to real data referent to a technology-intensive imaging equipment to illustrate its applicability;
v. To perform a sensitivity analysis to study the impact of parameters variations over the model output.

1.4. Literature Review

In this section, an overview of the main publications in the field of EWs and MSCs is provided. First, articles, which consider game theory and are the main references to the present work are discussed. Then, other articles, which use alternative solution tools are addressed.

Finally, papers that deal with the medical context inserted in the maintenance field are commented.

1.4.1. Game Theoretical Models

At the best of authors knowledge, the study of MSCs using stochastic models and game theory began with (MURTHY; YEUNG, 1995). In this paper, two different models were proposed to illustrate the problem of modelling MSCs, both considering two players, service agent and customer interacting according to a Stackelberg Game. One of the models is simpler and considers that there is always a spare part ready to use, while the other considers that after a maintenance service is executed a new spare part must be requested. The paper's main contribution was the use of Game Theory as a new solution tool to the problem of MSCs analysis.

In (Ashgarizadeh & Murthy, 2000), the service agent serves multiple customers, which are considered homogeneous regarding their attitude to risk. In order to model the failure and repairs of multiple equipment, Queueing Theory is used (Gross et al., 2008). As the service agent is a monopolist, a Stackelberg Game is used to model his interaction with the customers. After the optimal strategies are derived, the agent extracts all consumer surplus and leaves the customers with zero economic profit, what is consistent with the assumptions of a monopolistic market with complete and perfect information.

In (Murthy; Aasgharizadeh, 1999), besides servicing multiple customers, the agent is also able to repair multiple equipment in parallel, i.e. there are multiple service channels. Once again a monopolistic market is modeled using a Stackelberg Game. After the determination of the optimal strategies the customers are left with zero profit and the agent’s profit is higher than it was in (Ashgarizadeh; Murthy, 2000), what is expected due to the possibility of increasing the number of customers serviced due to the multiple service channels.

(Jackson & Pascual, 2008) innovate by. using a Nash Game, instead of a Stackelberg Game, to model the negotiation process. This kind of game is used in situations in which both players have the same bargain power, i.e. negotiate in equal conditions, differently from what happens in monopolistic markets. Preventive maintenance is also incorporated in their models, so they use a failure rate that increases linearly with time. They propose 2 models, the first one considers one service agent and one customer, while the other considers multiple customers and one service agent. After the optimal strategies are determined, the total surplus generated in each case is equally distributed by both set of players, in accordance with a Nash Game negotiation.

In (Esmaeili; Shamsi Gamchi & Asgharizadeh, 2014), the EWs and the MSCs are studied jointly. This is presented as a new trend in maintenance outsourcing, as the OEM and the service agent are possible providers of maintenance services to customers. Thus, a game formulation with one OEM, one service agent and customer is employed, in which the first offers an EW and the second an MSC. Three different models are proposed, in first a non-cooperative static game is used (Nash Game), i.e. all players intend to maximize their objective functions individually and make their decisions simultaneously. In second a non-cooperative sequential game is used (Stackelberg Game), i.e. the OEM has more bargain power than the agent and the agent has more bargain power than the customer. In third, a semi-cooperative game is used, i.e. OEM and agent cooperate and act as a single monopolist. After the optimal strategies are derived, the authors observed that the prices charged according to the third model are the highest while the prices charged in the first model are the lowest.

1.4.2. Alternative Models

Besides Game Theory, other strategies have been used to solve the problem of EWs and MSCs negotiation. A literature review of some of these works is presented here. (Rinsaka & Sandoh, 2006) propose a model in which EWs are studied. The Base Warranty (BW) is also incorporated to the model. As mentioned in Chapter 2, this kind of warranty is coupled with the sale of the product and its price is incorporated to the product's sale price. The model is solved through maximization of the objective functions of the manufacturer and the customer.

(Wang, 2010) develops a model for studying MSCs negotiation and optimization. Only one customer and one service provider (agent) are considered in the model, but it also encompasses the equipment's availability and reliability. The concept of delay time is used to model the equipment failures, it also allows the occurrence of defects that may be detected through inspections. In the model the agent may execute the repairs and inspections or only one of them, being the complementary actions executed by the customer. The use of the delay time concept in the context of MSCs is an important innovation, which has not been observed in other papers.

(Darghouth; Chelbi & Ait-Kadi, 2012) developed a model for the negotiation of EWs of a non-self-announcing failure equipment, i.e. equipment whose state may only be determined through inspections. It was developed to allow the OEM to predict the profit generated by an

EW offered to the customer and to plan maintenance activities covered by the EW. The model aims at determining the inspection instants and the manufacturer's profit per time unit over an infinite horizon, and is solved by a numerical iterative procedure.

(Huber & Spinler, 2012) present a model for pricing full-service repair contracts of small investment products for risk-averse customers. The main contributions of the paper are focused on two points: adopting a market perspective of multiple, heterogeneous customers and introducing a stochastic cost per repair event. In order to capture cost volatility, stochastic cost for single repair events is assumed, in addition to the stochastic failures commonly used in the literature. An important conclusion is pointed out by the authors: their model shows that service contract prices are strongly driven by the variance of the repair cost.

(Shamsi Gamchi; Esmaeili & Saniee Monfared, 2013) studied the negotiation of MSCs and EWs among three traders - manufacturer, agent and customer. Each customer may buy several products and incorporate a quantity discount offered by the manufacturer, what has not been presented in previous papers. They also make a distinction between two set of customers, risk-averse customers and risky customers, the first group is pessimistic and consider the worst possible scenario when purchasing a product; while the second group is optimistic and consider that the minimum number of failures would occur and thus, want to spend as little as possible on maintenance contracts. One important innovation brought by the authors is the consideration of the risk parameter without using an utility function, it is used directly in the customer's model.

Although MSCs and EWs have been widely discussed in literature, their study in the specific context of healthcare institutions is still scarce and, at the best of author’s knowledge, has only been found in (DE VIVO et al., 2004). Moreover, (Murthy & Yeung, 1995) analyzed MSCs using an adaptation of a Stackelberg Game and perfect repairs; (Ashgarizadeh & Murthy, 2000) brought Queueing Theory to the analysis; (Murthy & Asgharizadeh, 1999) turned the number of service channels into a decision variable of the service agent; (Jackson & Pascual, 2008) analyze MSCs using a Nash Game and minimal repairs; while (Esmaeili; Shamsi Gamchi & Asgharizadeh, 2014) analyze EWs and MSCs using non-cooperative and semi-cooperative games among manufacturer, service agent and customer. However, none of those papers focuses and properly characterizes one specific context to be modeled and to justify the game formulation employed, what leads to generic formulations.

Therefore, this master thesis proposes a novel approach based on Game and Queueing Theories to model the interaction between OEM and healthcare institutions in the context of outsourcing maintenance services for technology-intensive equipment. In this way, as the maintenance service market for such equipment is monopolistic, and the OEM has more bargain power then the hospital, a properly adapted Stackelberg Game formulation proposed in (Murthy & Yeung, 1995) may be employed. The interaction occurs between manufacturer and hospitals, that are split into two classes, class 1 and class 2.

After this introductory Chapter, this master thesis unfolds as follows: in Chapter 2 accomplishes some important theoretical aspects, regarding reliability, stochastic processes, queueing theory, game theory, etc.; in Chapter 3 the proposed model is presented and detailed; in Chapter 4 the results are presented and the sensitivity analysis is performed; and, in Chapter 5 the conclusions are pointed out, as well as some limitations and extension suggestions.

The present work proposes a novel approach based on Game and Queueing Theories to model the interaction between OEM and healthcare institutions in the context of outsourcing maintenance services for technology-intensive equipment. In this way, as the maintenance service market for such equipment is monopolistic, and the OEM has more bargain power then the hospital, a properly adapted Stackelberg Game formulation proposed in (Murthy & Yeung, 1995) may be employed. The interaction occurs between manufacturer and hospitals, that are split into two classes, class 1 and class 2.

Besides this introductory chapter, this Master's Thesis is structured as follows: Chapter 2 deals with important theories and concepts used to develop the proposed model, such as stochastic processes, queueing theory, as well as important economic concepts of Game Theory and Stackelberg Game. In Chapter 3 details about the proposed model are provided. In Chapter 4 an application example is presented and a complete sensitivity analysis is performed to investigate changes in the model output due to parameters variations. Finally, in Chapter 5, come concluding remarks are pointed out, as well as work limitations and suggestions for future work.

2. THEORETICAL BACKGROUND

The present chapter comments some important concepts behind the model formulated in the present master thesis. It begins discussing the most common maintenance policies and repair types, then it goes through the topics of medical equipment, medical maintenance and maintenance outsourcing. Finally, the more mathematical part of the theoretical background is discussed, with aspects of stochastic processes, queueing theory and game theory.

2.1. Perfect Repair

A repairable system is one that can be restored to operational state through any method other than the entire substitution. After a repair is executed the system may achieve one of the following possible states: 1) as good as new; 2). as bad as old; 3). better than old but worse than new; 4) better than new; and, 5). worse than old (Yanez; Joglar & Modarres, 2002). The first state will be focused as the proposed model supposes perfect repair.

When a system is subject to perfect or ideal repair, it is returned to ‘as good as new’ state, i.e. a failure occurs, and after the equipment is repaired it returns to operation with the same characteristics it had when it was new. The most common example of the perfect repair in practice is the total replacement of the failed item with a new identical one, (FINKELSTEIN, 2015). In order to model perfect repairs, the Renewal Process (RP) may be used.

RP is defined as a point process in which the different times to failure of a given system, X_i, are considered independently and identically distributed (iid) random variables. This is so due to the assumption that the system is restored to its original condition, as good as new, following an instantaneous repair action. As it represents a theoretical situation, this model has limited applications in the analysis of repairable systems, unless the system is composed only by replaceable components (YANEZ; JOGLAR; MODARRES, 2002).

2.2. Imaging Equipment

A significant growth has been observed in the medical imaging field over the last century. The use of imaging technology, previously used for other purposes, such as the defense sector, started to migrate to the health sector as computers with high processing power and lower costs were produced. This technology is used in a wide range of equipment, from the well-known xrays to virtual reality devices (BONZINO, 2000). This book will be the main reference in this section.

Some of these equipment may be relatively low-cost, such as an ultrasound, while others may easily cost millions of dollars, such as tomography equipment. However, even with the relatively fast development of the field, it still faces some threats, especially related to the mitigation of costs for the new technologies that are yet to be developed.

In section 4, failure data of an angiography will be used. It is a diagnostic equipment used to examine diseases of the circulatory system. In short, it works as follows: a radiopaque contrast is used to opacify the vessel of interest, then several radiographs of the contrast flowing through the vessel are obtained and are recorded using either film, in the older equipment, or video.

In Figure 2.2. a representation of an angiography is presented. Its basic components are the x-ray tube and generator, image intensifier, video camera, cine camera and digital image processor.

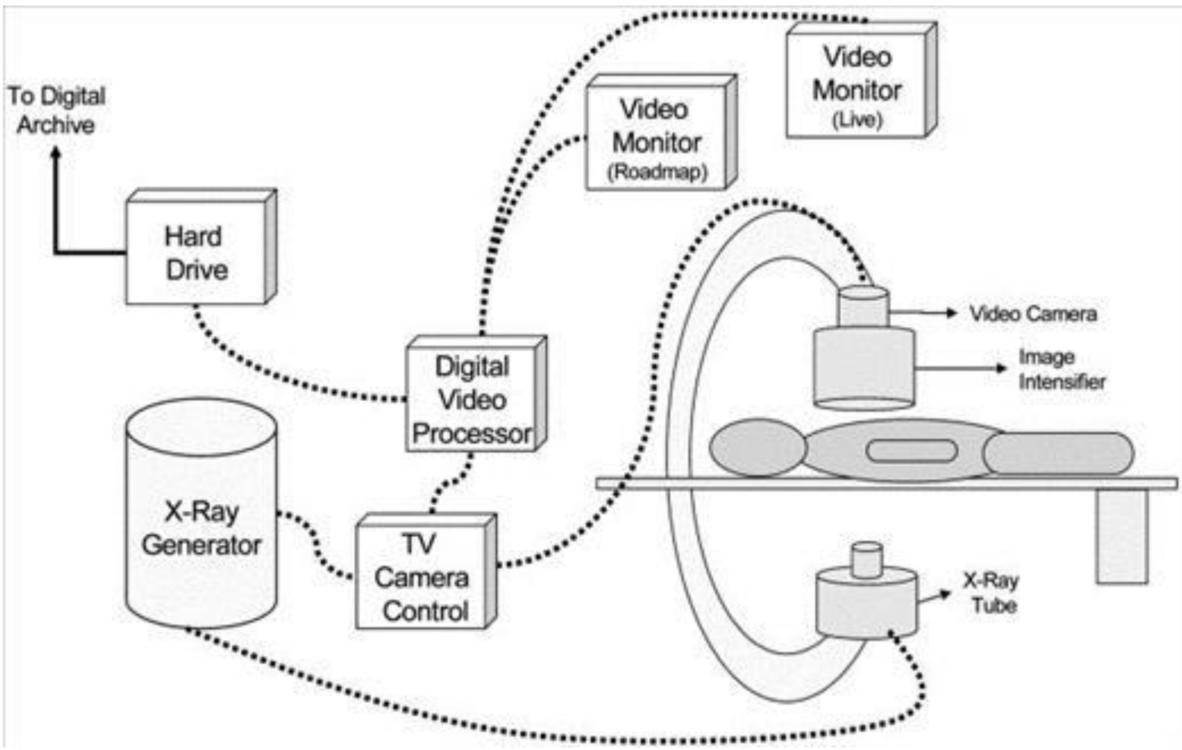


Figure 2.1. - Angiography representation
Source: (Bonzino, 2000, p.1128)

Such systems require a powerful and modern x-ray generation system to produce short and intense x-ray pulses required to form images of moving vessels. They use either a constant potential generator or a medium/high-frequency inverter, which is more common in recent versions of angiographic imaging systems. As the x-ray emissions cause high heat loads on the x-ray tube, this heat must be monitored using electronic heat computers and controlled using a cooling liquid, to avoid the damage of the x-ray tube.

Until the 1980's using film was the most common form of recording the images produced by the equipment, the so called cine angiography. However, nowadays, it has been replaced by digital angiography in most healthcare institutions. One application in which film is still in use is cardiac imaging. During cine angiography, x-ray pulses are synchronized with the cine camera shutter and the vertical retrace of the system video camera.

The image produced by the x-ray pulses goes then to the image intensifier, which produces a light image with enough brightness to allow the use of film and video cameras and an output image of convenient size to be stored for future use, allowing the image to be used in real-time image use as well as recording it. The image goes then from the image intensifier to the video camera through the optical distributor. Then it reaches the video system, which is integrated to a digital image processor and formed by many components, such as camera head, camera control unit (CCU), video monitors and video recording devices.

However, film angiography has been mostly replaced by digital angiography in almost all angiographic procedures, as digital image processing allows the manipulation of the image characteristics and provides immediate access to image data during each procedure. The digital image processor is a dedicated device to perform real-time video processing. The hardware is structured as a pipeline processor and algorithms using machine language are used to perform image subtraction, integration, spatial-filtration and temporal-filtration. After the image is processed it is ready to be stored in the image storage device, normally a regular hard drive (HD), in real-time.

The immediate access to video images is one of the most important advantages of digital angiography over film angiography, as the latter requires an extra procedure to develop the films. Another one is that it is possible to use specialized review software to better analyze the images processed. One hybrid form of angiography which is also employed, although in smaller scale, is the parallel cine, in which both digital and film images are recorded simultaneously.

2.3. Medical Technology Management

The management of medical technology is defined as a systematic process in which professionals with specific qualification, such as clinical engineers work together with hospital managers to form an interdisciplinary team that aims to ensure that the service will be provided according to the required quality standards established at the lowest possible cost, (DYRO, 2004). This interdisciplinary team is responsible for the processes of evaluation, acquisition, utilization and management of resources and for ensuring the quality of the services provided using medical equipment (BONZINO, 2000).

The medical equipment management team ensures that equipment used in the patients care are operational, safe, properly configured to meet the mission of the medical treatment facility and continue to function effectively in a good working condition. This is essential for providing good health services and saving resources. However, in addition to maintenance, medical equipment management involves other essential activities that vary according to the equipment's life-cycle (MUTIA; KIHIU & MARANGA, 2012).

Maintenance and repair activities are required to ensure that devices are kept working within the limits imposed by the test criteria and to return devices to operational state after a failure. Additionally, safety and performance testing is required in order to identify unsafe or incorrectly performing medical devices that could pose a risk to either patients or staff. Together, safety and performance testing and maintenance and repair form the basis for any medical device management program (DYRO, 2004).

In order to ensure that the medical equipment will be effectively managed it is important to consider the different requirements of the equipment along each stage of its life-cycle, (Oliveira, 2009) encompassing the tendering and commissioning of new equipment in conjunction with the end users, the training and education of all devices for staff on their operation and application, performing the necessary preventive maintenance, conducting repairs and finally device disposal, at which point the life cycle begins again, (MUTIA; KIHIU & MARANGA, 2012).

One question that pervades the maintenance activities/technical interventions is the decision of performing them in-house or outsourcing them. In order to make a correct decision it is important to consider which human resources and technological contribution will be necessary to perform the technical interventions. The relation between revenue and expense shall guide the decision. Hence, it is necessary to estimate the expected in-house maintenance costs and compare them to what is charged in market. Normally, it is economical to perform the maintenance of low-complexity equipment in-house and to outsource the maintenance of high-complexity equipment.

2.3.1. Maintenance Activities

Inside a hospital the maintenance activities extend through many different fields: buildings, elevators, generators, conditioning system and medical equipment. Medical maintenance encompasses areas such as electricity, mechanics, electronics, informatics, etc. In short, all those maintenance activities may be condensed in two groups: maintenance of medical equipment and maintenance of infrastructure.

An effective maintenance ensures that the equipment will be in good operational condition, while its downtime is minimized and its reliability is maximized (Antunes et al., 2002). The maintenance activities are divided into seven processes:

(1) Acquisition: Maintenance starts as soon as an equipment is acquired. All the maintenance costs should be considered negotiated by the acquisition process, such information may guide the choice for one manufacturer against the others.

(2) Equipment receipt: It is important to perform some verification activities such as analyze the physical state of the materials, perform technical and performance controls, ensure that the operation and maintenance manuals are delivered, etc.

(3) Technical installation: An equipment should be chosen considering the possible adaptation of the technical environment in which it will be inserted, considering the equipment requirements. During its installation it is important to ensure that the technical environment is suitable.

(4) Operators training: Manipulation errors cause a significant part of the equipment failures, increasing maintenance costs and downtime. The operators training should start during the installation process and extend through the whole useful life of the equipment.

(5) Preventive maintenance: All the preventive maintenance information should be given by the manufacturer, the set of activities to be performed, its periodicity, costs and procedures. It must also be negotiated during the acquisition process.

(6) Calibration: It is of difficult implementation due to the need of specific procedures, materials, measurement standards and knowledge. It is particularly difficult to keep

all the measurement instruments well calibrated in order to ensure that the adjustments are performed correctly.

(7) Corrective maintenance: It is the most costly maintenance and may interrupt treatments and diagnosis. An effective maintenance policy should be able to reduce the number of corrective interventions, minimizing the unscheduled interruption of the equipment operation.

2.3.2. Technical Interventions

One alternative classification to for the one presented in section 2.4.1 is presented by (BURMESTER; HERMINI & FERNANDES, 2013). In their view, the technical interventions are divided into six main activities:

(1) Technical inspections: Performed in order to verify the operational level at a given moment, trying to identify inefficiencies in the equipment. They should preferably be performed on-site in order to reduce the equipment downtime.

(2) Preventive maintenance: Interventions performed in order to avoid failures, improper operation or harm to the equipment by replacing wearing out components before a failure occurs.

(3) Predictive maintenance: Similar to preventive maintenance. It uses the measuring of a given parameter in order to determine whether a given component should be replaced or not.

(4) Corrective maintenance: Consists in returning the equipment to operational state after failure, according to the security and performance standards defined by the manufacturer and by the applicable legislation and regulation. According to (Burmester; Hermini & Fernandes, 2013), 75% of the critical breakdowns may be repaired by low complexity interventions, such as cables substitution. The medium complexity interventions require specialized tools and trained personnel. Sometimes it is necessary to have access to specific software or hardware keys in order to be able to execute a repair, however sometimes they are of exclusive use for the manufacturers, which restricts to them the ability of performing such repairs. Moreover, the high complexity of some equipment demands specific knowledge, technical skills and tools which are hardly ever available for maintenance workers which are not directly related to the manufacturer’s team. Those interventions which may only be performed by the OEM are the high complexity interventions.

(5) Performance regulation tests: Performed after an intervention to ensure that the equipment is working according to the pre-established standards, according to the specification provided by the manufacturer. Normally those tests are performed by an employee of the manufacturer and supervised by a qualified member of the healthcare institution.

(6) Calibration: Adjustments on the operational quality level to reach the specification provided by the manufacturer and by the applicable legislation and regulation.

2.3.3. Maintenance Quality Standards

Before maintenance and repair work is carried out, it is important to establish the regulatory framework under which such work is to be undertaken, to ensure compliance with local requirements. Requirements vary according to location, the type of equipment being managed, the nature of the health care organization’s operation, and possibly among different manufacturers, (DYRO, 2004).

In order to evaluate and attest the quality of the maintenance services in a healthcare institution, accreditation emerges as a manner to ensure that the service provided is in accordance with the quality standards required by a given medical equipment. In Brazil the hospital accreditation is provided by the National Accreditation Organization (ONA), that provides a quality certificate for the maintenance services. The organization is not responsible for inspecting such services, but to provide continued training for the companies that want to be accredited. Such adhesion comes along with the responsibility to ensure that the service provided is in accordance with all the standards defined by ONA, (OLIVEIRA, 2009).

2.4. Maintenance Services Outsourcing

Due to the increase in competition, companies started to notice that the maintenance function, neglected in the 1950’s, should receive more attention as the amount of resources invested in it was growing strongly and because ensuring the operation and the best regulation of their equipment was crucial to keep the quality standards of their products and processes (SLACK; CHAMBERS & JOHNSTON, 2009).

Over the years companies realized they could benefit from outsourcing certain services/processes which were not related to its main core. Outsourcing may be defined as the managed process of acquiring goods or services from an external agents under a contract rather than producing/providing them in-house (MURTHY & JACK, 2014). It would allow companies to achieve a higher level of production at lower costs, i.e., hiring an external agent to provide maintenance services would be less expensive than performing maintenance with an internal team and would also lead a higher and better production. However, to ensure that the service provided by the external agent reaches the minimal parameters expected by the contracting company, a legal agreement shall be celebrated by both parties. Moreover, this legal object would also be important to the hired company, as it would carefully describe its responsibilities, i.e., the hired company will only be responsible for failures that were not caused by misuse of equipment.

The legal agreement mentioned above may be an EW or an MSC. The former is celebrated between customer and manufacturer and the latter between customer and service agent. These concepts will be further discussed in Section 2.5.2.

2.4.1. Medical Equipment Maintenance Outsourcing

When the specific case of medical equipment maintenance is considered, the trend to outsource can still be observed. Medical devices maintenance increasingly demands larger sums from hospital budgets and is often outsourced (CRUZ; HAUGAN & RINCON, 2014). Moreover, according to (Dyro, 2004) a consolidated medical equipment management leads to considerable cost savings, especially in the case of technology-intensive equipment. In such cases, the annual expenditure with maintenance services may vary from of 5% to 12% of the equipment's purchase price.

It is important to observe that, in a hospital, not all maintenance services are normally outsourced. As pointed out by (Dyro, 2004), the in-house staff services 70%–80% of the equipment base, while the other 20%–30% are serviced by external agents, with their specialized expertise, tools and access to spare parts to service such equipment. However, these 20% represent up to 60% of the amount invested in hospital equipment (CALIL & TEIXEIRA, 1998). Considering the costs of maintenance outsourcing specifically, a substantial growth may be observed. In 1996, service contracts generated approximately US$ 10 billion in revenues; by 2015, the global medical device outsourcing market is projected to reach US$ 42.6 billion. Yet extreme inefficiencies are generated by the anti-competitive nature of this market, both in terms of cost and quality, suggesting that although the industry is growing, maintenance service quality is not keeping pace with rising costs and sales volumes (MIGUEL-CRUZ; RIOS-RINCÓN & HAUGAN, 2014)

Given that medical equipment may be serviced by in-house or external agents, it is relevant to analyze the reasons that lead to medical equipment maintenance outsourcing. The five most relevant reasons exposed by (Dyro, 2004) are: 1) Desire to reduce costs with employees who are directly on the hospital payroll; 2) Cost savings due to scale gains; 3) Fast access to resources that may not be easily available to the hospital (e.g. specialized staff and spare parts), especially in the case of the OEM providing maintenance services; 4) Short-term solution to a problematic in-house program, as it may be short-staffed and lack on expertise, thus hiring a well-trained external team may quickly solve this problem; 5) Reduced internal overhead related to invoice processing, i.e. much of the work of processing invoices individually may be aggregated to a single contract.

According to (Dyro, 2004), contract costs from the OEM have come down over the years and can range from 2% to 12% of the equipment acquisition cost, depending on equipment modality. Service staff provided are typically well trained, and they have access to all of the tools, software diagnostics, and spare parts designed by the OEM for supporting the equipment.

2.4.2. Base Warranties, Extended Warranties and Maintenance Service Contracts

It is important to make a differentiation about two warranty categories common in literature, BWs and the EWs. The former is coupled with the sale process, has its terms defined unilaterally by the manufacturer and its costs are already considered in the equipment's sale price; the latter, distinctly from the BW, may be acquired or not, has an additional cost related to its acquisition (the EW's price) and has its terms defined by manufacturer and customer through a negotiation process (MURTHY & JACK, 2014). This book will be the main reference in this subsection.

Besides the protection function inherent to the terms to which each warranty is committed, they also have two other functions: informational, related to the customer's view and is characterized by the fact of longer warranties indicating higher quality; and promotional, related to the manufacturer's view, who offers a longer warranty in order to differentiate his product and make it more attractive to potential customers.

A MSC is very similar to an EW and it is also a form of contractual relation between two companies in which one hires a given service and the other provides this service for a given time for a pre-established price. The basic difference between them is who provides the service, an external agent, in the case of an MSC and the manufacturer in the case of an EW.

2.5. Stochastic Processes

The theory of stochastic processes will be employed to model the failure-repair dynamics of the equipment. A stochastic process $\{N(t), t \epsilon T\}$ may be defined as a family of random variables such as each t contained in set T has an associated random variable $N(t)$ which represents the state of the process in t (normally interpreted as time) (ROSS, 1992). The set T is the parameter space, while the set of possible states assumed by $N(t)$ is the state space, denoted by E.

A given stochastic process $N(t)$ is a stationary process if for every n and for any set of time instants $\{t_i \epsilon T, i = 1,2, \dots, n\}$, the following statement is valid: $F_X(x_1, \dots, x_n; t_1, \dots, t_n) = F_X(x_1, \dots, x_n; t_1 + \tau, \dots, t_n + \tau)$, for any value of τ. Thus, the process $N(t)$ has the same distribution as the process $N(t + \tau)$ for any value of τ. A given stochastic process $N(t)$ is an independent process if $N(t_i)$ for $i = 1,2, \dots, n$ are independent random variables, so as $F_X(x_1, \dots, x_n; t_1, \dots, t_n) = \prod_{i=1}^{n} F_X(x_i; t_i)$.

A stochastic process $\{N(t), t \geq 0\}$ is of independent increments if for any n time instants $t_1 < t_2 < \dots < t_n$, $N(0), N(t_1) - N(0), N(t_2) - N(t_1), \dots, N(t_n) - N(t_{n-1})$ are independent random variables. If $\{N(t), t \geq 0\}$ has independent increments and $N(t) - N(s)$ follow the same distribution as $N(t + h) - N(s + h)$ for every $s, t, h \geq 0, s < t$, then the process is of independent and stationary increments.

2.5.1. Homogeneous Poisson Process

A stochastic process $\{N(t), t \geq 0\}$ is called a counting process when N(t) represents the total number of events which occur until time t, (ROSS, 1992). This process must satisfy the following properties:

1) $N(0) = 0$ and $N(t) \geq 0$;
2) $N(t)$ is an integer;
3) $N(s) \leq N(t)$ if $s < t$;
4) $N(t) - N(s)$ represents the number of events occurred in the interval (s, t).

If the events occurred in disjoint time intervals are independent, then the counting process $N(t)$ has independent increments. A counting process $N(t)$ has stationary increments if the number of events in the interval $(s + h, t + h)$ follows the same distribution of the number of events in (s, t), $\forall\ s < t$ and $h > 0$.

Among the most common used counting processes in literature are the Poisson Processes. According to (Ross, 1992), a Poisson Process follows three basic characteristics:

1) N(0) = 0;
2) {N(t), t ≥ 0} has independent increments;
3) The number of events in any interval t follows a Poisson distribution with mean λt, where λ is the arrival rate, i.e. the expected number of events within this interval is E[N(t)] = λt.

The Poisson Processes are divided into two main groups, the Homogeneous Poisson Process (HPP) and the Non-homogeneous Poisson Process (NHPP). The former is characterized by having stationary increments, i.e., it has a constant arrival rate (λ), the later, which is a generalized case of the HPP, is characterized by having an arrival rate that varies with time (λ(t)) (ROSS, 1992).

Using a more precise definition, a counting process $N(t)$ is an HPP with rate λ if:

1) $N(0) = 0$;
2) $N(t)$ has stationary and independent increments;
3) $\lim_{h \to 0} \frac{P[N(t+h) - N(t) = 1]}{h} = \lambda$;
4) $\lim_{h \to 0} \frac{P[N(t+h) - N(t) \geq 2]}{h} = 0$.

An HPP has $E[N(t)] = \lambda t$ and $Var[N(t)] = \lambda t$ thus the expected number of events in any unitary interval, for example $(1,2)$ is λ. Another important property is that the lengths of the time intervals $\{Z_n; n \geq 1\}$ of an HPP $N(t)$ are exponential random variables with rate λ independent from each other. Finally, the number of events that occur in a given interval of length t is a Poisson random variable with rate λt.

The HPP is equivalent to an RP, which may be defined as follows: suppose that a given experiment starts at $t = 0$, where t represents time, and that the i^{th} event occurs at time T_i, where T_i is a random variable, the values t_i assumed by the occurrences of T_i are the occurrence times. Let $Z_i = T_i - T_{i-1}$and $T_0 = 0$, thus Z_n represents the time between the $n-1$ previous events and the n^{th} event. If all random variables Z_n are independent and identically distributed, then $\{Z_n; n \geq 1\}$ is a renewal process.

2.6. Queueing Theory

Queues are common in production sites as well as in service providers, such as markets, banks or gas stations. Queueing Theory studies the relation between the demand on a given system and the delays faced by this systems' users, (ARENALES et al., 2007). Queues are generated when the demand is higher than the system's service capacity for a certain time, and Queueing Theory is used to optimize the costs to offer a certain service considering the delays faced by its users.

Queueing systems are usually composed by three basic elements: (1) Input source, i.e. population from which arrivals come (2) Queue, where customers wait before being served, and (3) Service mechanism, i.e. arrangement of servers. A basic queueing system is presented in Figure 2.2.

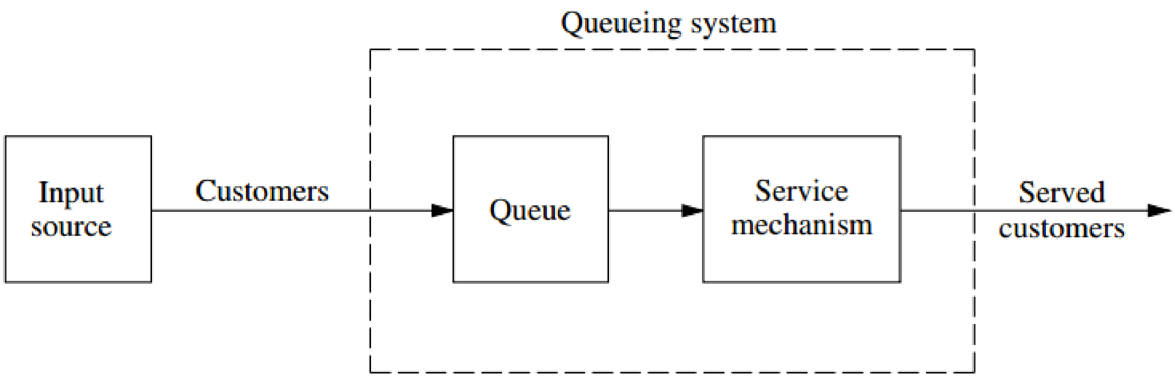


Figure 2.2 – Basic queueing process
Source: (Hillier & Lieberman, 2001, p.836).

In order to formally describe a queueing system it is necessary to provide information about: (1) The users' arrival process: described by the time interval between successive user arrivals. Normally it is supposed that no more than one user may arrive at a time and that the arrival process does not vary in time and with the number of users in the system. When considering priority queues, it is possible to split the users into different classes, inside which they are statistically identical; (2) The queue discipline: corresponds to the order in which users are selected to be serviced, some examples are first come, first served (FCFS), last come, first served (LCFS). When considering priority queues, it is possible to have cases with service interruption (preemption) and without service interruption (no preemption); and, (3) The service process: described by the service time per user. Normally it is supposed that no more than one user may be serviced at a time by a single server and that the service process does not vary in time and with the number of users in the system.

When studying a given queueing system, some performance measurements are commonly used in order to analyze its effectiveness (HILLIER & LIEBERMAN, 2001). Define: $t_a(i)$: arrival time of the i^{th} customer; $t_b(i)$: time i^{th} customer begins to be served; and $t_c(i)$: time i^{th} customer leaves the system. Now it is possible to define the following performance measurements:

$$X(i) = t_a(i) - t_a(i-1): i^{th} \text{ arrival interval between customers } i \text{ and } i-1; \quad (2.1)$$

$$S(i) = t_c(i) - t_b(i): \text{service time for user } i; \quad (2.2)$$

$$W_q(i) = t_b(i) - t_a(i): \text{waiting time in queue for user } i; \quad (2.3)$$

$$W(i) = W_q(i) + S(i) = t_c(i) - t_a(i): \text{total time in system for user } i \quad (2.4)$$

Usually it is supposed that $X(1), X(2), \dots, X(i)$ and $S(1), S(2), \dots, S(i)$ are independent and identically distributed continuous variables, thus the users' arrival rate to the system per time unit is:

$$\lambda = \frac{1}{E(X)} \quad (2.5)$$

and the service rate per time unit is:

$$\mu = \frac{1}{E(S)} \quad (2.6)$$

per server. If there are m parallel identical servers in a system, then the service rate per time unit is $m\mu$. The utilization factor ρ is the ratio between the users' arrival rate and the total service rate of the system:

$$\rho = \frac{\lambda}{m\mu} \quad (2.7)$$

The analysis of queueing systems through models often relies on the assumption that it will reach a steady-state after a considerably long operating time, i.e. such models are used to describe the long-term behavior of the queueing systems, considering that the arrival and service processes are stationary. In order to proceed with the analysis, define $L_s(t)$ as the number of users being serviced in time t; $L_q(t)$ as the number of users on queue in time t; and, $L(t) = L_s(t) + L_q(t)$ as the number of users in the system in time t.

For large values of t it may be assumed that $L(t), L_s(t)$ and $L_q(t)$ have independent and stationary probability density functions with means:

$$E(L) = \lim_{t\to\infty} E(L(t)),\ E(L_s) = \lim_{t\to\infty} E(L_s(t)) \text{ and } E(L_q) = \lim_{t\to\infty} E(L_q(t)) \quad (2.8)$$

Analogously, it may be assumed that for large values of i, $W(i)$ and $W_q(i)$ have independent probability density functions with means:

$$E(W) = \lim_{i\to\infty} E(W(i)) \text{ and } E(W_q) = \lim_{i\to\infty} E(W_q(i)) \quad (2.9)$$

An important relationship among $E(L)$, $E(L_s)$, $E(L_q)$, $E(W)$ and $E(W_q)$ was found by John Little in 1961 and became known as the Little's Formula:

$$E(L) = \bar{\lambda}E(W) \quad (2.10)$$

Where $\bar{\lambda}$ represents the average arrival rate over the long run. The other known relationships among the performance measurements are:

$$E(L_q) = \bar{\lambda}E(W_q),\ E(L) = \frac{\bar{\lambda}}{\mu} + E(L_q) \quad (2.11)$$

$$E(W) = \frac{1}{\mu} + E(W_q) \quad (2.12)$$

2.6.1. Birth and Death Process

Birth and death processes are queueing systems in which users arrive, are served and leave (HILLIER & LIEBERMAN, 2001). It is supposed that the time between consecutive arrivals and the service times are exponentially distributed and that the number of users that arrive in the system and are served are Poisson distributed. It is considered that users arrive with rate λ and are served with rate μ, the queue discipline is FCFS and $N(t)$ denotes the number of users in the queueing system on instant t.

Admitting that during a considerably short time interval $(t, t + \Delta t)$, no more than one arrival or one service conclusion may occur, the possible state transitions in the referred time interval are: (1) $P(N(t + \Delta t) = n + 1|N(t) = n) \approx \lambda_n \Delta t$: one user arrives; (2) $P(N(t + \Delta t) = n - 1|N(t) = n) \approx \mu_n \Delta t$: one user leaves; (3) $P(N(t + \Delta t) = n|N(t) = n) \approx 1 - \lambda_n \Delta t - \mu_n \Delta t$: there are no arrivals and no departures; and, (4) $P(N(t + \Delta t) = k|N(t) = n) \approx 0$, for $|k - n| > 1$: more than one user is served or leaves.

Now, defining $P_n = P(N(t) = n)$, and using the state transition probabilities showed above, it may be found that the probability that the system is in state n on instant $t + \Delta t$ is:

$$P_n(t + \Delta t) = P_{n+1}(t)\mu_{n+1}\Delta t + P_n(t)[1 - (\lambda_n + \mu_n)\Delta t] + P_{n-1}(t)\lambda_{n-1}\Delta t, \quad \text{for } n = 1,2,\ldots \quad (2.13)$$

$$P_0(t + \Delta t) = P_1(t)\mu_1\Delta t + P_0(t)(1 - \lambda_0\Delta t), \text{ for } n = 0 \quad (2.14)$$

(2.13) and (2.14) may also be rearranged to:

$$\frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} = -(\lambda_n + \mu_n)P_n(t) + \mu_{n+1}P_{n+1}(t) + \lambda_{n-1}P_{n-1}(t), \text{ for } n = 1,2,\ldots \quad (2.15)$$

$\frac{P_0(t+\Delta t)-P_0(t)}{\Delta \mathrm{t}} = -\lambda_0 P_0(t) + \mu_1 P_1(t)$, for $n = 0$ (2.16)

Taking the limit where $\Delta t \to 0$, it is found:

$\lim_{\Delta t \to 0} \frac{P_n(t+\Delta t)-P_n(t)}{\Delta \mathrm{t}} = \frac{dP_n(t)}{dt} = -(\lambda_n + \mu_n)P_n(t) + \mu_{n+1}P_{n+1}(t) + \lambda_{n-1}P_{n-1}(t)$, for $n =$ $1,2, \ldots;$ (2.17)

$\lim_{\Delta t \to 0} \frac{P_0(t+\Delta t)-P_0(t)}{\Delta \mathrm{t}} = -\lambda_0 P_0(t) + \mu_1 P_1(t)$, for $n = 0$ (2.18)

That system of equations describes the behavior of the queueing system along time, but as the analysis of the steady-state behavior is our objective, it is assumed that for large values of t, the probability $P_n(t)$ does not depend on time, thus:

$\lim_{t \to \infty} P_n(t) = P_n \Rightarrow \frac{dP_n(t)}{dt} = 0.$ (2.19)

Therefore, the infinite system of differential equations is reduced to a infinite system of linear equations on P_n:

$\lambda_0 P_0 = \mu_1 P_1$ for $n = 0$

$(\lambda_n + \mu_n)P_n = \lambda_{n-1}P_{n-1} + \mu_{n+1}P_{n+1}$ for $n = 1,2, \ldots$ (2.20)

That system of balance equations may be solved recursively in terms of P_0.

For $n = 0$: $\lambda_0 P_0 = \mu_1 P_1 \Rightarrow P_1 = \frac{\lambda_0}{\mu_1} P_0$.

For $n = 1$: $P_2 = \frac{\lambda_0 \lambda_1}{\mu_1 \mu_2} P_0$ (2.21)

Thus, by induction it is possible to find:

$P_n = \frac{\lambda_0 \lambda_1 \ldots \lambda_{n-2} \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_{n-1} \mu_n} P_0 = K_n P_0$, for $n = 1,2, \ldots$, where $K_n = \frac{\lambda_0 \lambda_1 \ldots \lambda_{n-2} \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_{n-1} \mu_n}$ (2.22)

As the sum of all probabilities $P_0 + P_1 + \cdots + P_n = 1$, it is possible to determine P_0 as follows:

$\sum_{n=0}^{\infty} P_n = P_0 + \sum_{n=1}^{\infty} K_n P_0 = 1 \Longrightarrow P_0 = \frac{1}{1+\sum_{n=1}^{\infty} K_n}$ (2.23)

Note that it is necessary that $\sum_{n=1}^{\infty} K_n$ converge to a finite value, so $P_0 > 0$, such convergence is a necessary condition for the system to reach a steady-state.

Figure 2.5. shows a state transition diagram in a birth and death process.

2.6.2. The M/M/1/GD/∞/∞ Model

The M/M/1/GD/∞/∞ model, (Hillier & Lieberman, 2001), admits that the interarrival intervals are exponentially distributed with mean

$E(X) = \frac{1}{\lambda}$ (2.24)

And that the service times are exponentially distributed with mean

$$E(S) = \frac{1}{\mu} \quad (2.25)$$

The systems' utilization factor is

$$\rho = \frac{\lambda}{m\mu} = \frac{\lambda}{\mu} \quad (2.26)$$

As there is only one server. As it is a system of unlimited capacity, every user that arrives may enter the system, so $\bar{\lambda} = \lambda$, moreover, the arrival rate and the service rate do not depend on the state of the system, thus $\lambda_n = \lambda$ and $\mu_n = \mu$ for every n.

Thus, the steady-state distribution for this particular birth and death process is given by:

$$P_0 = \frac{1}{1+\sum_{n=1}^{\infty} K_n} = \frac{1}{\sum_{n=0}^{\infty} \rho^n} \text{ for } n = 0;$$

$$P_n = K_n P_0 = \rho^n P_0 \text{ for } n = 1,2, \ldots. \quad (2.27)$$

Moreover, admitting that the system reaches a steady-state after a enough time, i.e. considering $\rho < 1$, it is possible to find:

$$P_0 = \frac{1}{\sum_{n=0}^{\infty} \rho^n} = 1 - \rho$$

$$P_n = \rho^n P_0 = \rho^n (1 - \rho) \text{ for } n = 0,1, \ldots \quad (2.28)$$

Note that, if $\lambda > \mu$, the system will never reach a steady-state and the queue will tend to grow indefinitely with time.

After some manipulation it is possible to show that the performance measurements for this specific birth and death process are:

$$E(L) = \frac{\rho}{1-\rho} \quad (2.29)$$

$$E(L_s) = \rho \quad (2.30)$$

$$E(L_q) = \frac{\rho^2}{1-\rho} \quad (2.31)$$

$$E(W) = \frac{\rho}{\lambda(1-\rho)} \quad (2.32)$$

$$E(W_q) = \frac{\rho}{\mu(1-\rho)} \quad (2.33)$$

All those results are independent of the queue discipline and also apply for a FCFS queue discipline. The M/M/1/GD/∞/∞ is the basis to understand the model M/M/1/GD/K/∞ presented in section 2.7.3.

2.6.3. The M/M/1/GD/K/∞ Model

The difference between the M/M/1/GD/K/∞ system from the M/M/1/GD/∞/∞ system is that the maximum number of users in the system is limited to K, (HILLIER & LIEBERMAN, 2001). Such system may be modeled according to a birth and death process as follows:

$$\lambda_n = \begin{cases} \lambda & n = 0,1,2 \ldots, K-1 \\ 0 & n = K, K+1, \ldots \end{cases} \tag{2.34}$$

$$\mu_n = \begin{cases} \mu & n = 1,2,3, \ldots K \\ 0 & n = K+1, K+2 \end{cases} \tag{2.35}$$

The steady-state distributions is given by:

$$P_0 = \begin{cases} \frac{1-\rho}{1-\rho^{K+1}} & \rho \neq 1 \\ \frac{1}{K+1} & \rho = 1 \end{cases} \tag{2.36}$$

$$P_n = \begin{cases} \frac{\rho^n(1-\rho)}{1-\rho^{K+1}} & \rho \neq 1 \\ \frac{1}{K+1} & \rho = 1 \end{cases} \quad n = 0,1,2, \ldots, K \tag{2.37}$$

Note that, in this model, the condition of $\rho < 1$ is not necessary for convergence, however, if $\rho \gg 1$, the number of users in the system tends to be K during most of the time, what leads to a considerable amount of users not being allowed to enter the system. In order to evaluate the effectiveness of the system, the performance measures shall be once again used. The mean number of users in the system is given by:

$$E(L) = \begin{cases} \frac{\rho[1-(K+1)\rho^K + K\rho^{K+1}]}{(1-\rho)(1-\rho^{K+1})}, & \rho \neq 1 \\ \frac{K}{2}, & \rho = 1 \end{cases} \tag{2.38}$$

All the other performance measures may be calculated using Little's Formula, to do so, use $\bar{\lambda}$, that may be found as follows:

$$\bar{\lambda} = \lambda(1 - P_K) \tag{2.39}$$

This specific birth and death process provides the framework which is considered in the model developed in Chapter 5.

2.6.4. The Priority Queues Model

Suppose that there are 2 classes of users, class 1 and class 2, each one with a given arrival rate, λ_1 and λ_2 respectively. The users of class 1 have priority over the users of class 2, thus, after a server finishes a service, the next user to be served belongs to the higher priority class.

If there are more than one user of the same priority class, they will be served according to the FCFS, LCFS or SIRO disciplines, for example (GROSS, et. al., 2008).

According to what has been defined previously, the aggregate arrival rate is:

$$\lambda = \lambda_1 + \lambda_2 \quad (2.40)$$

Define $P_{mnr} = P(m\ users\ of\ class\ 1\ are\ in\ the\ system,$

$n\ users\ of\ class\ 2\ are\ in\ the\ system, and$

$the\ user\ being\ served\ belongs\ to\ class\ r = 1\ or\ 2).$

It is assumed that the service rate for both classes is μ. So, the utilization factor for class 1 and for class 2 and the aggregate utilization factor is, respectively:

$$\rho_1 = \frac{\lambda_1}{\mu} \quad (2.41)$$

$$\rho_2 = \frac{\lambda_2}{\mu} \quad (2.42)$$

$$\rho = \rho_1 + \rho_2 = \frac{\lambda}{\mu} \quad (2.43)$$

The convergence condition of $\rho < 1$ needs to be valid in order to allow the steady-state analysis of the system, so, assuming it, it is possible to find the following rate-balance equations:

$$(\mu + \lambda)P_{mn2} = \lambda_1 P_{m-1,n,2} + \lambda_2 P_{m,n-1,2}\ (m \geq 1, n \geq 2)$$
$$(\mu + \lambda)P_{mn1} = \lambda_1 P_{m-1,n,1} + \lambda_2 P_{m,n-1,1} + \mu(P_{m+1,n,1} P_{m,n+1,2})\ (m \geq 2, n \geq 1)$$
$$(\mu + \lambda)P_{m12} = \lambda_1 P_{m-1,1,2}\ (m \geq 1)$$
$$(\mu + \lambda)P_{1n1} = \lambda_2 P_{1,n-1,1} + \mu\left(P_{2,n,1} + P_{1,n+1,2}\right)\ (n \geq 1)$$
$$(\mu + \lambda)P_{0n2} = \lambda_2 P_{0,n-1,2} + \mu\left(P_{1,n,1} + P_{0,n+1,2}\right)\ (n \geq 2)$$
$$(\mu + \lambda)P_{m01} = \lambda_1 P_{m-1,0,1} + \mu\left(P_{m+1,0,1} + P_{m,1,2}\right)\ (m \geq 2)$$
$$(\mu + \lambda)P_{012} = \lambda_2 P_0 + \mu(P_{1,1,1} + P_{0,2,2})$$
$$(\mu + \lambda)P_{101} = \lambda_1 P_0 + \mu(P_{2,0,1} + P_{1,2,2})$$
$$\lambda P_0 = \mu(P_{1,0,1} + P_{1,0,2}) \quad (2.44)$$

The utilization factor ρ represents the fraction of time the server is busy, or equivalently $P_0 = 1 - P$, thus, the fraction of time the server is serving a user of class r is P_r. Thus:

$$\sum_{m=1}^{\infty} \sum_{n=0}^{\infty} P_{mn1} = \rho_1 \quad (2.45)$$

$$\sum_{m=0}^{\infty} \sum_{n=1}^{\infty} P_{mn2} = \rho_2 \quad (2.46)$$

As the service rate for both classes is μ, the total number of users being served follows the same steady-state distribution as in the M/M/1 model. Thus:

$$P_n = \sum_{m=0}^{n-1}\left(P_{n-m,m,1} + P_{m,n-m,2}\right) = (1-\rho)\rho^n \ (n > 0) \tag{2.47}$$

However, it is difficult to find a solution for that system of steady-state equations due to the triple subscripts. In order to avoid that problem, the expected value using two-dimensional generating functions. Define:

$$P_{m1}(z) = \sum_{n=0}^{\infty} z^n \, P_{mn1} \ \ (m \geq 1) \tag{2.48}$$

$$P_{m2}(z) = \sum_{m=1}^{\infty} z^n P_{mn2} \ \ (m \geq 0) \tag{2.49}$$

$$H_1(y,z) = \sum_{m=1}^{\infty} y^m P_{m1}(z) \ (where\ H_1(1{,}1) = \rho_1) \tag{2.50}$$

$$H_2(y,z) = \sum_{m=1}^{\infty} y^m P_{m2}(z) \ (where\ H_2(1{,}1) = \rho_2) \tag{2.51}$$

$$H(y,z) = H_1(y,z) + \ H_2(y,z) + P_0$$

$$= \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} y^m z^n P_{mn1} + \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} y^m z^n P_{mn2} + P_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} y^m z^n (P_{mn1} + P_{mn2}) + \sum_{m=1}^{\infty} y^m P_{m01} + \sum_{n=1}^{\infty} z^n P_{0n2} + P_0 \tag{2.52}$$

Where $H(y,z)$ is the joint generating function of both classes, independent of which is being served. Note that $H(y,z) = P_0(1-P_y)$ [with $H(1{,}1) = 1$], since $H(y,z)$ turns into the generating function of a M/M/1 queue when $z = y$ and thus there is no priority distinction.

We also have that:

$$\left.\frac{\partial H(y,z)}{\partial y}\right|_{y=z=1} = L^{(1)} = L_q^{(1)} + \rho_1 = \lambda_1 W^{(1)} \tag{2.53}$$

$$\left.\frac{\partial H(y,z)}{\partial z}\right|_{y=z=1} = L^{(2)} = L_q^{(2)} + \rho_2 = \lambda_2 W^{(2)} \tag{2.54}$$

If we multiply the Equations in (2.44) by the appropriate powers of y and z and sum appropriately, it will be found that:

$$\left(1 + \rho - \rho_1 y - \rho_2 z - \frac{1}{y}\right) H_1(y,z) = \frac{H_2(y,z)}{z} + \rho_1 y P_0 - P_{11}(z) - \frac{P_{02}(z)}{z} \tag{2.55}$$

$$(1 + \rho - \rho_1 y - \rho_2 z) H_2(y,z) = P_{11}(z) + \frac{P_{02}(z)}{z} - (\rho - \rho_2 z) P_0 \tag{2.56}$$

In order to fully determine the generating functions H_1 and H_2, it is necessary to determine $P_{11}(z)$, $P_{02}(z)$, and P_0. To accomplish that, it is necessary to sum the equation in (2.44) that involves $P_{0n2}\ Z^n\ (n = 2, 3 \ldots)$ times in order to find an equation that relates $P_{11}(z)$, $P_{02}(z)$ and P_0, and then, using the last 3 equations in (2.44), obtain:

$$P_{11}(z) = \left(1 + \rho + \rho_2 z - \frac{1}{z}\right) P_{02}(z) + (\rho - \rho_2 z) P_0 \tag{2.57}$$

Using this equation in (2.55) and (2.56), it is possible to determine H_1 and H_2 as functions of P_0 and $P_{02}(z)$ and, consequently:

$$H(y,z) = H_1(y,z) + \ H_2(y,z) + P_0$$

$$= \frac{(1-y)P_0}{1-y-\rho y+\rho_1 y^2+\rho_2 yz} + \frac{(1+\rho-\rho z+\rho_1 z)(z-y)P_{02}(z)}{z(1+\rho-\rho_1 y-\rho_2 z)(1-y-\rho y+\rho_1 y^2+\rho_2 yz)} \quad (2.58)$$

Using the condition that $H(1,1) = 1$, it is possible to determine:

$$P_{02}(1) = \frac{P_2}{(1+P_1)} \quad (2.59)$$

Now, in order to determine $L^{(1)}$, the partial derivative $\frac{\partial H(y,z)}{\partial y}$ must be taken and evaluated at the point (1,1), however, as it may not be directly evaluated at that point, a limit should be taken. In order to make such analysis, the exact expression of $P_{02}(z)$ is not necessary, only the expression of $P_{02}(1)$ is needed. In order to determine $L^{(2)}$, the relation $L^{(1)} + L^{(2)} = p/(1-p)$ should be used, it shows that the total number of users in the system is the same as in the M/M/1 system, as the service rate is the same for both classes of users.

The other performance measures are:

$$L_q^{(1)} = \frac{\lambda_1 \rho}{\mu-\lambda_1} \quad (2.60)$$

$$L_q^{(2)} = \frac{\lambda_2 \rho}{(\mu-\lambda_1)(1-\rho)} \quad (2.61)$$

$$L_q = \frac{\rho^2}{1-\rho} \quad (2.62)$$

Where,

$$\rho = \frac{\lambda_1+\lambda_2}{\mu} \quad (2.63)$$

According to (Gross et al., 2008) some important notes about the mean value results are:

(1) Users of priority class 2 wait for a longer time in queue than users of priority class 1, what is showed by the expression below:

$$W_q^{(2)} = \frac{\rho}{(\mu-\lambda_1)(1-\rho)} = \frac{P/(\mu-\lambda_1)}{1-\rho} = \frac{W_q^{(1)}}{1-\rho} > W_q^{(1)}, \text{ for } (\rho < 1) \quad (2.64)$$

(2) When $\rho \to 1$, $L_q^{(2)}, W_q^{(2)}, W^{(2)}$ *and* $L^{(2)} \to \infty$. However, the equivalent measures for priority class 1 approach finite values, assuming that λ_1/μ is kept constant. Hence, it is possible that priority 1 users do not accumulate, even when a general steady-state is not reached.

(3) Although users of class 1 have priority, the presence of class 2 users causes delays for them. It is so due to the non-preemptive characteristics of the model, i.e. when a class 1 customer arrives and a class 2 user is being served, there is no service interruption and the class 1 user must wait until the class 2 user leaves.

(4) The mean number of users in queue is the same as in a M/M/1 system.

2.7. Discrete-Event Simulation

Discrete-event simulation is a set of techniques that generates sample paths (sequences) that describe the behavior of a given dynamical system under investigation (FISHMAN, 2001). Such method is often employed when the size and complexity of a system turns it difficult to obtain analytical solutions.

In discrete-events system, time is considered a discrete rather than continuous variable, thus, in such systems, one or more phenomena of interest change state at discrete points in time. Some examples are, manufacturing plants, transportation networks and delivery systems. In general, every discrete-event system encompasses the following concepts: 1) Work: items/customers that enter the system looking for service; 2) Resources: equipment/manpower that can provide service; 3) Routing: order in which services are to be provided; 4) Buffers: place where items/customers wait to be serviced, they may have finite or infinite capacity; 5) Scheduling: pattern of availability of resources; 6) Sequencing: order in which awaiting work is serviced, also called queueing discipline; and, 7) Performance: measured through time delay, number of waiting customers, resource utilization, etc.

According to (Ross, 2006), simulating a probabilistic model involves generating stochastic mechanisms to describe such model and observing its flow along time. As outputs the model will provide certain quantities of interest that one would be willing to determine. The key elements in discrete event simulation are variables and events, thus, three kinds are normally used: 1) Time variable t: amount of simulated time passed ; 2) Counter variables: counts the number of events of interest occurred; and, 3) System state variable: shows the state of the system in a given time t.

By using such variables, it is possible to model and follow a given system of interest. Their values are updated every time an event occurs and after that any data of interest may be collected or saved. The strategy of using discrete-event simulation is widely used to model the most diverse queueing systems. In this masters thesis, it will be employed to simulate a non-peemptive queueing system with two classes of customers in which the class one users have priority over the class 2 customers.

2.8. Game Theory

Game Theory is employed to model the strategic interactions and optimization process, it may be defined as a set of analytical tools developed to assist the understanding of observable phenomena while decision makers (players) interact. It presupposes some important assumptions: players are considered rational, i.e. they act in order to maximize their profits, and take into account the other players' decisions while making their own decision (OSBORNE; RUBINSTEIN, 1994).

2.8.1. Important Definitions

It is important to be familiar to some terms which are commonly used in Game Theory. A concise definition of each one of them is provided in Table 2.1:

Table 2.1 – Important definitions in Game Theory

Term	Definition
Game	A situation that involves interactions between rational agents who act strategically, i.e. a formal representation which allows the analysis of interest conflicts.
Players	Any individual, or group of individuals with autonomy to make decisions.
Action	A choice a player can make at a given time in a game.
Interactions	The actions of each player, taken individually, affect the other players involved in the game.
Rational agents	Make conclusions based on logic, consider premises based on rational arguments and use empirical evidences impartially while judging facts.
Strategic behavior	Each player, while making his decision, takes into account the interactions among all players and is aware that what he decides causes consequences to the other players and their decisions affect himself, i.e. while making a decision each player considers what he supposes the others will decide.
Strategy	Action plan which specifies, for a given player, what decision make in each of his decision points, i.e. his set of actions.
Pay-off	The reward each player obtains after a game is ended, according to their own decisions and the decision of others.
Game-tree	Representation of sequential games composed by nodes and lines.

Nodes	Represent players' decision moments.
Lines	Represent a player possible decisions at a given node.
Information set	Set composed by the nodes that a player judges may be reached when he needs to make a decision.

Source: (Fiani, 2009).

2.8.2. Games Classification

In order to model a conflict of interests many different games may be used, one that fits well the situation in which the negotiation is inserted shall be selected. Games may differ in many ways: whether players cooperate or not, whether there is information asymmetry or not, whether decisions are taken simultaneously or not, whether there is perfect information or not. Some of the possible classification categories are presented in Table 2.2:

Table 2.2 – Games Classification

Classification	Definition
Cooperative	When players are allowed to close formal agreements, with effective guarantees that they will act according to what has been previously established a game is said cooperative. In this kind of game the players act jointly in order to reach higher pay-offs.
Non-cooperative	When formal agreements may not be established, a game is said non-cooperative, in this kind of game the players intend to maximize their pay-offs individually.
Simultaneous	Games in which each player ignores the other players' decision while making his actions and does not take into account possible consequences of his decision in the future are classified as simultaneous games, i.e., it may be interpreted as a situation in which all players decide simultaneously. One limitation of this games is that they do not fit situations in which decisions are made following successive stages.

Sequential	Games in which players make their decisions in a predetermined order. Each player makes his decision in a given stage of the game development
Complete information	A game is said of complete information when the pay-offs of all players are common knowledge, e.g. in a two-player-game, player A is aware that player B knows his possible pay-offs and player B is aware that player A knows his possible pay-offs.
Incomplete information	A game is said of incomplete information when the pay-offs of all players are not common knowledge, e.g. in a two-player-game, player A is aware that player B knows his possible pay-offs, but player B is not aware that player A knows his possible pay-offs and vice-versa.
Perfect information	A game is said of perfect information when all players are aware of the complete previous history of the game at the moment they make a decision.
Imperfect information	A game is said of imperfect information if any player, in any moment of the game, needs to make a decision being unware of the previous history of the game until that point.

Source: (Fiani, 2009).

2.8.3. Stackelberg Game

The Stackelberg Game was proposed by Heinrich von Stackelberg in 1934 to model a duopoly, in which one firm (dominant or leader) moves first and the other firm (subordinate or follower) moves second (GIBBONS, 1992). This game was originally developed for situations in which both firms need to decide how much to produce in order to maximize their profits, considering that the leader firm decides first how much to produce and then the follower firm, observing that first move, decides how much to produce in order to maximize their profits.

The Stackelberg Game develops as follows: (1) the leader firm chooses a quantity $q_1 \geq 0$ to produce; (2) the follower firm observes it and decides to produce $q_2 \geq 0$; (3) the pay-offs of the leader and follower firms are given by:

$$\pi_1(q_1, q_2) = q_1(P(q_1 + q_2) - c) \text{ and} \tag{2.65}$$

$$\pi_2(q_1, q_2) = q_2(P(q_1 + q_2) - c), \tag{2.66}$$

respectively. Where $P(q_1 + q_2) = a - (q_1 + q_2)$ represents the market clearing price of the product when the aggregate production is $(q_1 + q_2)$ and c represents the marginal production costs (when the fixed costs are zero).

To solve sequential games, the technique of backward-induction is normally used. It is applied as follows. Suppose a game with 2 players, player 1 and player 2. Given that both players are rational, player 2 will react to his decision problem with the best response possible given the decision previously taken by player 1, a unique solution denoted by R_2. As it is a game of perfect and complete information, player 1 can anticipate this future decision R_2 and take it into account while making his own decision in stage 1, which will also be the best and unique solution for his optimization problem. Following these steps the optimal decision set may be determined by backward-induction, and it will be a unique solution for the game.

Thus, by backward-induction, first the problem of the follower is solved, by computing his reaction function for a given quantity q_1 produced by the leader

$$R_2(q_1) = \max_{q_2 \geq 0} \pi_2(q_1, q_2) = \max_{q_2 \geq 0} q_2[a - q_1 - q_2 - c] \quad (2.67)$$

Moreover, as it is a game of perfect and complete information, the leader is able to predict the output q_2 of the function $R_2(q_1)$, so the leader's problem is:

$$\max_{q_1 \geq 0} \pi_1(q_1, R_2(q_1)) = \max_{q_1 \geq 0} q_1(a - q_1 - R_2(q_1) - c) \quad (2.68)$$

What means that he decides how much to produce taking into account how much the follower will decide to produce next.

This solution framework was then adapted to the context of contracts analysis and was employed in (ASHGARIZADEH & MURTHY, 2000; MURTHY & ASGHARIZADEH, 1999; MURTHY & YEUNG, 1995; SHAMSI GAMCHI; ESMAEILI & SANIEE MONFARED, 2013). In that context, the leader is able to predict the follower's maximum willingness to pay for the MSC and for maintenance interventions, and thus, he is also able to force the follower to choose the option which maximizes his profit. It is a valid assumption as this game is used to model a monopolistic market, in which the leader has more bargain/negotiation power than the follower.

Considering the specific case of technology-intensive medical devices maintenance, it is a common practice that the OEM adopts some protectionist measures in order to guard the market for himself, such as, not providing training to possible external maintenance service providers and not providing sufficient spare parts to the market, which makes it difficult to external maintenance providers to repair such failed units, as mentioned by (DE VIVO et al., 2004). Therefore, OEM becomes the only feasible agent to execute maintenance services to the equipment manufactured by him, granting him monopoly power in the maintenance services market and a higher bargain power than the customer. This provides enough evidence for the use of such game to model the interaction process among the OEM and his customers.

The present chapter intended to provide the necessary theoretical background necessary to understanding the proposed model. Therefore, concepts about perfect repairs, medical imaging equipment and technology management, maintenance outsourcing, stochastic processes, specifically HPP, used to model the equipment failures and repairs, Queueing Theory, especially the Priority Queues Model, Discrete Event Simulation, used to simulate the model and Game Theory, used to model the interaction OEM-customer were presented.

3. PROPOSED MODEL

This chapter presents and details the proposed model. Besides the considerations regarding the game which rules the interaction between the OEM and the hospital, other model's assumptions related to the medical equipment and the players' decision problems are pointed out. The players' optimal strategies are also determined by means of a discrete-event simulation algorithm.

The proposed model deals specifically with the problem of analyzing EWs for medical equipment, considering two classes of customers, where priority customers, normally big hospitals, belong to class 1, while non-priority customers, normally smaller hospitals belong to class 2. We consider class 1 customers have priority over class 2 customers. The interaction among manufacturer and each customer will be modeled according to a Stackelberg Game formulation, (Osborne & Rubinstein, 1994), with perfect and complete information, in which the OEM acts as a leader and the customer as follower. Game Theory is commonly employed to model conflicts and specifically the Stackelberg Game is used to model situations in which one of the players is a monopolist, i.e., has more bargain power than the other does, which fits well the context described of medical equipment maintenance.

The OEM will be considered risk neutral and will pursue profit maximization through the sale of the medical equipment, selling of EWs and the performance of maintenance interventions on demand. The customers will be considered risk averse and may belong to one of the two different classes above mentioned. As in (Boom, 1998), a monopolist supplies risk-averse customers with products. To model risk aversion, it is used a utility function, which is a function of the profit obtained by the use of the medical equipment and a risk aversion parameter.

The game develops in three stages: first, the OEM observes the class of the customer (s)he is dealing with, secondly the leader (OEM) defines the EW's and maintenance intervention prices, and thirdly the follower (customer) decides whether or not to buy the equipment and whether or not to hire the EW. The EW declares that every failure that occurs during its length should be repaired at no additional cost for the customer. If no EW is hired, the customer will pay a fixed price for each intervention when necessary. As stated by (Lutz & Padmanabhan, 1998), the customer makes the decision that maximizes his expected utility. Furthermore, as we consider multiple customers, and consequently multiple equipment to be repaired, we will use

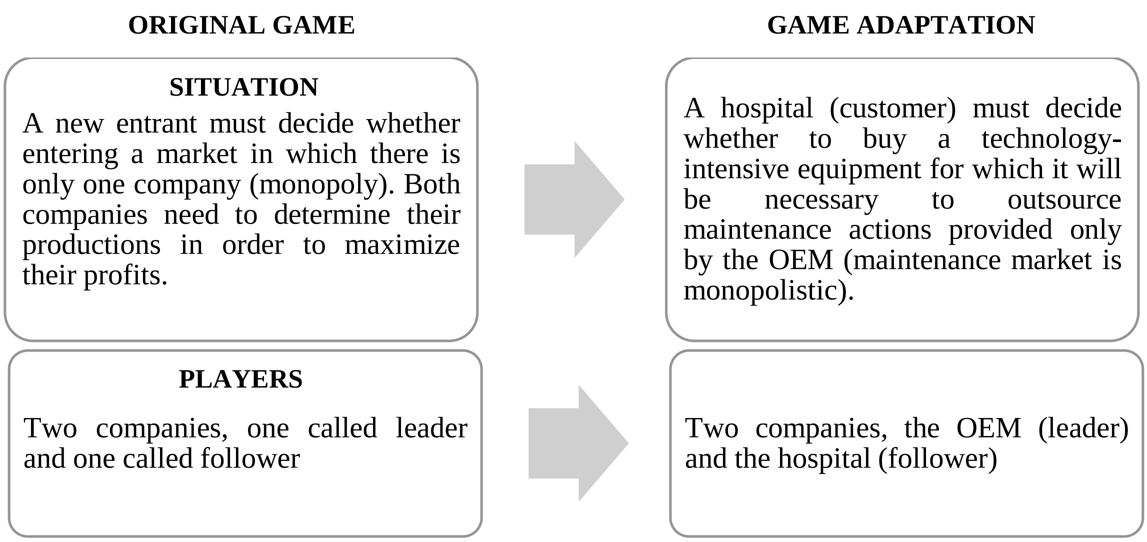
Queueing Theory (Gross et al., 2008) to model the dynamic of equipment failing, waiting for repair and being returned to operational state.

3.1. Game Characteristics

In the proposed approach, the players are considered rational, thus they intend to maximize their objective functions, i.e. expected profit, for the OEM, and expected utility, for the customer. Given that, a priority queues formulation will be used as described in Section 2.2, and the customers may belong to two different priority classes: class 1 (higher priority) and class 2 (lower priority).

The manufacturer, which decides first, must determine the optimal EW price for class i (P_{wi}), the optimal maintenance service price (C_{si}) and the optimal number of customers of each class to service (M_1 and M_2). Then, the customer must decide among options: A_1 - hiring the priority EW; A_2 - hiring the standard EW; A_3 - not hiring an EW, and thus paying for maintenance interventions on demand; and A_0: not buying the equipment. Generally, the priority EWs are offered to great size hospitals, while the standard EWs are offered to smaller ones, as it has been observed as a common practice in the medical equipment maintenance market.

The players interact in accordance with a leader-follower game with perfect and complete information, so it will be a static, non-cooperative, sequential game in which the OEM will act as a leader, as he is a monopolist in the maintenance service market and has more bargain power than the healthcare institutions, which will act as a follower.



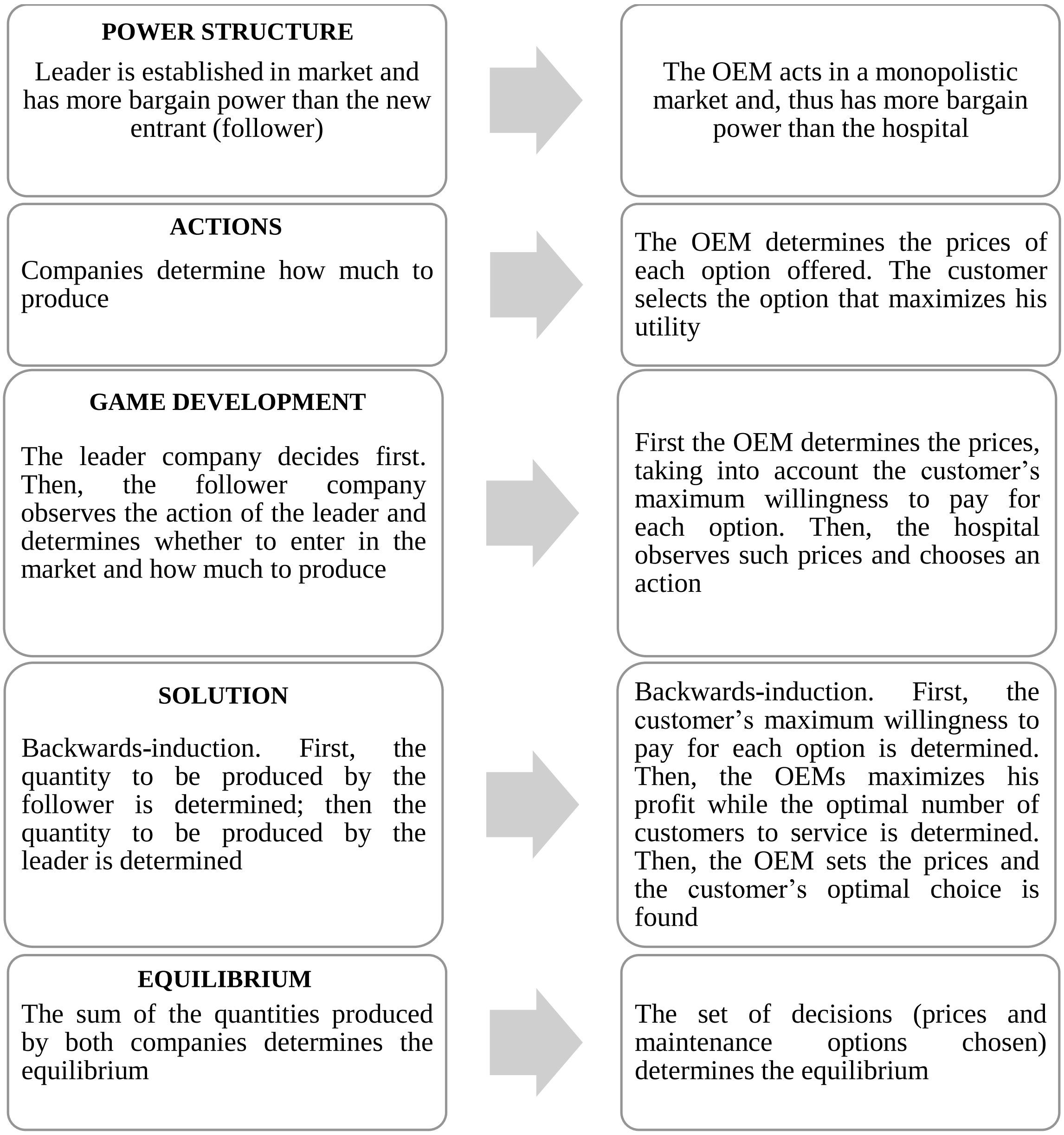


Figure 3.1 – Stackelberg Game adaptation
Source: this masters thesis (2015)

3.2. Equipment Failures and Repairs

By using the referred equipment, the hospital generates revenue of R per time unit while it is in operational state. It is considered equipment has a useful life L and its purchase price is C_b. The time to first failure follows an Exponential distribution with mean $1/\lambda$. We assume that all equipment are identical regarding their reliability and that M represents the total number of customers of both classes serviced by the OEM. All customers, in a given class, are considered homogeneous regarding their risk aversion, although there are two possible profiles.

The OEM offers three repair options to the customer. A_1) EW with priority: for a fixed price P_{w1}, all failures occurred over [0,L) are repaired at no additional cost to the customer. If the repair takes more than a time τ_1 (after failure) to be executed, the manufacturer is charged a penalty of $\alpha_1(y - \tau_1)$ if $y > \tau_1$ and zero otherwise, where y is the time to repair. A_2) EW with no priority: for a fixed price P_{w2}, all failures occurred over [0,L) are repaired at no additional cost to the customer. If the repair takes more than a time τ_2 (after failure) to be executed, the manufacturer is charged a penalty of $\alpha_2(y - \tau_2)$ if $y > \tau_2$ and zero otherwise. Note that, as class 1 customers have priority over class 2 customers, $\tau_1 < \tau_2$ and $\alpha_1 > \alpha_2$; A_3) No EW: each failure is repaired at a cost C_{si}. As there is no legal agreement between the two parties in this option, there is no penalty regarding the time to put the equipment back into operation. The total repair cost under this option is a random variable. The option of not buying the equipment will be referred as A_0.

3.3. Customer's Decision Problem

Utility functions are widely used to model risk aversion. We consider here the customer is risk averse and intends to maximize his utility given by (3.1):

$$U(w) = \frac{1-e^{-\beta_i w}}{\beta_i}, \tag{3.1}$$

where w is the wealth, β_i is the risk aversion parameter, and β_i = 0 represents risk neutrality, and the agent becomes more averse to risk as β_i grows. A similar utility function has been used by (ASHGARIZADEH & MURTHY, 2000). Figure 2 shows the behavior of the utility function (1) for different values of β_i. It may be observed that a higher risk aversion implies in a higher utility for the same wealth w. Other examples of utility functions, such as the Cobb-Douglas Utility Function, used by (Glickman & Berger, 1976) in the warranty context, or the CES Utility Function may be found in (VARIAN, 1992).

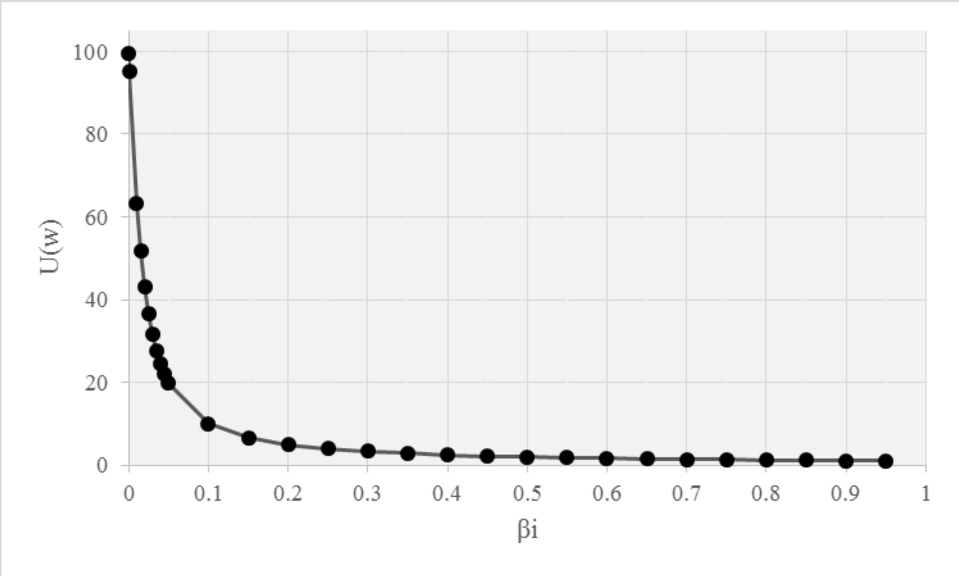


Figure 3.2 – Utility function behavior for different values of the risk parameter
Source: This masters thesis (2016)

The expected return $w(A_k)$ for $k = 0,1,2,3$ and jth $(1 \le j \le M)$ customer is given as:

$$w(A_0) = 0 \tag{3.2}$$

$$w(A_1) = R\left(\sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) + \alpha_1\left(\sum_{i=0}^{N_j} \max\{0, (y_{ji} - \tau_1)\}\right) - C_b - P_{w1} \tag{3.3}$$

$$w(A_2) = R\left(\sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) + \alpha_2\left(\sum_{i=0}^{N_j} \max\{0, (y_{ji} - \tau_2)\}\right) - C_b - P_{w2} \tag{3.4}$$

$$w(A_3) = R\left(\sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) - C_b - C_s N_j \tag{3.5}$$

where N_j is the number of failures occurred over $[0, L)$; X_{ji} is the time to the i^{th} $(0 \le i \le N_j)$ failure after the $(i - 1)^{th}$ repair; $\tilde{X}_j$ is the time for which the equipment was in operational state at the end of its useful life subsequent to being returned to operational state after the last repair. Note that $\tilde{X}_j = 0$ if the equipment is in failed state when it reaches the end of its useful life; y_{ji} $(0 \le i \le N_j)$ is the time to repair the equipment after the i^{th} failure; α is the penalty parameter; τ_1, τ_2 = maximal time in contract to return equipment to operational state for classes 1 and 2; C_b = equipment sale price; P_{w1}, P_{w2} are the EW prices for classes 1 and 2; C_s is the maintenance service price.

Thus, let $U(A_k;\, P_{wi}, C_{si}, M_i)$ represent the customer's expected utility when option A_k $(0 \le k \le 3)$ is chosen by customers of class i, and it can be obtained using (3.2), (3.3), (3.4) or (3.5) in (3.1). The customer's optimal choice A^* is the one from the set $\{A_0, A_1, A_2, A_3\}$

which yields the maximum expected utility. Note that $A(P_{wi}, C_{si}, M_i)$ is a function of the manufacturer's decision variables.

3.4. Manufacturer's Decision Problem

We assume that the manufacturer is risk neutral. He may offer his product and services to class 1 and class 2 hospitals. We also consider that in a given class, all customers are homogeneous, and thus they choose the same action with similar prices. Let the manufacturer's profit to a given action A_k chosen by a given customer be denoted by $\pi\ (P_{wi}, C_s;\ A_k)$. Thus,

$$\pi(P_{wi}, C_s; A_0) = 0 \tag{3.6}$$

$$\pi(P_{wi}, C_s; A_1) = P_{w1} + C_b - C_r N_j - \alpha_1\left(\sum_{i=0}^{Nj} \max\{0, (y_{ji} - \tau_1)\}\right) \tag{3.7}$$

$$\pi(P_{wi}, C_s; A_2) = P_{w2} + C_b - C_r N_j - \alpha_2\left(\sum_{i=0}^{Nj} \max\{0, (y_{ji} - \tau_2)\}\right) \tag{3.8}$$

$$\pi(P_{wi}, C_s; A_3) = C_b + (C_s - C_r) N_j, \tag{3.9}$$

where C_r = repair cost.

The manufacturer's optimal choice for the decision variables P_{wi}, C_{si} and M_i is obtained by maximizing his expected profit, taking into account the optimal choice done by the customers, A^*.

3.5. Model Simplifying Assumptions

In order to make the model tractable, we make some simplifying assumptions:

(i) Items are repaired in a 2-class non-preemptive priority queueing system, where class 1 has priority over class 2;

(ii) The model supposes perfect repairs.

3.6. Model Resolution

This formulation follows a Markovian queue with finite population (M_i), where users are split into two classes and class 1 has priority over class 2 with no preemption. If there are more than one user of a given class at a time the queue follows a FCFS service rule. For k failed units, the arrival rate is $\lambda_k = (M - k)\lambda$ for $0 \le k \le M$, $\lambda_k = 0$ for $k > M$. As we considered there is only one service crew, the service rate is μ. We assume that $M_i\lambda_i < \mu$, otherwise the queue would grow over time and the equipment waiting time in the system would increase.

In order to formulate the proposed model and determine the customer's and manufacturer's optimal strategies discrete-event simulation has been employed, following (Hill; Beall & Blischke, 1991; Murthy; Iskandar & Wilson, 1992) that used such approach while studying warranties. The algorithm was implemented using the language C++ and the software Dev-C++. It is divided into 3 main modules. The priority queues system module can be viewed in Figure 3. Note that: (1) T_d, time of departure, is set to infinity so that it will never be smaller than T_{a1} and T_{a2}, which refer to the times of arrival of class 1 and class 2 customers respectively when there is no equipment being repaired; (2) the algorithm generates two lists (one for each class) containing information about time of arrival (failure), repair duration and departure time. The indexes are based on the arrival order at the system, thus each list contains information about the failure occurrences for one priority class and their order. The priority queueing system is simulated 10.000 times, the number of samples considered in the application example in Section 5.

Initially the first arrival times are generated, for classes 1 and 2, then it is checked if the first customer that arrived belongs to class 1 or class 2, as the system will be empty, the customer will be served immediately. As the next events occur, it is checked whether it is an arrival or a departure, based on which of the times T_{a1}, T_{a2} and T_d is the smallest. If it is an arrival of a class 2 customer and the system is not empty, he will proceed to queue, if it is an arrival of a class 1 customer, he will use his priority and stay in front of the class 2 customers and stay in queue until all the other class 1 customers is served or until the server finishes his current service. During this process all important information is saved, i.e. time the server is idle, time to repair the equipment, total delay time (greater than τ) to repair the equipment. This process is repeated until the campaign time is reached.

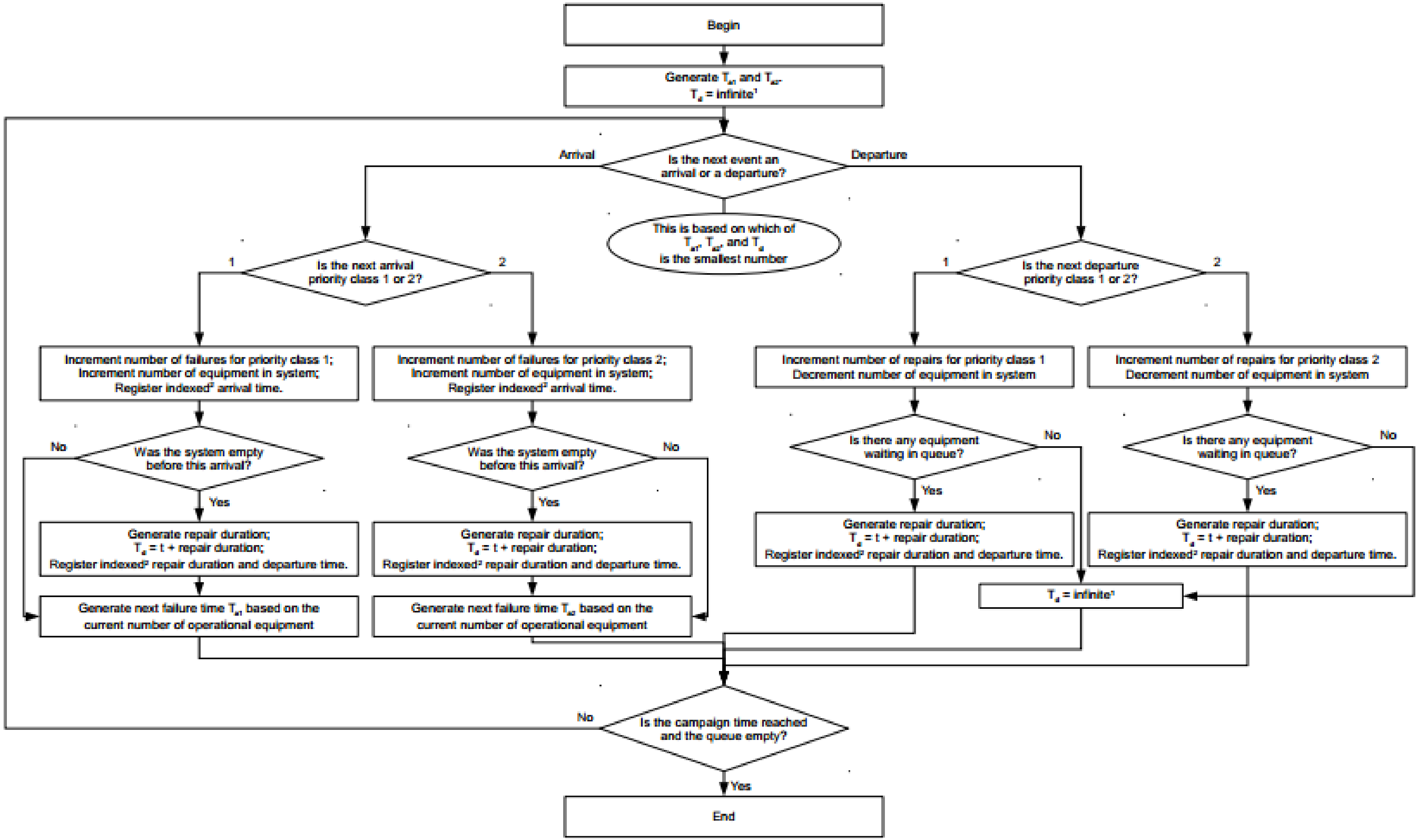


Figure 3.3 – Priority queue system module.
Source: This masters thesis (2016)

After the queue is simulated it is necessary to determine the customer's optimal strategy. To this end, it is necessary to determine the maximum willingness to pay of the customer for each option offered by the OEM. Such variables are denoted by $P_{w1}(max)$, $P_{w2}(max)$, and $C_{si}(max)$, i.e. the maximum willingness to pay for the EW for classes 1 and 2 and for maintenance services on demand, respectively, which are given by the expressions (3.10), (3.11) and (3.12). Note that, as stated by (CHUN; TANG, 1995), how much the customer is willing to pay for each of the options offered depends on the magnitude of his risk aversion.

$$P_{w1}(max) = R\left(\sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) - C_b - \frac{\ln(E[Penalty_1])}{\beta_1} \tag{3.10}$$

$$P_{w2}(max) = R\left(\sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) - C_b - \frac{\ln(E[Penalty_2])}{\beta_2} \tag{3.11}$$

$$\ln E\left[e^{\beta C_{si}(max)N}\right] = \beta_i * [R\left(\sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) - C_b] \tag{3.12}$$

As these expressions denote the customer's maximum willingness to pay for each option, they are calculated by equalizing his expected utility to zero. Note that it is difficult to determine $C_{si}(max)$ analytically by using (12), thus a numerical method has been used in order to determine it. Such method may be viewed in Figure 4. Note that: (1) the initial candidate for $C_{si}(max)$ is an arbitrary real value; (2) Y is generated from a continuous uniform probability distribution whose limits must include the desired value of $C_{si}(max)$ for this method to converge and find the optimal value, which will turn the expected utility sufficiently close to zero, in this case, a tolerance of 10^{-10} is used.

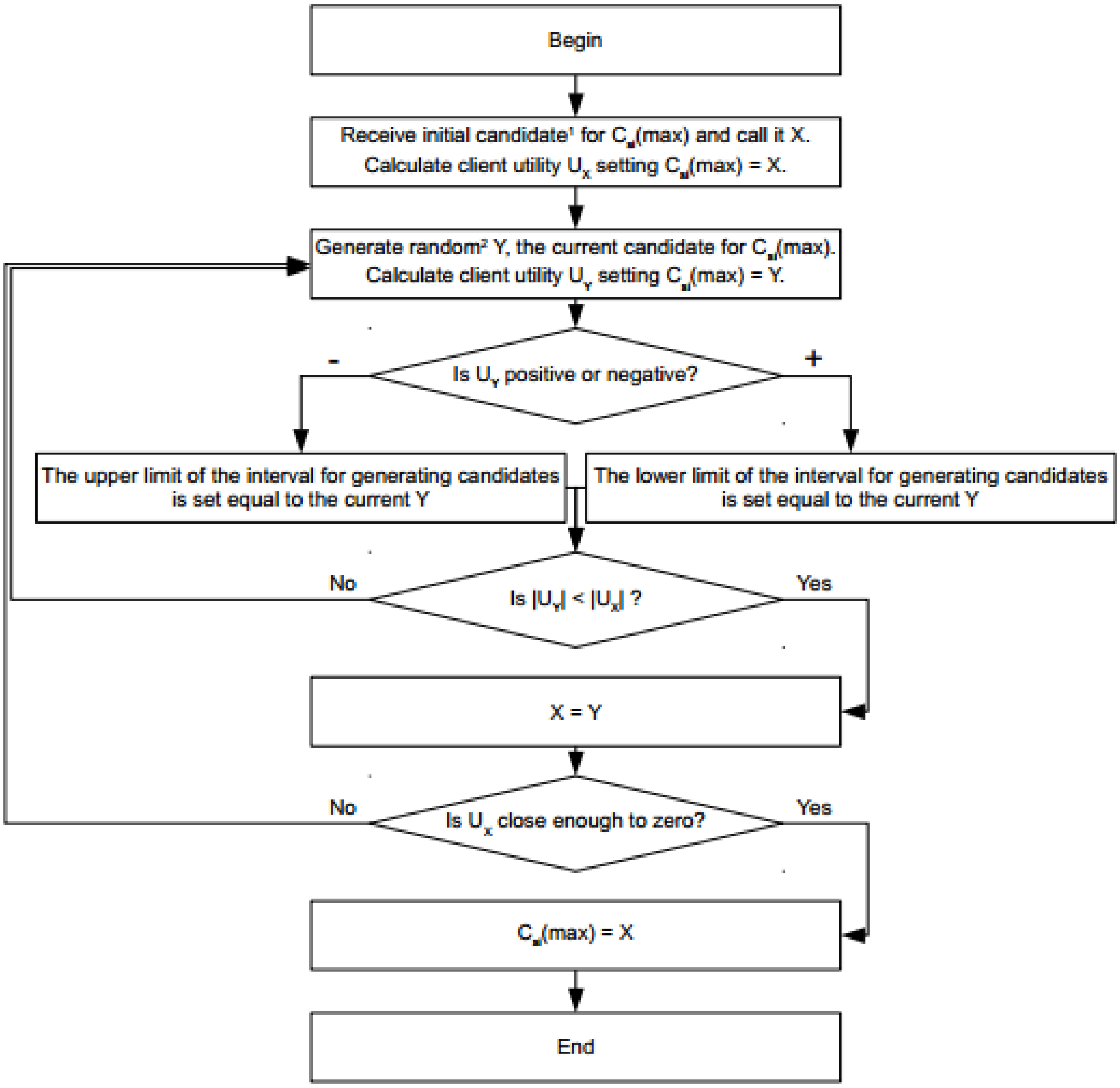


Figure 3.4 – Method for Csi(max) determination.
Source: This masters thesis (2016)

3.7. Results

3.7.1. Customer's Optimal Strategy

In order to determine the customer's optimal strategy it is necessary to evaluate the expected utility for each of the possible decisions. The expected utilities for the decisions A_0, A_1, A_2 and A_3 are given by the expressions (3.13), (3.14), (3.15) and (3.16) respectively:

$$E[U(A_0; P_{wi}, C_s)] = 0 \tag{3.13}$$

$$E[U(A_1; P_{w1}, C_s)] = \frac{1}{\beta}(1 - e^{-\beta\left[R\left(\sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) - C_b - P_{w1}\right]} E[Penalty_1] \qquad (3.14)$$

$$E[U(A_2; P_{w2}, C_s)] = \frac{1}{\beta}(1 - e^{-\beta\left[R\left(\sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) - C_b - P_{w2}\right]} E[Penalty_2]) \qquad (3.15)$$

$$E[U(A_3; P_{wi}, C_s)] = \frac{1}{\beta}(1 - e^{-\beta\left[R\left(\sum_{i=0}^{N_j} X_{ji} + \tilde{X}_j\right) - C_b\right]} E\left[e^{\beta N C_{si}}\right]) \qquad , \qquad (3.16)$$

where the terms $E[Penalty_1]$ and $E[Penalty_2]$ represent the expected values of the amount to be refunded to the customer by the manufacturer due to delays in returning the equipment to operational state for classes 1 and 2 respectively. They are calculated using some outputs of the queueing system described in Figure 3, more specifically, the arrival, repair and departure times are checked and the part that is greater than τ_i is aggregated in a variable delay time for each class of customers. Then, the total delay time is divided by the number of customers of each class and the mean delay time for each class is found. Finally, these delay times are used to calculate the expected penalties, multiplying them by α_1 and α_2. In case the customer decides not to buy the equipment, his expected utility will be $E[U(A_0;\ P_{wi}, C_{si})] = 0$. For a given P_{w1}, C_{s1}, for the class 1 customers, a comparison between the expected utilities of each of their options will define the class 1 customer's optimal choice, while for a given P_{w2}, C_{s2} for the class 2 customers, a comparison between the expected utilities of each of their options will define the class 2 customer's optimal choice.

3.7.2. Manufacturer's Optimal Strategy

The manufacturer's optimal strategy may be obtained as follows. After setting the EW prices for each class of customers and the price for maintenance interventions on demand the manufacturer must determine the optimal number of customers of each class to service. To this end, it is necessary to compare the expected profit of each combination of customers, from classes 1 and 2, by exhaustive enumeration.

It is possible to show that for given M_1 and M_2, the manufacturer should choose $P_{wi} > \bar{P}_{wi}$ and $C_{si} = C_{si}(max)$ (to force customers to decide for maintenance interventions on demand) or $P_{wi} = \bar{P}_{wi}$ and $C_{si} > C_{si}(max)$ (to force customers to decide for acquiring an EW) in order to maximize his expected profit. This is because $\bar{P}_{wi}$ and $C_{si}(max)$ represent the maximal prices the customer is willing to pay for each option. Thus, the manufacturer's expected profit for each choice of the customers is given by (3.17):

$$E[\pi(P_{w1}, P_{w2}, C_{s1}, C_{s2}, M_1^*, M_2^*; A_1{}^*, A_2{}^*)] = \begin{cases} 0, if\ C_{si} > C_{si}(max), P_{wi} > P_{w1}(max) \\ M_1\big(P_{w1}(max) + C_b - C_r N_j - E[penalty_1]\big), if\ C_{s1} > C_{s1}(max), P_{w1} = P_{w1}(max) \\ M_2\big(P_{w2}(max) + C_b - C_r N_j - E[penalty_2]\big), if\ C_{s2} > C_{s2}(max), P_{w2} = P_{w2}(max) \\ M_1\big(C_b + (C_{s1} - C_r)N_j\big), if\ C_{s1} = C_{s1}(max), P_{w1} > P_{w1}(max) or\ M_2(C_b + (C_{s2} - C_r)N_j), if\ C_{s2} = C_{s2}(max), P_{w2} > P_{w2}(max) \end{cases} \quad (3.17)$$

The optimal strategies are given as follows:

$C_{s1} > C_{s1}(max)$ and $P_{w1} = P_{w1}(max)$, the optimal strategy for class 1 customers is A_1;

$C_{s2} > C_{s2}(max)$ and $P_{w2} = P_{w2}(max)$, the optimal strategy for class 2 customers is A_2;

$P_{wi} > P_{wi}(max)$ and $C_{si} = C_{si}(max)$, $(i = 1,2)$ the customer’s optimal strategy is A_3;

$P_{wi} > P_{wi}(max)$ and $C_{si} > C_{si}(max)$, $(i = 1,2)$ the customer’s optimal strategy is A_0.

Now that the model has been fully presented, it is important to have an overview of the game development. Figure 3.5 presents the full game tree that shows all possible actions of each player and the order in which they make their decisions.

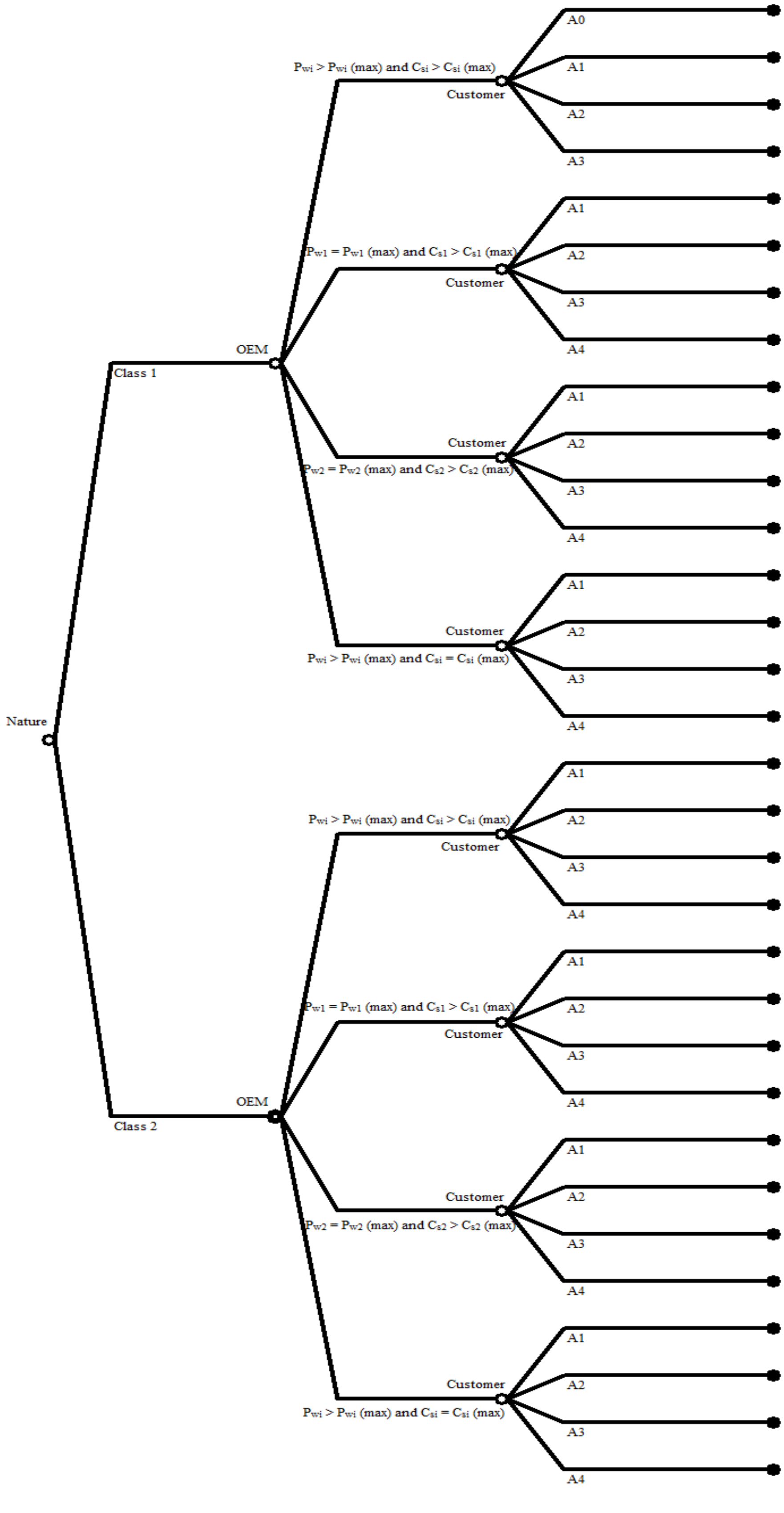


Figure 3.5 – Full game tree for a given customer j
Source: This masters thesis (2016)

4. APPLICATION EXAMPLE

4.1. Imaging Equipment (Angiography)

In order to illustrate the proposed methodology, we apply it to real data provided by a Brazilian hospital. The equipment here considered is an angiography, a technology-intensive item used to view blood vessels after injecting them with a contrast that traces them on x-ray. It may be used to examine arteries in many areas of the body, including the brain, neck, heart and chest, among others (Suri & Laxminarayan, 2003), see Figure 5. The model parameters considered are: $\lambda = 0.0015$ (per hour); $\mu = 0.25$ (per hour); $\alpha_1 = 1.5$ (10^3 \$ per hour); $\alpha_2 = 0.5$ (10^3 \$ per hour); $\beta_1 = 0.3$; $\beta_2 = 0.1$; $\tau_1 = 12$ (hours); $\tau_2 = 16$ (hours); $C_b = 1476.5$ (10^3 \$); $L = 8640$ (hours); $C_r = 5.4$ (10^3 \$); $R = 0.185$ (10^3 \$ per hour); samples = 10000; tolerance = 10^{-10}.

Figure 4.1 – Angiography equipment
Source: (Suri; Laxminarayan, 2003, p.14)

It has been considered that the maximum number of customers to service was 167, considering both classes, as it may be less than $\mu/\lambda \approx 167$, otherwise the queue would grow too fast and the expected penalties to be refunded by the manufacturer would increase too much, as mentioned by (ASHGARIZADEH & MURTHY, 2000). Therefore, in order to determine the optimal number of customers of each class to service, we have used simulation to compare the expected profit for each possible combination varying exhaustively the number of customers between 1 and 167.

According to the simulation performed, the optimal number of customers to be serviced is M = 103, being M_1 = 3 of class 1 and M_2= 100 of class 2. Under such conditions, , it is necessary to analyze which set of possible actions maximizes the manufacturer's profit, (i) offering EWs to both classes; (ii) offering EWs to class 1 and services on demand to class 2; (iii) offering services on demand to class 1 and EWs to class 2; or (iv) offering services on demand to both classes. After comparing the manufacturer's expected profit for each of those combinations it was found that the optimal strategy to the manufacturer is to offer EWs for class 1 customers (A_1) and services on demand to class 2 customers (A_3), as it is the strategy that maximizes the manufacturer's expected profit. Thus, the manufacturer sets the prices as follows: $P_{w1} = P_{w1}(max)$ = \$ 115,563.00 and $C_{s1} > C_{s1}(max)$ = \$ 8,287.00 for class 1; and $P_{w2} > P_{w2}(max)$ = \$ 109,769.00 and $C_{s2} = C_{s2}(max)$ = \$ 7,7,791.00 for class 2. Such strategy generates an expected profit $E[\pi]$ of \$ 3,204,450.00.

The adoption of different strategies by each class of customers may have been caused by 2 factors: (i) class 2 customers are less averse to risk than class 1 customers, as $\beta_2 < \beta_1$, and the degree of risk aversion is directly related to the magnitude of the risk aversion parameter. So, class 2 customers are less likely to hire an EW; and, (ii) as there are many customers in class 2 the expected penalty tends to grow and the manufacturer prefers to offer services on demand as there is no penalty associated to them.

In order to enrich the analysis, the output of the 2-class-priority system was compared to the output of a single-class system in which all customers were similar to the customers belong to class 1. In this situation the optimal number of customers to be serviced is M = 77, a reduction of 25% in the number of customers to be serviced; additionally, considering the maximum prices that could be charged for each class and maintenance option, the prices are set as follows: P_w > 125,026.00 (8% higher than $P_{w1}(max)$ and 13% higher than $P_{w2}(max)$) and C_s = 8,149.00(2% lower than $C_{s1}(max)$ and 4,5% higher than $C_{s2}(max)$) and the customers must choose the action A_3 and pay for services on demand, as it is the option that maximizes the customers profit. Then, the OEM yields an expected profit $E[\pi]$ = 2,678,520.00 (11% lower than the expected profit considering 2 classes) see Table 4.1.

Table 4.1. Comparison considering 2-class and single-class systems

	Class 1	Class 2	Single Class System
M_i	3	100	77
P_{wi}(max) $	115,563	109,769	125,026
C_{si}(max) $	8,287	7,791	8,149
S_i	A_1	A_3	A_3
E[π] $	181,000	3,023,450	2,678,520

Source: this masters thesis (2016)

The performance of the queueing system is another important source of information. First, considering the 2-class priority queueing system, it has been found that the server's mean idle time is 3,484.41h (40% of *L*), the mean time the equipment stays in the system, i.e. time since the equipment fails until it is repaired is 54.80h for class 1 and 122.86 for class 2 . Additionally, the mean delay time (greater than τ) is 5.88h for class 1 and 15.25h for class 2 (see Table 4). Within such arrangement, the total amount the manufacturer pays in penalties is $ 26,500 for class 1 customers and $ 762,715 for class 2 customers.

Considering now the queueing system with only one class it has been found that the server's mean idle time is 4,745.88h (55% of L and 36% higher than the 2-class priority case), the mean time the equipment stays in the system is 90.95h (65% more than class 1 and 26% less than class 2). Yet, the mean delay time (greater than τ) is 13.95h (137% more than class 1 and 8,5% less than class 2); see Table 4.2.

Table 4.2 Performance differences between priority and non-priority systems

	2-class model		Single-class model
Mean idle time (h)	3,484.41		4,745.88
	Class 1	Class 2	
Mean time in system (h)	54.80	122.86	90.95
Mean delay time (h)	5.88	15.25	13.95

Source: this masters thesis (2016)

Note that comparing the priority and the non-priority models it may be observed that the class 1 customers face a much better situation regarding the mean time in system and the mean delay time, while the non-priority customers, class 2, need to wait for a much longer time to get their equipment restored.

In order to investigate how the optimal values change due to marginal variations of some parameters values, we perform a sensitivity analysis. In Table 4.3, it is presented how the optimal solution changes due to variations in the failure rate (λ). As λ decreases, the equipment is less likely to fail, so there is a tendency of maintenance expenses to diminish, along with the expected penalties due to delays in repairing the equipment. As λ increases, the equipment is more likely to fail and the maintenance expenses tend to grow, as well as the expected penalties due to delays in returning the equipment to operational state. As a consequence, it may be observed that the optimal number of customers to service decreases, tied up with the manufacturer's expected profit. Furthermore, as the failure rate reaches 0.0020 the optimal strategy of class 2 customers changes from A_3 to A_2, and it becomes better to hire an EW than to pay for maintenance services on demand.

Table 4.3. Optimal solution changes due to λ variations

λ	M	M_1	P_{w1}(max) $10^3	C_{s1}(max) $10^3	S_1	M_2	P_{w2}(max) $10^3	C_{s2}(max) $10^3	S_2	E[π] $10^3
$1.0*10^{-3}$	186	3	117.569	12.581	A_1	183	116.382	11.698	A_3	10,005.89
$1.5*10^{-3}$	103	3	115.563	8.287	A_1	100	109.769	7.791	A_3	3,204.45
$2.0*10^{-3}$	48	3	114.107	7.016	A_1	45	106.370	6.105	A_2	685.104
$2.5*10^{-3}$	4	2	114.324	6.698	A_1	2	113.596	8.322	A_2	208.801
$3.0*10^{-3}$	4	2	112.831	5.945	A_1	2	111.975	7.077	A_2	154.581

Source: this masters thesis (2016)

Table 4.4 shows how the optimal solution changes due to variation in the service rate (μ). As μ decreases, the service team is able to repair less items per time unit, as a consequence, the optimal number of customers serviced decreases, as well as the maximum prices for the EW and services on demand the customers are willing to pay. On the other hand, when μ increases the service team is able to repair more equipment per time unit and the optimal number of customers serviced increases, tied up with the prices charged by the manufacturer. The optimal strategies for each class of customers did not change when μ varied.

Table 4.4. Optimal solution changes due to μ variations

μ	M	M_1	P_{w1}(max) 10^3	C_{s1}(max) 10^3	S_1	M_2	P_{w2}(max) 10^3	C_{s2}(max) 10^3	S_2	E[π] 10^3
$1.5*10^{-1}$	49	2	114.648	9.041	A_1	47	112.024	7.435	A_3	1,297.023
$2.0*10^{-1}$	74	2	115.629	9.042	A_1	72	110.367	7.688	A_3	2,204.398
$2.5*10^{-1}$	103	3	115.563	8.287	A_1	100	109.769	7.791	A_3	3,204,450
$3.0*10^{-1}$	129	2	115.997	8.774	A_1	127	109.288	7.955	A_3	4,271.885
$3.5*10^{-1}$	157	3	115.357	9.290	A_1	154	109.201	8.049	A_3	5,390.437
$3.5*10^{-1}$	185	3	115.633	9.154	A_1	182	109.227	8.138	A_3	6,551.599

Source: this masters thesis (2016)

In Table 4.5, it is presented how the optimal solution changes due to variations in the class 1 risk aversion parameter rate (β_1). As β_1 increases, the class 1 customers become more averse to risk and it causes their expected utilities to reach lower values faster, thus they tend to pay lower prices for the EW and for the services on demand, it causes the manufacturer's expected profit to decrease. Note that when β_1 reaches 0.5 it becomes uneconomical (M_1 = 0) for the manufacturer to service class 1 customers, as the price they are willing to pay would cause prejudice to the manufacturer, thus only class 2 customers are serviced, i.e. class 1 customers choose the action A_0, and class 2 customers choose the action A_3.

Table 4.5. Optimal solution changes due to β_1 variations

β_1	M	M_1	P_{w1}(max) \$$10^3$	C_{s1}(max) \$$10^3$	S_1	M_2	P_{w2}(max) \$$10^3$	C_{s2}(max) \$$10^3$	S_2	E[π] \$$10^3$
0.1	103	5	118.679	9.671	A_1	98	109.926	7.784	A_3	3,214.600
0.2	102	2	117.010	10.700	A_1	100	109.708	7.818	A_3	3,207.097
0.3	103	3	115.563	8.287	A_1	100	109.769	7.791	A_3	3,204.450
0.4	102	2	114.749	8.439	A_1	100	109.707	7.825	A_3	3,209.391
0.5	100	0	0	0	A_0	100	109.657	7.822	A_3	3,063.760

Source: this masters thesis (2016)

In Table 4.6, it is presented how the optimal solution changes due to variations in the class 2 risk aversion parameter rate (β_2). Similarly to what was presented in Table 7, as β_2 increases, the risk aversion of class 2 customers grows and they tend to pay lower prices for the EW and for services on demand, what causes the manufacturer's expected profit to decrease. Moreover, when β_2 reaches the value of 0.4, the optimal strategy for class 2 customers changes to A_2. Furthermore, when β_2 reaches the value of 0.5 the optimal strategy for class 1 customers change to A_3, so they prefer to pay for maintenance services on demand and the optimal strategy for class 2 customers change to A_0, so they choose not to buy the equipment.

Table 4.6. Optimal solution changes due to β_2 variations

β_2	M	M_1	P_{w1}(max) \$$10^3$	C_{s1}(max) \$$10^3$	S_1	M_2	P_{w2}(max) \$$10^3$	C_{s2}(max) \$$10^3$	S_2	$E[\pi]$ \$$10^3$
0.1	103	3	115.563	8.287	A_1	100	109.769	7.791	A_3	3,204.450
0.2	102	3	115.620	8.773	A_1	99	108.840	7.759	A_3	3,135.514
0.3	100	2	115.828	9.930	A_1	98	108.124	7.758	A_3	3,069.269
0.4	93	2	116.012	9.470	A_1	91	107.915	7.853	A_2	3,010.271
0.5	89	89	129.089	7.957	A_3	0	0	0	A_0	2,883.220

Source: this masters thesis (2016)

Table 4.7 and Table 4.8 present the optimal solution changes due to variations in the penalty per time unit parameters for classes 1 and 2, respectively. The number of customers serviced do not change significantly. The same pattern is observed for the prices charged by the manufacturer for each class of customers. However, when α_2 reaches 0.5 the optimal strategy for class 2 customers changes to A3, it may happen due to the increase in the penalties to be refunded by the manufacturer caused by the increment in α2. As a consequence, it becomes more profitable to provide services on demand, as they have no penalty associated, than EWs.

Table 4.7. Optimal solution changes due to α_1 variations

α_1	M	M_1	P_{w1}(max) \$$10^3$	C_{s1}(max) \$$10^3$	S_1	M_2	P_{w2}(max) \$$10^3$	C_{s2}(max) \$$10^3$	S_2	E[π] \$$10^3$
0.5	103	3	112.847	8.284	A_1	100	109.793	7.793	A_3	3,215.984
1.0	103	3	114.486	9.081	A_1	100	109.795	7.794	A_3	3,213.242
1.5	103	3	115.563	8.287	A_1	100	109.769	7.791	A_3	3,204.45
2.0	102	2	116.311	9.513	A_1	100	109.670	7.823	A_3	3,206,72
2.5	102	2	116.588	8.947	A_1	100	109.650	7.822	A_3	3,200.566

Source: this masters thesis (2016)

Table 4.8. Optimal solution changes due to α_2 variations

α_2	M	M_1	P_{w1}(max) \$$10^3$	C_{s1}(max) \$$10^3$	S_1	M_2	P_{w2}(max) \$$10^3$	C_{s2}(max) \$$10^3$	S_2	E[π] \$$10^3$
0.25	101	3	115.603	9.045	A_1	98	104.741	7.841	A_2	3,225.349
0.5	103	3	115.563	8.287	A_1	100	109.769	7.791	A_3	3,204.450
0.75	104	2	115.832	9.764	A_1	102	114.869	7.770	A_3	3,203.623
1.0	105	2	115.836	8.950	A_1	103	120.195	7.748	A_3	3,203.491

Source: this masters thesis (2016)

Tables 4.9 and 4.10 show how the optimal solution changes due to variations in the maximum time to repair an equipment parameter for class 1 and class 2, respectively τ_1 and τ_2. One more time the number of customers to be serviced do not change significantly, as well as the maximum prices charged for maintenance services. Note that τ_1 and τ_2 are related only to P_{w1} and P_{w2} and both of them increase when the maximum time to repair is reduced and decrease when the maximum time to repair is extended. Note that the optimal strategies for both classes of customers do not change.

Table 4.9. Optimal solution changes due to τ_1 variations

τ_1	M	M_1	P_{w1}(max) \$$10^3$	C_{s1}(max) \$$10^3$	S_1	M_2	P_{w2}(max) \$$10^3$	C_{s2}(max) \$$10^3$	S_2	E[π] \$$10^3$
8	103	2	119.723		A_1	101	109.743	7.804	A_3	3,206.327
10	103	2	117.489		A_1	101	109.709	7.797	A_3	3,203.719
12	103	3	115.563		A_1	100	109.769	7.791	A_3	3,204,450
14	101	3	113.910		A_1	98	109.696	7.841	A_3	3,210.953
16	103	3	112.747		A_1	100	109.763	7.792	A_3	3,213.282

Source: this masters thesis (2016)

Table 4.10. Optimal solution changes due to τ_2 variations

τ_2	M	M_1	P_{w1}(max) \$$10^3$	C_{s1}(max) \$$10^3$	S_1	M_2	P_{w2}(max) \$$10^3$	C_{s2}(max) \$$10^3$	S_2	E[π] \$$10^3$
12	102	3	115.552	8.527	A_1	99	115.398	7.819	A_3	3,208.543
14	102	4	115.725	8.662	A_1	98	112.340	7.813	A_3	3,204.595
16	103	3	115.563	8.287	A_1	100	109.769	7.791	A_3	3,204.450

18	102	2	115.825	9.758	A_1	100	107.650	7.819	A_3	3,204.951
20	104	3	115.605	9.096	A_1	101	105.970	7.769	A_3	3,205.794

Source: this masters thesis (2016)

This example was developed to show how the model may be employed to determine, for a given set of parameters, the customer's and manufacturer's optimal strategies as well as the manufacturer's maximum expected profit. Moreover, it was presented how the optimal solutions change when each of the parameters vary, more specifically, the number of customers of each class that is serviced, the maximum prices each class of customers is willing to pay for the EW and the maintenance services on demand, as well as the customers optimal strategy and the manufacturer's maximum expected profit.

5. CONCLUSIONS

The model developed that was developed here is an extension of the approach presented by (Ashgarizadeh & Murthy, 2000), and illustrates the situation in which a healthcare institution needs to acquire and maintain a technology-intensive equipment in order to service its patients. The extensions were: to study the specific field of a medical equipment, with healthcare institutions dealing with the OEM to maintain an equipment for which it may be difficult to find other service providers, and to consider a 2-class priority queueing system. More specifically, using a 2-class priority queueing system it was possible to formulate the situation in which the manufacturer offers 2 kinds of EWs, an EW with priority for class 1 customers, normally big hospitals, and a non-priority EW for class 2 customers, normally small hospitals, where class 1 has priority over class 2, which is a common practice in real world. Customers of a given class are considered homogeneous regarding their attitude to risk, thus they make similar decisions, but they are not homogeneous between classes.

In this way, the manufacturer and the customer interact in a Stackelberg Game formulation in which the former, acting as a leader, intends to maximize his expected profit and needs to set the prices of the EWs and of the maintenance interventions, and the latter, acting as a follower, intends to maximize his expected utility and needs to decide among the options of not buying the equipment, buying the equipment and hiring an EW (with priority for class 1, or without priority for class 2), or paying for each maintenance intervention on demand.

Due to the game characteristics the OEM is able to extract all the customer's surplus and leaves him with utility zero. The option that maximizes his profit is to offer EWs with priority for 3 customers of class 1 and services on demand for 100 customers of class 2. Then, 103 customers should be serviced. With such arrangement the manufacturer yields an expected profit of $ 3,204,450. Comparing it with the model without priority distinctions it was noted that the number of customers serviced decreased to 77 and that the manufacturer's expected profit decreased $ 2,678,520.

Analyzing some performance measures it was observed that the server's mean idle time is 3,484.41h, 40% of the time horizon considered, the mean time the equipment stays in the system is 54.80h and 122.86h for class 1 and class 2 respectively. Moreover, the mean delay time (greater than τ) is 5.88h and 15.25h for class 1 and class 2 respectively. When the 2-class model and the single-class model are compared it may be observed that the class 1 customers face a much better situation regarding the mean time in system and the mean delay time, e.g.

the priority system brings clear benefits to the priority class. On the other hand, non-priority customers, class 2, need to wait for a much longer time to get their equipment restored.

Some limitations of the present work are, the use of perfect repairs, although it tends to underestimate the expected number of failures of a given equipment, the consideration of only one service team, although it is common that a maintenance provider has numerous service team, and assuming the risk-aversion parameter as given, although it is a parameter which is difficult to estimate.

The model can be extended in several ways. Some of them are: (i) using a principal-agent game formulation (Jiang; Pang & Savin, 2012; Jin; Tian & Xie, 2015) to consider information asymmetry, and (ii) considering heterogeneous customers as (Padmanabhan & Rao, 1993) to turn the negotiation process more realistic; (iii) considering two-dimensional warranties (Samatli-Paç & Taner, 2009) to take two performance parameters such as equipment reliability and availability into account in the negotiation process instead of only one, when a one dimensional warranty is considered; (iv) considering the possibility of renewing the EW and analyzing a time period longer than one year and (v) incorporating preventive maintenance actions to reduce warranty costs, as pointed out by (Yun; Murthy & Jack, 2008) and (vi) relax the assumption of perfect repair and consider imperfect repair, as brought to the warranties context by (Yeo & Yuan, 2009), using the Generalized Renewal Process proposed by (YANEZ; JOGLAR & MODARRES, 2002).

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